

Report: Signals and Systems Lab
Assignment 4
Bilkent University Electrical and Electronics Department
EE321-01

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Part 2:

Derivation can be found in the figure below.

The figure shows a handwritten derivation on a piece of paper. It starts with the definition of the unit impulse signal $\delta[m, n]$ as 1 when $m, n = 0$ and 0 otherwise. This is used to express the input signal $x[m, n]$ as a sum over k of $x[m, k]$ multiplied by $\delta[n-k]$. This is then expanded into a double sum over l and k of $x[l, k]$ multiplied by $\delta[n-k]$ and $\delta[m-l]$. This is further simplified to a double sum of $x[l, k]$ multiplied by $\delta[m-l, n-k]$. Finally, the output signal $y[m, n]$ is expressed as a double sum of $x[l, k]$ multiplied by $h[m-l, n-k]$. The derivation concludes with the statement that this can be written as $y[m, n] = x[m, n] * h[m, n]$.

$$\begin{aligned}\delta[m, n] &= \begin{cases} 1, & \text{if } m, n = 0 \\ 0 & \text{else} \end{cases} \\ \Rightarrow x[m, n] &= \sum_{k=-\infty}^{\infty} x[m, k] \delta[n-k] = \sum_{l=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x[l, k] \delta[n-k] \delta[m-l] \\ &= \sum_{l=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x[l, k] \delta[m-l, n-k] \\ \Rightarrow y[m, n] &= \sum_{l=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x[l, k] h[m-l, n-k] \\ \text{which can be written as } &\Rightarrow y[m, n] = x[m, n] * h[m, n]\end{aligned}$$

Figure 1: Convolution in LSI system.

Part 3:

The code for the Part 3:

```
x=[1,0,2;
   -1,3,1;
   -2,4,0];

h=[1,-1;
   0,2];

y=DSL SI2D(h,x)

function [y]=DSL SI2D(h,x)
    [Mh,Nh] = size(h);
    [Mx,Nx] = size(x);
    y= zeros(Mh+Mx-1,Nh+Nx-1);
    for k=0:Mh-1
        for l=0:Nh-1
            y(k+1:k+Mx,l+1:l+Nx)=y(k+1:k+Mx,l+1:l+Nx)+h(k+1,l+1)*x;
        end
    end
end
```

Derivation for Part 3 can be found in the figure below.

$$y[m,n] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} x[k,l] h[m-k, n-l]$$

$$\Rightarrow y[m,n] = \sum_{k=0}^{M_h-1} \sum_{l=0}^{N_h-1} h[k,l] x[m-k, n-l]$$

for FTI

$x[m,n]$ can take non-zero values between $0 \leq m \leq M_x-1$
 $0 \leq n \leq N_x-1$

$x[m-k, n-l]$ can take non-zero values between $0 \leq m-k \leq M_x-1$
 $0 \leq n-l \leq N_x-1$

Maximum M_h value for $k = M_h-1$

then, $m-k \leq M_x-1 \Rightarrow m - (M_h-1) \leq M_x-1 \Rightarrow m \leq M_x + M_h - 2$

Similarly $\Rightarrow n \leq N_x + N_h - 2$

We showed max values that $y[m,n]$ can be non-zero, therefore;

$M_y = M_x + M_h - 1$
 $N_y = N_x + N_h - 1$

Figure 2: Boundaries for the y.

Part 4:

The code for the Part 4:

```
x=ReadMyImage('Part4.bmp');
D = rem(22103132, 7);

h = zeros(30+D,30+D);
for k=1:30+D
    for l=1:30+D
        h(k,l)=sinc(0.7*(k-1-((29+D)/2)))*sinc(0.7*(l-1-((29+D)/2)));
    end
end

h2 = zeros(30+D,30+D);
for k=1:30+D
    for l=1:30+D
        h2(k,l)=sinc(0.4*(k-1-((29+D)/2)))*sinc(0.4*(l-1-((29+D)/2)));
    end
end

h3 = zeros(30+D,30+D);
for k=1:30+D
    for l=1:30+D
        h3(k,l)=sinc(0.1*(k-1-((29+D)/2)))*sinc(0.1*(l-1-((29+D)/2)));
    end
end

y=DSL SI2D(h,x);
y2=DSL SI2D(h2,x);
y3=DSL SI2D(h3,x);

figure;

subplot(3,1,1);
DisplayMyImage(y)
title('image with B=0.7')

subplot(3,1,2);
DisplayMyImage(y2)
title('image with B=0.4')
```

```

subplot(3,1,3);
DisplayMyImage(y3)
title('image with B=0.1')

function []=DisplayMyImage(Image)
    Image=Image-min(min(Image));
    imshow(uint8(255*Image/max(max(abs(Image))))));
end

```

image with B=0.7

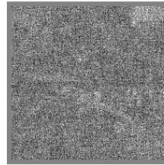


image with B=0.4

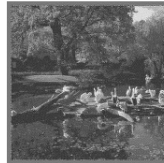


image with B=0.1



Figure 3: Image after low-pass filter.

Since low-frequency content determines the characteristics of the image, high-frequency content usually determines the details or the noise. By applying a low-pass filter, we were able to eliminate the noise. As B increases, the band-pass region of the low-pass filter also increases; therefore, the larger it is, the noisier the output. However, the narrower the bandwidth, the more the image becomes blurry since only very low-frequency content is left.

