Data structures and algorithms
(2018/19)
Written exam 23. prosinca 2019

This written exam must be taken individually. Any and all literature may be used while taking this test. In your answers be precise, and: (i) answer the questions *as they were asked*; and (ii) answer *all* tasks – if you will be answering to all tasks you might get bonus points.

Time: 60 minutes.

We wish you a lot of success - veliko uspeha!

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DATUM: ______________________

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1. naloga: Basics. Let us define the function

```
int FooBar(c, n):
  int p= 2**n
  result= 1
  while (p > 0):
    result= result * c
    p= p-1
  return result
```

QUESTIONS:

A) What is the time complexity of the function `FooBar` in terms of n? Justify your answer.

B) Can you speed up the function `FooBar`? If yes, how and what is the time complexity now, and if not, why not.

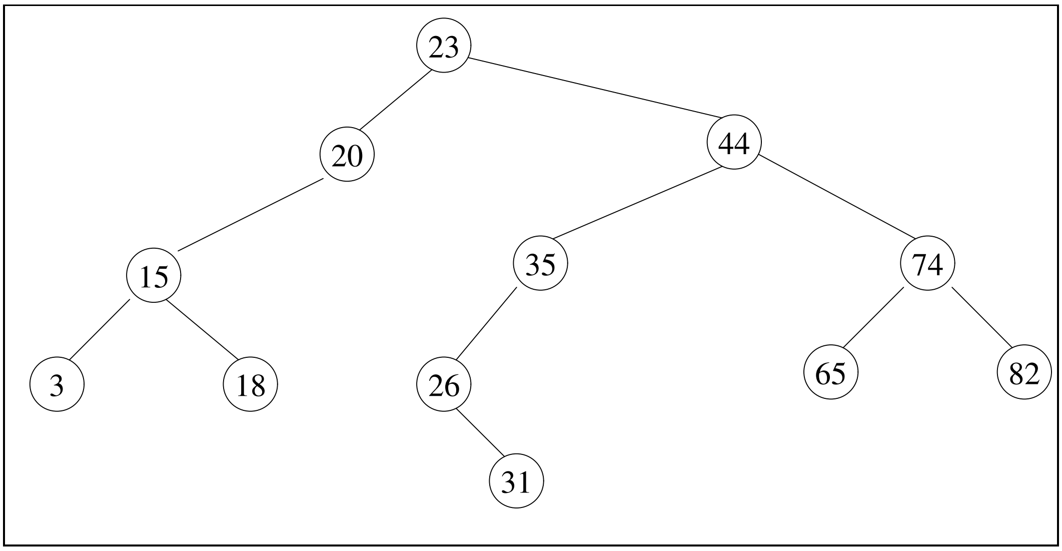
C) Prove that

$$n \ln n = O(n^{3/2})$$

and

$$n \ln n = \Omega(n^{1/2}) \ .$$

2. naloga: Peter Puzzle found a small paper where a tree as in Figure 1 was drawn.



Slika 1: Search tree.

QUESTIONS:

A) (i.) Is the tree an AVL tree? Justify your answer. (ii.) Can you colour it so that it becomes a red-black tree? Justify your answer. (iii.) Augment the picture of a tree by adding to each vertex the information about the number of elements in the subtree of that vertex including the vertex itself.

B) We define the function `RangeNum(a,b)`, which returns the number of elements in a tree that are contained in the interval $[a, b]$, including a and b. (i.) What does the function `RangeNum(18,45)` return on the tree from Figure 1?

(ii.) The augmented tree has the following definition of a node:

```
node:
  int root
  int num
  subtree left, right
```

Using this definition write a (pseudo)code of the function `RangeNum(a,b)`.

C) What is the time and space complexity of your function, if there are n elements in a tree and k of them lie in the interval $[a, b]$? Justify your answer.

3. naloga: Tries.

QUESTIONS:

A) For a (PATRICIA) tree it holds that paths are compressed by height, that is, each node has always two successors, if the alphabet is $\Sigma = \{0, 1\}$. Show that a Patricia tree with n elements has precisely $n - 1$ internal nodes.

B) We have a Patricia tree, where we always add elements at leaves[1]. In such a tree insert the following elements and draw the corresponding tree after each insertion: `01010011`, `0000101` and `10101`.

HINT: When drawing do not forget that internal nodes have additional information. Which one?

C) Suppose that we have now a ternary alphabet $\Sigma = \{0, 1, X\}$ and n elements. What is the greatest possible height of a Patricia tree and what the smallest possible? Justify your answer.

[1] Indices of letters used in internal nodes do not necessary decrease from the root.

4. naloga: Graph algorithms and dynamic programming.

QUESTIONS:

A) Throughout our lectures, we have described different spanning trees. Draw a graph that has different minimum spanning tree and spanning tree of shortest paths from some vertex to all other vertices.

B) Represent the drawn graph with the incidence matrix and the adjacency matrix.

C) Shortest paths between a pair of vertices can be found by dynamic programming (Floyd-Warshal algorithm). It is defined by the follwing formula:

$$\delta_{i,j}^{(n)} = \begin{cases} w_{i,j} & \text{if } n = 0 \\ \min(\delta_{i,j}^{(n-1)}, \delta_{i,n}^{(n-1)} + \delta_{n,j}^{(n-1)}) & \text{otherwise} \end{cases},$$

where $w_{i,j}$ is the cost of the edge (v_i, v_j) and $\delta_{i,j}^{(m)}$ is the weight of a shortest path $v_i \rightsquigarrow v_j$ such that for all intermediate vertices v_k on the path (excluding initial and terminal vertex) we have $k \leq m$. (i.) Write down an algorithm by using this recursive definition and use memoization. (ii.) What is the time and space complexity of your algorithm? Justify your answer.