

We introduce the structure constants of  $sl_2$   
We show the Killing-Cartan metric

```
-8.00 0.00 0.00
0.00 8.00 0.00
0.00 0.00 8.00
```

and its determinant  
-512.0

In what follows we expand  $sl_2$  with all the abelian semigroups of order 4 having zero element and a resonance; and identify the ones that leads to a semisimple algebra.

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A semisimple algebra has been found, expanding with the semigroup #88

```
1 1 1 1
1 2 2 2
1 2 3 4
1 2 4 3
```

which has the zero element 1 and the resonance:

S0: 1 2 3

S1: 1 2 4

The metric of the reduction of the resonant subalgebra is:

```
-8.00 -8.00 0.00 0.00 0.00 0.00
-8.00 -16.00 0.00 0.00 0.00 0.00
0.00 0.00 8.00 8.00 0.00 0.00
0.00 0.00 8.00 16.00 0.00 0.00
0.00 0.00 0.00 0.00 8.00 8.00
0.00 0.00 0.00 0.00 8.00 16.00
```

and its determinant  
262144.0

\*\*\*\*\*

There are 32 semigroups of order 4 with zero element and at least one resonant decomposition.

In total, there are 92 different resonant decompositions.

And 1 of them leads to a semisimple algebra.

There are 0 semigroups with zero element, having more than one resonance and leading to semisimple algebras with all those resonances.