

This is the output of the program II\_findAllResonances\_ord4.java, which gives the list of semigroups of order 4 with resonant decompositions. The number of semigroups with at least one resonance and the total number of different resonances is given at the end of the file.

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The semigroup #1

1 1 1 1

1 1 1 1

1 1 1 1

1 1 1 1

has #12 resonances.

Resonance #1

S0: 1 4

S1: 1 2 3

Resonance #2

S0: 1 3

S1: 1 2 4

Resonance #3

S0: 1 2

S1: 1 3 4

Resonance #4

S0: 1 3 4

S1: 1 2

Resonance #5

S0: 1 2 4

S1: 1 3

Resonance #6

S0: 1 2 3

S1: 1 4

Resonance #7

S0: 1 3 4

S1: 1 2 4

Resonance #8

S0: 1 3 4

S1: 1 2 3

Resonance #9

S0: 1 2 4

S1: 1 3 4

Resonance #10

S0: 1 2 4

S1: 1 2 3

Resonance #11

S0: 1 2 3

S1: 1 3 4

Resonance #12

S0: 1 2 3

S1: 1 2 4

\*\*\*\*\*

The semigroup #2

1 1 1 1

1 1 1 1

1 1 1 1

1 1 1 2

has #6 resonances.

Resonance #1

S0: 1 2

S1: 1 3 4

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Resonance #2
S0: 1 2 4
S1: 1 3
Resonance #3
S0: 1 2 3
S1: 1 4
Resonance #4
S0: 1 2 4
S1: 1 2 3
Resonance #5
S0: 1 2 3
S1: 1 3 4
Resonance #6
S0: 1 2 3
S1: 1 2 4
*****

```

The semigroup #3

```

1 1 1 1
1 1 1 1
1 1 1 1
1 1 1 4
has #7 resonances.

```

Resonance #1

```

S0: 1 4
S1: 1 2 3

```

Resonance #2

```

S0: 1 3 4
S1: 1 2

```

Resonance #3

```

S0: 1 2 4
S1: 1 3

```

Resonance #4

```

S0: 1 3 4
S1: 1 2 4

```

Resonance #5

```

S0: 1 3 4
S1: 1 2 3

```

Resonance #6

```

S0: 1 2 4
S1: 1 3 4

```

Resonance #7

```

S0: 1 2 4
S1: 1 2 3

```

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The semigroup #10

```

1 1 1 1
1 1 1 1
1 1 1 2
1 1 2 1

```

has #5 resonances.

Resonance #1

```

S0: 1 4
S1: 1 2 3

```

Resonance #2

```

S0: 1 3
S1: 1 2 4

```

Resonance #3

```

S0: 1 2
S1: 1 3 4
Resonance #4
S0: 1 2 4
S1: 1 2 3
Resonance #5
S0: 1 2 3
S1: 1 2 4
*****
The semigroup #11
1 1 1 1
1 1 1 1
1 1 1 2
1 1 2 2
has #3 resonances.
Resonance #1
S0: 1 2
S1: 1 3 4
Resonance #2
S0: 1 2 4
S1: 1 2 3
Resonance #3
S0: 1 2 3
S1: 1 2 4
*****
The semigroup #12
1 1 1 1
1 1 1 1
1 1 1 2
1 1 2 3
has #2 resonances.
Resonance #1
S0: 1 3
S1: 1 2 4
Resonance #2
S0: 1 2 3
S1: 1 2 4
*****
The semigroup #13
1 1 1 1
1 1 1 1
1 1 1 3
1 1 3 4
has #5 resonances.
Resonance #1
S0: 1 4
S1: 1 2 3
Resonance #2
S0: 1 3 4
S1: 1 2
Resonance #3
S0: 1 2 4
S1: 1 3
Resonance #4
S0: 1 3 4
S1: 1 2 3
Resonance #5

