Report: Signals and Systems Lab Assignment 2

Bilkent University Electrical and Electronics Department
EE321-01

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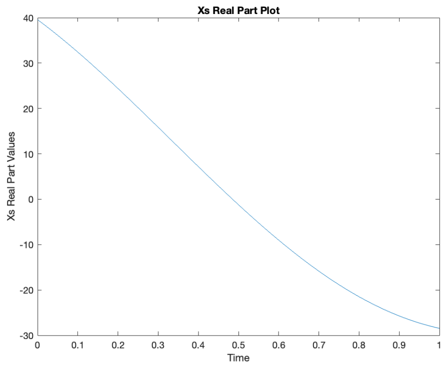
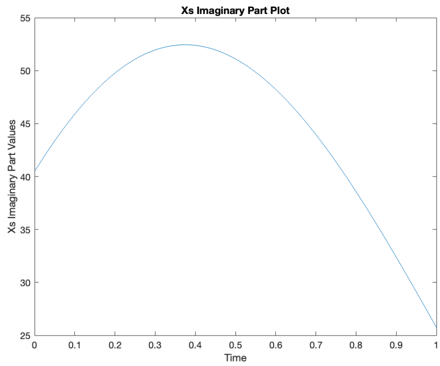
Part 1:

```
% ------------------------------------ %
% VECTORS
% ------------------------------------ %

t = [0:0.001:1];
n = mod(22103132, 41);
A = 3*rand(n, 1, 'like', 1i);
omega = pi*rand(n, 1);
xs = SUMCS(t, A, omega);

% ------------------------------------ %
% PLOTS
% ------------------------------------ %

figure(1);
plot(t, real(xs));
title('Xs Real Part Plot');
ylabel('Xs Real Part Values');
xlabel('Time');
figure(2);
plot(t, imag(xs));
title('Xs Imaginary Part Plot');
ylabel('Xs Imaginary Part Values');
xlabel('Time');
```

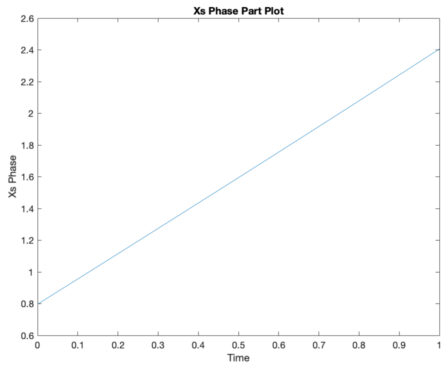
```matlab
figure(3);
plot(t, angle(xs));
title('Xs Phase Part Plot');
ylabel('Xs Phase');
xlabel('Time');
figure(4);
plot(t, abs(xs));
title('Xs Magnitude Plot');
ylabel('Magnitude of Xs');
xlabel('Time');

% ------------------------------------ %
% FUNCTION
% ------------------------------------ %

function[xs]=SUMCS(t,A,omega)
    % t: 1xN vector
    % A: 1XM complex-valued vector
    % omega: 1xM vector
    xs = 0;
    if length(A)==length(omega)
        for ii = 1:1:length(omega)
            xs=xs+A(ii)*exp(1i*omega(ii)*t);
        end
    end
end
```



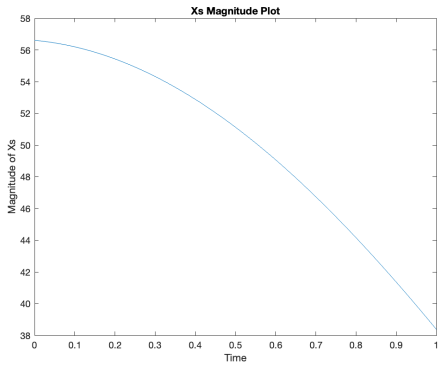
(a) Real part.



(b) Imagery part.



(c) Phase part.



(d) Magnitude part.

Figure 1: Graphs of the function.

