

#### PRELIMINAR OBSERVATIONS:

This is the output of the example program II\_SExpStructConst\_sl2\_S770.java.

It performs expansions of the  $sl(2)$  algebra,

$$[X_{\{1\}}, X_{\{2\}}] = -2 X_{\{3\}} ,$$

$$[X_{\{1\}}, X_{\{3\}}] = 2 X_{\{2\}} ,$$

$$[X_{\{2\}}, X_{\{3\}}] = 2 X_{\{1\}} ,$$

with the semigroup  $S_{\{5\}}^{\{770\}}$ , whose multiplication table is given by:

1 1 1 1 1

1 2 1 1 5

1 1 3 4 1

1 1 4 3 1

1 5 1 1 2

The resonance that will be considered is:

$$S_0 = \{1, 2, 3\}, \quad S_1 = \{1, 4, 5\},$$

and the zero element is: 1.

It gives the structure constants  $C_{\{(i,a)(j,b)\}^{\{(k,c)\}}}$  of:

- 1) Expanded algebra
- 2) Resonant subalgebra
- 3) Reduced algebra
- 4) Reduction of the resonant subalgebra

#### NOTATION:

Using  $i, j, k=1, \dots, n$  and  $a, b, c=1, \dots, m$ , the structure constants of the expanded algebra will be given as follows:

We first give  $m$  matrices  $C_{\{(1,a)(j,b)\}^{\{(k,c)\}}}$

$$C_{\{(1,1)(j,b)\}^{\{(k,c)\}}}, C_{\{(1,2)(j,b)\}^{\{(k,c)\}}}, \dots, C_{\{(1,m)(j,b)\}^{\{(k,c)\}}}$$

Then the  $m$  matrices  $C_{\{(2,a)(j,b)\}^{\{(k,c)\}}}$

$$C_{\{(2,1)(j,b)\}^{\{(k,c)\}}}, C_{\{(2,2)(j,b)\}^{\{(k,c)\}}}, \dots, C_{\{(2,m)(j,b)\}^{\{(k,c)\}}}$$

and thus we continue until giving the  $m$  boxes  $C_{\{(n,a)(j,b)\}^{\{(k,c)\}}}$

$$C_{\{(n,1)(j,b)\}^{\{(k,c)\}}}, C_{\{(n,2)(j,b)\}^{\{(k,c)\}}}, \dots, C_{\{(n,m)(j,b)\}^{\{(k,c)\}}}$$

In a similar way we give the selectors of the reduced algebra, resonant subalgebra and reduction of

the resonant subalgebra, omitting the rows and columns  $(i,a)$  that are not in the corresponding algebra.

The range where the indices  $(i,a)$  are running is indicated for each case 1-4.

Finally, we remind that the method 'setStructureConstant()' reads the non-vanishing structure constants  $C_{\{ij\}^{\{k\}}}$

in such a way that  $i, j, k=0, 1, \dots, n-1$ . They are introduced as follows:

name.setStructureConstant( i , j , k ,  $C_{\{ij\}^{\{k\}}}$  )

Similarly  $a, b, c=0, 1, \dots, m-1$  in the functions  $C_{\{(i,a)(j,b)\}^{\{(k,c)\}}}$ .

However, the outputs will be given in such a way that  $i, j, k=1, \dots, n$  and  $a, b, c=1, \dots, m$ .

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We introduce the structure constants of  $sl_2$ .

Remind that if a non vanishing structure constant  $C_{\{ij\}^{\{k\}}}$  has the

value  $V$ , then we introduce it as: name.setStructureConstant( i-1 , j-1 , k-1 ,  $V$  )

Show its Killing-Cartan metric

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-8.00 0.00 0.00
0.00 8.00 0.00
0.00 0.00 8.00

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whose determinant is:  
-512.0

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NOTATION for the Expanded algebra:

n = 3 , Dimension of the original Lie algebra.  
m = 5 , Order of the semigroup.

To print the structure constants notice that for (i,a) fixed,  
the quantities  $C_{\{(i,a)(j,b)\}^{\{(k,c)\}}}=M_{\{A,B\}}$  are elements  
of a matrix M whose indices have the following values:  
A,B = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,  
Or equivalently,  
A,B = (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5), (3,1),  
(3,2), (3,3), (3,4), (3,5),

Here we print the m tables  $C_{\{(1,a)(j,b)\}^{\{(k,c)\}}}$ , with a=1,...,m.  
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C_{(1,1)} (j,b)^{(k,c)}
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
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0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

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C_{(1,2)} (j,b)^{(k,c)}
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -2.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0 -0.0
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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -0.0 -2.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0

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 $C_{\{(1,3)(j,b)\}^{\{(k,c)\}}}$





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[illegible]

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[illegible]

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[illegible]

[illegible][illegible][illegible]

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NOTATION for the Resonant Subalgebra:

n = 3 , Dimension of the original Lie algebra.  
m = 5 , Order of the semigroup.

To print the structure constants notice that for (i,a) fixed,  
the quantities  $C_{\{(i,a)(j,b)\}^{\{(k,c)\}}} = M_{\{A,B\}}$  are elements  
of a matrix M whose indices have the following values:  
A,B = 1, 2, 3, 6, 9, 10, 11, 14, 15,  
Or equivalently,  
A,B = (1,1), (1,2), (1,3), (2,1), (2,4), (2,5), (3,1), (3,4), (3,5),

Here we print the matrices  $C_{\{(1,a)(j,b)\}^{\{(k,c)\}}}$ , with the double indices having  
the values described above.

