Sets

CST 5/L-Automata
and Language Theory

What is a Set?

- A **set** is a collection of distinct objects, called **elements** of the set
- A set can be defined by describing the contents, or by listing the elements of the set, enclosed in curly brackets.

Examples:

1. Vowels in the English alphabet	V = { a, e, i, o, u }
2. First seven prime numbers.	X = { 2, 3, 5, 7, 11, 13, 17 }
3. Set of designer bags.	B={Gucci, Prada, Dior, Chanel}

Notations: Roster vs. Set Builder Notation

There are two methods of representing a set :

(i) Roster or tabular form

- In roster form, all the elements of a set are listed, the elements are being separated by commas and are enclosed within braces { }.

For Example:

Z=thesetofallintegers={…,−3,−2,−1,0,1,2,3,…}

Notations: Roster vs. Set Builder Notation

(ii) Set-builder form.

- In the set builder form, all the elements of the set, must possess a single property to become the member of that set.

For Example:

Z={*x : x* is an integer}

- You can read Z={x:x is an integer}
  Z={x:x is an integer} as "The set Z equals all the values of x such that x is an integer."

Notations: Roster vs. Set Builder Notation

Set builder form

Example 2:

M={x|x>3}

This last notation means "all real numbers xx such that x is greater than 3 ."

Basic Set Notation

- An object that belongs to a set is called an **element (or a member)** of that set. We use special notation to indicate whether an element belongs to a set, as shown below.

Symbol	Meaning
$\in$	is an element of
$\notin$	is not an element of

Examples

Set	Notation	Meaning
A = {2, 4, 6, 8}	$2 \in A$	2 is an element of A
	$5 \notin A$	5 is not an element of A
B = {a, e, i, o, u}	$e \in B$	e is an element of B
	$w \notin B$	w is not an element of B
C = {1, 3, 5, 7, 9}	$7 \in C$	7 is an element of C
	$2 \notin C$	2 is not an element of C
D = {-3, -2, -1, 0, 1, 2, 3}	$-2 \in D$	-2 is an element of D
	$\frac{1}{2} \notin D$	One-half is not an

Determine if the given item is an element of the set.

Determine if the given item is an element of the set.		
Set	Item	Is an element?
R = {2, 4, 6, 8}	10	$10 \notin R$
S = {2, 4, 6, 8, 10}	10	$10 \in S$
D = {English alphabet}	m	
D = {English alphabet}	Σ	$\Sigma \notin D$
X = {prime numbers less than 10}	9	$9 \notin X$
A = {even numbers}	8	$8 \in A$

Equality

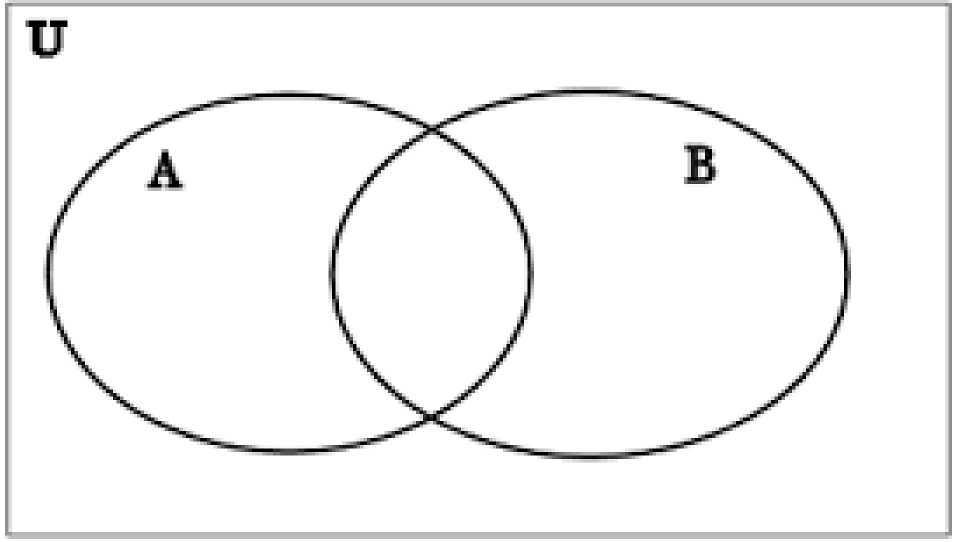
- Definition: Two sets are equal if and only if they have the same elements.
- Example: $\{1,2,3\} = \{3,1,2\} = \{1,2,1,3,2\}$
- Note: Duplicates don't contribute anything new to a set, so remove them. The order of the elements in a set doesn't contribute anything new.
- Example: Are $\{1,2,3,4\}$ and $\{1,2,2,4\}$ equal? No.