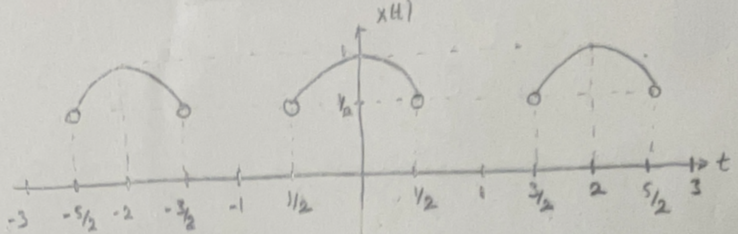
Part 2:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j\frac{2\pi k t}{T}}$$

$$x(t) = \begin{cases} 1-2t^2 & \text{if } -\frac{w}{2} < t < \frac{w}{2} \\ 0 & \text{, otherwise} \end{cases}$$

$$X_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi k t}{T}}\, dt$$

$T=2,\ w=1$ $\quad x(t)$ over $-3<t<3$



$$X_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi k t}{T}}\, dt = \frac{1}{T}\int_{-w/2}^{w/2} (1-2t^2)\, e^{-j\frac{2\pi k t}{T}}\, dt = \frac{1}{T}\Bigg[\underbrace{\int_{-w/2}^{w/2} e^{-j\frac{2\pi k t}{T}} dt}_{I_1} - 2\underbrace{\int_{-w/2}^{w/2} t^2 e^{-j\frac{2\pi k t}{T}} dt}_{I_2}\Bigg]$$

$$I_1 = \frac{e^{-j\frac{2\pi k t}{T}}}{j\frac{2\pi k}{T}}\Bigg|_{-w/2}^{w/2} = \frac{1}{j\frac{2\pi k}{T}}\left(e^{j\frac{\pi k w}{T}} - e^{-j\frac{\pi k w}{T}}\right)$$

$$I_2 = \int_{-w/2}^{w/2} t^2 e^{-j\frac{2\pi k t}{T}} dt \underset{\substack{\uparrow \\ w = t^2 \\ dv = e^{-j\frac{2\pi k t}{T}}}}{=} \frac{t^2 e^{-j\frac{2\pi k t}{T}}}{j\frac{2\pi k}{T}}\Bigg|_{-w/2}^{w/2} - \int_{-w/2}^{w/2} \frac{2t e^{-j\frac{2\pi k t}{T}}}{-j\frac{2\pi k}{T}} dt \underset{\substack{\uparrow \\ u = t \\ dv = e^{-j\frac{2\pi k t}{T}}}}{=}$$

$$= \frac{t^2 e^{-j\frac{2\pi k t}{T}}}{j\frac{2\pi k}{T}}\Bigg|_{-w/2}^{w/2} - \frac{t e^{-j\frac{2\pi k t}{T}}}{\frac{2\pi^2 k^2}{T^2}}\Bigg|_{-w/2}^{w/2} - \int_{-w/2}^{w/2} \frac{e^{-j\frac{2\pi k t}{T}}}{\frac{-2\pi^2 k^2}{T^2}} dt = \frac{t^2 e^{-j\frac{2\pi k t}{T}}}{j\frac{2\pi k}{T}}\Bigg|_{-w/2}^{w/2} - \frac{t e^{-j\frac{2\pi k t}{T}}}{\frac{2\pi^2 k^2}{T^2}}\Bigg|_{-w/2}^{w/2} - \frac{e^{-j\frac{2\pi k t}{T}}}{\frac{4j\pi^3 k^3}{T^3}}\Bigg|_{-w/2}^{w/2}$$

$$X_k = \frac{1}{T}(I_1 + I_2) = \frac{1}{T}\left[\frac{\sin\left(\frac{\pi k w}{T}\right)\cdot T}{\pi k} - 2\left(\frac{w^2 T}{4\pi k}\sin\left(\frac{\pi k w}{T}\right) + \frac{w T^2}{2\pi^2 k^2}\cos\left(\frac{\pi k w}{T}\right) + \frac{T^3}{2\pi^3 k^3}\sin\left(\frac{\pi k w}{T}\right)\right)\right]$$

$$= \frac{(2-w^2)\pi^2 k^2 - 2T^2}{2\pi^3 k^3}\sin\left(\frac{\pi k w}{T}\right) - \frac{wT}{\pi^2 k^2}\cos\left(\frac{\pi k w}{T}\right)$$

