

PRELIMINAR OBSERVATIONS:

This is the output of the example program II_SExp_sl2_S990.java, which performs expansions of the $sl(2)$ algebra with the semigroup $S_{\{5\}^{990}}$, whose multiplication table is given by:

```
1 1 1 1 1
1 2 3 4 5
1 3 2 5 4
1 4 5 2 3
1 5 4 3 2
```

The resonance is:

$S_0 = \{1,2,5\}$, $S_1 = \{1,3,4\}$ and zero element is 1.

It gives the commutation relations of:

- 1) Expanded algebra
- 2) Resonant subalgebra
- 3) Reduced algebra
- 4) Reduction of the resonant subalgebra

NOTATION:

The original algebra and the expanded one, GXS , are given by

$$[X_{\{i\}}, X_{\{j\}}] = C_{\{ij\}}^{\{k\}} X_{\{k\}}, \quad (1)$$

$$[X_{\{i,a\}}, X_{\{j,b\}}] = C_{\{(i,a)(j,b)\}}^{\{(k,c)\}} X_{\{k,c\}}, \quad (2)$$

where $i,j,k=1,\dots,n$ and $a,b,c=1,\dots,m$.

As the first position of an array $[][]$ is usually $[0][0]$, the method 'setStructureConstant()' reads the non-vanishing structure constants $C_{\{ij\}}^{\{k\}}$ in such a way that $i,j,k=0,1,\dots,n-1$. They are introduced as follows:

```
name.setStructureConstant( i , j , k , C_{\{ij\}}^{\{k\}} )
```

Similarly $a,b,c=0,1,\dots,m-1$ in the functions $C_{\{(i,a)(j,b)\}}^{\{(k,c)\}}$.

However, the outputs will be given in such a way that $i,j,k=1,\dots,n$ and $a,b,c=1,\dots,m$.

We introduce the structure constants of sl_2 .

Remind that if a non vanishing structure constant $C_{\{ij\}}^{\{k\}}$ has the value V , then we introduce it as:

```
name.setStructureConstant( i-1 , j-1 , k-1 , V )
```

Show its Killing-Cartan metric

```
-8.00 0.00 0.00
0.00 8.00 0.00
0.00 0.00 8.00
```

whose determinant is:

-512.0

METHOD: showCommut()

Non vanishing commutators of the 'Expanded algebra'

n = 3 , Dimension of the original Lie algebra.
m = 5 , Order of the semigroup.

With the notation: $X_{\{i,a\}} = X_{\{i\}} \lambda_{\{a\}}$, the generators of the 'Expanded algebra' are given by:

$Y_{\{1\}} = X_{\{1,1\}}$
 $Y_{\{2\}} = X_{\{1,2\}}$
 $Y_{\{3\}} = X_{\{1,3\}}$
 $Y_{\{4\}} = X_{\{1,4\}}$
 $Y_{\{5\}} = X_{\{1,5\}}$
 $Y_{\{6\}} = X_{\{2,1\}}$
 $Y_{\{7\}} = X_{\{2,2\}}$
 $Y_{\{8\}} = X_{\{2,3\}}$
 $Y_{\{9\}} = X_{\{2,4\}}$
 $Y_{\{10\}} = X_{\{2,5\}}$
 $Y_{\{11\}} = X_{\{3,1\}}$
 $Y_{\{12\}} = X_{\{3,2\}}$
 $Y_{\{13\}} = X_{\{3,3\}}$
 $Y_{\{14\}} = X_{\{3,4\}}$
 $Y_{\{15\}} = X_{\{3,5\}}$

The non vanishing commutators of the 'Expanded algebra' are given by:

$[X_{\{1,1\}}, X_{\{2,1\}}] = -2.0 X_{\{3,1\}}$
 $[X_{\{1,1\}}, X_{\{2,2\}}] = -2.0 X_{\{3,1\}}$
 $[X_{\{1,1\}}, X_{\{2,3\}}] = -2.0 X_{\{3,1\}}$
 $[X_{\{1,1\}}, X_{\{2,4\}}] = -2.0 X_{\{3,1\}}$
 $[X_{\{1,1\}}, X_{\{2,5\}}] = -2.0 X_{\{3,1\}}$
 $[X_{\{1,1\}}, X_{\{3,1\}}] = 2.0 X_{\{2,1\}}$
 $[X_{\{1,1\}}, X_{\{3,2\}}] = 2.0 X_{\{2,1\}}$
 $[X_{\{1,1\}}, X_{\{3,3\}}] = 2.0 X_{\{2,1\}}$
 $[X_{\{1,1\}}, X_{\{3,4\}}] = 2.0 X_{\{2,1\}}$
 $[X_{\{1,1\}}, X_{\{3,5\}}] = 2.0 X_{\{2,1\}}$
 $[X_{\{1,2\}}, X_{\{2,1\}}] = -2.0 X_{\{3,1\}}$
 $[X_{\{1,2\}}, X_{\{2,2\}}] = -2.0 X_{\{3,2\}}$
 $[X_{\{1,2\}}, X_{\{2,3\}}] = -2.0 X_{\{3,3\}}$
 $[X_{\{1,2\}}, X_{\{2,4\}}] = -2.0 X_{\{3,4\}}$
 $[X_{\{1,2\}}, X_{\{2,5\}}] = -2.0 X_{\{3,5\}}$
 $[X_{\{1,2\}}, X_{\{3,1\}}] = 2.0 X_{\{2,1\}}$
 $[X_{\{1,2\}}, X_{\{3,2\}}] = 2.0 X_{\{2,2\}}$
 $[X_{\{1,2\}}, X_{\{3,3\}}] = 2.0 X_{\{2,3\}}$
 $[X_{\{1,2\}}, X_{\{3,4\}}] = 2.0 X_{\{2,4\}}$
 $[X_{\{1,2\}}, X_{\{3,5\}}] = 2.0 X_{\{2,5\}}$
 $[X_{\{1,3\}}, X_{\{2,1\}}] = -2.0 X_{\{3,1\}}$
 $[X_{\{1,3\}}, X_{\{2,2\}}] = -2.0 X_{\{3,3\}}$
 $[X_{\{1,3\}}, X_{\{2,3\}}] = -2.0 X_{\{3,2\}}$
 $[X_{\{1,3\}}, X_{\{2,4\}}] = -2.0 X_{\{3,5\}}$
 $[X_{\{1,3\}}, X_{\{2,5\}}] = -2.0 X_{\{3,4\}}$
 $[X_{\{1,3\}}, X_{\{3,1\}}] = 2.0 X_{\{2,1\}}$
 $[X_{\{1,3\}}, X_{\{3,2\}}] = 2.0 X_{\{2,3\}}$
 $[X_{\{1,3\}}, X_{\{3,3\}}] = 2.0 X_{\{2,2\}}$
 $[X_{\{1,3\}}, X_{\{3,4\}}] = 2.0 X_{\{2,5\}}$
 $[X_{\{1,3\}}, X_{\{3,5\}}] = 2.0 X_{\{2,4\}}$
 $[X_{\{1,4\}}, X_{\{2,1\}}] = -2.0 X_{\{3,1\}}$
 $[X_{\{1,4\}}, X_{\{2,2\}}] = -2.0 X_{\{3,4\}}$
 $[X_{\{1,4\}}, X_{\{2,3\}}] = -2.0 X_{\{3,5\}}$

$$\begin{aligned}
[X_{\{1,4\}}, X_{\{2,4\}}] &= -2.0 X_{\{3,2\}} \\
[X_{\{1,4\}}, X_{\{2,5\}}] &= -2.0 X_{\{3,3\}} \\
[X_{\{1,4\}}, X_{\{3,1\}}] &= 2.0 X_{\{2,1\}} \\
[X_{\{1,4\}}, X_{\{3,2\}}] &= 2.0 X_{\{2,4\}} \\
[X_{\{1,4\}}, X_{\{3,3\}}] &= 2.0 X_{\{2,5\}} \\
[X_{\{1,4\}}, X_{\{3,4\}}] &= 2.0 X_{\{2,2\}} \\
[X_{\{1,4\}}, X_{\{3,5\}}] &= 2.0 X_{\{2,3\}} \\
[X_{\{1,5\}}, X_{\{2,1\}}] &= -2.0 X_{\{3,1\}} \\
[X_{\{1,5\}}, X_{\{2,2\}}] &= -2.0 X_{\{3,5\}} \\
[X_{\{1,5\}}, X_{\{2,3\}}] &= -2.0 X_{\{3,4\}} \\
[X_{\{1,5\}}, X_{\{2,4\}}] &= -2.0 X_{\{3,3\}} \\
[X_{\{1,5\}}, X_{\{2,5\}}] &= -2.0 X_{\{3,2\}} \\
[X_{\{1,5\}}, X_{\{3,1\}}] &= 2.0 X_{\{2,1\}} \\
[X_{\{1,5\}}, X_{\{3,2\}}] &= 2.0 X_{\{2,5\}} \\
[X_{\{1,5\}}, X_{\{3,3\}}] &= 2.0 X_{\{2,4\}} \\
[X_{\{1,5\}}, X_{\{3,4\}}] &= 2.0 X_{\{2,3\}} \\
[X_{\{1,5\}}, X_{\{3,5\}}] &= 2.0 X_{\{2,2\}} \\
[X_{\{2,1\}}, X_{\{3,1\}}] &= 2.0 X_{\{1,1\}} \\
[X_{\{2,1\}}, X_{\{3,2\}}] &= 2.0 X_{\{1,1\}} \\
[X_{\{2,1\}}, X_{\{3,3\}}] &= 2.0 X_{\{1,1\}} \\
[X_{\{2,1\}}, X_{\{3,4\}}] &= 2.0 X_{\{1,1\}} \\
[X_{\{2,1\}}, X_{\{3,5\}}] &= 2.0 X_{\{1,1\}} \\
[X_{\{2,2\}}, X_{\{3,1\}}] &= 2.0 X_{\{1,1\}} \\
[X_{\{2,2\}}, X_{\{3,2\}}] &= 2.0 X_{\{1,2\}} \\
[X_{\{2,2\}}, X_{\{3,3\}}] &= 2.0 X_{\{1,3\}} \\
[X_{\{2,2\}}, X_{\{3,4\}}] &= 2.0 X_{\{1,4\}} \\
[X_{\{2,2\}}, X_{\{3,5\}}] &= 2.0 X_{\{1,5\}} \\
[X_{\{2,3\}}, X_{\{3,1\}}] &= 2.0 X_{\{1,1\}} \\
[X_{\{2,3\}}, X_{\{3,2\}}] &= 2.0 X_{\{1,3\}} \\
[X_{\{2,3\}}, X_{\{3,3\}}] &= 2.0 X_{\{1,2\}} \\
[X_{\{2,3\}}, X_{\{3,4\}}] &= 2.0 X_{\{1,5\}} \\
[X_{\{2,3\}}, X_{\{3,5\}}] &= 2.0 X_{\{1,4\}} \\
[X_{\{2,4\}}, X_{\{3,1\}}] &= 2.0 X_{\{1,1\}} \\
[X_{\{2,4\}}, X_{\{3,2\}}] &= 2.0 X_{\{1,4\}} \\
[X_{\{2,4\}}, X_{\{3,3\}}] &= 2.0 X_{\{1,5\}} \\
[X_{\{2,4\}}, X_{\{3,4\}}] &= 2.0 X_{\{1,2\}} \\
[X_{\{2,4\}}, X_{\{3,5\}}] &= 2.0 X_{\{1,3\}} \\
[X_{\{2,5\}}, X_{\{3,1\}}] &= 2.0 X_{\{1,1\}} \\
[X_{\{2,5\}}, X_{\{3,2\}}] &= 2.0 X_{\{1,5\}} \\
[X_{\{2,5\}}, X_{\{3,3\}}] &= 2.0 X_{\{1,4\}} \\
[X_{\{2,5\}}, X_{\{3,4\}}] &= 2.0 X_{\{1,3\}} \\
[X_{\{2,5\}}, X_{\{3,5\}}] &= 2.0 X_{\{1,2\}}
\end{aligned}$$

