Report: Signals and Systems Lab Assignment 3

Bilkent University Electrical and Electronics Department
EE321-01

Semih Akkoç - 22103132

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Part 1:

Code for the Transmitter part, namely, DTMF Transmitter:

```
Number = [5 3 5 0 4 6 0 3 4 6];

x = DTMFT(Number);

soundsc(x, 8192);

function[x] = DTMFT(Number)

    frequency = [941 1336;
                 697 1209;
                 697 1336;
                 697 1477;
                 770 1209;
                 770 1336;
                 770 1477;
                 852 1209;
                 852 1336;
                 852 1477];

    number_size = size(Number);
    character = [0 1 2 3 4 5 6 7 8 9];
    duration = 0:1/8192:0.25;
    x = zeros(1, number_size(1) * 2049);
```

```
    i = 0;

    for digit = Number
        index = find(character == digit);
        if digit <= 9 && digit >= 0
            digit_freq = frequency(index, :);
            signal =
                cos(2*pi*digit_freq(1)*duration)+cos(2*pi*digit_freq(2)*duration);
            i = i + 1;
            x = horzcat(x, signal); % horzcat() horizantal
                concatination of arrays
        end
    end
end
```

The sound reminded me of dialing a phone number on my phone.

Code for the Receiver part, namely, DTMF Receiver:

```
Number = [3 1 3 2 1];
x = DTMFTRA(Number);
X=FT(x);
omega = linspace(-8192*pi, 8192*pi, 10241);
omega = omega(1:10241);
figure(1);
plot(omega, abs(X))

function x= DTMFTRA(Number)
    x=0;
    t=linspace(0,1,10241);
    num_size=length(Number);
    for ii = 1:1:num_size
        % using step function to create a rectangular signal
        rect = heaviside(t - 0.25*(ii-1)) - heaviside(t - 0.25*ii);
        if Number(ii)==0
            x = x + (cos(t*2*pi*1336)+cos(t*2*pi*941)).*(rect);
        elseif Number(ii)==1
            x = x + (cos(t*2*pi*697)+cos(t*2*pi*1209)).*(rect);
        elseif Number(ii)==2
            x = x + (cos(t*2*pi*697)+cos(t*2*pi*1336)).*(rect);
        elseif Number(ii)==3
            x = x + (cos(t*2*pi*697)+cos(t*2*pi*1477)).*(rect);
        elseif Number(ii)==4
```

```
            x = x + (cos(t*2*pi*770)+cos(t*2*pi*1209)).*(rect);
        elseif Number(ii)==5
            x = x + (cos(t*2*pi*770)+cos(t*2*pi*1336)).*(rect);
        elseif Number(ii)==6
            x = x + (cos(t*2*pi*770)+cos(t*2*pi*1477)).*(rect);
        elseif Number(ii)==7
            x = x + (cos(t*2*pi*852)+cos(t*2*pi*1209)).*(rect);
        elseif Number(ii)==8
            x = x + (cos(t*2*pi*852)+cos(t*2*pi*1336)).*(rect);
        elseif Number(ii)==9
            x = x + (cos(t*2*pi*852)+cos(t*2*pi*1477)).*(rect);
        end
    end
end
```

By examining this figure alone, we can discern the dialed numbers. Without this ability, converting the frequency domain into these functions would serve no purpose. However, it's important to note that the numeric values in the figure represent angular frequency, not standard frequency. Therefore, to compare the frequency of each number accurately, these values need to be divided by 2π.