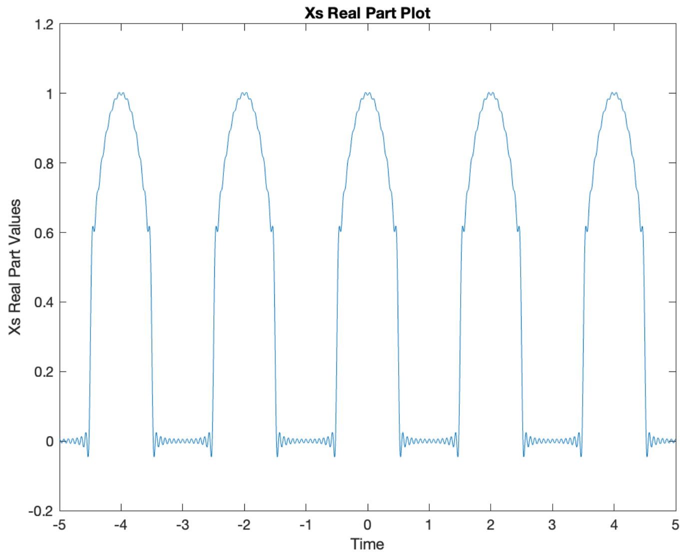
Figure 2: Derivations for Part 2.

Part 3:

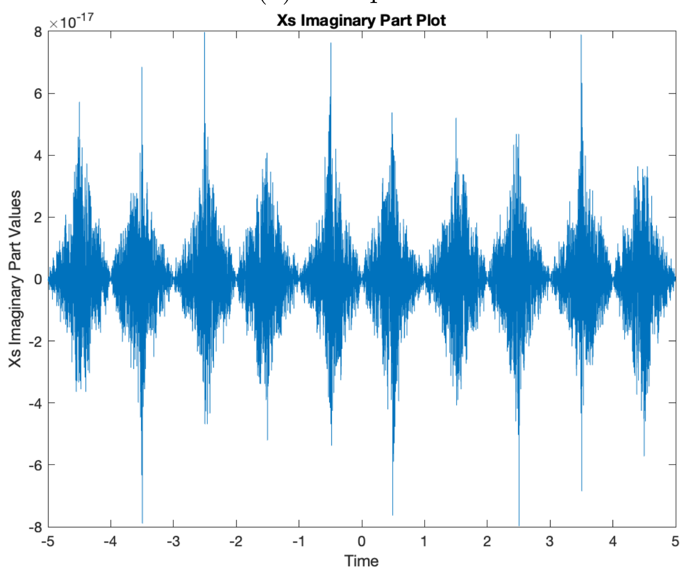
```
D11=mod(22103132,11);
D5=mod(22103132,5);
T=2;
W=1;
K=20+D11;
t=[-5:0.001:5];
xs=FSWave(t, K, T, W);

figure(1);
plot(t, real(xs));
title('Xs Real Part Plot');
ylabel('Xs Real Part Values');
xlabel('Time');
figure(2);
plot(t, imag(xs));
title('Xs Imaginary Part Plot');
ylabel('Xs Imaginary Part Values');
xlabel('Time');

K=2+D5;
xs=FSWave(t, K, T, W);
figure(3);
plot(t, real(xs));
title('Xs Real Part Plot');
ylabel('Xs Real Part Values');
xlabel('Time');

K=7+D5;
xs=FSWave(t, K, T, W);
figure(4);
plot(t, real(xs));
title('Xs Real Part Plot');
ylabel('Xs Real Part Values');
xlabel('Time');

K=15+D5;
xs=FSWave(t, K, T, W);
figure(5);
plot(t, real(xs));
title('Xs Real Part Plot');
ylabel('Xs Real Part Values');
```

```
xlabel('Time');

K=50+D5;
xs=FSWave(t, K, T, W);
figure(6);
plot(t, real(xs));
title('Xs Real Part Plot');
ylabel('Xs Real Part Values');
xlabel('Time');

K=100+D5;
xs=FSWave(t, K, T, W);
figure(7);
plot(t, real(xs));
title('Xs Real Part Plot');
ylabel('Xs Real Part Values');
xlabel('Time');


function[xt]=FSWave(t,K,T,W)
    % t is time grid
    % xt is the values of x~t over t
    % K, T, W are parameters
    syms x;
    integral1=0;
    A=zeros(2*K,1);
    omega=zeros(2*K,1);
    for ii=0:1:2*K
        f=(1-2*x*x)*exp(-1i*2*pi*(ii-K)*x/T);
        integral1=int(f,x,-W/2, W/2)/T;
        A(ii+1)=integral1;
        omega(ii+1)=2*pi*(ii-K)/T;
    end
    xt=SUMCS(t,A,omega);
end

function[xs]=SUMCS(t,A,omega)
    % t: 1xN vector
    % A: 1XM complex-valued vector
    % omega: 1xM vector
    xs = 0;
    if length(A)==length(omega)
        for ii = 1:1:length(omega)
```

```
            xs=xs+A(ii)*exp(1i*omega(ii)*t);
        end
    end
end
```

