



Vavuniya Campus of the University of Jaffna
First Examination in Information and Communication
Technology - 2017
First Semester - September/October 2018
ICT1113 Discrete Structures
Answer Five Questions Only

Time Allowed : Three hours

1. (a) In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories? [20%]
- (b) Let A, B and C are sets, using the set identities prove that $(A^c \cup B)^c \cap A^c = \emptyset$ [10%]
- (c) Let the set $A = \{ 1, 2, 3, 4 \}$. A relation on A is defined by:
- i. $R_1 = \{ (2,2), (3,3), (4,4), (1,4), (2,4), (3,4), (4,2), (3,2) \}$
 - ii. $R_2 = \{ (1,4), (4,2) \}$
 - iii. $R_3 = \{ (1,1), (2,2), (3,3), (2,3), (3,1), (1,3), (3,2) \}$
- Identify which of the above relations are reflexive, symmetric and transitive, justify your answer. [30%]

[This question is continued on the next page]

(d) Which of the following functions defined on a set of real numbers are *one-to-one* and which are *onto*? Justify your answer.

i. $f(x) = 5 - 3x$

ii. $f(x) = x^2 + 9$

iii. $f(x) = x^3 + 1$

[30%]

(e) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 3x + 5$ and $g(x) = 2x^2 + 2$. Find the formula for the composite function $g \circ f$.

[10%]

2. (a) Write each of the following statements in a symbolic form:

i. If Mr.Ram goes home early then Mrs.Ram feels happy.

ii. You can take the flight if and only if you buy a ticket.

iii. He is neither rich nor happy.

iv. Shali is either kind or innocent.

[20%]

(b) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.

[25%]

(c) Construct the truth table for each of the following propositions:

i. $(p \wedge q) \vee (\neg p \wedge \neg q)$

ii. $(p \oplus q) \vee (\neg p \oplus \neg q)$

[20%]

(d) State the *converse*, *contrapositive*, and *inverse* of the following conditional statement.

"If this program is correct, then it produces the correct answer." [15%]

(e) Symbolize the following by using quantifiers, predicates and logical connectives:

i. There exists an integer such that it is even and prime.

ii. For any integers there exists an integer such that their sum is 0.

iii. Every student in a university has a computer or has a friend who has a computer.

iv. All mathematics books published in India has blue cover.

[20%]

3. (a) Convert each of the following numbers into an equivalent octal number system:

i. 17.265625_{10}

ii. 10111011_2

iii. $2.A7_{16}$

[15%]

(b) Convert the following binary IP address into a dotted-decimal notation:

$$11001101\ 10010011\ 10000001\ 11011111$$

[15%]

(c) Subtract 3 from -12 using two's complement method in an 4-bit word size computer system.

[10%]

(d) Decide if each of the following scenarios describes an arithmetic or geometric sequence. Then, write the formula for the sequence.

i. Round 1 of a tennis tournament starts with 128 players. After each round, half the players have lost and are eliminated from the tournament. Therefore, in round 2 there are 64 players, in round 3 there are 32 players and so on.

ii. Paul has Rs 6800.00 in a saving account. He makes a deposit after he receives each paycheck. After one month he has Rs 7580.00 in the account. The next month the balance is Rs 8360.00. The balance after the third month is Rs 9140.00.

[20%]

(e) Show that the sequence $\{a_n\}$ is a solution of the recurrence relation

$$a_n = -3a_{n-1} + 4a_{n-2} \text{ if}$$

i. $a_n = (-4)^n$

ii. $a_n = 2(-4)^n + 3$

[20%]

(f) Given the two terms of an arithmetic sequence, find the recursive formula, and the first term of each of the following sequences:

i. $a_{18} = 97$ and $a_{40} = 229$

ii. $a_{12} = -43/8$ and $a_{36} = -139/8$

[20%]

4. (a) Suppose that a group of 5 boys and 3 girls is to be photographed.
- Find how many ways can they be arranged in one row.
 - Find how many ways can 5 boys and 3 girls be arranged with the girls in front row and the boys in the back row. [10%]
- (b) i. Find how many different eight letter words are possible using the letters of the word SYLLABUS.
- ii. If a word is chosen at random, find the probability that the word:
- Contains the two S's together
 - begins and ends with L [15%]
- (c) Find how many bit strings of length eight either start with the three bits 111 or end with the two bits 00. [15%]
- (d) In a class, there are 21 boys and 15 girls. Three boys and five girls wear glasses. A pupil is picked at random from the class.
- Find the probability that the pupil is a boy.
 - Find the probability that the pupil wears glasses.
 - Find the probability that the pupil is a boy who wears glasses.
 - Suppose a girl is picked at random from the class, find what is the probability that she wears glasses.
 - Suppose a pupil wearing glasses is picked at random from the class, find what is the probability that it is a boy. [30%]
- (e) There are fourteen juniors and twenty-three seniors in the Service Club. The club is to send four representatives to the state conference.
- Find how many different ways are there to select a group of four students to attend the conference from the 37 Service Club members.
 - Find how many ways are there to select exactly two juniors.
 - Find how many ways are there to select exactly two seniors.

