PRELIMINAR OBSERVATIONS:

This is the output of the example program II_SExp_sl2_S968.java,
which performs expansions of the sl(2) algebra with the
semigroup S_{5}^{968}, whose multiplication table is given by:
1 1 1 1 1
1 2 2 4 4
1 2 3 4 5
1 4 4 2 2
1 4 5 2 3
The resonance is:
S0 = {1,2,3}, S1 = {1,4,5} and zero element is 1.

It gives the commutation relations of:
1) Expanded algebra
2) Resonant subalgebra
3) Reduced algebra
4) Reduction of the resonant subalgebra

NOTATION:

The original algebra and the expanded one, GxS, are given by
[X_{i} , X_{j}] = C_{ij}^{k} X_{k} ,   (1)
[X_{i,a} , X_{j,b}] = C_{(i,a)(j,b)}^{(k,c)} X_{k,c)} ,   (2)
where i,j,k=1,...,n and a,b,c=1,...,m.

As the first possition of an array [][] is usually [0][0], the method
'setStructureConstant()' reads the non-vanishing structure constants C_{ij}^{k}
in such a way that i,j,k=0,1...,n-1. They are introduced as follows:

name.setStructureConstant(i , j , k , C_{ij}^{k})

Similarly a,b,c=0,1,...,m-1 in the functions C_{(i,a)(j,b)}^{(k,c)}.
However, the outputs will be given in such a way that i,j,k=1,...,n and
a,b,c=1,...,m.

We introduce the structure constants of sl2.
Remind that if a non vanishing structure constant C_{ij}^{k} has the
value V, then we introduce it as:
name.setStructureConstant(i-1 , j-1 , k-1 , V)

Show its Killing-Cartan metric

 -8.00 0.00 0.00
 0.00 8.00 0.00
 0.00 0.00 8.00

whose determinant is:
-512.0

METHOD: showCommut()
Non vanishing commutators of the 'Expanded algebra'

n = 3 , Dimension of the original Lie algebra.
m = 5 , Order of the semigroup.

With the notation: X_{i,a}= X_{i} lambda_{a}, the generators of the 'Expanded algebra' are given by:
 Y_{1} = X_{1,1}
 Y_{2} = X_{1,2}
 Y_{3} = X_{1,3}
 Y_{4} = X_{1,4}
 Y_{5} = X_{1,5}
 Y_{6} = X_{2,1}
 Y_{7} = X_{2,2}
 Y_{8} = X_{2,3}
 Y_{9} = X_{2,4}
 Y_{10} = X_{2,5}
 Y_{11} = X_{3,1}
 Y_{12} = X_{3,2}
 Y_{13} = X_{3,3}
 Y_{14} = X_{3,4}
 Y_{15} = X_{3,5}