Part 5:

The code for the Part 5:

```
x=ReadMyImage('Part5.bmp');
figure;
subplot(2,2,1);
DisplayMyImage(x);
title('image itself')

h1 = [0.5 -0.5;0 0];

h2 = [0.5 0;-0.5 0];

subplot(2,2,2);
[y1]=DSL2D(h1,x);
s1 = y1.*y1;
DisplayMyImage(s1);
title('image convolved with h1')

subplot(2,2,3);
[y2]=DSL2D(h2,x);
s2 = y2.*y2;
DisplayMyImage(s2);
title('image convolved with h2')

subplot(2,2,4);
h3=0.5*h2+0.5*h1;
[y3]=DSL2D(h3,x);
s3 = y3.*y3;
DisplayMyImage(s3);
title('image convolved with h3')

function []=DisplayMyImage(Image)
Image=Image-min(min(Image));
imshow(uint8(255*Image/max(max(abs(Image)))));
end
```

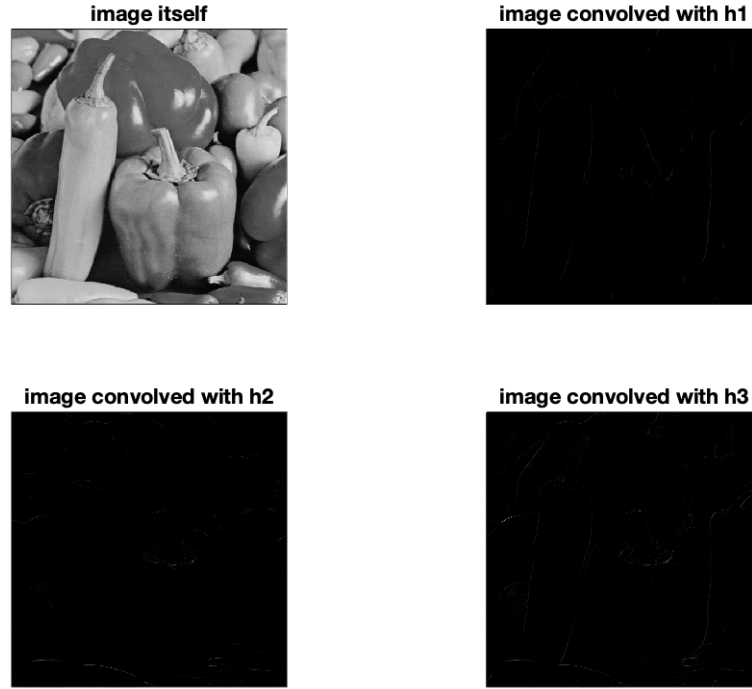


Figure 4: Edge detection.

For the first $h_1[m, n]$ it finds the horizontal edges, for the $h_2[m, n]$ it finds the vertical edges. Since $h_3[m, n]$ is a linear combination of both impulse responses it detects both vertical and horizontal edges.

Part 6:

The code for the Part 6:

```
x=ReadMyImage('Part6x.bmp');
DisplayMyImage(x);

h=ReadMyImage('Part6h.bmp');
DisplayMyImage(h);

[y]=DSL2D(h,x);

y = abs(y);

DisplayMyImage(y);

y= y.^(3);

DisplayMyImage(y);

[y]=DSL2D(h,x);

y= abs(y);
y= y.^(5);

DisplayMyImage(y);
```

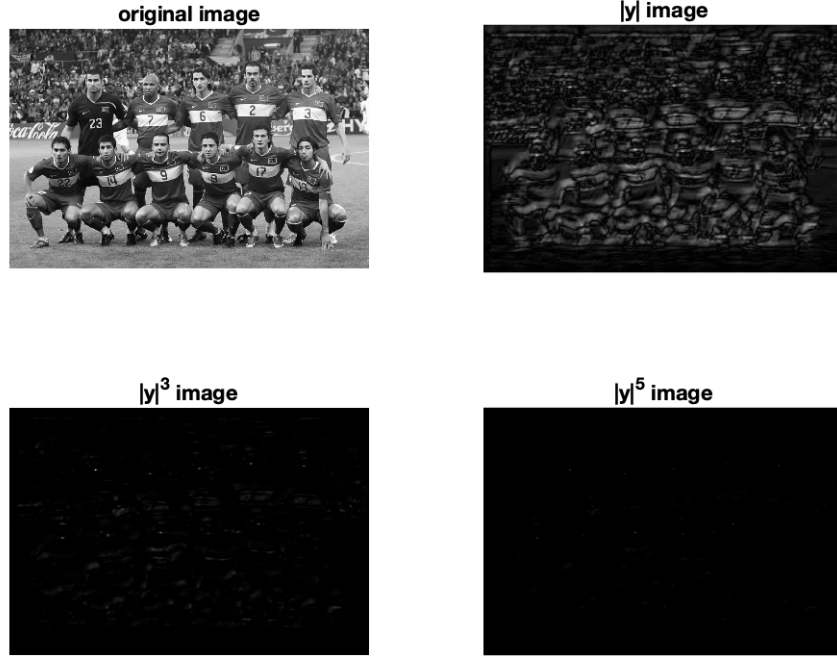


Figure 5: Pattern recognition.

Evidently, a distinct highlight is seen at the center of the face in Subplot 3 $|y|^3$ and Subplot 4 $|y|^5$ within Figure 5. Nevertheless, the utilization of $|y|$ yields sub-optimal results on the whole. Hence, it is advisable to use the higher powers of the $|y|$. Notably, the facial region of Volkan Oge stands out as the brightest, thereby establishing his photograph as the reference for $h[m, n]$.