Data structures and algorithms
(2019/20)
Written exam 9. rožnika 2020

This written exam must be taken individually. Any and all literature may be used while taking the test. In your answers be precise, and: (i) answer the questions *as they were asked*; and (ii) answer *all* tasks – if you will be answering to all tasks you might get bonus points.

Time: 120 minutes.

We wish you a lot of success - veliko uspeha!

TASK	POINTS	OF POINTS	TASK	POINTS	OF POINTS
1			4		
2			5		
3					

IME IN PRIIMEK: _____

ŠTUDENTSKA ŠTEVILKA: _____

DATUM: _____

PODPIS: _____

1. naloga: This time Peter was called by the organizers of the New York Marathon, who have a special desire. Namely, they want to set up a website through which they would be able to review the current order of runners. They want that Peter's solution supports the following functions as efficiently as possible:

- `RaceStart()` – the race has started;
- `RaceEnded()` – the race has ended;
- `CurrentTime(who, time)` – which corrects the current time of the person `who` [1]; and
- `Place(who)` – which returns the current position of the competitor `who`.

Of course, the website must still work after the competition.

QUESTIONS:

1. We used augmented structures to compute rank and select functions. Describe how the values in nodes change when using single and double rotation of the AVL tree. Draw the structure before insertion, after insertion but before rotation, and after rotation.

2. Describe the data structure that will solve the above problem and provide a description or pseudocode of the individual operations. The more efficient your solution, the more points you will earn.

3. What is the time complexity of each operation? Justify your answer.

2. naloga: We have a set $\{1, ..., 10\}$, from which we form 10 one-element disjoint sets using an efficient data structure from lectures.

QUESTIONS:

1. The following operations are performed over such formed sets:

   ```
   F 5, U 3 5, U 1 5, U 7 1, F 5
   ```

   where `F x` returns the name of the set to which the element `x` belongs, and `U x y` forms the union of the sets to which the elements `x` and `y` belong.

   For each of the above operations, draw how the data structure changes and count and write down the number of comparisons made at each operation.

2. Write down the pseudocode of the operation (function) `F`.

   HINT: Write down the definition of the data structure first, otherwise the pseudocode will be a big mess.

[1] You can assume that both `who` and `time` are integers – starting number and time in seconds.

3. Suppose we are dealing with the set $\{1, ..., n\}$, from which we again form n pairwise disjoint non-empty sets. Over these sets we perform n operations `F` and n operations `U` in an arbitrary order. How many comparisons shall we do altogether after all $2n$ operations are completed? Justify your answer.

   HINT: For almost all points, I do not expect completely precise answer. Think about the best and the worst case.

3. naloga: Tries.

QUESTIONS:

1. First, (i.) write down the pseudocode for inserting into a Patricia tree and (ii.) insert into a Patricia tree over the alphabet $\Sigma = \{0, 1\}$ the following keys
   $$0010, 100, 010, 100011, 111001, 0011, 000010$$
   and draw a tree after each insertion.

2. Sometimes we are dealing with texts whose letters are from an arbitrary large alphabet – so far our alphabet has always been finite. How would you effeciently implement an individual node in this case?

3. Suppose we have a text $t = a_1a_2a_3...a_n$ from which we can form n prefixes $p_1 = a_1$, $p_2 = a_2a_1$, $p_3 = a_3a_2a_1$, ..., $p_i = a_i...a_2a_1$, ... $p_n = a_na_{n-1}a_{n-2}...a_1$. Now we insert all prefixes p_i insert into a trie. Does such a tree help us with an efficient search of the pattern $v = v_1v_2...v_m$ in the text t? How – justify the answer.

4. naloga: At the primary school competition *Beaver* there was the following task:

> Cooking is not as easy with beavers as it is with people. The mother Vanda prepares two dishes from four ingredients - meat, tomatoes, fish and broccoli. She mixes the fish and broccoli and cook for five minutes (S1). Also, she cooks tomatoes and meat for five minutes (S2). Then, she divides tomatoes and meat in three parts; the first two parts are cooked separately for another five minutes (S5 and S9), the second part is mixed with half of the mixture of broccoli and fish, and cooked again for five minutes (S4). The whole process is shown in Figure 1, where each circle represents five minutes of cooking.

QUESTIONS:

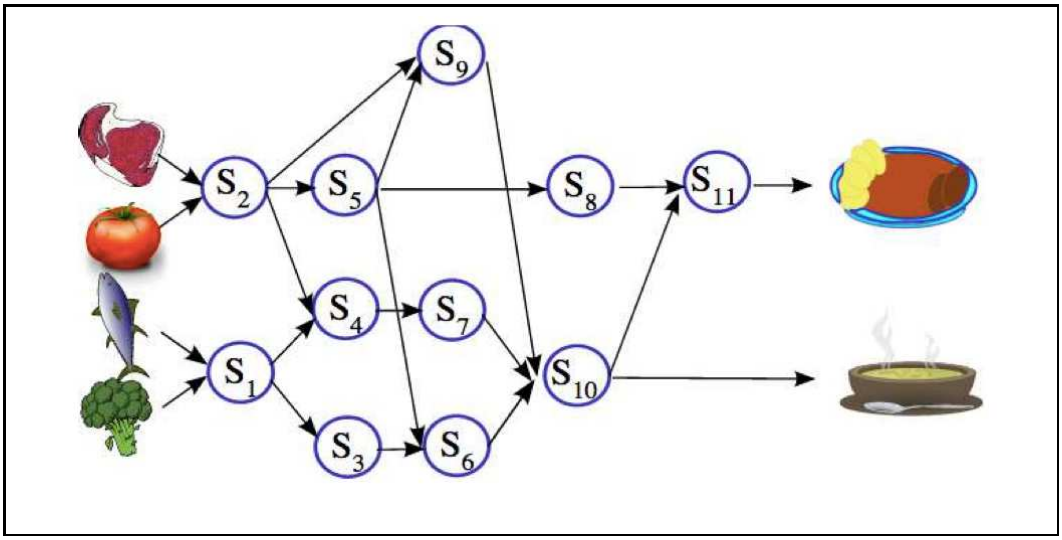

Figure 1: Cooking with the mother Vanda.

1. How long does it take the mother Vanda to cook lunch if she has (i.) unlimited number of pots; (ii.) only two pots? Justify the answer!

2. In general, any recipe for preparing a dish, or any procedure for carrying out some work consisting of tasks, can be modelled with a graph $G(V, E)$.

   (i.) Describe how to construct, in general, a graph for describing a process consisting of tasks.

   HINT: Describe what the sets V and E look like.

   Let us assume that we have a directed graph $G(V, E)$ that describes the implementation of individual tasks of the procedure. (ii.) Does the graph G always describe a process that is feasible? Justify the answer and, if not, write down the algorithm that would check the feasibility of the procedure.

3. Write down an algorithm that for a given directed graph $G(V, E)$ which describes the feasible procedure, calculates the shortest execution time.

bonus Consider and describe the algorithm for calculating the shortest time if only k jobs can be run at the same time.