The non vanishing commutators of the 'Expanded algebra' are given by:
 [X_{1,1} , X_{2,1}] = -2.0 X_{3,1}
 [X_{1,1} , X_{2,2}] = -2.0 X_{3,1}
 [X_{1,1} , X_{2,3}] = -2.0 X_{3,1}
 [X_{1,1} , X_{2,4}] = -2.0 X_{3,1}
 [X_{1,1} , X_{2,5}] = -2.0 X_{3,1}
 [X_{1,1} , X_{3,1}] = 2.0 X_{2,1}
 [X_{1,1} , X_{3,2}] = 2.0 X_{2,1}
 [X_{1,1} , X_{3,3}] = 2.0 X_{2,1}
 [X_{1,1} , X_{3,4}] = 2.0 X_{2,1}
 [X_{1,1} , X_{3,5}] = 2.0 X_{2,1}
 [X_{1,2} , X_{2,1}] = -2.0 X_{3,1}
 [X_{1,2} , X_{2,2}] = -2.0 X_{3,2}
 [X_{1,2} , X_{2,3}] = -2.0 X_{3,2}
 [X_{1,2} , X_{2,4}] = -2.0 X_{3,4}
 [X_{1,2} , X_{2,5}] = -2.0 X_{3,4}
 [X_{1,2} , X_{3,1}] = 2.0 X_{2,1}
 [X_{1,2} , X_{3,2}] = 2.0 X_{2,2}
 [X_{1,2} , X_{3,3}] = 2.0 X_{2,2}
 [X_{1,2} , X_{3,4}] = 2.0 X_{2,4}
 [X_{1,2} , X_{3,5}] = 2.0 X_{2,4}
 [X_{1,3} , X_{2,1}] = -2.0 X_{3,1}
 [X_{1,3} , X_{2,2}] = -2.0 X_{3,2}
 [X_{1,3} , X_{2,3}] = -2.0 X_{3,3}
 [X_{1,3} , X_{2,4}] = -2.0 X_{3,4}
 [X_{1,3} , X_{2,5}] = -2.0 X_{3,5}
 [X_{1,3} , X_{3,1}] = 2.0 X_{2,1}
 [X_{1,3} , X_{3,2}] = 2.0 X_{2,2}
 [X_{1,3} , X_{3,3}] = 2.0 X_{2,3}
 [X_{1,3} , X_{3,4}] = 2.0 X_{2,4}
 [X_{1,3} , X_{3,5}] = 2.0 X_{2,5}
 [X_{1,4} , X_{2,1}] = -2.0 X_{3,1}
 [X_{1,4} , X_{2,2}] = -2.0 X_{3,4}
 [X_{1,4} , X_{2,3}] = -2.0 X_{3,4}

```
[ X_{1,4} , X_{2,4} ] = -2.0 X_{3,2}
[ X_{1,4} , X_{2,5} ] = -2.0 X_{3,2}
[ X_{1,4} , X_{3,1} ] = 2.0 X_{2,1}
[ X_{1,4} , X_{3,2} ] = 2.0 X_{2,4}
[ X_{1,4} , X_{3,3} ] = 2.0 X_{2,4}
[ X_{1,4} , X_{3,4} ] = 2.0 X_{2,2}
[ X_{1,4} , X_{3,5} ] = 2.0 X_{2,2}
[ X_{1,5} , X_{2,1} ] = -2.0 X_{3,1}
[ X_{1,5} , X_{2,2} ] = -2.0 X_{3,4}
[ X_{1,5} , X_{2,3} ] = -2.0 X_{3,5}
[ X_{1,5} , X_{2,4} ] = -2.0 X_{3,2}
[ X_{1,5} , X_{2,5} ] = -2.0 X_{3,3}
[ X_{1,5} , X_{3,1} ] = 2.0 X_{2,1}
[ X_{1,5} , X_{3,2} ] = 2.0 X_{2,4}
[ X_{1,5} , X_{3,3} ] = 2.0 X_{2,5}
[ X_{1,5} , X_{3,4} ] = 2.0 X_{2,2}
[ X_{1,5} , X_{3,5} ] = 2.0 X_{2,3}
[ X_{2,1} , X_{3,1} ] = 2.0 X_{1,1}
[ X_{2,1} , X_{3,2} ] = 2.0 X_{1,1}
[ X_{2,1} , X_{3,3} ] = 2.0 X_{1,1}
[ X_{2,1} , X_{3,4} ] = 2.0 X_{1,1}
[ X_{2,1} , X_{3,5} ] = 2.0 X_{1,1}
[ X_{2,2} , X_{3,1} ] = 2.0 X_{1,1}
[ X_{2,2} , X_{3,2} ] = 2.0 X_{1,2}
[ X_{2,2} , X_{3,3} ] = 2.0 X_{1,2}
[ X_{2,2} , X_{3,4} ] = 2.0 X_{1,4}
[ X_{2,2} , X_{3,5} ] = 2.0 X_{1,4}
[ X_{2,3} , X_{3,1} ] = 2.0 X_{1,1}
[ X_{2,3} , X_{3,2} ] = 2.0 X_{1,2}
[ X_{2,3} , X_{3,3} ] = 2.0 X_{1,3}
[ X_{2,3} , X_{3,4} ] = 2.0 X_{1,4}
[ X_{2,3} , X_{3,5} ] = 2.0 X_{1,5}
[ X_{2,4} , X_{3,1} ] = 2.0 X_{1,1}
[ X_{2,4} , X_{3,2} ] = 2.0 X_{1,4}
[ X_{2,4} , X_{3,3} ] = 2.0 X_{1,4}
[ X_{2,4} , X_{3,4} ] = 2.0 X_{1,2}
