Data Structures and Algorithms
(2021/22)
Written exam February 18th, 2022

Written exam must be taken individually. Any and all literature may be used while taking the exam. In your answers be precise, and: (i) answer the questions *as they were asked*; (ii) answer *all* tasks – if you will be answering to *all* tasks you might get bonus points; and (iii) when you are asked to provide the justification of correctness of your answer provide it as otherwise the points will be deducted.

Time: 105 minutes.

We wish you a lot of success – veliko uspeha!

TASK	POINTS	OF POINTS	TASK	POINTS	OF POINTS
1			3		
2			4		

IME IN PRIIMEK: _____

ŠTUDENTSKA ŠTEVILKA: _____

DATUM: _____

PODPIS: _____

1. task: We have the following program (let the first element of the array have index 0):

```
public int foobar(int[] a, int b, int c, int d) {
  if c < 0    return d;
  if b > a[c] d++;
  return foobar(a, b, c-1, d)
}
```

QUESTIONS:

A) Suppose we have the array `a` $= [1, 101, 12, 13, 65, 14, 17]$. What is the value returned by the call `foobar(a, 20, 4, 7)`? Justify your answer.

B) What does the function really do? So, according to the description what is probably the most correct value of the parameter `d` on the first call of the function? Justify the answer.

C) What is the time complexity of the algorithm? Prove your claim.

2. task: Balanced trees.

QUESTIONS:

A) First inserting in an AVL tree. (i.) Show that when inserting an element at most one (single or double) rotation is needed. (ii.) Now insert the following integers in an AVL tree:

$$24, 18, 84, 24, 45, 85, 62, 24, 84. \tag{1}$$

Draw a tree after each insertion.

B) What about the red-black (RB) tree? (i.) Is one rotation enough when inserting an element in a RB tree? Prove your claim. (ii.) Insert integers (1) in a 2-3-4 tree and draw the final tree.

C) Let us return to an AVL tree, which is of hight h. (i.) At most how many elements are in the tree? Prove your claim. (ii.) And at least how many elements are in the tree? Prove your claim.

3. task: The Butale edit distance between the strings s and t is defined as the sum of the costs of operations that convert the string s to the string t. Let the strings consist of letters of the alphabet $\Sigma = \{\mathrm{A}, \mathrm{C}, \mathrm{G}, \mathrm{T}\}$. We also have the following operations: `insert`(a) with the cost c_I, `replace`(a, b) with the cost c_R, če $a \neq b$ and 0 otherwise ter `insert`(a) with the cost c_D.

QUESTIONS:

A) Let $|s| = n$ and $s = t$. *At most how large* is the Butale edit distance between the strings s and t? Justify your answer.

HINT: The questions is asking for the *largest* Butale distance, which means, that there is no other sequence of operations that transforms s into t and it is more expensive.

B) Is the problem of translating s to t solvable with only two of the above operations? Justify the answer.

HINT: There are three cases - in each we omit the other operation. If you think the case is still solvable write down a translation algorithm and, if you think it is unsolvable, an example of strings when it is unsolvable and why no.

C) From the Butale distance to the Levenstein distance. But this time, we have, in addition to the above three operations, a fourth operation `dDelete`(aa), which deletes two letters if they are the same and has the cost c_{dD}. Write a recursive formula that finds the cheapest translation of s to t for the mentioned *four* operations.

4. task: Let $G = (V, E)$ be a weighted directed graph, where $|V| = n$ and $|E| = m$.

QUESTIONS:

A) (i.) Write down an algorithm that checks whether there exist a negative directed cycle in G. (ii.) Justify the correctness of your algorithm. (iii.) What is the time complexity of the algorithm? Justify the answer.

B) (i.) Extend your algorithm so that if there is a negative cycle in the graph, it prints the nodes of one of the cycles. (ii.) Justify the correctness of your algorithm. (iii.) What is the time complexity of the algorithm? Justify the answer.

C) Butale is a really special place and residents of Butale take care to stay special. Thus, all roads are one-way. Roads connect their residences, business premises and municipal administration. The road system can be modeled as the set E of edges of a weighted directed graph, while residences, business premises and municipal administration as the set V of the same graph. The residents of Butale have been fighting for quite some time, who lives closest and who is a little less close to the municipal administration. (i.) Write an algorithm that sorts the nodes in V by distance from the node s, which represents the municipal administration. (ii.) What is the time complexity of the algorithm? Justify the answer.