Special sets

- The universal set is denoted by U: the set of all objects under the consideration.
- The empty set is denoted as $\varnothing$ or { }.

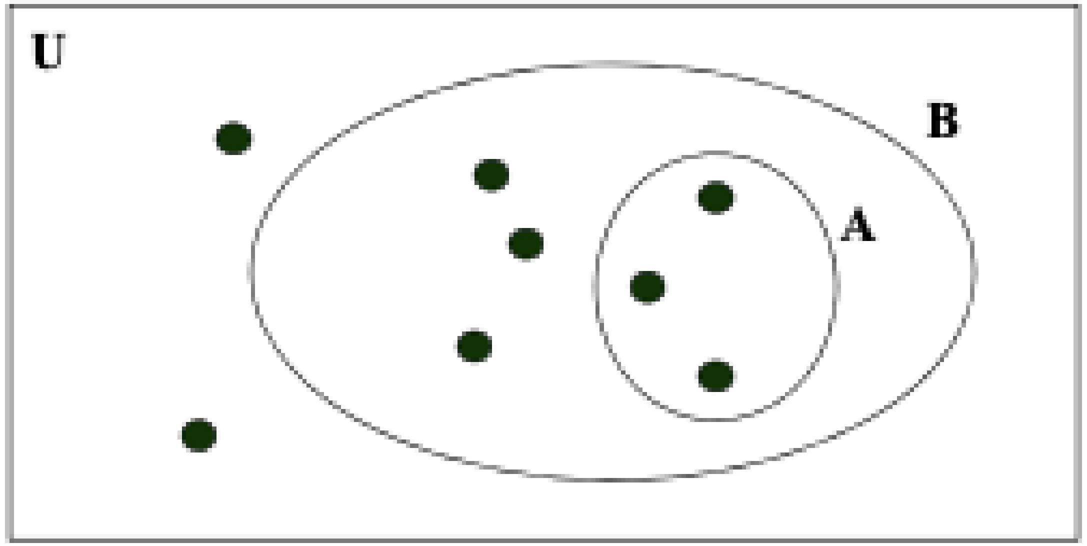
Venn diagrams

- Sets are represented in a Venn diagram by circles drawn inside a rectangle representing the universal set.



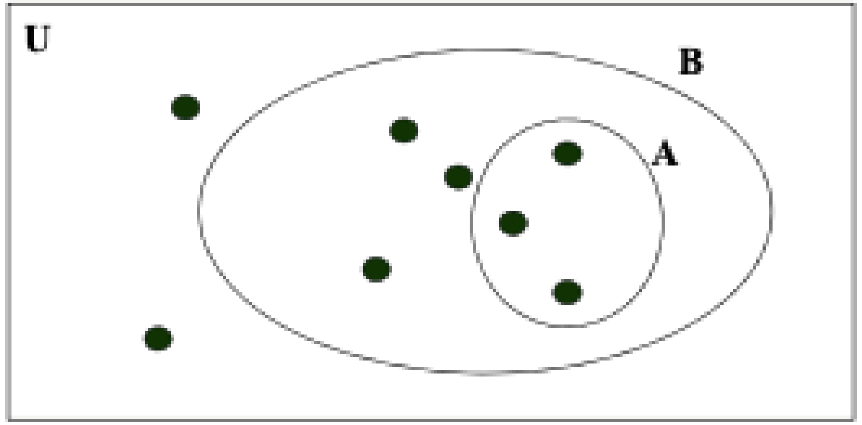
A Subset

- **Definition**: A set A is said to be a subset of B if and only if every element of A is also an element of B. We use A $\subseteq$ B to indicate **A is a subset of B.**



A proper subset

Definition: A **set A** is said to be a **proper subset** of B if and only if **$A \subseteq B$** and **$A \neq B$**. We denote that A is a proper subset of B with the notation $A \subset B$.



Example: $A=\{1,2,3\}$ $B=\{1,2,3,4,5\}$

Is: $A \subset B$? Yes.