[ X_{2,4} , X_{3,5} ] = 2.0 X_{1,2}
[ X_{2,5} , X_{3,1} ] = 2.0 X_{1,1}
[ X_{2,5} , X_{3,2} ] = 2.0 X_{1,4}
[ X_{2,5} , X_{3,3} ] = 2.0 X_{1,5}
[ X_{2,5} , X_{3,4} ] = 2.0 X_{1,2}
[ X_{2,5} , X_{3,5} ] = 2.0 X_{1,3}

METHOD: showSC()
Non vanishing structure constants of the Expanded algebra:
 C_{(1,1)(2,1)}^{(3,1)} = -2.0
 C_{(1,1)(2,2)}^{(3,1)} = -2.0
 C_{(1,1)(2,3)}^{(3,1)} = -2.0
 C_{(1,1)(2,4)}^{(3,1)} = -2.0
 C_{(1,1)(2,5)}^{(3,1)} = -2.0
 C_{(1,1)(3,1)}^{(2,1)} = 2.0
 C_{(1,1)(3,2)}^{(2,1)} = 2.0
 C_{(1,1)(3,3)}^{(2,1)} = 2.0
 C_{(1,1)(3,4)}^{(2,1)} = 2.0
 C_{(1,1)(3,5)}^{(2,1)} = 2.0
```

```
C_{(1,2)(2,1)}^{(3,1)} = -2.0
C_{(1,2)(2,2)}^{(3,2)} = -2.0
C_{(1,2)(2,3)}^{(3,2)} = -2.0
C_{(1,2)(2,4)}^{(3,4)} = -2.0
C_{(1,2)(2,5)}^{(3,4)} = -2.0
C_{(1,2)(3,1)}^{(2,1)} = 2.0
C_{(1,2)(3,2)}^{(2,2)} = 2.0
C_{(1,2)(3,3)}^{(2,2)} = 2.0
C_{(1,2)(3,4)}^{(2,4)} = 2.0
C_{(1,2)(3,5)}^{(2,4)} = 2.0
C_{(1,3)(2,1)}^{(3,1)} = -2.0
C_{(1,3)(2,2)}^{(3,2)} = -2.0
C_{(1,3)(2,3)}^{(3,3)} = -2.0
C_{(1,3)(2,4)}^{(3,4)} = -2.0
C_{(1,3)(2,5)}^{(3,5)} = -2.0
C_{(1,3)(3,1)}^{(2,1)} = 2.0
C_{(1,3)(3,2)}^{(2,2)} = 2.0
C_{(1,3)(3,3)}^{(2,3)} = 2.0
C_{(1,3)(3,4)}^{(2,4)} = 2.0
C_{(1,3)(3,5)}^{(2,5)} = 2.0
C_{(1,4)(2,1)}^{(3,1)} = -2.0
C_{(1,4)(2,2)}^{(3,4)} = -2.0
C_{(1,4)(2,3)}^{(3,4)} = -2.0
C_{(1,4)(2,4)}^{(3,2)} = -2.0
C_{(1,4)(2,5)}^{(3,2)} = -2.0
C_{(1,4)(3,1)}^{(2,1)} = 2.0
C_{(1,4)(3,2)}^{(2,4)} = 2.0
C_{(1,4)(3,3)}^{(2,4)} = 2.0
C_{(1,4)(3,4)}^{(2,2)} = 2.0
C_{(1,4)(3,5)}^{(2,2)} = 2.0
C_{(1,5)(2,1)}^{(3,1)} = -2.0
C_{(1,5)(2,2)}^{(3,4)} = -2.0
C_{(1,5)(2,3)}^{(3,5)} = -2.0
C_{(1,5)(2,4)}^{(3,2)} = -2.0
C_{(1,5)(2,5)}^{(3,3)} = -2.0
C_{(1,5)(3,1)}^{(2,1)} = 2.0
C_{(1,5)(3,2)}^{(2,4)} = 2.0
C_{(1,5)(3,3)}^{(2,5)} = 2.0
C_{(1,5)(3,4)}^{(2,2)} = 2.0
C_{(1,5)(3,5)}^{(2,3)} = 2.0
C_{(2,1)(3,1)}^{(1,1)} = 2.0
C_{(2,1)(3,2)}^{(1,1)} = 2.0
C_{(2,1)(3,3)}^{(1,1)} = 2.0
C_{(2,1)(3,4)}^{(1,1)} = 2.0
C_{(2,1)(3,5)}^{(1,1)} = 2.0
C_{(2,2)(3,1)}^{(1,1)} = 2.0
C_{(2,2)(3,2)}^{(1,2)} = 2.0
C_{(2,2)(3,3)}^{(1,2)} = 2.0
C_{(2,2)(3,4)}^{(1,4)} = 2.0
C_{(2,2)(3,5)}^{(1,4)} = 2.0
C_{(2,3)(3,1)}^{(1,1)} = 2.0
C_{(2,3)(3,2)}^{(1,2)} = 2.0
C_{(2,3)(3,3)}^{(1,3)} = 2.0
C_{(2,3)(3,4)}^{(1,4)} = 2.0
C_{(2,3)(3,5)}^{(1,5)} = 2.0
```

```
C_{(2,4)(3,1)}^{(1,1)} = 2.0
C_{(2,4)(3,2)}^{(1,4)} = 2.0
C_{(2,4)(3,3)}^{(1,4)} = 2.0
C_{(2,4)(3,4)}^{(1,2)} = 2.0
C_{(2,4)(3,5)}^{(1,2)} = 2.0
C_{(2,5)(3,1)}^{(1,1)} = 2.0
C_{(2,5)(3,2)}^{(1,4)} = 2.0
C_{(2,5)(3,3)}^{(1,5)} = 2.0
C_{(2,5)(3,4)}^{(1,2)} = 2.0
C_{(2,5)(3,5)}^{(1,3)} = 2.0
```

