Techniek, Ontwerpen en Informatica / Engineering, Design and Computing
Bedrijfswiskunde / Information Technology

Front page exam	
Student - Surname	
Student - First name	
Studentnumber	
Group:	
Exam for group(s):	2IT
Exam Code:	3713IT232A
Exam Title:	Probability Theory
Period:	1
Date of Exam:	October 30, 2015
Time of Exam:	14:00-16:00
Number of questions	10
Number of pages: (title page excluded)	6
Additional means to be allowed:	none
Remarks / Particulars:	Return of exam paper: Yes
Name of lecturer:	Koos van Tubergen

Below are 10 exercises, each exercise weighted at 12 point max.
However you get ONLY points, if you add a motivation by calculation, by theorem / theory, by graph etc.
The grade is the total divided by 12.

Good luck!

1. [Definition of Conditional Probability]

Give the definition of Conditional Probability.

2. [Domino]

a) Calculate the Probability, that a stone has at least one blanco side.
b) Calculate the Probability, that a stone has both odd sides.
c) Calculate the Probability, that a stone has at least one 4, given that both sides are even.

3. [Conditional probability]

Vase I contains 3 white and 4 black balls, and Vase II contains 2 white and 6 black balls.

a) You pick a ball randomly from Vase I and place it in Vase II. Next you pick a ball randomly from Vase II. What is the probability that the ball is black?

b) This time, you pick a Vase at random, each of the two Vases being picked with probability ½ , and you pick a ball at random from the chosen Vase. Given the ball is black, what is the probability you picked Vase I ?

4. [Vase]

A Vase contains 1 red, 2 white, 3 blue balls. Someone picks out 3 balls.

a) Calculate the Probability, that he has got one of each color.

b) Calculate the Probability, that the balls have at least 2 different colors.

5. [Probability mass function]

Someone throws a fair die.
Let the stochast $\underline{x}$ = the number of dots in one throw.

a) Make a graph of the (probability) mass function $p(x)$

b) Make a graph of the (cumulative) distribution function $F(x)$

c) Calculate E$\underline{x}$, var ($\underline{x}$), and σ ($\underline{x}$)

6. [Uniform distribution]

A stochast $\underline{x}$ has an Uniform distribution with density function

$$f(x) = \begin{cases} \frac{1}{5} \; if \; 3 < x < 8 \\ 0 \; otherwise \end{cases}$$

a) Make a graph of the (cumulative) distribution function $F(x)$.

b) Calculate E $\underline{x}^2$

c) Calculate $P\left(\left|\underline{x}\right| \geq 6\right)$ by Chebyshev's inequality

7. [Correlation coefficient]

Let's look at Domino again.
Let the stochast $\underline{x}$ = the maximum of both sides of dots of one stone
and let the stochast $\underline{y}$ = the minimum of both sides of dots of one stone.
Further let the difference stochast $\underline{d}$ = $\underline{x}$ - $\underline{y}$

a) Calculate $E\underline{x}$
b) Calculate $var(\ \underline{x}\ )$
c) Calculate $\rho(\ \underline{x}\ ,\ \underline{y}\ )$
d) Calculate $var(\ \underline{d}\ )$

8. [Central limit theorem]

A fair die is thrown 12000 times.
Let the stochast $\underline{s}$ = the total number of sixes thrown.
Use the central limit theorem to find values of a and b such that

$$P\left(1900 < \underline{s} < 2200\right) \approx \int_a^b \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}x^2}\, dx$$

9. [Markov chain with a Die]

Someone throws a die many times.
Let the stochast $\underline{x}_n$ = the greatest number shown in the first n throws of a fair six-sided die.
He starts counting at n=0 (first throw).

a) Calculate the initial distribution.
b) Calculate the transition matrix.
c) Calculate E $\underline{x}_0$
d) Calculate E $\underline{x}_1$

10. [Two-state Markov Chain]

A two-state Markov chain has a transition matrix:

$$P = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

a) Calculate P^2
b) Calculate P^3
c) What will be $\lim_{n\to\infty} P^n$? (no calculation)