

PRELIMINAR OBSERVATIONS:

This is the output of the example program II_SExpStructConst_sl2_S990.java.

It performs expansions of the $sl(2)$ algebra,

$$[X_{\{1\}}, X_{\{2\}}] = -2 X_{\{3\}} ,$$

$$[X_{\{1\}}, X_{\{3\}}] = 2 X_{\{2\}} ,$$

$$[X_{\{2\}}, X_{\{3\}}] = 2 X_{\{1\}} ,$$

with the semigroup $S_{\{5\}}^{\{990\}}$, whose multiplication table is given by:

1 1 1 1 1

1 2 3 4 5

1 3 2 5 4

1 4 5 2 3

1 5 4 3 2

The resonance that will be considered is:

$$S_0 = \{1, 2, 5\}, \quad S_1 = \{1, 3, 4\},$$

and the zero element is: 1.

It gives the structure constants $C_{\{(i,a)(j,b)\}^{\{(k,c)\}}}$ of:

- 1) Expanded algebra
- 2) Resonant subalgebra
- 3) Reduced algebra
- 4) Reduction of the resonant subalgebra

NOTATION:

Using $i, j, k=1, \dots, n$ and $a, b, c=1, \dots, m$, the structure constants of the expanded algebra will be given as follows:

We first give m matrices $C_{\{(1,a)(j,b)\}^{\{(k,c)\}}}$

$$C_{\{(1,1)(j,b)\}^{\{(k,c)\}}}, C_{\{(1,2)(j,b)\}^{\{(k,c)\}}}, \dots, C_{\{(1,m)(j,b)\}^{\{(k,c)\}}}$$

Then the m matrices $C_{\{(2,a)(j,b)\}^{\{(k,c)\}}}$

$$C_{\{(2,1)(j,b)\}^{\{(k,c)\}}}, C_{\{(2,2)(j,b)\}^{\{(k,c)\}}}, \dots, C_{\{(2,m)(j,b)\}^{\{(k,c)\}}}$$

and thus we continue until giving the m boxes $C_{\{(n,a)(j,b)\}^{\{(k,c)\}}}$

$$C_{\{(n,1)(j,b)\}^{\{(k,c)\}}}, C_{\{(n,2)(j,b)\}^{\{(k,c)\}}}, \dots, C_{\{(n,m)(j,b)\}^{\{(k,c)\}}}$$

In a similar way we give the selectors of the reduced algebra, resonant subalgebra and reduction of

the resonant subalgebra, omitting the rows and columns (i,a) that are not in the corresponding algebra.

The range where the indices (i,a) are running is indicated for each case 1-4.

Finally, we remind that the method 'setStructureConstant()' reads the non-vanishing structure constants $C_{\{ij\}^k}$

in such a way that $i, j, k=0, 1, \dots, n-1$. They are introduced as follows:

name.setStructureConstant(i , j , k , $C_{\{ij\}^k}$)

Similarly $a, b, c=0, 1, \dots, m-1$ in the functions $C_{\{(i,a)(j,b)\}^{\{(k,c)\}}}$.

However, the outputs will be given in such a way that $i, j, k=1, \dots, n$ and $a, b, c=1, \dots, m$.

We introduce the structure constants of sl_2 .

Remind that if a non vanishing structure constant $C_{\{ij\}^k}$ has the

value V , then we introduce it as: name.setStructureConstant(i-1 , j-1 , k-1 , V)

Show its Killing-Cartan metric

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-8.00 0.00 0.00
0.00 8.00 0.00
0.00 0.00 8.00

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whose determinant is:
-512.0

NOTATION for the Expanded algebra:

n = 3 , Dimension of the original Lie algebra.
m = 5 , Order of the semigroup.

To print the structure constants notice that for (i,a) fixed,
the quantities $C_{\{(i,a)(j,b)\}^{\{(k,c)\}}} = M_{\{A,B\}}$ are elements
of a matrix M whose indices have the following values:
A,B = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,
Or equivalently,
A,B = (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5), (3,1),
(3,2), (3,3), (3,4), (3,5),

Here we print the m tables $C_{\{(1,a)(j,b)\}^{\{(k,c)\}}}$, with a=1,...,m.