METHOD: cartanKillingMetric()
The Killing-Cartan Metric of the Expanded algebra is:

```
 -8  -8  -8  -8  -8  0  0  0  0  0  0  0  0  0  0
 -8 -24 -24  -8  -8  0  0  0  0  0  0  0  0  0  0
 -8 -24 -40  -8  -8  0  0  0  0  0  0  0  0  0  0
 -8  -8  -8 -24 -24  0  0  0  0  0  0  0  0  0  0
 -8  -8  -8 -24 -40  0  0  0  0  0  0  0  0  0  0
  0   0   0   0   0  8  8  8  8  8  0  0  0  0  0
  0   0   0   0   0  8 24 24  8  8  0  0  0  0  0
  0   0   0   0   0  8 24 40  8  8  0  0  0  0  0
  0   0   0   0   0  8  8  8 24 24  0  0  0  0  0
  0   0   0   0   0  8  8  8 24 40  0  0  0  0  0
  0   0   0   0   0  0  0  0  0  0  8  8  8  8  8
  0   0   0   0   0  0  0  0  0  0  8 24 24  8  8
  0   0   0   0   0  0  0  0  0  0  8 24 40  8  8
  0   0   0   0   0  0  0  0  0  0  8  8  8 24 24
  0   0   0   0   0  0  0  0  0  0  8  8  8 24 40
```