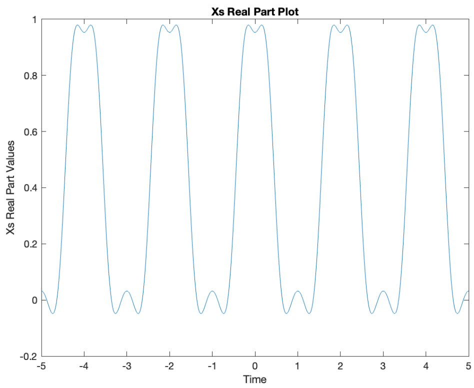


(a) Real part.

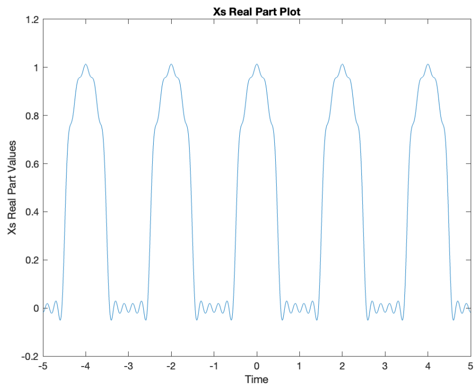


(b) Imagery part and it is almost zero.

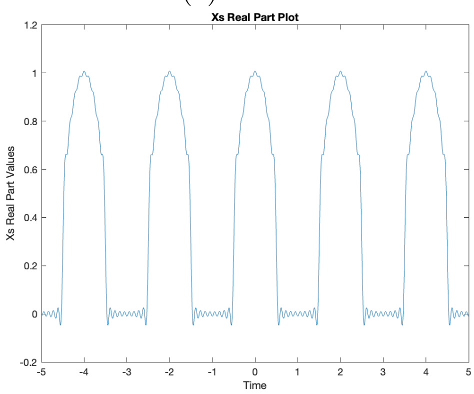
Figure 3: Real and Imaginary Parts of FSWave Signal.



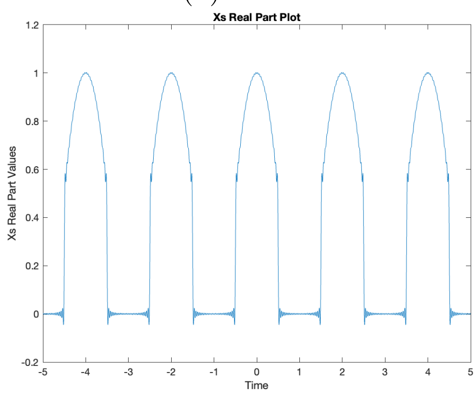
(a) K=4.



