

We introduce the structure constants of sl2  
We show the Killing-Cartan metric

```
-8.00 0.00 0.00
0.00 8.00 0.00
0.00 0.00 8.00
```

and its determinant  
-512.0

In what follows we expand sl2 with all the abelian semigroups of order 4 having zero element and identify the ones that leads to a semisimple algebra.

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A semisimple algebra has been found, expanding with the semigroup #59

```
1 1 1 1
1 2 1 1
1 1 3 1
1 1 1 4
```

whose zero element is: 1.

The metric of the reduced algebra is:

```
-8.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 8.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 8.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 8.00
```

and its determinant

-1.34217728E8

\*\*\*\*\*

A semisimple algebra has been found, expanding with the semigroup #62

```
1 1 1 1
1 2 1 1
1 1 3 3
1 1 3 4
```

whose zero element is: 1.

The metric of the reduced algebra is:

```
-8.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 -8.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 -8.00 -16.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 8.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 8.00 16.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 8.00 8.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 8.00 16.00
```

and its determinant

-1.34217728E8

\*\*\*\*\*

A semisimple algebra has been found, expanding with the semigroup #64

1 1 1 1

1 2 1 1

1 1 3 4

1 1 4 3

whose zero element is: 1.

The metric of the reduced algebra is:

```
-8.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 -16.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 -16.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 16.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 16.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 16.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 16.00
```

and its determinant

-8.589934592E9

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A semisimple algebra has been found, expanding with the semigroup #68

1 1 1 1

1 2 1 2

1 1 3 3

1 2 3 4

whose zero element is: 1.

The metric of the reduced algebra is:

```
-8.00 0.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 -8.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
-8.00 -8.00 -24.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 0.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 8.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 8.00 24.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 0.00 8.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 8.00 8.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 8.00 24.00
```

and its determinant

-1.34217728E8

\*\*\*\*\*

A semisimple algebra has been found, expanding with the semigroup #83

1 1 1 1

1 2 2 2

1 2 3 2

1 2 2 4

whose zero element is: 1.

The metric of the reduced algebra is:

```

-8.00 -8.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
-8.00 -16.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
-8.00 -8.00 -16.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 8.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 16.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 8.00 16.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 8.00 8.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 16.00 8.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 8.00 16.00

```

and its determinant

-1.34217728E8

\*\*\*\*\*

A semisimple algebra has been found, expanding with the semigroup #86

1 1 1 1

1 2 2 2

1 2 3 3

1 2 3 4

whose zero element is: 1.

The metric of the reduced algebra is:

```

-8.00 -8.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
-8.00 -16.00 -16.00 0.00 0.00 0.00 0.00 0.00 0.00
-8.00 -16.00 -24.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 8.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 16.00 16.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 16.00 24.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 8.00 8.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 16.00 16.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 16.00 24.00

```

and its determinant

-1.34217728E8

\*\*\*\*\*

A semisimple algebra has been found, expanding with the semigroup #88

1 1 1 1

1 2 2 2

1 2 3 4

1 2 4 3

whose zero element is: 1.

The metric of the reduced algebra is:

```

-8.00 -8.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
-8.00 -24.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
-8.00 -8.00 -24.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 8.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 24.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 8.00 24.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 8.00 8.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 24.00 8.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 8.00 24.00

```

and its determinant

-8.589934592E9

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A semisimple algebra has been found, expanding with the semigroup #93

1 1 1 1  
1 2 2 4  
1 2 3 4  
1 4 4 2

whose zero element is: 1.

The metric of the reduced algebra is:

```
-16.00 -16.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
-16.00 -24.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 -16.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 16.00 16.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 16.00 24.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 16.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 16.00 16.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 16.00 24.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 16.00
```

and its determinant

-8.589934592E9

\*\*\*\*\*

A semisimple algebra has been found, expanding with the semigroup #99

1 1 1 1  
1 2 3 4  
1 3 4 2  
1 4 2 3

whose zero element is: 1.

The metric of the reduced algebra is:

```
-24.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 -24.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 -24.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 24.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 24.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 24.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 24.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 24.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 24.00 0.00
```

and its determinant

2.641807540224E12

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There are 39 semigroups of order 4 with zero element.  
And 9 of them leads to a semisimple algebra.