The determinant of the Killing-Cartan Metric of the Expanded algebra is:
-1.44115188075855872E17

METHOD: showCommutRes()
Non vanishing commutators of the 'Resonant Subalgebra'

n = 3 , Dimension of the original Lie algebra.
m = 5 , Order of the semigroup.

With the notation: X_{i,a}= X_{i} lambda_{a}, the generators of the 'Resonant Subalgebra' are given by:
 Y_{1} = X_{1,1}
 Y_{2} = X_{1,2}
 Y_{3} = X_{1,3}
 Y_{6} = X_{2,1}
 Y_{9} = X_{2,4}
 Y_{10} = X_{2,5}
 Y_{11} = X_{3,1}
 Y_{14} = X_{3,4}
 Y_{15} = X_{3,5}

The non vanishing commutators of the 'Resonant Subalgebra' are given by:
 [X_{1,1} , X_{2,1}] = -2.0 X_{3,1}
 [X_{1,1} , X_{2,4}] = -2.0 X_{3,1}

```
[ X_{1,1} , X_{2,5} ] = -2.0 X_{3,1}
[ X_{1,1} , X_{3,1} ] = 2.0 X_{2,1}
[ X_{1,1} , X_{3,4} ] = 2.0 X_{2,1}
[ X_{1,1} , X_{3,5} ] = 2.0 X_{2,1}
[ X_{1,2} , X_{2,1} ] = -2.0 X_{3,1}
[ X_{1,2} , X_{2,4} ] = -2.0 X_{3,4}
[ X_{1,2} , X_{2,5} ] = -2.0 X_{3,4}
[ X_{1,2} , X_{3,1} ] = 2.0 X_{2,1}
[ X_{1,2} , X_{3,4} ] = 2.0 X_{2,4}
[ X_{1,2} , X_{3,5} ] = 2.0 X_{2,4}
[ X_{1,3} , X_{2,1} ] = -2.0 X_{3,1}
[ X_{1,3} , X_{2,4} ] = -2.0 X_{3,4}
[ X_{1,3} , X_{2,5} ] = -2.0 X_{3,5}
[ X_{1,3} , X_{3,1} ] = 2.0 X_{2,1}
[ X_{1,3} , X_{3,4} ] = 2.0 X_{2,4}
[ X_{1,3} , X_{3,5} ] = 2.0 X_{2,5}
[ X_{2,1} , X_{3,1} ] = 2.0 X_{1,1}
[ X_{2,1} , X_{3,4} ] = 2.0 X_{1,1}
[ X_{2,1} , X_{3,5} ] = 2.0 X_{1,1}
[ X_{2,4} , X_{3,1} ] = 2.0 X_{1,1}
[ X_{2,4} , X_{3,4} ] = 2.0 X_{1,2}
[ X_{2,4} , X_{3,5} ] = 2.0 X_{1,2}
[ X_{2,5} , X_{3,1} ] = 2.0 X_{1,1}
[ X_{2,5} , X_{3,4} ] = 2.0 X_{1,2}
[ X_{2,5} , X_{3,5} ] = 2.0 X_{1,3}
```

METHOD: showSCRes()
Non vanishing structure constants of the 'Resonant Subalgebra' are given by:
```
 C_{(1,1)(2,1)}^{(3,1)} = -2.0
 C_{(1,1)(2,4)}^{(3,1)} = -2.0
 C_{(1,1)(2,5)}^{(3,1)} = -2.0
 C_{(1,1)(3,1)}^{(2,1)} = 2.0
 C_{(1,1)(3,4)}^{(2,1)} = 2.0
 C_{(1,1)(3,5)}^{(2,1)} = 2.0
 C_{(1,2)(2,1)}^{(3,1)} = -2.0
 C_{(1,2)(2,4)}^{(3,4)} = -2.0
 C_{(1,2)(2,5)}^{(3,4)} = -2.0
 C_{(1,2)(3,1)}^{(2,1)} = 2.0
 C_{(1,2)(3,4)}^{(2,4)} = 2.0
 C_{(1,2)(3,5)}^{(2,4)} = 2.0
 C_{(1,3)(2,1)}^{(3,1)} = -2.0
 C_{(1,3)(2,4)}^{(3,4)} = -2.0
 C_{(1,3)(2,5)}^{(3,5)} = -2.0
 C_{(1,3)(3,1)}^{(2,1)} = 2.0
 C_{(1,3)(3,4)}^{(2,4)} = 2.0
 C_{(1,3)(3,5)}^{(2,5)} = 2.0
 C_{(2,1)(3,1)}^{(1,1)} = 2.0
 C_{(2,1)(3,4)}^{(1,1)} = 2.0
 C_{(2,1)(3,5)}^{(1,1)} = 2.0
 C_{(2,4)(3,1)}^{(1,1)} = 2.0
 C_{(2,4)(3,4)}^{(1,2)} = 2.0
 C_{(2,4)(3,5)}^{(1,2)} = 2.0
 C_{(2,5)(3,1)}^{(1,1)} = 2.0
 C_{(2,5)(3,4)}^{(1,2)} = 2.0
 C_{(2,5)(3,5)}^{(1,3)} = 2.0
```

METHOD: cartanKillingMetricPretty()
The Killing-Cartan Metric of the Resonant subalgebra is:

```
 -8 -8 -8 0 0 0 0 0 0
 -8 -16 -16 0 0 0 0 0 0
 -8 -16 -24 0 0 0 0 0 0
 0 0 0 8 8 8 0 0 0
 0 0 0 8 16 16 0 0 0
 0 0 0 8 16 24 0 0 0
 0 0 0 0 0 0 8 8 8
 0 0 0 0 0 0 8 16 16
 0 0 0 0 0 0 8 16 24
```

The determinant of the Killing-Cartan Metric of the Resonant subalgebra is:
-1.34217728E8

METHOD: showCommutRed()
Non vanishing commutators of the 'Reduced algebra'

n = 3 , Dimension of the original Lie algebra.
m = 5 , Order of the semigroup.