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S0: 1 2 4
S1: 1 2 3
*****
The semigroup #16
1 1 1 1
1 1 1 1
1 1 2 1
1 1 1 2
has #5 resonances.
Resonance #1
S0: 1 2
S1: 1 3 4
Resonance #2
S0: 1 2 4
S1: 1 3
Resonance #3
S0: 1 2 3
S1: 1 4
Resonance #4
S0: 1 2 4
S1: 1 2 3
Resonance #5
S0: 1 2 3
S1: 1 2 4
*****
The semigroup #17
1 1 1 1
1 1 1 1
1 1 2 1
1 1 1 4
has #3 resonances.
Resonance #1
S0: 1 2 4
S1: 1 3
Resonance #2
S0: 1 2 4
S1: 1 3 4
Resonance #3
S0: 1 2 4
S1: 1 2 3
*****
The semigroup #20
1 1 1 1
1 1 1 1
1 1 2 2
1 1 2 2
has #3 resonances.
Resonance #1
S0: 1 2
S1: 1 3 4
Resonance #2
S0: 1 2 4
S1: 1 2 3
Resonance #3
S0: 1 2 3
S1: 1 2 4
*****

```

The semigroup #21

1 1 1 1

1 1 1 1

1 1 3 1

1 1 1 4

has #3 resonances.

Resonance #1

S0: 1 3 4

S1: 1 2

Resonance #2

S0: 1 3 4

S1: 1 2 4

Resonance #3

S0: 1 3 4

S1: 1 2 3

\*\*\*\*\*

The semigroup #24

1 1 1 1

1 1 1 1

1 1 3 3

1 1 3 3

has #3 resonances.

Resonance #1

S0: 1 3 4

S1: 1 2

Resonance #2

S0: 1 3 4

S1: 1 2 3

Resonance #3

S0: 1 2 3

S1: 1 3 4

\*\*\*\*\*

The semigroup #25

1 1 1 1

1 1 1 1

1 1 3 3

1 1 3 4

has #2 resonances.

Resonance #1

S0: 1 3 4

S1: 1 2

Resonance #2

S0: 1 3 4

S1: 1 2 3

\*\*\*\*\*

The semigroup #28

1 1 1 1

1 1 1 1

1 1 3 4

1 1 4 3

has #4 resonances.

Resonance #1

S0: 1 3

S1: 1 2 4

Resonance #2

S0: 1 3 4

S1: 1 2

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Resonance #3
S0: 1 2 3
S1: 1 4
Resonance #4
S0: 1 2 3
S1: 1 2 4
*****
The semigroup #40
1 1 1 1
1 1 1 2
1 1 1 2
1 2 2 4
has #2 resonances.
Resonance #1
S0: 1 4
S1: 1 2 3
Resonance #2
S0: 1 2 4
S1: 1 2 3
*****
The semigroup #42
1 1 1 1
1 1 1 2
1 1 1 3
1 2 3 4
has #5 resonances.
Resonance #1
S0: 1 4
S1: 1 2 3
Resonance #2
S0: 1 3 4
S1: 1 2
Resonance #3
S0: 1 2 4
S1: 1 3
Resonance #4
S0: 1 3 4
S1: 1 2 3
Resonance #5
S0: 1 2 4
S1: 1 2 3
*****
The semigroup #43
1 1 1 1
1 1 1 2
1 1 2 3
1 2 3 4
has #2 resonances.
Resonance #1
S0: 1 2 4
S1: 1 3
Resonance #2
S0: 1 2 4
S1: 1 2 3
*****
The semigroup #44
1 1 1 1

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1 1 1 2
1 1 3 1
1 2 1 4
has #2 resonances.
Resonance #1
S0: 1 3 4
S1: 1 2
Resonance #2
S0: 1 3 4
S1: 1 2 3
*****
The semigroup #45
1 1 1 1
1 1 1 2
1 1 3 3
1 2 3 4
has #2 resonances.
Resonance #1
S0: 1 3 4
S1: 1 2
Resonance #2
S0: 1 3 4
S1: 1 2 3
*****
The semigroup #55
1 1 1 1
1 1 2 2
1 2 3 3
1 2 3 3
has #1 resonances.
Resonance #1
S0: 1 3 4
S1: 1 2
*****
The semigroup #56
1 1 1 1
1 1 2 2
1 2 3 3
1 2 3 4
has #1 resonances.
Resonance #1
S0: 1 3 4
S1: 1 2
*****
The semigroup #58
1 1 1 1
1 1 2 2
1 2 3 4
1 2 4 3
has #2 resonances.
Resonance #1
S0: 1 3 4
S1: 1 2
Resonance #2
S0: 1 2 3
S1: 1 2 4
*****

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The semigroup #61
1 1 1 1
1 2 1 1
1 1 3 3
1 1 3 3
has #1 resonances.
Resonance #1
S0: 1 2 3
S1: 1 3 4
*****

The semigroup #64
1 1 1 1
1 2 1 1
1 1 3 4
1 1 4 3
has #2 resonances.
Resonance #1
S0: 1 2 3
S1: 1 4
Resonance #2
S0: 1 2 3
S1: 1 2 4
*****

The semigroup #77
1 1 1 1
1 2 2 2
1 2 2 2
1 2 2 2
has #2 resonances.
Resonance #1
S0: 1 2 4
S1: 1 2 3
Resonance #2
S0: 1 2 3
S1: 1 2 4
*****

The semigroup #78
1 1 1 1
1 2 2 2
1 2 2 2
1 2 2 3
has #1 resonances.
Resonance #1
S0: 1 2 3
