Stochastic Processes First Exam
(15 points)
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1. **(4 points)**Given Z distributed with Ø, where Z is a gaussian normal random variable with pdf:

$$\emptyset(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$$

    Taking into account that :

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du = 1$$

    Find the moment-generating function for $\emptyset(z)$

    Hint: For moment-generating function, the limits of integral is defined from minus infinity to infinity.

2. **(3 points)**A dice is rolled 11 times.Calculate the expected sum of the 11 rolls.

3. **(3 points)**A player of games throws simultaneously a dice and a coin. If the coin land tail, then the player wins twice, and if heads, the one-half of the value that appears on the dice. Build the pdf for the problem, and explain the expected value.

4. **(4 points)** Given an Earlang Ditribution as follows:
$$p(x|\theta) = \theta^2 x e^{-\theta x} I(x)_{x>0}$$

    - **(1 points)** Find the Maximum Likelihood function and logLikelihood.
    - **(1 points)** Plot the likelihood function and compare with loglikelihood.
    - **(1 points)** Find the Maximum Likelihood Estimator for θ.
    - **(1 points)** Explain the value θ parameter.

5. **(3 Additional points)** Choose one picture and calculate the invariant hu moments using R-Language. Explain each invariant Hu moment w.r.t selected picture and relationship between them.