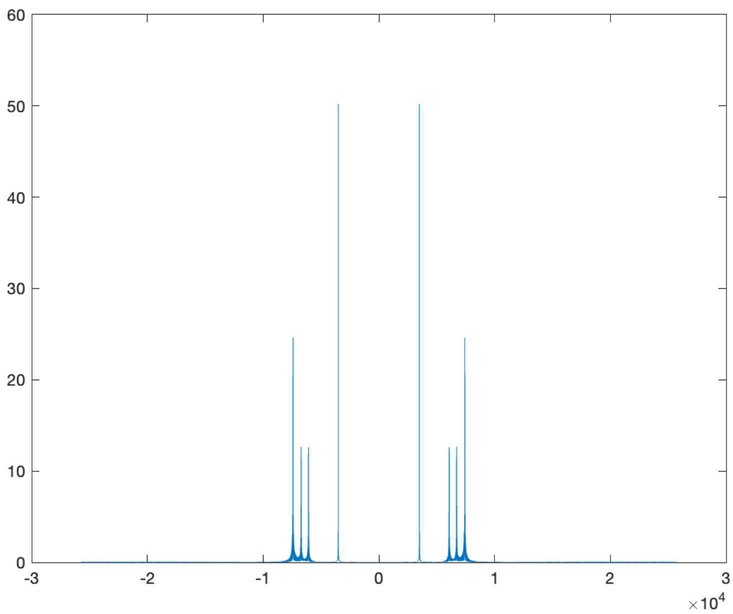
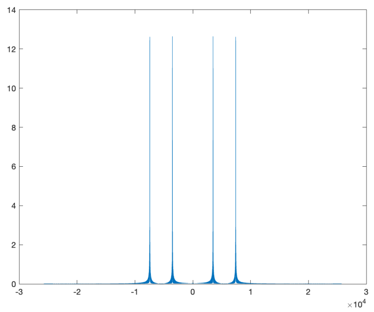
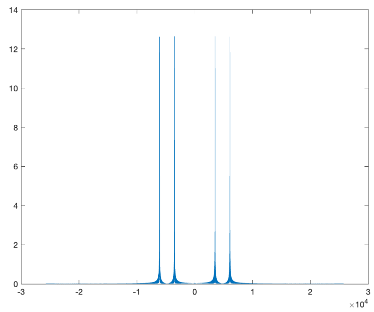


Figure 1: Fourier transform of $x(t)$.

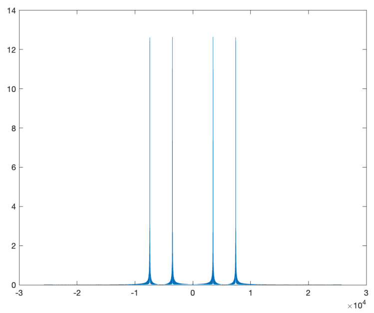
Now we will look at each $x_1(t)$ to $x_5(t)$ separately,



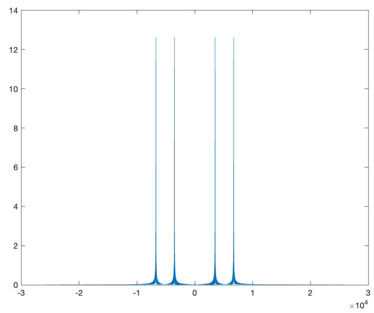
(a) Fourier transform of $x_1(t)$, has 697Hz and 1477Hz as frequency component.



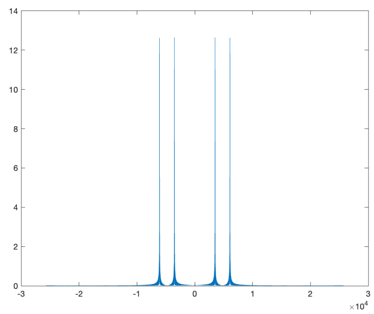
(b) Fourier transform of $x_2(t)$, has 697Hz and 1209Hz as frequency component.



(c) Fourier transform of $x_3(t)$, has 697Hz and 1477Hz as frequency component.



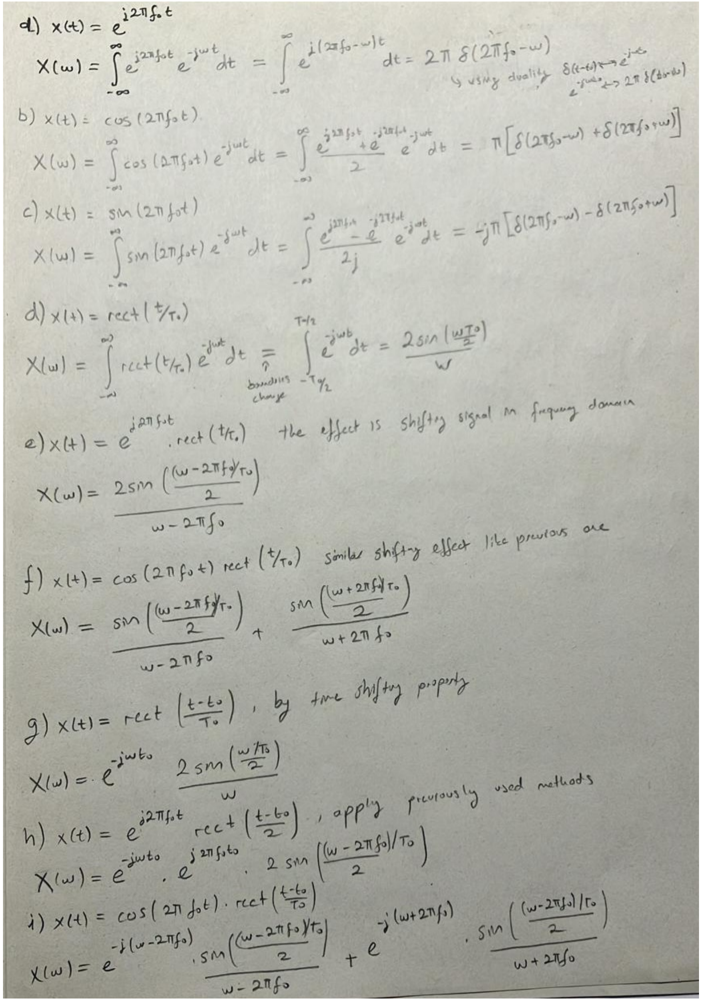
(d) Fourier transform of $x_4(t)$, has 697Hz and 1336Hz as frequency component.