METHOD: showSC()

Non vanishing structure constants of the Expanded algebra:

$$\begin{aligned}
C_{\{(1,1)(2,1)\}}^{\{(3,1)\}} &= -2.0 \\
C_{\{(1,1)(2,2)\}}^{\{(3,1)\}} &= -2.0 \\
C_{\{(1,1)(2,3)\}}^{\{(3,1)\}} &= -2.0 \\
C_{\{(1,1)(2,4)\}}^{\{(3,1)\}} &= -2.0 \\
C_{\{(1,1)(2,5)\}}^{\{(3,1)\}} &= -2.0 \\
C_{\{(1,1)(3,1)\}}^{\{(2,1)\}} &= 2.0 \\
C_{\{(1,1)(3,2)\}}^{\{(2,1)\}} &= 2.0 \\
C_{\{(1,1)(3,3)\}}^{\{(2,1)\}} &= 2.0 \\
C_{\{(1,1)(3,4)\}}^{\{(2,1)\}} &= 2.0 \\
C_{\{(1,1)(3,5)\}}^{\{(2,1)\}} &= 2.0
\end{aligned}$$

$C_{\{(1,2)(2,1)\}}^{\{(3,1)\}} = -2.0$
 $C_{\{(1,2)(2,2)\}}^{\{(3,2)\}} = -2.0$
 $C_{\{(1,2)(2,3)\}}^{\{(3,3)\}} = -2.0$
 $C_{\{(1,2)(2,4)\}}^{\{(3,4)\}} = -2.0$
 $C_{\{(1,2)(2,5)\}}^{\{(3,5)\}} = -2.0$
 $C_{\{(1,2)(3,1)\}}^{\{(2,1)\}} = 2.0$
 $C_{\{(1,2)(3,2)\}}^{\{(2,2)\}} = 2.0$
 $C_{\{(1,2)(3,3)\}}^{\{(2,3)\}} = 2.0$
 $C_{\{(1,2)(3,4)\}}^{\{(2,4)\}} = 2.0$
 $C_{\{(1,2)(3,5)\}}^{\{(2,5)\}} = 2.0$
 $C_{\{(1,3)(2,1)\}}^{\{(3,1)\}} = -2.0$
 $C_{\{(1,3)(2,2)\}}^{\{(3,3)\}} = -2.0$
 $C_{\{(1,3)(2,3)\}}^{\{(3,2)\}} = -2.0$
 $C_{\{(1,3)(2,4)\}}^{\{(3,5)\}} = -2.0$
 $C_{\{(1,3)(2,5)\}}^{\{(3,4)\}} = -2.0$
 $C_{\{(1,3)(3,1)\}}^{\{(2,1)\}} = 2.0$
 $C_{\{(1,3)(3,2)\}}^{\{(2,3)\}} = 2.0$
 $C_{\{(1,3)(3,3)\}}^{\{(2,2)\}} = 2.0$
 $C_{\{(1,3)(3,4)\}}^{\{(2,5)\}} = 2.0$
 $C_{\{(1,3)(3,5)\}}^{\{(2,4)\}} = 2.0$
 $C_{\{(1,4)(2,1)\}}^{\{(3,1)\}} = -2.0$
 $C_{\{(1,4)(2,2)\}}^{\{(3,4)\}} = -2.0$
 $C_{\{(1,4)(2,3)\}}^{\{(3,5)\}} = -2.0$
 $C_{\{(1,4)(2,4)\}}^{\{(3,2)\}} = -2.0$
 $C_{\{(1,4)(2,5)\}}^{\{(3,3)\}} = -2.0$
 $C_{\{(1,4)(3,1)\}}^{\{(2,1)\}} = 2.0$
 $C_{\{(1,4)(3,2)\}}^{\{(2,4)\}} = 2.0$
 $C_{\{(1,4)(3,3)\}}^{\{(2,5)\}} = 2.0$
 $C_{\{(1,4)(3,4)\}}^{\{(2,2)\}} = 2.0$
 $C_{\{(1,4)(3,5)\}}^{\{(2,3)\}} = 2.0$
 $C_{\{(1,5)(2,1)\}}^{\{(3,1)\}} = -2.0$
 $C_{\{(1,5)(2,2)\}}^{\{(3,5)\}} = -2.0$
 $C_{\{(1,5)(2,3)\}}^{\{(3,4)\}} = -2.0$
 $C_{\{(1,5)(2,4)\}}^{\{(3,3)\}} = -2.0$
 $C_{\{(1,5)(2,5)\}}^{\{(3,2)\}} = -2.0$