S1: 1 2 4
*****

The semigroup #79
1 1 1 1
1 2 2 2
1 2 2 2
1 2 2 4
has #1 resonances.
Resonance #1
S0: 1 2 4
S1: 1 2 3
*****

The semigroup #82

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1 1 1 1
1 2 2 2
1 2 2 3
1 2 3 4
has #1 resonances.
Resonance #1
S0: 1 2 4
S1: 1 2 3
*****

The semigroup #88
1 1 1 1
1 2 2 2
1 2 3 4
1 2 4 3
has #1 resonances.
Resonance #1
S0: 1 2 3
S1: 1 2 4
*****

The semigroup #91
1 1 1 1
1 2 2 4
1 2 2 4
1 4 4 2
has #1 resonances.
Resonance #1
S0: 1 2 3
S1: 1 4
*****

The semigroup #93
1 1 1 1
1 2 2 4
1 2 3 4
1 4 4 2
has #1 resonances.
Resonance #1
S0: 1 2 3
S1: 1 4
*****

The semigroup #96
1 1 1 1
1 2 3 3
1 3 2 2
1 3 2 2
has #1 resonances.
Resonance #1
S0: 1 2
S1: 1 3 4
*****

The semigroup #103
1 1 1 4
1 1 1 4
1 1 1 4
4 4 4 1
has #3 resonances.
Resonance #1
S0: 1 2 3

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S1: 4
Resonance #2
S0: 1 3 4
S1: 1 2 4
Resonance #3
S0: 1 2 4
S1: 1 3 4
*****
The semigroup #104
1 1 1 4
1 1 1 4
1 1 2 4
4 4 4 1
has #2 resonances.
Resonance #1
S0: 1 2 3
S1: 4
Resonance #2
S0: 1 2 4
S1: 1 3 4
*****
The semigroup #105
1 1 1 4
1 1 1 4
1 1 3 4
4 4 4 1
has #2 resonances.
Resonance #1
S0: 1 2 3
S1: 4
Resonance #2
S0: 1 3 4
S1: 1 2 4
*****
The semigroup #107
1 1 1 4
1 1 2 4
1 2 3 4
4 4 4 1
has #2 resonances.
Resonance #1
S0: 1 2 3
S1: 4
Resonance #2
S0: 1 3 4
S1: 1 2 4
*****
The semigroup #108
1 1 1 4
1 2 1 4
1 1 3 4
4 4 4 1
has #1 resonances.
Resonance #1
S0: 1 2 3
S1: 4
*****

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The semigroup #109
1 1 1 4
1 2 2 4
1 2 2 4
4 4 4 1
has #1 resonances.
Resonance #1
S0: 1 2 3
S1: 4
*****

The semigroup #110
1 1 1 4
1 2 2 4
1 2 3 4
4 4 4 1
has #1 resonances.
Resonance #1
S0: 1 2 3
S1: 4
*****

The semigroup #112
1 1 1 4
1 2 3 4
1 3 2 4
4 4 4 1
has #2 resonances.
Resonance #1
S0: 1 2 3
S1: 4
Resonance #2
S0: 1 2 4
S1: 1 3 4
*****

The semigroup #113
1 1 3 3
1 1 3 3
3 3 1 1
3 3 1 1
has #3 resonances.
Resonance #1
S0: 1 2
S1: 3 4
Resonance #2
S0: 1 3 4
S1: 1 2 3
Resonance #3
S0: 1 2 3
S1: 1 3 4
*****

The semigroup #114
1 1 3 3
1 1 3 3
3 3 1 1
3 3 1 2
has #2 resonances.
Resonance #1
S0: 1 2

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```

S1: 3 4
Resonance #2
S0: 1 2 3
S1: 1 3 4
*****
The semigroup #115
1 1 3 3
1 2 3 3
3 3 1 1
3 3 1 1
has #2 resonances.
Resonance #1
S0: 1 2
S1: 3 4
Resonance #2
S0: 1 2 3
S1: 1 3 4
*****
The semigroup #117
1 1 3 3
1 2 3 4
3 3 1 1
3 4 1 1
has #2 resonances.
Resonance #1
S0: 1 2
S1: 3 4
Resonance #2
S0: 1 2 3
S1: 1 3 4
*****
The semigroup #118
1 1 3 3
1 2 3 4
3 3 1 1
3 4 1 2
has #2 resonances.
Resonance #1
S0: 1 2
S1: 3 4
Resonance #2
S0: 1 2 3
S1: 1 3 4
*****
The semigroup #123
1 2 2 2
2 1 1 1
2 1 1 1
2 1 1 1
has #3 resonances.
Resonance #1
S0: 1
S1: 2 3 4
Resonance #2
S0: 1 2 4
S1: 1 2 3
Resonance #3

```

```

S0: 1 2 3
S1: 1 2 4
*****

The semigroup #125
1 2 3 4
2 1 4 3
3 4 1 2
4 3 2 1
has #3 resonances.
Resonance #1
S0: 1 4
S1: 2 3
Resonance #2
S0: 1 3
S1: 2 4
Resonance #3
S0: 1 2
S1: 3 4
*****

The semigroup #126
1 2 3 4
2 1 4 3
3 4 2 1
4 3 1 2
has #1 resonances.
Resonance #1
S0: 1 2
S1: 3 4
*****

There are 48 semigroups with at least one resonance and there are in
total 124 different resonances.

```