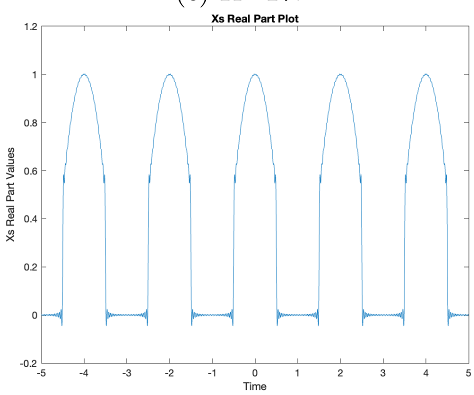
(b) K=9.



(c) K=17.



(d) K=52.



(e) K=102.

Figure 4: The outcome of the signal for various K values.

Answers to the questions for Part 3:

Question 1: Why do you think that the imaginary part is not perfectly zero?

Answer 1: MATLAB can calculate up to some precision while doing calculations, which is 16-bit precision, therefore imaginary part is not perfectly zero.

It can be verified since that imaginary part should be zero since the function is real-valued.

Question 2: What do you observe as K gets larger?

Answer 2: As K gets larger and larger the plots resemble more of the function itself. For small K's it is a rough approximation of the function but as they get larger the function is better approximated.

Question 3: Do you observe some irregularities or oscillations within the neighborhood of the discontinuities?

Answer 3: There are some irregularities and oscillations within the neighborhood of the discontinuities as one can see by observing the graphs.

Part 4:

Part a) When $Y_k = X_{-k}$,

```
function[xt]=FSWave1(t,K,T,W)
    % t is time grid
    % xt is the values of x~t over t
    % K, T, W are parameters
    syms x;
    integral1=0;
    A=zeros(2*K,1);
    omega=zeros(2*K,1);
    for ii=0:1:2*K
        f=(1-2*x*x)*exp(-1i*2*pi*(K-ii)*x/T);
        integral1=int(f,x,-W/2, W/2)/T;
        A(ii+1)=integral1;
        omega(ii+1)=2*pi*(ii-K)/T;
    end
    xt=SUMCS(t,A,omega);
end
```

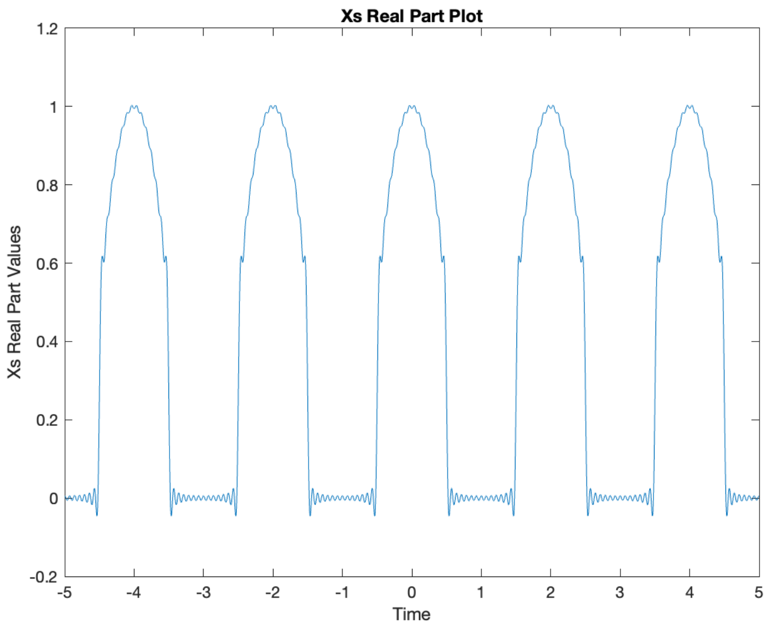


Figure 5: Imagery part and it is almost zero.

Reversing the sign of k in X_k yields $y(t)$ being the symmetric version of $x(t)$ around the $t = 0$ axis.