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C_{(1,1)} (j,b)^{(k,c)}
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0 -0.0
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0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

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C_{(1,2)} (j,b)^{(k,c)}
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -2.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -2.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -2.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -0.0 -2.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0

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C_{(1,3)} (j,b)^{(k,c)}

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[illegible]

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[illegible]

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* * * * *

[illegible]

[illegible]

[illegible][illegible][illegible]

NOTATION for the Resonant Subalgebra:

n = 3 , Dimension of the original Lie algebra.
m = 5 , Order of the semigroup.

To print the structure constants notice that for (i,a) fixed,
the quantities $C_{\{(i,a)(j,b)\}^{\{(k,c)\}}}=M_{\{A,B\}}$ are elements
of a matrix M whose indices have the following values:
A,B = 1, 2, 5, 6, 8, 9, 11, 13, 14,
Or equivalently,
A,B = (1,1), (1,2), (1,5), (2,1), (2,3), (2,4), (3,1), (3,3), (3,4),

Here we print the matrices $C_{\{(1,a)(j,b)\}^{\{(k,c)\}}}$, with the double indices having
the values described above.

$C_{\{(1,1)(j,b)\}^{\{(k,c)\}}}$
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0

$C_{\{(1,2)(j,b)\}^{\{(k,c)\}}}$
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -2.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -2.0
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0

$C_{\{(1,5)(j,b)\}^{\{(k,c)\}}}$
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -2.0
0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -2.0 -0.0
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0

Here we print the matrices $C_{\{(2,a)(j,b)\}^{\{(k,c)\}}}$, with the double indices having
the values described above.

$C_{\{(2,1)(j,b)\}^{\{(k,c)\}}}$
0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0


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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
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NOTATION for the Reduced Algebra:

n = 3 , Dimension of the original Lie algebra.
m = 5 , Order of the semigroup.

To print the structure constants notice that for (i,a) fixed,
the quantities $C_{\{(i,a)(j,b)\}^{\{(k,c)\}}=M_{\{A,B\}}$ are elements
of a matrix M whose indices have the following values:

A,B = 2, 3, 4, 5, 7, 8, 9, 10, 12, 13, 14, 15,

Or equivalently,

A,B = (1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5), (3,2), (3,3), (3,4),
(3,5),

Here we print the matrices $C_{\{(1,a)(j,b)\}^{\{(k,c)\}}}$, with the double indices having
the values described above.

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$C_{\{(1,2)(j,b)\}^{\{(k,c)\}}}$

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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -2.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -2.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -2.0
0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0
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0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0

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$C_{\{(1,3)(j,b)\}^{\{(k,c)\}}}$

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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -2.0 -0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0
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0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0

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$C_{\{(1,4)(j,b)\}^{\{(k,c)\}}}$

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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -2.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -2.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -2.0 -0.0 -0.0

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0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0

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$C_{\{(1,5)\}}(j,b)^{\{(k,c)\}}$

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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -0.0 -2.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -0.0 -2.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -0.0 -2.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 -2.0 -0.0 -0.0 -0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

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Here we print the matrices $C_{\{(2,a)\}}(j,b)^{\{(k,c)\}}$, with the double indices having the values described above.

$C_{\{(2,2)\}}(j,b)^{\{(k,c)\}}$

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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

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$C_{\{(2,3)\}}(j,b)^{\{(k,c)\}}$

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0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 2.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

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$C_{\{(2,4)\}}(j,b)^{\{(k,c)\}}$

```

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 2.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0

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0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0

```

$C_{\{(2,5)\}}(j,b)^{\{(k,c)\}}$

```

0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  2.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  2.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  2.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  2.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0

```

Here we print the matrices $C_{\{(3,a)\}}(j,b)^{\{(k,c)\}}$, with the double indices having the values described above.