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$C_{\{(1,1)(j,b)\}^{\{(k,c)\}}}$   
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0  
0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0  
0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0  
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0

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$C_{\{(1,2)(j,b)\}^{\{(k,c)\}}}$   
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0  
0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0  
0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -2.0  
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0

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$C_{\{(1,3)(j,b)\}^{\{(k,c)\}}}$   
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0  
0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -2.0 -0.0  
0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0  
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0

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Here we print the matrices  $C_{\{(2,a)(j,b)\}^{\{(k,c)\}}}$ , with the double indices having  
the values described above.

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$C_{\{(2,1)(j,b)\}^{\{(k,c)\}}}$   
0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0  
2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0





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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
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NOTATION for the Reduced Algebra:

n = 3 , Dimension of the original Lie algebra.  
m = 5 , Order of the semigroup.

To print the structure constants notice that for (i,a) fixed,  
the quantities  $C_{\{(i,a)(j,b)\}^{\{(k,c)\}}=M_{\{A,B\}}$  are elements  
of a matrix M whose indices have the following values:

A,B = 2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15,

Or equivalently,

A,B = (1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5), (3,2), (3,3), (3,4),  
(3,5),

Here we print the matrices  $C_{\{(1,a)(j,b)\}^{\{(k,c)\}}}$ , with the double indices having  
the values described above.

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$C_{\{(1,2)(j,b)\}^{\{(k,c)\}}}$

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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -2.0
0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0

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$C_{\{(1,3)(j,b)\}^{\{(k,c)\}}}$

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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -2.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -2.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

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$C_{\{(1,4)(j,b)\}^{\{(k,c)\}}}$

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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -2.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -2.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -0.0

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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

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$C_{\{(1,5)\}}(j,b)^{\{(k,c)\}}$

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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -2.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

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Here we print the matrices  $C_{\{(2,a)\}}(j,b)^{\{(k,c)\}}$ , with the double indices having the values described above.

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$C_{\{(2,2)\}}(j,b)^{\{(k,c)\}}$

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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

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$C_{\{(2,3)\}}(j,b)^{\{(k,c)\}}$

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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

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$C_{\{(2,4)\}}(j,b)^{\{(k,c)\}}$

```

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

```



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-0.0  -2.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0

```

\*\*\*\*\*

$C_{\{(3,5)\}}(j,b)^{\{(k,c)\}}$

```

0.0  0.0  0.0  0.0  -0.0  -0.0  -0.0  -2.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -0.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -0.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -2.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0
-0.0  -0.0  -0.0  -2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-2.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0

```

\*\*\*\*\*

-----

NOTATION for the Reduction of the Resonant Subalgebra:

$n = 3$  , Dimension of the original Lie algebra.

$m = 5$  , Order of the semigroup.

To print the structure constants notice that for  $(i,a)$  fixed,  
the quantities  $C_{\{(i,a)\}}(j,b)^{\{(k,c)\}} = M_{\{A,B\}}$  are elements  
of a matrix  $M$  whose indices have the following values:

$A,B = 2, 3, 9, 10, 14, 15,$

Or equivalently,

$A,B = (1,2), (1,3), (2,4), (2,5), (3,4), (3,5),$

Here we print the matrices  $C_{\{(1,a)\}}(j,b)^{\{(k,c)\}}$ , with the double indices having  
the values described above.

\*\*\*\*\*

$C_{\{(1,2)\}}(j,b)^{\{(k,c)\}}$

```

0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -0.0  -0.0
0.0  0.0  0.0  0.0  -0.0  -2.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  2.0  0.0  0.0

```

\*\*\*\*\*

$C_{\{(1,3)\}}(j,b)^{\{(k,c)\}}$

```

0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -2.0  -0.0
0.0  0.0  0.0  0.0  -0.0  -0.0
0.0  0.0  2.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0

```

\*\*\*\*\*

Here we print the matrices  $C_{\{(2,a)\}}(j,b)^{\{(k,c)\}}$ , with the double indices having  
the values described above.

\*\*\*\*\*

```

C_{(2,4)} (j,b)^{(k,c)}
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  2.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  2.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0

```

\*\*\*\*\*

```

C_{(2,5)} (j,b)^{(k,c)}
0.0  0.0  0.0  0.0  0.0  2.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
2.0  0.0  0.0  0.0  0.0  0.0

```

\*\*\*\*\*

Here we print the matrices  $C_{(3,a)} (j,b)^{(k,c)}$ , with the double indices having the values described above.

\*\*\*\*\*

```

C_{(3,4)} (j,b)^{(k,c)}
0.0  0.0  -0.0  -0.0  0.0  0.0
0.0  0.0  -2.0  -0.0  0.0  0.0
-0.0  -2.0  0.0  0.0  0.0  0.0
-0.0  -0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0

```

\*\*\*\*\*

```

C_{(3,5)} (j,b)^{(k,c)}
0.0  0.0  -0.0  -2.0  0.0  0.0
0.0  0.0  -0.0  -0.0  0.0  0.0
-0.0  -0.0  0.0  0.0  0.0  0.0
-2.0  -0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0

```

\*\*\*\*\*

-----