A proper subset

Example:

- A proper subset of a set A is a subset of A that is not equal to A. In other words, if B is a proper subset of A, then all elements of B are in A but A contains at least one element that is not in B.
- For example, if A={1,3,5} then B={1,5} is a proper subset of A.
- The set C={1,3,5} is a subset of A, but it is not a proper subset of A since C=A.
- The set D={1,4} is not even a subset of A, since 4 is not an element of A.

Definition: Let S be a set. If there are exactly n distinct elements in S, where n is a nonnegative integer, we say S is a finite set and that n is the **cardinality of S**. The cardinality of S is denoted by **| S |.**

Examples:

- **V={1 2 3 4 5}**
  | V | = 5

- **A={1,2,3,4, …, 20}**
  |A| =20

- **| ∅ | = 0**

Cardinality

Cardinality

- The cardinality of a set is a measure of a set's size, meaning the number of elements in the set.
- For instance, the set A={1,2,4} has a cardinality of 3 for the three elements that are in it.
- The cardinality of a set is denoted by vertical bars, like absolute value signs; for instance, for a set A its cardinality is denoted |A|.
- When A is finite, |A|is simply the number of elements in A. When A is infinite, |A|is represented by a cardinal number.

Cartesian Product

Definition: Let S and T be sets. The **Cartesian product of S and T**, denoted by **S x T,** is the set of all ordered pairs (s,t), where s $\in$ S and t $\in$ T. Hence,

- $S \times T = \{ (s,t) \mid s \in S \wedge t \in T \}$.

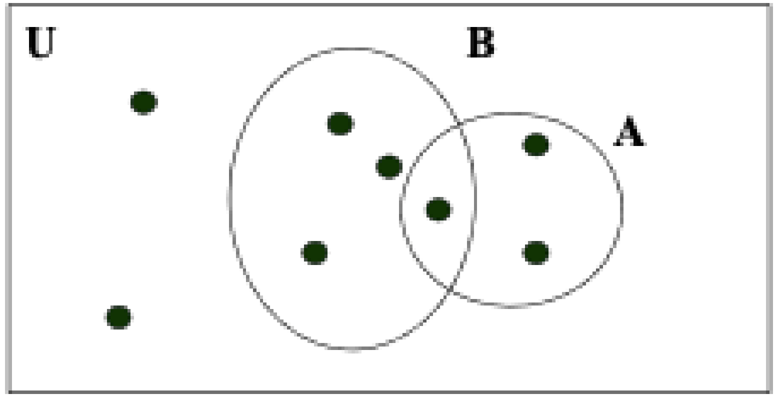
Examples:

- $S = \{1,2\}$ and $T = \{a,b,c\}$
- $S \times T = \{ (1,a), (1,b), (1,c), (2,a), (2,b), (2,c) \}$
- $T \times S = \{ (a,1), (a, 2), (b,1), (b,2), (c,1), (c,2) \}$
- Note: $S \times T \neq T \times S$!!!!

Set Operations

Definition: Let A and B be sets. The **union of A and B**, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

- Alternate: $A \cup B = \{ x \mid x \in A \lor x \in B \}$.

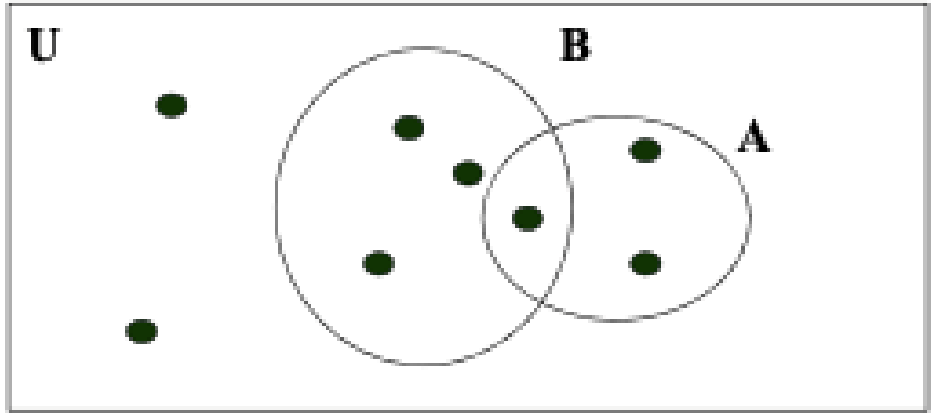


- **Example:**
- $A = \{1,2,3,6\}$ $B = \{ 2,4,6,9\}$
- $A \cup B = \{ 1,2,3,4,6,9 \}$

Union

Definition: Let A and B be sets. The **intersection of A and B**, denoted by $A \cap B$, is the set that contains those elements that a in both A and B.