(e) Fourier transform of $x_5(t)$, has 697Hz and 1209Hz as frequency component.

Figure 2: Fourier transform of the each number's addition to x(t).

In the below figure, the derivations for given $x(t)$'s can be found:


a) $x(t) = e^{j2\pi f_0 t}$

$X(\omega) = \int_{-\infty}^{\infty} e^{j2\pi f_0 t} e^{-j\omega t} dt = \int_{-\infty}^{\infty} e^{j(2\pi f_0 - \omega)t} dt = 2\pi \delta(2\pi f_0 - \omega)$ ↳ using duality

b) $x(t) = \cos(2\pi f_0 t)$

$X(\omega) = \int_{-\infty}^{\infty} \cos(2\pi f_0 t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} e^{-j\omega t} dt = \pi[\delta(2\pi f_0 - \omega) + \delta(2\pi f_0 + \omega)]$

c) $x(t) = \sin(2\pi f_0 t)$

$X(\omega) = \int_{-\infty}^{\infty} \sin(2\pi f_0 t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} \frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j} e^{-j\omega t} dt = -j\pi[\delta(2\pi f_0 - \omega) - \delta(2\pi f_0 + \omega)]$

d) $x(t) = \text{rect}(t/T_0)$

$X(\omega) = \int_{-\infty}^{\infty} \text{rect}(t/T_0) e^{-j\omega t} dt \underset{\text{boundaries change}}{=} \int_{-T/2}^{T/2} e^{-j\omega t} dt = \frac{2\sin(\frac{\omega T_0}{2})}{\omega}$

e) $x(t) = e^{j2\pi f_0 t} \cdot \text{rect}(t/T_0)$ the effect is shifting signal in frequency domain

$X(\omega) = \frac{2\sin\left(\frac{(\omega - 2\pi f_0)T_0}{2}\right)}{\omega - 2\pi f_0}$

f) $x(t) = \cos(2\pi f_0 t)\, \text{rect}(t/T_0)$ similar shifting effect like previous one

$X(\omega) = \frac{\sin\left(\frac{(\omega - 2\pi f_0)T_0}{2}\right)}{\omega - 2\pi f_0} + \frac{\sin\left(\frac{(\omega + 2\pi f_0)T_0}{2}\right)}{\omega + 2\pi f_0}$

g) $x(t) = \text{rect}\left(\frac{t - t_0}{T_0}\right)$, by time shifting property

$X(\omega) = e^{-j\omega t_0} \frac{2\sin\left(\frac{\omega T_0}{2}\right)}{\omega}$

h) $x(t) = e^{j2\pi f_0 t}\, \text{rect}\left(\frac{t - t_0}{2}\right)$, apply previously used methods

$X(\omega) = e^{-j\omega t_0} \cdot e^{j2\pi f_0 t_0} \cdot 2\sin\left(\frac{(\omega - 2\pi f_0)/T_0}{2}\right)$

i) $x(t) = \cos(2\pi f_0 t) \cdot \text{rect}\left(\frac{t - t_0}{T_0}\right)$

$X(\omega) = e^{-j(\omega - 2\pi f_0)} \cdot \frac{\sin\left(\frac{(\omega - 2\pi f_0)T_0}{2}\right)}{\omega - 2\pi f_0} + e^{-j(\omega + 2\pi f_0)} \cdot \frac{\sin\left(\frac{(\omega - 2\pi f_0)/T_0}{2}\right)}{\omega + 2\pi f_0}$

Figure 3: $X(\omega)$ for different $x(t)$'s.