Part b) When $Y_k = X_k e^{\frac{2\pi k t_0}{T}}$,

```
function[xt]=FSWave2(t,K,T,W,t0)
    % t is time grid
    % xt is the values of x~t over t
    % K, T, W are parameters
    syms x;
    integral1=0;
    A=zeros(2*K,1);
    omega=zeros(2*K,1);
    for ii=0:1:2*K
        f=(1-2*x*x)*exp(-1i*2*pi*(K-ii)*x/T);
        integral1=int(f,x,-W/2, W/2)/T;
        A(ii+1)=integral1*exp(1i*2*pi*(ii-K)*t0/T);
        omega(ii+1)=2*pi*(ii-K)/T;
    end
    xt=SUMCS(t,A,omega);
end
```

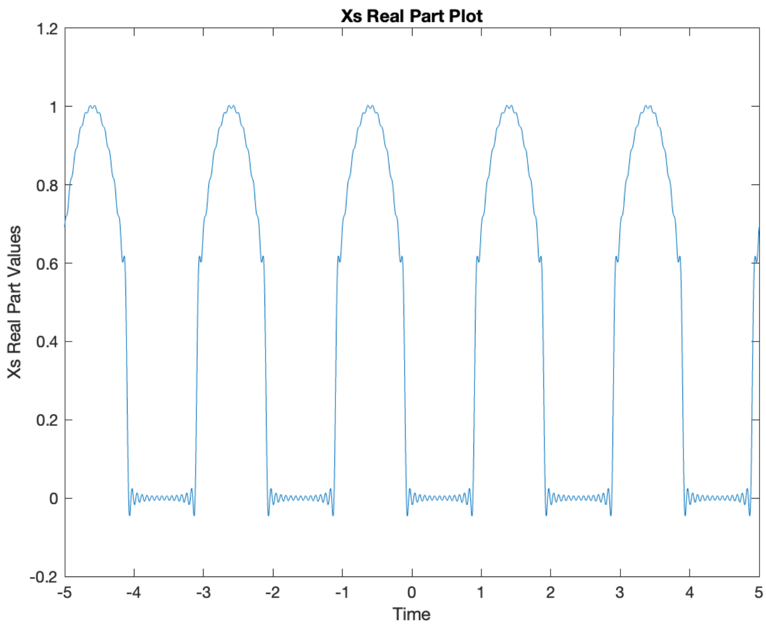


Figure 6: Imagery part and it is almost zero.

Adding t_0 to the signal causes the new signal $y(t)$ to be a left-shifted version of the original signal $x(t)$ by t_0 units.

Part c) When $Y_k = \frac{jk2\pi}{T} X_k$,

```
function[xt]=FSWave3(t,K,T,W)
    % t is time grid
    % xt is the values of x~t over t
    % K, T, W are parameters
    syms x;
    integral1=0;
    A=zeros(2*K,1);
    omega=zeros(2*K,1);
    for ii=0:1:2*K
        f=(1-2*x*x)*exp(-1i*2*pi*(ii-K)*x/T);
        integral1=int(f,x,-W/2, W/2)/T;
        A(ii+1)=integral1*1i*2*pi*(ii-K)/T;
        omega(ii+1)=2*pi*(ii-K)/T;
    end
    xt=SUMCS(t,A,omega);
end
```