Alternate: $A \cap B = \{ x \mid x \in A \land x \in B \}$.



Example:

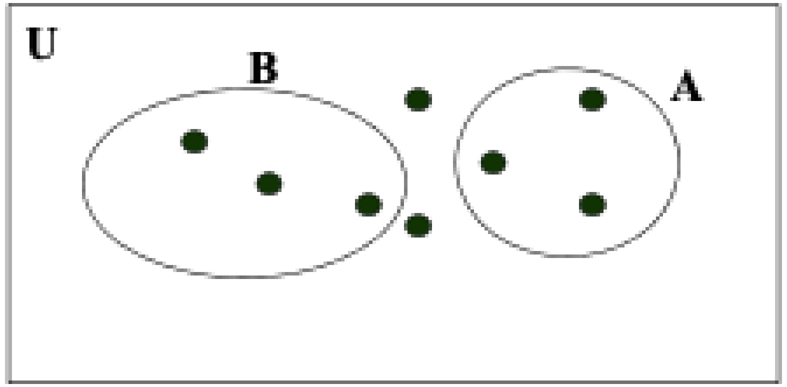
$A = \{1,2,3,6\}$ $B = \{ 2, 4, 6, 9\}$

$A \cap B = \{ 2, 6 \}$

Intersection

Definition: Two sets are called **disjoint** if their intersection is empty.

- Alternate: A and B are disjoint **if and only if** $A \cap B = \varnothing$.



Example:

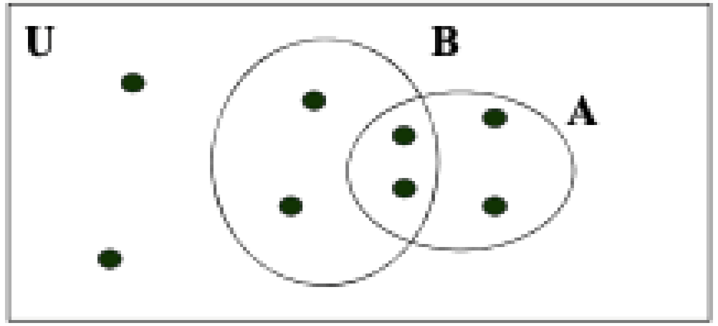
- $A=\{1,2,3,6\}$ $B=\{4,7,8\}$ Are these disjoint?
- Yes.
- $A \cap B = \varnothing$

Disjoint Sets

Set difference

Definition: Let A and B be sets. The **difference of A and B**, denoted by **A - B**, is the set containing those elements that are in A but not in B. The difference of A and B is also called the complement of B with respect to A.

- Alternate: $A - B = \{ x \mid x \in A \wedge x \notin B \}$.



Example: A= {1,2,3,5,7} B = {1,5,6,8}

- A - B ={2,3,7}

Set Difference

Activity

1. **Which of the following is a true statement for set R?**

R = {liquid, gas, solid, plasma}

- gas $\notin$ R
- solid $\notin$ R
- liquid $\in$ R
- None of the above.

2. **Which of the following is true for set G?**

G = {1, 3, 5, 7, 9}

- ○ $5 \notin G$
- ○ $7 \in G$
- ○ $3 \notin G$
- ○ All of the above.

3. **Which of the following statements is true about set B?**

B = {US flag colors}

- ◯ red $\in B$
- ◯ blue $\in B$
- ◯ white $\in B$
- ◯ All of the above.

Which of the following elements is not a member of set X?

X = {tiger, lion, puma, cheetah, leopard, cougar, ocelot}

- ○ cougar
- ○ bobcat
- ○ puma
- ○ tiger

5. **Which of the following elements is not a member of set A?**

A = {states in the US}

- Guam
- Haiti
- Philippines
- All of the above.

Create a set builder and roster notation based on what is given:

- The set of numbers divisible by 5 from 20-60.
- The set of courses offered in UM that are situated in UM Visayan Campus.