With the notation: $X_{i,a}= X_{i} \lambda_{a}$, the generators of the 'Reduced algebra' are given by:
```
 Y_{2} =   X_{1,2}
 Y_{3} =   X_{1,3}
 Y_{4} =   X_{1,4}
 Y_{5} =   X_{1,5}
 Y_{7} =   X_{2,2}
 Y_{8} =   X_{2,3}
 Y_{9} =   X_{2,4}
 Y_{10} =  X_{2,5}
 Y_{12} =  X_{3,2}
 Y_{13} =  X_{3,3}
 Y_{14} =  X_{3,4}
 Y_{15} =  X_{3,5}
```

The non vanishing commutators of the 'Reduced algebra' are given by:
```
 [ X_{1,2} , X_{2,2} ] = -2.0 X_{3,2}
 [ X_{1,2} , X_{2,3} ] = -2.0 X_{3,2}
 [ X_{1,2} , X_{2,4} ] = -2.0 X_{3,4}
 [ X_{1,2} , X_{2,5} ] = -2.0 X_{3,4}
 [ X_{1,2} , X_{3,2} ] = 2.0 X_{2,2}
 [ X_{1,2} , X_{3,3} ] = 2.0 X_{2,2}
 [ X_{1,2} , X_{3,4} ] = 2.0 X_{2,4}
 [ X_{1,2} , X_{3,5} ] = 2.0 X_{2,4}
 [ X_{1,3} , X_{2,2} ] = -2.0 X_{3,2}
 [ X_{1,3} , X_{2,3} ] = -2.0 X_{3,3}
 [ X_{1,3} , X_{2,4} ] = -2.0 X_{3,4}
 [ X_{1,3} , X_{2,5} ] = -2.0 X_{3,5}
 [ X_{1,3} , X_{3,2} ] = 2.0 X_{2,2}
 [ X_{1,3} , X_{3,3} ] = 2.0 X_{2,3}
 [ X_{1,3} , X_{3,4} ] = 2.0 X_{2,4}
```

```
[ X_{1,3} , X_{3,5} ] = 2.0 X_{2,5}
[ X_{1,4} , X_{2,2} ] = -2.0 X_{3,4}
[ X_{1,4} , X_{2,3} ] = -2.0 X_{3,4}
[ X_{1,4} , X_{2,4} ] = -2.0 X_{3,2}
[ X_{1,4} , X_{2,5} ] = -2.0 X_{3,2}
[ X_{1,4} , X_{3,2} ] = 2.0 X_{2,4}
[ X_{1,4} , X_{3,3} ] = 2.0 X_{2,4}
[ X_{1,4} , X_{3,4} ] = 2.0 X_{2,2}
[ X_{1,4} , X_{3,5} ] = 2.0 X_{2,2}
[ X_{1,5} , X_{2,2} ] = -2.0 X_{3,4}
[ X_{1,5} , X_{2,3} ] = -2.0 X_{3,5}
[ X_{1,5} , X_{2,4} ] = -2.0 X_{3,2}
[ X_{1,5} , X_{2,5} ] = -2.0 X_{3,3}
[ X_{1,5} , X_{3,2} ] = 2.0 X_{2,4}
[ X_{1,5} , X_{3,3} ] = 2.0 X_{2,5}
[ X_{1,5} , X_{3,4} ] = 2.0 X_{2,2}
[ X_{1,5} , X_{3,5} ] = 2.0 X_{2,3}
[ X_{2,2} , X_{3,2} ] = 2.0 X_{1,2}
[ X_{2,2} , X_{3,3} ] = 2.0 X_{1,2}
[ X_{2,2} , X_{3,4} ] = 2.0 X_{1,4}
[ X_{2,2} , X_{3,5} ] = 2.0 X_{1,4}
[ X_{2,3} , X_{3,2} ] = 2.0 X_{1,2}
[ X_{2,3} , X_{3,3} ] = 2.0 X_{1,3}
[ X_{2,3} , X_{3,4} ] = 2.0 X_{1,4}
