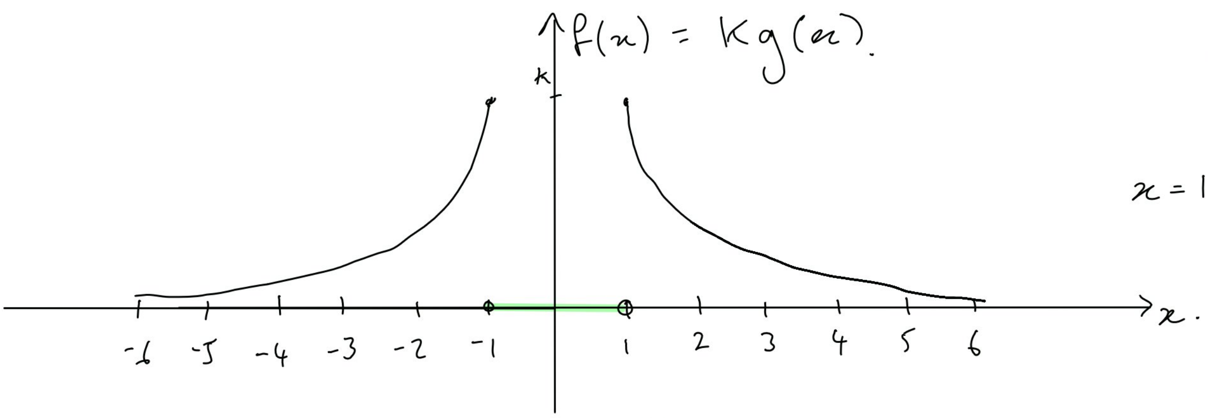
(3) $\quad X \propto g(x) \qquad g(x) = \begin{cases} |x|^{-n} & \text{if } |x| \geq 1 \\ 0 & \text{otherwise.} \end{cases} \quad , \ n \geq 2 .$

X has density function $\quad f(x) = Kg(x) = \begin{cases} k|x|^{-n} & \text{if } |X| \geq 1 \\ 0 & \text{otherwise.} \end{cases}$



$x = 1 \Rightarrow f(x) = K.$

density function $\Rightarrow \displaystyle\int_{-\infty}^{\infty} f(x)\, dx = 1$

Since the function is symmetric about $x=0$, we need

$$2\int_0^\infty f(x)\,dx = 1 = 2\int_1^\infty f(x)\,dx = 1.$$

So, we need to find K s.t.

$$2\int_1^\infty K\cdot|x|^{-n}\,dx = 1.$$

$$\Rightarrow\ 2K\int_1^\infty x^{-n}\,dx = 1$$

$$\Rightarrow\ K = \frac{1}{2\int_1^\infty x^{-n}\,dx} \qquad (n\geq 2).$$

$$\Rightarrow\ K = \frac{1}{2\cdot\left[\frac{x^{1-n}}{1-n}\right]_1^\infty} = \frac{1-n}{2\left(\lim_{x\to\infty}\frac{1}{x^{n-1}} - 1\right)}$$

$$= \frac{1-n}{-2}$$

$$= \frac{n-1}{2}.$$

check:

$$2\cdot\frac{(n-1)}{2}\int_1^\infty x^{-n}\,dx$$

$$= (n-1)\left[\frac{x^{1-n}}{1-n}\right]_1^\infty$$

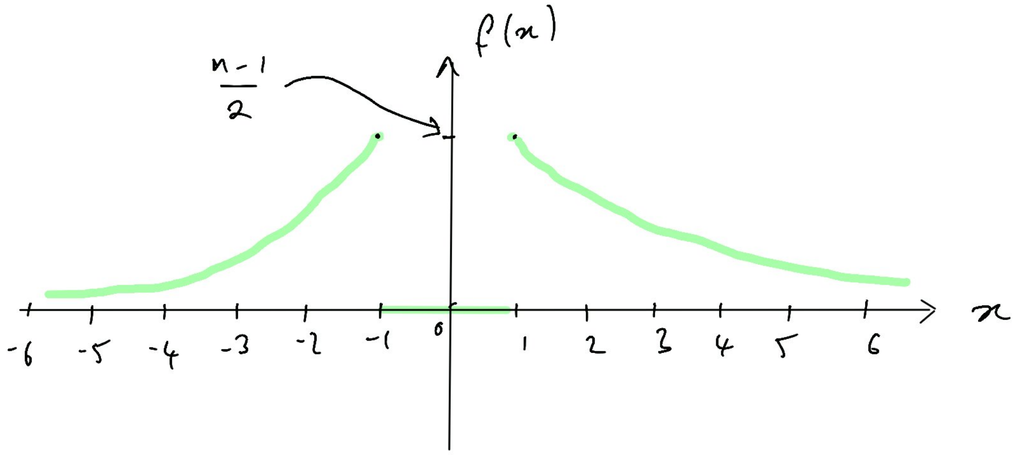
$$= -\left(\lim_{n\to\infty}\frac{1}{x^{n-1}} - 1\right)$$

$$= -(-1) = 1$$

✓ checks out!

So! $$f(x) = Kg(x) = \frac{n-1}{2}g(x) = \begin{cases}\frac{n-1}{2}|x|^{-n} & \text{if } |x|\geq 1\\ 0 & \text{otherwise.}\end{cases} \qquad (n\geq 2)$$

So, density function of X is.



mean $\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$

$$= 2\int_1^{\infty} x \cdot x^{-n}\, dx.$$

$$= 2\int_1^{\infty} x^{1-n}\, dx = \frac{2}{2-n}\left[x^{2-n}\right]_1^{\infty}$$

$$= \frac{2}{2-n}\left(\lim_{x\to\infty} \frac{1}{x^{n-2}} - 1\right)$$

$$= \frac{2}{n-2} \quad \text{only if } n \geq 3.$$

variance: $var(X) = E(X^2) - E(X)^2$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx = 2\int_1^{\infty} x^2 x^{-n}\, dx$$

$$= 2\int_1^{\infty} x^{2-n}\, dx = \frac{2}{3-n}\left[x^{3-n}\right]_1^{\infty}$$

$$= \frac{2}{3-n}\left[\lim_{n\to\infty} \frac{1}{x^{n-3}} - 1\right].$$

$$= \frac{2}{n-3} \quad \text{only if } n \geq 4.$$

therefore, mean & variance exist iff $n \geq 4$.