 $C_{\{(1,5)(3,1)\}}^{\{(2,1)\}} = 2.0$
 $C_{\{(1,5)(3,2)\}}^{\{(2,5)\}} = 2.0$
 $C_{\{(1,5)(3,3)\}}^{\{(2,4)\}} = 2.0$
 $C_{\{(1,5)(3,4)\}}^{\{(2,3)\}} = 2.0$
 $C_{\{(1,5)(3,5)\}}^{\{(2,2)\}} = 2.0$
 $C_{\{(2,1)(3,1)\}}^{\{(1,1)\}} = 2.0$
 $C_{\{(2,1)(3,2)\}}^{\{(1,1)\}} = 2.0$
 $C_{\{(2,1)(3,3)\}}^{\{(1,1)\}} = 2.0$
 $C_{\{(2,1)(3,4)\}}^{\{(1,1)\}} = 2.0$
 $C_{\{(2,1)(3,5)\}}^{\{(1,1)\}} = 2.0$
 $C_{\{(2,2)(3,1)\}}^{\{(1,1)\}} = 2.0$
 $C_{\{(2,2)(3,2)\}}^{\{(1,2)\}} = 2.0$
 $C_{\{(2,2)(3,3)\}}^{\{(1,3)\}} = 2.0$
 $C_{\{(2,2)(3,4)\}}^{\{(1,4)\}} = 2.0$
 $C_{\{(2,2)(3,5)\}}^{\{(1,5)\}} = 2.0$
 $C_{\{(2,3)(3,1)\}}^{\{(1,1)\}} = 2.0$
 $C_{\{(2,3)(3,2)\}}^{\{(1,3)\}} = 2.0$
 $C_{\{(2,3)(3,3)\}}^{\{(1,2)\}} = 2.0$
 $C_{\{(2,3)(3,4)\}}^{\{(1,5)\}} = 2.0$
 $C_{\{(2,3)(3,5)\}}^{\{(1,4)\}} = 2.0$

```

C_{(2,4)(3,1)}^{(1,1)} = 2.0
C_{(2,4)(3,2)}^{(1,4)} = 2.0
C_{(2,4)(3,3)}^{(1,5)} = 2.0
C_{(2,4)(3,4)}^{(1,2)} = 2.0
C_{(2,4)(3,5)}^{(1,3)} = 2.0
C_{(2,5)(3,1)}^{(1,1)} = 2.0
C_{(2,5)(3,2)}^{(1,5)} = 2.0
C_{(2,5)(3,3)}^{(1,4)} = 2.0
C_{(2,5)(3,4)}^{(1,3)} = 2.0
C_{(2,5)(3,5)}^{(1,2)} = 2.0

```

METHOD: cartanKillingMetric()

The Killing-Cartan Metric of the Expanded algebra is:

```

-8 -8 -8 -8 -8 0 0 0 0 0 0 0 0 0
-8 -40 -8 -8 -8 0 0 0 0 0 0 0 0 0
-8 -8 -40 -8 -8 0 0 0 0 0 0 0 0 0
-8 -8 -8 -40 -8 0 0 0 0 0 0 0 0 0
-8 -8 -8 -8 -40 0 0 0 0 0 0 0 0 0
0 0 0 0 0 8 8 8 8 8 0 0 0 0
0 0 0 0 0 8 40 8 8 8 0 0 0 0
0 0 0 0 0 8 8 40 8 8 0 0 0 0
0 0 0 0 0 8 8 8 40 8 0 0 0 0
0 0 0 0 0 8 8 8 8 40 0 0 0 0
0 0 0 0 0 0 0 0 0 8 8 8 8 8
0 0 0 0 0 0 0 0 0 8 40 8 8 8
0 0 0 0 0 0 0 0 0 8 8 40 8 8
0 0 0 0 0 0 0 0 0 8 8 8 40 8
0 0 0 0 0 0 0 0 0 8 8 8 8 40

```

The determinant of the Killing-Cartan Metric of the Expanded algebra is:

-5.9029581035870565E20

METHOD: showCommutRes()

Non vanishing commutators of the 'Resonant Subalgebra'

n = 3 , Dimension of the original Lie algebra.

m = 5 , Order of the semigroup.