[ X_{2,3} , X_{3,5} ] = 2.0 X_{1,5}
[ X_{2,4} , X_{3,2} ] = 2.0 X_{1,4}
[ X_{2,4} , X_{3,3} ] = 2.0 X_{1,4}
[ X_{2,4} , X_{3,4} ] = 2.0 X_{1,2}
[ X_{2,4} , X_{3,5} ] = 2.0 X_{1,2}
[ X_{2,5} , X_{3,2} ] = 2.0 X_{1,4}
[ X_{2,5} , X_{3,3} ] = 2.0 X_{1,5}
[ X_{2,5} , X_{3,4} ] = 2.0 X_{1,2}
[ X_{2,5} , X_{3,5} ] = 2.0 X_{1,3}

METHOD: showSCRed()
Non vanishing structure constants of the 'Reduced algebra' are given by:
 C_{(1,2)(2,2)}^{(3,2)} = -2.0
 C_{(1,2)(2,3)}^{(3,2)} = -2.0
 C_{(1,2)(2,4)}^{(3,4)} = -2.0
 C_{(1,2)(2,5)}^{(3,4)} = -2.0
 C_{(1,2)(3,2)}^{(2,2)} = 2.0
 C_{(1,2)(3,3)}^{(2,2)} = 2.0
 C_{(1,2)(3,4)}^{(2,4)} = 2.0
 C_{(1,2)(3,5)}^{(2,4)} = 2.0
 C_{(1,3)(2,2)}^{(3,2)} = -2.0
 C_{(1,3)(2,3)}^{(3,3)} = -2.0
 C_{(1,3)(2,4)}^{(3,4)} = -2.0
 C_{(1,3)(2,5)}^{(3,5)} = -2.0
 C_{(1,3)(3,2)}^{(2,2)} = 2.0
 C_{(1,3)(3,3)}^{(2,3)} = 2.0
 C_{(1,3)(3,4)}^{(2,4)} = 2.0
 C_{(1,3)(3,5)}^{(2,5)} = 2.0
 C_{(1,4)(2,2)}^{(3,4)} = -2.0
 C_{(1,4)(2,3)}^{(3,4)} = -2.0
 C_{(1,4)(2,4)}^{(3,2)} = -2.0
```

```
C_{(1,4)(2,5)}^{(3,2)} = -2.0
C_{(1,4)(3,2)}^{(2,4)} = 2.0
C_{(1,4)(3,3)}^{(2,4)} = 2.0
C_{(1,4)(3,4)}^{(2,2)} = 2.0
C_{(1,4)(3,5)}^{(2,2)} = 2.0
C_{(1,5)(2,2)}^{(3,4)} = -2.0
C_{(1,5)(2,3)}^{(3,5)} = -2.0
C_{(1,5)(2,4)}^{(3,2)} = -2.0
C_{(1,5)(2,5)}^{(3,3)} = -2.0
C_{(1,5)(3,2)}^{(2,4)} = 2.0
C_{(1,5)(3,3)}^{(2,5)} = 2.0
C_{(1,5)(3,4)}^{(2,2)} = 2.0
C_{(1,5)(3,5)}^{(2,3)} = 2.0
C_{(2,2)(3,2)}^{(1,2)} = 2.0
C_{(2,2)(3,3)}^{(1,2)} = 2.0
C_{(2,2)(3,4)}^{(1,4)} = 2.0
C_{(2,2)(3,5)}^{(1,4)} = 2.0
C_{(2,3)(3,2)}^{(1,2)} = 2.0
C_{(2,3)(3,3)}^{(1,3)} = 2.0
C_{(2,3)(3,4)}^{(1,4)} = 2.0
C_{(2,3)(3,5)}^{(1,5)} = 2.0
C_{(2,4)(3,2)}^{(1,4)} = 2.0
C_{(2,4)(3,3)}^{(1,4)} = 2.0
C_{(2,4)(3,4)}^{(1,2)} = 2.0
C_{(2,4)(3,5)}^{(1,2)} = 2.0
C_{(2,5)(3,2)}^{(1,4)} = 2.0
C_{(2,5)(3,3)}^{(1,5)} = 2.0
C_{(2,5)(3,4)}^{(1,2)} = 2.0
C_{(2,5)(3,5)}^{(1,3)} = 2.0
```