$C_{\{(3,2)\}}(j,b)^{\{(k,c)\}}$

```

0.0  0.0  0.0  0.0  0.0  -2.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -0.0  -2.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -0.0  -0.0  -2.0  -0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -0.0  -0.0  -0.0  -2.0  0.0  0.0  0.0  0.0  0.0
-2.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -2.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -0.0  -2.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -0.0  -0.0  -2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0

```

$C_{\{(3,3)\}}(j,b)^{\{(k,c)\}}$

```

0.0  0.0  0.0  0.0  -0.0  -2.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -2.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -0.0  -0.0  -0.0  -2.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -0.0  -0.0  -2.0  -0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -2.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-2.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -0.0  -0.0  -2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -0.0  -2.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0

```

$C_{\{(3,4)\}}(j,b)^{\{(k,c)\}}$

```

0.0  0.0  0.0  0.0  -0.0  -0.0  -2.0  -0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -0.0  -0.0  -2.0  -0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -2.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -0.0  -2.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -0.0  -2.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -0.0  -0.0  -2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0

```

```

-2.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -2.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0   0.0   0.0   0.0   0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0

```

$C_{\{(3,5)\}}(j,b)^{\{(k,c)\}}$

```

0.0  0.0  0.0  0.0  -0.0  -0.0  -0.0  -2.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -0.0  -0.0  -2.0  -0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -0.0  -2.0  -0.0  -0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -2.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0
-0.0  -0.0  -0.0  -2.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -0.0  -2.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-0.0  -2.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
-2.0  -0.0  -0.0  -0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0

```

NOTATION for the Reduction of the Resonant Subalgebra:

$n = 3$, Dimension of the original Lie algebra.

$m = 5$, Order of the semigroup.

To print the structure constants notice that for (i,a) fixed, the quantities $C_{\{(i,a)\}}(j,b)^{\{(k,c)\}} = M_{\{A,B\}}$ are elements of a matrix M whose indices have the following values:

$A, B = 2, 5, 8, 9, 13, 14,$

Or equivalently,

$A, B = (1,2), (1,5), (2,3), (2,4), (3,3), (3,4),$

Here we print the matrices $C_{\{(1,a)\}}(j,b)^{\{(k,c)\}}$, with the double indices having the values described above.

$C_{\{(1,2)\}}(j,b)^{\{(k,c)\}}$

```

0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -2.0  -0.0
0.0  0.0  0.0  0.0  -0.0  -2.0
0.0  0.0  2.0  0.0  0.0  0.0
0.0  0.0  0.0  2.0  0.0  0.0

```

$C_{\{(1,5)\}}(j,b)^{\{(k,c)\}}$

```

0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  -0.0  -2.0
0.0  0.0  0.0  0.0  -2.0  -0.0
0.0  0.0  0.0  2.0  0.0  0.0
0.0  0.0  2.0  0.0  0.0  0.0

```

Here we print the matrices $C_{\{(2,a)\}}(j,b)^{\{(k,c)\}}$, with the double indices having the values described above.

```

C_{(2,3)}(j,b)^{(k,c)}
0.0  0.0  0.0  0.0  2.0  0.0
0.0  0.0  0.0  0.0  0.0  2.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
2.0  0.0  0.0  0.0  0.0  0.0
0.0  2.0  0.0  0.0  0.0  0.0

```

```

C_{(2,4)}(j,b)^{(k,c)}
0.0  0.0  0.0  0.0  0.0  2.0
0.0  0.0  0.0  0.0  2.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  2.0  0.0  0.0  0.0  0.0
2.0  0.0  0.0  0.0  0.0  0.0

```

Here we print the matrices $C_{(3,a)}(j,b)^{(k,c)}$, with the double indices having the values described above.

```

C_{(3,3)}(j,b)^{(k,c)}
0.0  0.0  -2.0  -0.0  0.0  0.0
0.0  0.0  -0.0  -2.0  0.0  0.0
-2.0  -0.0  0.0  0.0  0.0  0.0
-0.0  -2.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0

```

```

C_{(3,4)}(j,b)^{(k,c)}
0.0  0.0  -0.0  -2.0  0.0  0.0
0.0  0.0  -2.0  -0.0  0.0  0.0
-0.0  -2.0  0.0  0.0  0.0  0.0
-2.0  -0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0

```