With the notation: $X_{i,a} = X_i \lambda_a$, the generators of the 'Resonant Subalgebra' are given by:

```

Y_{1} = X_{1,1}
Y_{2} = X_{1,2}
Y_{5} = X_{1,5}
Y_{6} = X_{2,1}
Y_{8} = X_{2,3}
Y_{9} = X_{2,4}
Y_{11} = X_{3,1}
Y_{13} = X_{3,3}
Y_{14} = X_{3,4}

```

The non vanishing commutators of the 'Resonant Subalgebra' are given by:

```

[ X_{1,1} , X_{2,1} ] = -2.0 X_{3,1}
[ X_{1,1} , X_{2,3} ] = -2.0 X_{3,1}

```

```

[ X_{1,1} , X_{2,4} ] = -2.0 X_{3,1}
[ X_{1,1} , X_{3,1} ] = 2.0 X_{2,1}
[ X_{1,1} , X_{3,3} ] = 2.0 X_{2,1}
[ X_{1,1} , X_{3,4} ] = 2.0 X_{2,1}
[ X_{1,2} , X_{2,1} ] = -2.0 X_{3,1}
[ X_{1,2} , X_{2,3} ] = -2.0 X_{3,3}
[ X_{1,2} , X_{2,4} ] = -2.0 X_{3,4}
[ X_{1,2} , X_{3,1} ] = 2.0 X_{2,1}
[ X_{1,2} , X_{3,3} ] = 2.0 X_{2,3}
[ X_{1,2} , X_{3,4} ] = 2.0 X_{2,4}
[ X_{1,5} , X_{2,1} ] = -2.0 X_{3,1}
[ X_{1,5} , X_{2,3} ] = -2.0 X_{3,4}
[ X_{1,5} , X_{2,4} ] = -2.0 X_{3,3}
[ X_{1,5} , X_{3,1} ] = 2.0 X_{2,1}
[ X_{1,5} , X_{3,3} ] = 2.0 X_{2,4}
[ X_{1,5} , X_{3,4} ] = 2.0 X_{2,3}
[ X_{2,1} , X_{3,1} ] = 2.0 X_{1,1}
[ X_{2,1} , X_{3,3} ] = 2.0 X_{1,1}
[ X_{2,1} , X_{3,4} ] = 2.0 X_{1,1}
[ X_{2,3} , X_{3,1} ] = 2.0 X_{1,1}
[ X_{2,3} , X_{3,3} ] = 2.0 X_{1,2}
[ X_{2,3} , X_{3,4} ] = 2.0 X_{1,5}
[ X_{2,4} , X_{3,1} ] = 2.0 X_{1,1}
[ X_{2,4} , X_{3,3} ] = 2.0 X_{1,5}
[ X_{2,4} , X_{3,4} ] = 2.0 X_{1,2}

```

METHOD: showSCRes()

Non vanishing structure constants of the 'Resonant Subalgebra' are given by:

```

C_{(1,1)(2,1)}^{(3,1)} = -2.0
C_{(1,1)(2,3)}^{(3,1)} = -2.0
C_{(1,1)(2,4)}^{(3,1)} = -2.0
C_{(1,1)(3,1)}^{(2,1)} = 2.0
C_{(1,1)(3,3)}^{(2,1)} = 2.0
C_{(1,1)(3,4)}^{(2,1)} = 2.0
C_{(1,2)(2,1)}^{(3,1)} = -2.0
C_{(1,2)(2,3)}^{(3,3)} = -2.0
C_{(1,2)(2,4)}^{(3,4)} = -2.0
C_{(1,2)(3,1)}^{(2,1)} = 2.0
C_{(1,2)(3,3)}^{(2,3)} = 2.0
C_{(1,2)(3,4)}^{(2,4)} = 2.0
C_{(1,5)(2,1)}^{(3,1)} = -2.0
C_{(1,5)(2,3)}^{(3,4)} = -2.0
C_{(1,5)(2,4)}^{(3,3)} = -2.0
C_{(1,5)(3,1)}^{(2,1)} = 2.0
C_{(1,5)(3,3)}^{(2,4)} = 2.0
C_{(1,5)(3,4)}^{(2,3)} = 2.0
C_{(2,1)(3,1)}^{(1,1)} = 2.0
C_{(2,1)(3,3)}^{(1,1)} = 2.0
C_{(2,1)(3,4)}^{(1,1)} = 2.0
C_{(2,3)(3,1)}^{(1,1)} = 2.0
C_{(2,3)(3,3)}^{(1,2)} = 2.0
C_{(2,3)(3,4)}^{(1,5)} = 2.0
C_{(2,4)(3,1)}^{(1,1)} = 2.0
C_{(2,4)(3,3)}^{(1,5)} = 2.0
C_{(2,4)(3,4)}^{(1,2)} = 2.0

```

METHOD: cartanKillingMetricPretty()

The Killing-Cartan Metric of the Resonant subalgebra is:

```
-8 -8 -8 0 0 0 0 0 0
-8 -24 -8 0 0 0 0 0 0
-8 -8 -24 0 0 0 0 0 0
0 0 0 8 8 8 0 0 0
0 0 0 8 24 8 0 0 0
0 0 0 8 8 24 0 0 0
0 0 0 0 0 0 8 8 8
0 0 0 0 0 0 8 24 8
0 0 0 0 0 0 8 8 24
```

The determinant of the Killing-Cartan Metric of the Resonant subalgebra is:

-8.589934592E9

METHOD: showCommutRed()

Non vanishing commutators of the 'Reduced algebra'

n = 3 , Dimension of the original Lie algebra.

m = 5 , Order of the semigroup.