Part 2:

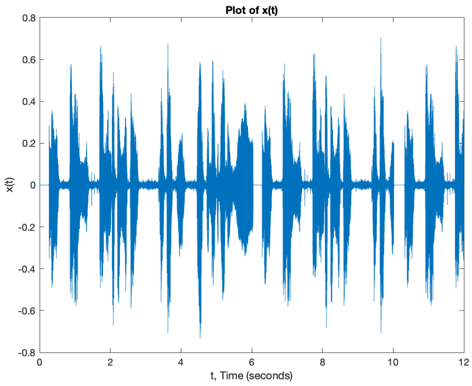
The code for the Part 2:

```
[x,fs] = audioread('reinput.wav');
x_10sec = x(1:98304); % only taking a certain length of input
t = 0:1/8192:12-1/8192;
soundsc(x,8192)

% creating delayed versions of the signal with circular shift
```

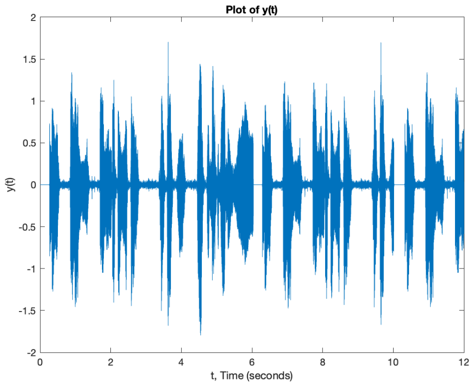
```
A = [0.65 0.5 0.3 0.22 0.15 0.1];
t_shift = [0.25 0.75 1 1.25 2 3.25];
len_xt = size(x_10sec);
echos = zeros(len_xt(2),6);
for ii = 1:6
    echo = circshift(x_10sec, t(ii)*8192);
    echo(1:t(ii)*8192-1)=zeros;
    echos(:, ii) = A(ii)*echo;
end

y = x_10sec;
% adding echos to y
for jj = 1:6
    y = y + echos(:, jj)';
end

omega = linspace(-8192*pi, 8192*pi, 98305);
omega = omega(1:98304);

Y = FT(y);
X_10sec = FT(x_10sec);

H=ones(1, length(omega));
% defining the H(jw) by using hand calculations
for ii = 1:6
    H = H + A(ii)*exp(-1i*omega*t_shift(ii));
end

h = IFT(H);

Xe = Y./H;
xe = IFT(Xe);


% plot of x_10sec
figure(1);
plot(t, x_10sec);
title('Plot of x(t)');
xlabel('t, Time (seconds)');
ylabel('x(t)');

% plot of y
```

```
figure(2);
plot(t, y);
title('Plot of y(t)');
xlabel('t, Time (seconds)');
ylabel('y(t)');

% plot of frequency components of H(w)
figure(3);
plot(omega, abs(H));
title('Plot of H(w)');
xlabel('w, Frequency');
ylabel('H(w)');

% plot of frequency components of h(t)
figure(4);
plot(t, h);
title('Plot of h(t)');
xlabel('t, Time (seconds)');
ylabel('h(t)');

% plot of frequency components of Xe(w)
figure(5);
plot(omega, abs(Xe));
title('Plot of Xe(w)');
xlabel('w, Frequency');
ylabel('Xe(w)');

% plot of frequency components of xe(t)
figure(6);
plot(t, xe);
title('Plot of xe(t)');
xlabel('t, Time (seconds)');
ylabel('xe(t)');

% to listen the xe
soundsc(xe, 8192)
```

In the below figures plot $x(t)$ and $y(t)$ can be found, $y(t)$ sounded like an intensely echoed version of the initial given input audio.



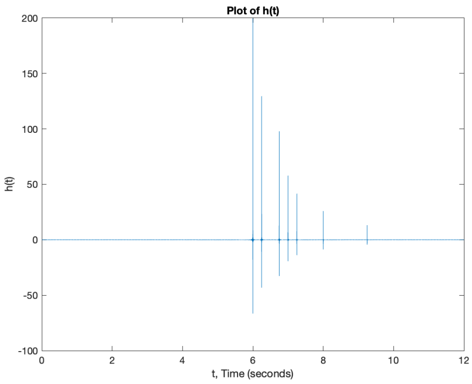
(a) Plot of $x(t)$.



