

Data structures and algorithms
(2020/21)
Written exam January 27th, 2021

This written exam must be taken individually. Any and all literature may be used while taking this test. In your answers be precise, and: (i) answer the questions *as they were asked*; and (ii) answer *all* tasks – if you will be answering to all tasks you might get bonus points.

Time: 120 minutes.

We wish you a lot of success - veliko uspeha!

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IME IN PRIIMEK: _____

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DATUM: _____

PODPIS: _____

1. task: Basics. We are given an array S of integers indexed from 0. The function `len()` returns the length of an array. Given

$$S = [80, 20, 32, 99, 81],$$

`len(S)` returns 5. In addition, we have the following part of the code

```
int FooBar(S, k):
    if k >= len(S) return MAXINT;
    tmp= FooBar(S, k+1)
    if tmp < S[k] return tmp
    else return S[k]
```

QUESTIONS:

- A) (i.) What does the call `FooBar(S, len(S)+2)` return? Justify your answer. (ii.) What does the call `FooBar(S, 2)` return?
- B) (i.) What the call of `FooBar(S, 0)` actually does? (ii.) Formally prove your claim.
- C) Let $n = \text{len}(S)$. (i.) Precisely how many comparisons between the elements of the array S does the call of the function `FooBar(S, 0)` make – what is the time complexity considering comparisons? Justify your answer. (ii.) Due to the architecture of the computer, when accessing an element $S[i]$, the elements from $S[i]$ to $S[i+d]$ are transferred from the main memory to the cache. One transfer from the main memory to the cache is called an access (probe). What is the time complexity of the call of the function `FooBar(S, 0)` considering accesses (probes)? Justify your answer.

HINT: The function will probably depend on n and d .

2. task: Dictionary – basics. Suppose we have the following sorted integers: 1, 4, 19, 19, 19, 24, 39, 40, 42, 46, 50, 53, 56, 58, 59, 61, 66, 69, 74, 77, 79 and 90.

QUESTIONS:

- A) Draw the shallowest ordered binary tree that contains all the above elements.
- B) (i) Write an algorithm that, given n *different* sorted integers in the array A , builds the shallowest ordered binary tree. Justify that your tree is indeed the shallowest. (ii) What is the time complexity of your algorithm? Justify your answer.

- C) Suggest a data structure (or structures) that takes the minimum time for the following sequence of operations: (i) we first insert n elements (`Insert`); and (ii) then we make n^2 queries (`Find`). How much time does your structure (or structures) take on all operations? Justify your answer.

HINT: The better your solution, the more points you get.

3. task: We have the following data with keys $((key, data))$:

(111, A), (01001, B), (0111000, C), (01101101, D),
(10100100, E), (010010, F), (1100, G), (011011111,
H), (101011, I),

where the keys consist of digits from the alphabet $\Sigma = \{0, 1\}$.

QUESTIONS:

- A) Insert the above data into a PATRICIA trie.
- B) Transform the resulting tree into a tree that will be compressed by layers such that the lowest degree of fullness of each node will be at least $\alpha = 0,524$.
- C) A special form of tries compressed by paths are suffix trees. For a given text t of length $n = |t|$, the corresponding suffix tree can be build in $O(n)$ time if the size of the alphabet is $|\Sigma| = O(1)$ (Ukkonen). We have a text t . Describe the process that in $O(n)$ time finds the longest palindrome. Palindromes are texts which read the same backward as forward. For example, in the text below, there is such a palindrome underlined:

10010101001001010001.

4. task: Graphs. Tabel 1 gives the distances of road sections in km between major places in the district of Butale. Rows contain names of places with labels, while columns contain only labels. The residents of Butale, Butalci, are orderly people and so is the width of all roads standardized and the same. The Butale parliament has provided quite a bit of budget funds for resurfacing roads in 2021, but not enough to resurface all.

QUESTIONS:

- A) (i.) At least how many km of road sections must be resurfaced by Butalci so that after resurfacing it will be possible to come from any place to any other place. In addition, the following condition should be met. The mayor

	(S)	(P)	(R)	(H)	(V)	(O)	(B)	(Z)
(S) Smrečje		61	104	70				
(P) Podlipa	61		92		200			
(R) Ribje	104	92		120	110			
(H) Huda rida	70		120			125		
(V) Veliki tolmun		200	110			103	85	114
(O) Oni kraj				125	103		101	
(B) Bolšje					85	101		120
(Z) Zadnja vas					114		120	

Table 1: The distances of road sections in km between major places in Butale.

of Butale, who lives in *Oni kraj*, should have the shortest possible route after the restored roads to any other place. Justify your answer. (ii.) There are in fact 9557 places in the district of Butale. How many road sections must be resurfaced if the roads are to be resurfaced as it is defined above? Justify your answer.

- B) TOB, *Tourist Organization of Butale*, is preparing a new product – *Discover the beauties of rural Butale*. The product will offer the guest a sequence (list) of road sections between the major places along which the guest should travel, and be introduced to the beauties of Butale.¹ To not make the visit too long, they want to make the list so that each road section on it appears exactly once. (i.) Peter looked at the list of possible sections in Table 1 and immediately found that it is not possible to prepare such a list. Why? (ii.) But at the same time, Peter suggested that if Butalci add some road sections in Table 1, such a list can be made. At least how many and which sections should they add? Justify your answer.

HINT: It is not necessary that the entire path of a guest starts and ends at the same place.

- C) After the great success of the product *Discover the beauties of rural Butale*, TOB has started preparing the next product - *Meet places of Butale*. This time they want to offer the guest a visit to all the major places in Butale. To this end, they want to make again a list of road sections along which the guest should travel to make any major place visited exactly once. (i.) This time Peter does not know whether such a path exists. Does it exist? Justify your answer. (ii.) What if we would want to visit all the places in Butale? Do

¹Of course, a sequence of sections means that the consequent sections in the list should be of the form (u, v) and (v, w) .

we have any assurance that we will compute the list in practical time? Justify your answer.