With the notation: $X_{\{i,a\}} = X_{\{i\}} \lambda_{\{a\}}$, the generators of the 'Reduced algebra' are given by:

```
Y_{2} = X_{1,2}
Y_{3} = X_{1,3}
Y_{4} = X_{1,4}
Y_{5} = X_{1,5}
Y_{7} = X_{2,2}
Y_{8} = X_{2,3}
Y_{9} = X_{2,4}
Y_{10} = X_{2,5}
Y_{12} = X_{3,2}
Y_{13} = X_{3,3}
Y_{14} = X_{3,4}
Y_{15} = X_{3,5}
```

The non vanishing commutators of the 'Reduced algebra' are given by:

```
[ X_{1,2} , X_{2,2} ] = -2.0 X_{3,2}
[ X_{1,2} , X_{2,3} ] = -2.0 X_{3,3}
[ X_{1,2} , X_{2,4} ] = -2.0 X_{3,4}
[ X_{1,2} , X_{2,5} ] = -2.0 X_{3,5}
[ X_{1,2} , X_{3,2} ] = 2.0 X_{2,2}
[ X_{1,2} , X_{3,3} ] = 2.0 X_{2,3}
[ X_{1,2} , X_{3,4} ] = 2.0 X_{2,4}
[ X_{1,2} , X_{3,5} ] = 2.0 X_{2,5}
[ X_{1,3} , X_{2,2} ] = -2.0 X_{3,3}
[ X_{1,3} , X_{2,3} ] = -2.0 X_{3,2}
[ X_{1,3} , X_{2,4} ] = -2.0 X_{3,5}
[ X_{1,3} , X_{2,5} ] = -2.0 X_{3,4}
[ X_{1,3} , X_{3,2} ] = 2.0 X_{2,3}
[ X_{1,3} , X_{3,3} ] = 2.0 X_{2,2}
[ X_{1,3} , X_{3,4} ] = 2.0 X_{2,5}
```

$$\begin{aligned}
[X_{\{1,3\}} , X_{\{3,5\}}] &= 2.0 X_{\{2,4\}} \\
[X_{\{1,4\}} , X_{\{2,2\}}] &= -2.0 X_{\{3,4\}} \\
[X_{\{1,4\}} , X_{\{2,3\}}] &= -2.0 X_{\{3,5\}} \\
[X_{\{1,4\}} , X_{\{2,4\}}] &= -2.0 X_{\{3,2\}} \\
[X_{\{1,4\}} , X_{\{2,5\}}] &= -2.0 X_{\{3,3\}} \\
[X_{\{1,4\}} , X_{\{3,2\}}] &= 2.0 X_{\{2,4\}} \\
[X_{\{1,4\}} , X_{\{3,3\}}] &= 2.0 X_{\{2,5\}} \\
[X_{\{1,4\}} , X_{\{3,4\}}] &= 2.0 X_{\{2,2\}} \\
[X_{\{1,4\}} , X_{\{3,5\}}] &= 2.0 X_{\{2,3\}} \\
[X_{\{1,5\}} , X_{\{2,2\}}] &= -2.0 X_{\{3,5\}} \\
[X_{\{1,5\}} , X_{\{2,3\}}] &= -2.0 X_{\{3,4\}} \\
[X_{\{1,5\}} , X_{\{2,4\}}] &= -2.0 X_{\{3,3\}} \\
[X_{\{1,5\}} , X_{\{2,5\}}] &= -2.0 X_{\{3,2\}} \\
[X_{\{1,5\}} , X_{\{3,2\}}] &= 2.0 X_{\{2,5\}} \\
[X_{\{1,5\}} , X_{\{3,3\}}] &= 2.0 X_{\{2,4\}} \\
[X_{\{1,5\}} , X_{\{3,4\}}] &= 2.0 X_{\{2,3\}} \\
[X_{\{1,5\}} , X_{\{3,5\}}] &= 2.0 X_{\{2,2\}} \\
[X_{\{2,2\}} , X_{\{3,2\}}] &= 2.0 X_{\{1,2\}} \\
[X_{\{2,2\}} , X_{\{3,3\}}] &= 2.0 X_{\{1,3\}} \\
[X_{\{2,2\}} , X_{\{3,4\}}] &= 2.0 X_{\{1,4\}} \\
[X_{\{2,2\}} , X_{\{3,5\}}] &= 2.0 X_{\{1,5\}} \\
[X_{\{2,3\}} , X_{\{3,2\}}] &= 2.0 X_{\{1,3\}} \\
[X_{\{2,3\}} , X_{\{3,3\}}] &= 2.0 X_{\{1,2\}} \\
[X_{\{2,3\}} , X_{\{3,4\}}] &= 2.0 X_{\{1,5\}} \\
[X_{\{2,3\}} , X_{\{3,5\}}] &= 2.0 X_{\{1,4\}} \\
[X_{\{2,4\}} , X_{\{3,2\}}] &= 2.0 X_{\{1,4\}} \\
[X_{\{2,4\}} , X_{\{3,3\}}] &= 2.0 X_{\{1,5\}} \\
[X_{\{2,4\}} , X_{\{3,4\}}] &= 2.0 X_{\{1,2\}} \\
[X_{\{2,4\}} , X_{\{3,5\}}] &= 2.0 X_{\{1,3\}} \\
[X_{\{2,5\}} , X_{\{3,2\}}] &= 2.0 X_{\{1,5\}} \\
[X_{\{2,5\}} , X_{\{3,3\}}] &= 2.0 X_{\{1,4\}} \\
[X_{\{2,5\}} , X_{\{3,4\}}] &= 2.0 X_{\{1,3\}} \\
[X_{\{2,5\}} , X_{\{3,5\}}] &= 2.0 X_{\{1,2\}}
\end{aligned}$$

