Low-field electron mobility analytical models

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The following analytical models for low-field electron mobility (μ) calculations from different scattering mechanism are based on the Reference-[1, 2] and the references therein.

$$\mu = \left(\frac{m^*}{e} \sum_i \frac{1}{\tau_i} \right)^{-1}$$

where τ_i's are the momentum relaxation rate from different scattering mechanisms presented below.

1 Parameters

Parameters name	Symbol	SI Unit
Electron elementary charge	e	C
Boltzmann constant	k_B	$\mathrm{J\,K^{-1}}$
Temperature	T	K
Mass density	ρ	$\mathrm{kg\,m^{-3}}$
Lattice parameter	a_0	Å
Lattice parameter	c_0	Å
Unit cell volume	Ω_0	$\mathrm{Å}^3$
Bandgap energy	E_g	eV
Alloy-disordered scattering potential	U_0	eV
Electron effective mass	m^*	m_0
Electron rest mass	m_0	kg
Static dielectric constant	ε_s	ε_0
High frequency dielectric constant	ε_h	ε_0
Vacuum permittivity	ε_0	$\mathrm{C\,V^{-1}\,m^{-1}}$
Longitudinal acoustic phonon velocity	v_{LA}	$\mathrm{m\,s^{-1}}$
Transversal acoustic phonon velocity	v_{TA}	$\mathrm{m\,s^{-1}}$
Deformation potential	E_D	eV
Polar optical phonon energy	E_{POP}	eV
Electromechanical coupling coefficient	K^2	-
Scattering wave vector	k	$\mathrm{m^{-1}}$
Fermi wave vector	k_F	$\mathrm{m^{-1}}$
Thomas-Fermi wave vector	q_{TF}	$\mathrm{m^{-1}}$
Fang-Howard variational WF parameter	b	$\mathrm{m^{-1}}$
Fang-Howard WF form factor	G, F	-
POP wave vector	k_0	$\mathrm{m^{-1}}$
2D electron gas density	n_{2D}	$\mathrm{m^{-2}}$
Interface RMS roughness	Δ	m
Interface roughness correlation length	L	m
Dislocation density	N_{DIS}	$\mathrm{m^{-2}}$
Dislocation occupancy	f_{DIS}	-

Note: Proper unit conversions for all parameters have been implemented within the equations.

2 Scattering mechanisms

$$b = \left[\frac{33m^*e^2n_{_{2D}}}{8\varepsilon_{_s}\hbar^2}\right]^{\frac{1}{3}} \quad (1)$$

$$k = 2k_{_F}u \quad (2)$$

$$k_{_F} = \sqrt{2\pi n_{_{2D}}} \quad (3)$$

$$q_{_{TF}} = \frac{m^*e^2}{2\pi\varepsilon_{_s}\hbar^2} \quad (4)$$

2.1 Structural related scattering

2.1.1 Interface roughness momentum relaxation rate

$$\frac{1}{\tau_{_{IFR}}} = \frac{m^*e^4(\Delta Ln_{_{2D}})^2}{8\varepsilon_s^2\hbar^3} \times \int_0^1 \frac{u^4\exp\left(-(Lk_{_F}u)^2\right)}{\left(u+\frac{q_{_{TF}}G(u)}{2k_{_F}}\right)^2\sqrt{1-u^2}}du \quad (5)$$

$$G(k) = \frac{2\eta(k)^3+3\eta(k)^2+3\eta(k)}{8} \quad (6)$$

$$\eta(k) = \frac{b}{b+k} \implies \eta(u) = \frac{b}{b+2k_{_F}u} \quad (7)$$

2.1.2 Dislocation momentum relaxation rate

$$\frac{1}{\tau_{_{DIS}}} = \frac{m^*e^4N_{_{DIS}}f_{_{DIS}}^2}{4\pi k_{_F}^4c_0^2\varepsilon_s^2\hbar^3} \times \int_0^1 \frac{1}{\left(u+\frac{q_{_{TF}}}{2k_{_F}}\right)^2\sqrt{1-u^2}}du \quad (8)$$

2.1.3 Alloy-disordered momentum relaxation rate

$$\frac{1}{\tau_{_{AD}}} = \frac{m^*\Omega_0U_0^2x(1-x)}{\hbar^3} \times \int_{-\infty}^{\infty}\Psi(z)^4dz = \frac{m^*\Omega_0U_0^2x(1-x)}{\hbar^3}\frac{3b}{16} \quad (9)$$

2.2 Phonon related scattering

2.2.1 Deformation potential momentum relaxation rate

$$\frac{1}{\tau_{_{DP}}} = \frac{6m^*E_{_D}^2 k_{_B}Tk_{_F}^2 b}{2\pi\rho v_{_{LA}}^2\hbar^3} \times \int_0^1 \frac{u^4}{\left(2k_{_F}u + q_{_{TF}}F(u)\right)^2\sqrt{1-u^2}}du \quad (10)$$

2.2.2 Piezoelectric effect momentum relaxation rate

$$\frac{1}{\tau_{_{PE}}} = \frac{4m^*e^2K^2k_{_B}Tk_{_F}}{\pi\varepsilon_s\hbar^3} \times \int_0^1 \frac{u^3F(u)}{\left(2k_{_F}u + q_{_{TF}}F(u)\right)^2\sqrt{1-u^2}}du \quad (11)$$

$$F(k) = \eta(k)^3 \quad (12)$$

$$\eta(k) = \frac{b}{b+k} \implies \eta(u) = \frac{b}{b+2k_{_F}u} \quad (13)$$

2.2.3 Acoustic phonon scattering relaxation rate

$$\frac{1}{\tau_{_{AP}}} = \frac{1}{\tau_{_{DP}}} + \frac{1}{\tau_{_{PE}}} \quad (14)$$

2.2.4 Polar optical phonon momentum relaxation rate

$$\frac{1}{\tau_{_{POP}}} = \frac{m^*e^2E_{_{POP}}G(k_0)}{2\varepsilon^*k_0\hbar^3} \times \frac{y\left(1+y-e^{-y}\right)^{-1}}{\exp\left(\frac{E_{_{POP}}}{k_{_B}T}\right)-1} \quad (15)$$

$$G(k_0) = \frac{2\eta(k_0)^3 + 3\eta(k_0)^2 + 3\eta(k_0)}{8} \quad (16)$$

$$\eta(k_0) = \frac{b}{b+k_0} \quad (17)$$

$$k_0 = \sqrt{\frac{2m^*E_{_{POP}}}{\hbar^2}} \quad (18)$$

$$\frac{1}{\varepsilon^*} = \frac{1}{\varepsilon_h} - \frac{1}{\varepsilon_s} \quad (19)$$

$$y = \frac{\pi\hbar^2 n_{_{2D}}}{m^*k_{_B}T} \quad (20)$$

References

[1] Julien Bassaler, Jash Mehta, Idriss Abid, Leszek Konczewicz, Sandrine Juillaguet, Sylvie Contreras, Stéphanie Rennesson, Sebastian Tamariz, Maud Nemoz, Fabrice Semond, Julien Pernot, Farid Medjdoub, Yvon Cordier, and Philippe Ferrandis. Al-rich algan channel high electron mobility transistors on silicon: A relevant approach for high temperature stability of electron mobility. *Advanced Electronic Materials*, page 2400069, 2024.

[2] JinFeng Zhang, Yue Hao, JinCheng Zhang, and JinYu Ni. The mobility of two-dimensional electron gas in AlGaN/GaN heterostructures with varied al content. *Science in China Series F: Information Sciences*, 51(6):780–789, 2008.