

We introduce the structure constants of sl_2
We show the Killing-Cartan metric

```
-8.00 0.00 0.00
0.00 8.00 0.00
0.00 0.00 8.00
```

and its determinant
-512.0

In what follows we expand sl_2 with all the abelian semigroups of order 3
and identify the ones that leads to a semisimple algebra

A semisimple algebra has been found, expanding with the semigroup #7

```
1 1 1
1 2 1
1 1 3
```

The metric of the expanded algebra is:

```
-8.00 -8.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
-8.00 -16.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
-8.00 -8.00 -16.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 8.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 16.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 8.00 16.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 8.00 8.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 16.00 8.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 8.00 16.00
```

whose determinant is: -1.34217728E8

A semisimple algebra has been found, expanding with the semigroup #10

```
1 1 1
1 2 2
1 2 3
```

The metric of the expanded algebra is:

```
-8.00 -8.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
-8.00 -16.00 -16.00 0.00 0.00 0.00 0.00 0.00 0.00
-8.00 -16.00 -24.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 8.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 16.00 16.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 16.00 24.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 8.00 8.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 16.00 16.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 16.00 24.00
```

whose determinant is: -1.34217728E8

A semisimple algebra has been found, expanding with the semigroup #12

```
1 1 1
1 2 3
1 3 2
```

The metric of the expanded algebra is:

```

-8.00 -8.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
-8.00 -24.00 -8.00 0.00 0.00 0.00 0.00 0.00 0.00
-8.00 -8.00 -24.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 8.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 24.00 8.00 0.00 0.00 0.00
0.00 0.00 0.00 8.00 8.00 24.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 8.00 8.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 24.00 8.00
0.00 0.00 0.00 0.00 0.00 0.00 8.00 8.00 24.00

```

whose determinant is: -8.589934592E9

A semisimple algebra has been found, expanding with the semigroup #16

1 1 3

1 2 3

3 3 1

The metric of the expanded algebra is:

```

-16.00 -16.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
-16.00 -24.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 -16.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 16.00 16.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 16.00 24.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 16.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 16.00 16.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 16.00 24.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 16.00

```

whose determinant is: -8.589934592E9

A semisimple algebra has been found, expanding with the semigroup #18

1 2 3

2 3 1

3 1 2

The metric of the expanded algebra is:

```

-24.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 -24.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 -24.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 24.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 24.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 24.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 24.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 24.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 24.00 0.00

```

whose determinant is: 2.641807540224E12

There are 12 commutative semigroups of order 3

And 5 expansions that give a semisimple algebra