METHOD: showSCRed()

Non vanishing structure constants of the 'Reduced algebra' are given by:

$$\begin{aligned}
C_{\{(1,2)(2,2)\}^{\{(3,2)\}}} &= -2.0 \\
C_{\{(1,2)(2,3)\}^{\{(3,3)\}}} &= -2.0 \\
C_{\{(1,2)(2,4)\}^{\{(3,4)\}}} &= -2.0 \\
C_{\{(1,2)(2,5)\}^{\{(3,5)\}}} &= -2.0 \\
C_{\{(1,2)(3,2)\}^{\{(2,2)\}}} &= 2.0 \\
C_{\{(1,2)(3,3)\}^{\{(2,3)\}}} &= 2.0 \\
C_{\{(1,2)(3,4)\}^{\{(2,4)\}}} &= 2.0 \\
C_{\{(1,2)(3,5)\}^{\{(2,5)\}}} &= 2.0 \\
C_{\{(1,3)(2,2)\}^{\{(3,3)\}}} &= -2.0 \\
C_{\{(1,3)(2,3)\}^{\{(3,2)\}}} &= -2.0 \\
C_{\{(1,3)(2,4)\}^{\{(3,5)\}}} &= -2.0 \\
C_{\{(1,3)(2,5)\}^{\{(3,4)\}}} &= -2.0 \\
C_{\{(1,3)(3,2)\}^{\{(2,3)\}}} &= 2.0 \\
C_{\{(1,3)(3,3)\}^{\{(2,2)\}}} &= 2.0 \\
C_{\{(1,3)(3,4)\}^{\{(2,5)\}}} &= 2.0 \\
C_{\{(1,3)(3,5)\}^{\{(2,4)\}}} &= 2.0 \\
C_{\{(1,4)(2,2)\}^{\{(3,4)\}}} &= -2.0 \\
C_{\{(1,4)(2,3)\}^{\{(3,5)\}}} &= -2.0 \\
C_{\{(1,4)(2,4)\}^{\{(3,2)\}}} &= -2.0
\end{aligned}$$


```

C_{(1,4)(2,5)}^{(3,3)} = -2.0
C_{(1,4)(3,2)}^{(2,4)} = 2.0
C_{(1,4)(3,3)}^{(2,5)} = 2.0
C_{(1,4)(3,4)}^{(2,2)} = 2.0
C_{(1,4)(3,5)}^{(2,3)} = 2.0
C_{(1,5)(2,2)}^{(3,5)} = -2.0
C_{(1,5)(2,3)}^{(3,4)} = -2.0
C_{(1,5)(2,4)}^{(3,3)} = -2.0
C_{(1,5)(2,5)}^{(3,2)} = -2.0
C_{(1,5)(3,2)}^{(2,5)} = 2.0
C_{(1,5)(3,3)}^{(2,4)} = 2.0
C_{(1,5)(3,4)}^{(2,3)} = 2.0
C_{(1,5)(3,5)}^{(2,2)} = 2.0
C_{(2,2)(3,2)}^{(1,2)} = 2.0
C_{(2,2)(3,3)}^{(1,3)} = 2.0
C_{(2,2)(3,4)}^{(1,4)} = 2.0
C_{(2,2)(3,5)}^{(1,5)} = 2.0
C_{(2,3)(3,2)}^{(1,3)} = 2.0
C_{(2,3)(3,3)}^{(1,2)} = 2.0
C_{(2,3)(3,4)}^{(1,5)} = 2.0
C_{(2,3)(3,5)}^{(1,4)} = 2.0
C_{(2,4)(3,2)}^{(1,4)} = 2.0
C_{(2,4)(3,3)}^{(1,5)} = 2.0
C_{(2,4)(3,4)}^{(1,2)} = 2.0
C_{(2,4)(3,5)}^{(1,3)} = 2.0
C_{(2,5)(3,2)}^{(1,5)} = 2.0
C_{(2,5)(3,3)}^{(1,4)} = 2.0
C_{(2,5)(3,4)}^{(1,3)} = 2.0
C_{(2,5)(3,5)}^{(1,2)} = 2.0

