Stochastic Processes First Exam
(15 points)
Roger Guzmán Avendaño

1. **(4 points)** Given Z distributed with $\emptyset$, where Z is a gaussian normal random variable with pdf:

$$\emptyset(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$$

Taking into account that :

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du = 1$$

Find the moment-generating function for $\emptyset(z)$

Hint: For moment-generating function, the limits of integral is defined from minus infinity to infinity.

2. **(3 points)** A dice is rolled 1000 times. Calculate the expected sum of the 1000 rolls.

3. **(3 points)** A player of games throws simultaneously a dice and a coin. If the coin land tail, then the player wins twice, and if heads, the one-half of the value that appears on the dice. Build the pdf for the problem, and explain the expected value.

4. **(4 points)** Given an Earlang Ditribution as follows:
$$p(x|\theta) = \theta^2 x e^{-\theta x} I(x)_{x>0}$$

   - **(1 points)** Find the Maximum Likelihood function and logLikelihood.
   - **(1 points)** Plot the likelihood function and compare with loglikelihood.
   - **(1 points)** Find the Maximum Likelihood Estimator for θ.
   - **(1 points)** Explain the value θ parameter.

5. **(3 Additional points)** Choose one picture and calculate the invariant hu moments using R-Language. Explain each invariant Hu moment w.r.t selected picture and relationship between them.