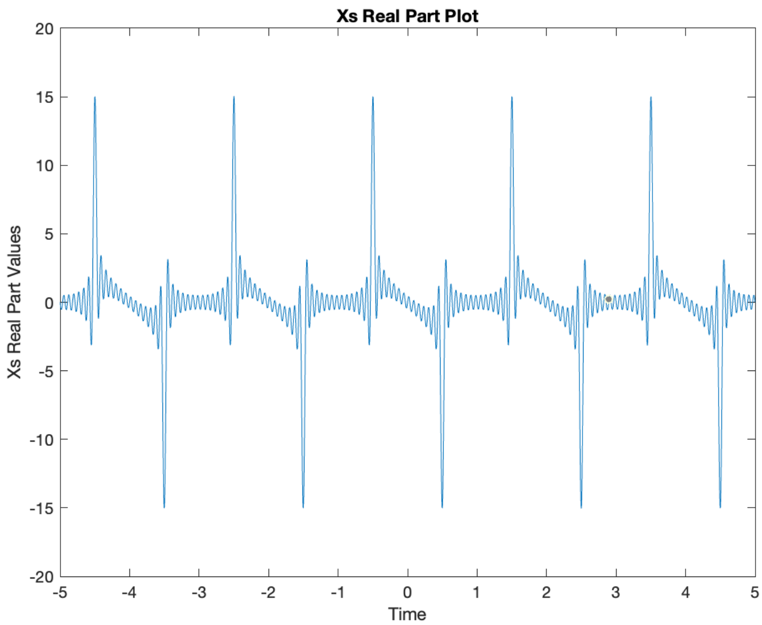


Figure 7: Imagery part and it is almost zero.

Multiplying X_k by $j2\pi k/T$ has the result of transforming the signal $y(t)$ into the derivative of $x(t)$.

Part d)

```
function[xt]=FSWave4(t,K,T,W)
    % t is time grid
    % xt is the values of x~t over t
    % K, T, W are parameters
    syms x;
    integral1=0;
    A=zeros(2*K,1);
    omega=zeros(2*K,1);
    for ii=0:1:2*K
        if ii<K
            f=(1-2*x*x)*exp(1i*2*pi*(-1-ii)*x/T);
            integral1=int(f,x,-W/2,W/2)/T;
            A(ii+1)=integral1;
        elseif i==K
            f=(1-2*x*x)*exp(1i*2*pi*(ii-K)*x/T);
            integral1=int(f,x,-W/2,W/2)/T;
            A(ii+1)=integral1;
        else
            f=(1-2*x*x)*exp(1i*2*pi*(2*K+1-ii)*x/T);
            integral1=int(f,x,-W/2,W/2)/T;
```

```
            A(ii+1)=integral1;
        end
        omega(ii+1)=2*pi*(ii-K)/T;
    end
    xt=SUMCS(t,A,omega);
end
```

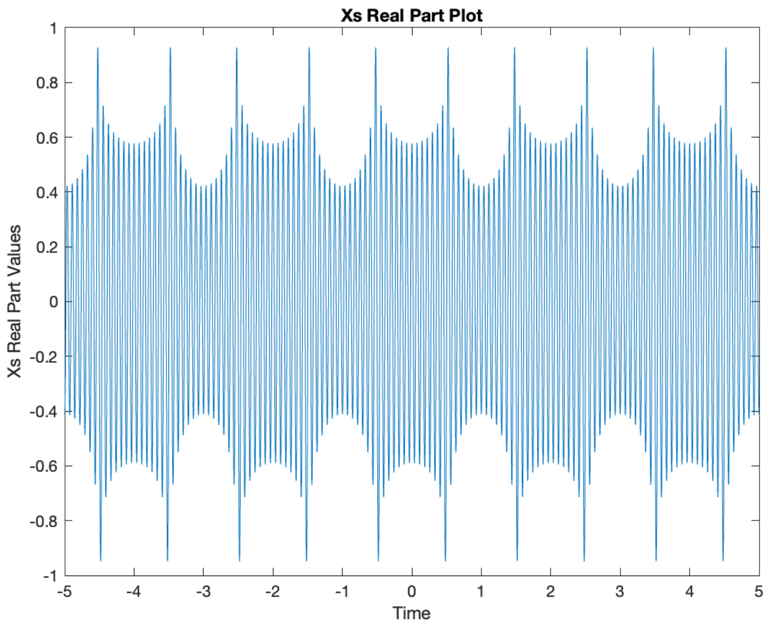


Figure 8: Imagery part and it is almost zero.

The operation's result is that the new signal $y(t)$ has essentially interchanged the sensitivity of high-frequency signals with low-frequency signals, and vice versa.