```

METHOD: cartanKillingMetricPretty()

The Killing-Cartan Metric of the Reduced algebra is:

```

-32 0 0 0 0 0 0 0 0 0 0
0 -32 0 0 0 0 0 0 0 0 0
0 0 -32 0 0 0 0 0 0 0 0
0 0 0 -32 0 0 0 0 0 0 0
0 0 0 0 32 0 0 0 0 0 0
0 0 0 0 0 32 0 0 0 0 0
0 0 0 0 0 0 32 0 0 0 0
0 0 0 0 0 0 0 32 0 0 0
0 0 0 0 0 0 0 0 32 0 0
0 0 0 0 0 0 0 0 0 32 0
0 0 0 0 0 0 0 0 0 0 32

```

The determinant of the Killing-Cartan Metric of the Reduced algebra is:
1.15292150460684698E18

METHOD: showCommutResRed()

Non vanishing commutators of the 'Reduction of the Resonant Subalgebra'

n = 3 , Dimension of the original Lie algebra.

m = 5 , Order of the semigroup.

With the notation: $X_{\{i,a\}} = X_{\{i\}} \lambda_{\{a\}}$, the generators of the 'Reduction of the Resonant Subalgebra' are given by:

$$\begin{aligned} Y_{\{2\}} &= X_{\{1,2\}} \\ Y_{\{5\}} &= X_{\{1,5\}} \\ Y_{\{8\}} &= X_{\{2,3\}} \\ Y_{\{9\}} &= X_{\{2,4\}} \\ Y_{\{13\}} &= X_{\{3,3\}} \\ Y_{\{14\}} &= X_{\{3,4\}} \end{aligned}$$

The non vanishing commutators of the 'Reduction of the Resonant Subalgebra' are given by:

$$\begin{aligned} [X_{\{1,2\}}, X_{\{2,3\}}] &= -2.0 X_{\{3,3\}} \\ [X_{\{1,2\}}, X_{\{2,4\}}] &= -2.0 X_{\{3,4\}} \\ [X_{\{1,2\}}, X_{\{3,3\}}] &= 2.0 X_{\{2,3\}} \\ [X_{\{1,2\}}, X_{\{3,4\}}] &= 2.0 X_{\{2,4\}} \\ [X_{\{1,5\}}, X_{\{2,3\}}] &= -2.0 X_{\{3,4\}} \\ [X_{\{1,5\}}, X_{\{2,4\}}] &= -2.0 X_{\{3,3\}} \\ [X_{\{1,5\}}, X_{\{3,3\}}] &= 2.0 X_{\{2,4\}} \\ [X_{\{1,5\}}, X_{\{3,4\}}] &= 2.0 X_{\{2,3\}} \\ [X_{\{2,3\}}, X_{\{3,3\}}] &= 2.0 X_{\{1,2\}} \\ [X_{\{2,3\}}, X_{\{3,4\}}] &= 2.0 X_{\{1,5\}} \\ [X_{\{2,4\}}, X_{\{3,3\}}] &= 2.0 X_{\{1,5\}} \\ [X_{\{2,4\}}, X_{\{3,4\}}] &= 2.0 X_{\{1,2\}} \end{aligned}$$

METHOD: showSCResRed()

Non vanishing structure constants of the 'Reduction of the Resonant Subalgebra' are given by:

$$\begin{aligned} C_{\{(1,2)(2,3)\}^{\{(3,3)\}}} &= -2.0 \\ C_{\{(1,2)(2,4)\}^{\{(3,4)\}}} &= -2.0 \\ C_{\{(1,2)(3,3)\}^{\{(2,3)\}}} &= 2.0 \\ C_{\{(1,2)(3,4)\}^{\{(2,4)\}}} &= 2.0 \\ C_{\{(1,5)(2,3)\}^{\{(3,4)\}}} &= -2.0 \\ C_{\{(1,5)(2,4)\}^{\{(3,3)\}}} &= -2.0 \\ C_{\{(1,5)(3,3)\}^{\{(2,4)\}}} &= 2.0 \\ C_{\{(1,5)(3,4)\}^{\{(2,3)\}}} &= 2.0 \\ C_{\{(2,3)(3,3)\}^{\{(1,2)\}}} &= 2.0 \\ C_{\{(2,3)(3,4)\}^{\{(1,5)\}}} &= 2.0 \\ C_{\{(2,4)(3,3)\}^{\{(1,5)\}}} &= 2.0 \\ C_{\{(2,4)(3,4)\}^{\{(1,2)\}}} &= 2.0 \end{aligned}$$

METHOD: cartanKillingMetricPretty()

The Killing-Cartan Metric of the Reduced algebra is:

$$\begin{pmatrix} -16 & 0 & 0 & 0 & 0 & 0 \\ 0 & -16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 & 0 & 0 \\ 0 & 0 & 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 16 \end{pmatrix}$$

The determinant of the Killing-Cartan Metric of the Reduced algebra is:
1.6777216E7