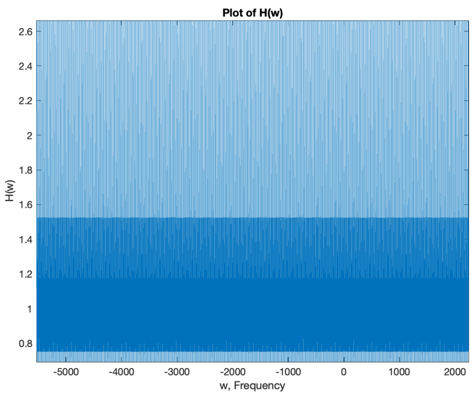
(b) Plot of $y(t)$, it contains shifted and scaled versions of $x(t)$.

Figure 4: Plots of $x(t)$ and $y(t)$.

In the following figures the impulse response of the system $h(t)$ and its frequency response $H(\omega)$'s plots can be found. As expected $h(t)$ is made of scaled and shifted versions of $\delta(t)$, namely impulse function. Due to $H(\omega)$ being made of cosines, its plot seems crowded.



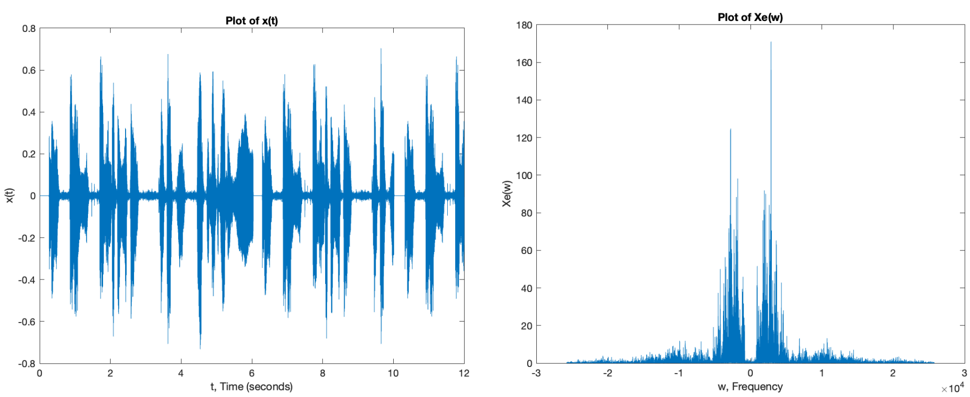
(a) Plot of $h(t)$.



(b) Plot of $H(\omega)$.

Figure 5: Plots of $h(t)$ and $H(\omega)$.

The figures provided below depict both $x_e(t)$ and $X_e(\omega)$. It is evident that the graph of $x_e(t)$ closely resembles that of $x(t)$. Moreover, the audio produced by $x_e(t)$ is nearly indistinguishable from that of $x(t)$.



(a) Plot of $x_e(t)$.

(b) Plot of $X_e(\omega)$.

Figure 6: Plots of $x_e(t)$ and $X_e(\omega)$.

The figure below has the answers asked in the Part 2:

a) To find the impulse response of the system we use $\delta(t)$ as input.

$$y(t) = x(t) + \sum_{i=1}^{M} A_i x(t-t_i) \xrightarrow[x(t)=\delta(t)]{} h(t) = \delta(t) + \sum_{i=1}^{M} A_i \delta(t-t_i)$$

b)

$$H(\omega) = FT\{h(t)\} = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} \left[\delta(t) + \sum_{i=1}^{M} A_i \delta(t-t_i)\right] e^{-j\omega t} dt$$

$$= 1 + \sum_{i=1}^{M} A_i e^{-j\omega t_i}$$

c)

$$y(t) = h(t) * x(t) \xrightarrow{FT} Y(\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(z) x(z-t) dz \, e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} h(z) \int_{-\infty}^{\infty} \underset{\text{shifted } x}{x(z-t)} e^{-j\omega t} \, dt \, dz \quad \phi = z-t \quad = \underbrace{\int_{-\infty}^{\infty} h(z) e^{-j\omega z} dz}_{H(\omega)} \underbrace{\int_{-\infty}^{\infty} x(\phi) e^{-j\omega \phi} d\phi}_{X(\omega)}$$

$$= H(\omega) X(\omega)$$

d) Using the previous relation we get

$$X(\omega) = \frac{Y(\omega)}{H(\omega)}$$

Figure 7: Derivations for Part 2.