[This question is continued on the next page]

- iv. If the members of the club decide to send two juniors and two seniors, find how many different groupings are possible.
- v. What is the probability that two juniors and two seniors are selected to attend the conference? [30%]

5. (a) Using the properties of Boolean algebra, simplify the boolean expression:

$$xyz + xy\bar{z} + x\bar{y}z + \bar{x}yz + x\bar{y}\bar{z} \quad [15\%]$$

- (b) Reduce the expression $\Sigma m(0,2,3,4,5,6)$ using Karnaugh map. [15%]

- (c) A system used 3 switches P, Q and R; a combination of switches determines whether an alarm, X, sounds: If switch P or switch Q are in the ON position and if switch R is in the OFF position then a signal to sound an alarm, X is produced. Design a circuit that produced alarm X sound. [20%]

- (d) Use a Karnaugh map to simplify each of the following Boolean expressions:

i. $F(x,y,z,t) = xyz t + xy\bar{z}t + \bar{x}yz t + \bar{x}y\bar{z}t$

ii. $F(x,y,z,t) = xyz t + \bar{x}y\bar{z}t + x\bar{y}z t + xy\bar{z}t + x\bar{y}\bar{z}t + \bar{x}\bar{y}z t + \bar{x}yzt + \bar{x}\bar{y}\bar{z}t$

iii. $F(x,y,z,t) = x\bar{y} + xyz + \bar{x}\bar{y}\bar{z} + \bar{x}yz\bar{t}$ [30%]

- (e) Design a combinational logic circuit with four inputs and one output. The output of the circuit will be logic 1 when the input value represented by the four inputs is prime number (e.g, 2=0010 where 0,0,1,0 are four inputs) and logic 0 otherwise. [20%]

6. (a) Consider the following graph in Figure 1:

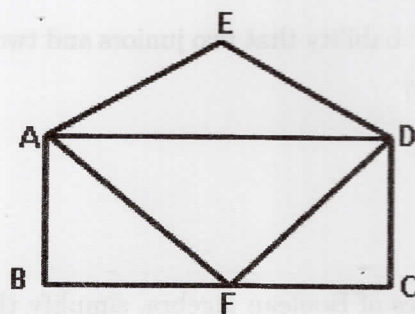


Figure 1

- i. Find the degree of each vertex of the graph. [05%]
- ii. Represent the graph in an adjacency matrix. [10%]
- iii. Find the Euler circuit from vertex A to A. [10%]

- (b) Consider the following graph in Figure 2:

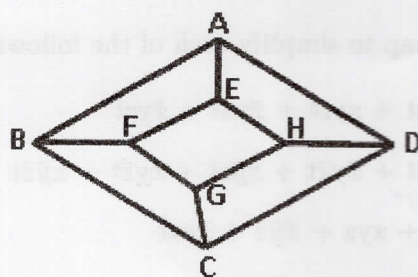


Figure 2

- i. Represent the graph in an adjacency list. [10%]
- ii. Find the Hamiltonian path from vertex A to vertex D. [10%]

[This question is continued on the next page]

(c) Draw the state diagram for the finite-state machines with the following state table:

State	<i>f</i>		<i>g</i>	
	Input		Input	
	0	1	0	1
s_0	s_0	s_4	1	1
s_1	s_0	s_3	0	1
s_2	s_0	s_2	0	0
s_3	s_1	s_1	1	1
s_4	s_1	s_0	1	0

[15%]

(d) Consider the state diagram for the Finite State Machine M as shown in Figure 3:

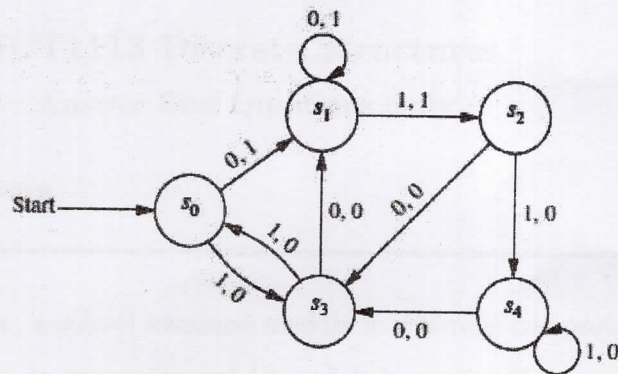


Figure 3: State Diagram

- Construct the state table for the Finite State Machine M.
- Find the output for each of the following strings given as input to the finite state machine in the above Figure 3. Justify your answer.

- 101011
- 101010
- 11011100010

[30%]