METHOD: cartanKillingMetricPretty()
The Killing-Cartan Metric of the Reduced algebra is:

```
-16 -16 0 0 0 0 0 0 0 0 0 0
-16 -32 0 0 0 0 0 0 0 0 0 0
 0 0 -16 -16 0 0 0 0 0 0 0 0
 0 0 -16 -32 0 0 0 0 0 0 0 0
 0 0 0 0 16 16 0 0 0 0 0 0
 0 0 0 0 16 32 0 0 0 0 0 0
 0 0 0 0 0 0 16 16 0 0 0 0
 0 0 0 0 0 0 16 32 0 0 0 0
 0 0 0 0 0 0 0 0 16 16 0 0
 0 0 0 0 0 0 0 0 16 32 0 0
 0 0 0 0 0 0 0 0 0 0 16 16
 0 0 0 0 0 0 0 0 0 0 16 32
```

The determinant of the Killing-Cartan Metric of the Reduced algebra is:
2.81474976710656E14

METHOD: showCommutResRed()
Non vanishing commutators of the 'Reduction of the Resonant Subalgebra'

n = 3 , Dimension of the original Lie algebra.
m = 5 , Order of the semigroup.

With the notation: X_{i,a}= X_{i} lambda_{a}, the generators of the 'Reduction of the Resonant Subalgebra' are given by:
 Y_{2} = X_{1,2}
 Y_{3} = X_{1,3}
 Y_{9} = X_{2,4}
 Y_{10} = X_{2,5}
 Y_{14} = X_{3,4}
 Y_{15} = X_{3,5}

The non vanishing commutators of the 'Reduction of the Resonant Subalgebra' are given by:
 [X_{1,2} , X_{2,4}] = -2.0 X_{3,4}
 [X_{1,2} , X_{2,5}] = -2.0 X_{3,4}
 [X_{1,2} , X_{3,4}] = 2.0 X_{2,4}
 [X_{1,2} , X_{3,5}] = 2.0 X_{2,4}
 [X_{1,3} , X_{2,4}] = -2.0 X_{3,4}
 [X_{1,3} , X_{2,5}] = -2.0 X_{3,5}
 [X_{1,3} , X_{3,4}] = 2.0 X_{2,4}
 [X_{1,3} , X_{3,5}] = 2.0 X_{2,5}
 [X_{2,4} , X_{3,4}] = 2.0 X_{1,2}
 [X_{2,4} , X_{3,5}] = 2.0 X_{1,2}
 [X_{2,5} , X_{3,4}] = 2.0 X_{1,2}
 [X_{2,5} , X_{3,5}] = 2.0 X_{1,3}

METHOD: showSCResRed()
Non vanishing structure constants of the 'Reduction of the Resonant Subalgebra' are given by:
 C_{(1,2)(2,4)}^{(3,4)} = -2.0
 C_{(1,2)(2,5)}^{(3,4)} = -2.0
 C_{(1,2)(3,4)}^{(2,4)} = 2.0
 C_{(1,2)(3,5)}^{(2,4)} = 2.0
 C_{(1,3)(2,4)}^{(3,4)} = -2.0
 C_{(1,3)(2,5)}^{(3,5)} = -2.0
 C_{(1,3)(3,4)}^{(2,4)} = 2.0
 C_{(1,3)(3,5)}^{(2,5)} = 2.0
 C_{(2,4)(3,4)}^{(1,2)} = 2.0
 C_{(2,4)(3,5)}^{(1,2)} = 2.0
 C_{(2,5)(3,4)}^{(1,2)} = 2.0
 C_{(2,5)(3,5)}^{(1,3)} = 2.0

METHOD: cartanKillingMetricPretty()
The Killing-Cartan Metric of the Reduced algebra is:

 -8 -8 0 0 0 0
 -8 -16 0 0 0 0
 0 0 8 8 0 0
 0 0 8 16 0 0
 0 0 0 0 8 8
 0 0 0 0 8 16

The determinant of the Killing-Cartan Metric of the Reduced algebra is:
262144.0