Vavuniya Campus of the University of Jaffna

First Examination in Information and Communication Technology - 2015

First Semester - August/September 2016

ICT1113 Discrete Structures

Answer Five Questions Only

Time Allowed : **Three hours**

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1. (a) In Figure 1, A is the set of people who go to a resort area for vacation, B is the set of people who take a cruise for vacation, and C is the set of people who go to a national park to vacation. The numbers in the figure represent the number of people in that region. Suppose that $|A|=150$, $|B|=100$, and $|C|=300$.

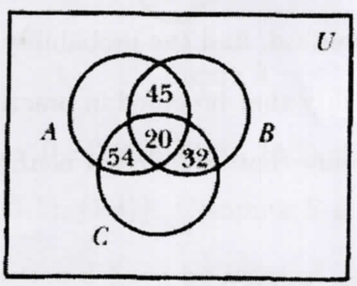

Figure 1

   i. How many people only go to a resort area for vacation?

   ii. How many people only take a cruise for vacation?

   iii. How many people only go to a national park for vacation?

[*This question is continued on the next page*]

iv. How many people either go to a resort area for vacation or take a cruise for vacation?

v. How many people use one of the three methods to take a vacation?

(b) Let A and B be two non empty sets. Using the set identities show that $(A^c \cup B)^c \cap A^c = \phi$.

(c) Let $A = \{ x \in \mathbb{R} \mid 1 < x \leq 5\}$ and $B = \{ x \in \mathbb{R} \mid 3 \leq x \leq 8 \}$. Find $A \cup B$, $A \cap B$, $A - B$, and $B - A$.

(d) A school basketball team play 20 matches each year. The probability for win any match is 3/5:

i. What is the probability that they lose a match?

ii. How many matches can they expect to win each year?

(e) A sequence of 8 bits is randomly generated. What is the probability that at least one of these bits is 1?

(f) In Department of Physical Science, 10% of the students failed in Practical's (P), 5% failed in Theory's (T), and 3% failed in both Practical's and Theories. A student is selected randomly:

i. If he failed in theory, find the probability that he also failed in practical.

ii. If he failed in practical, find the probability that he also failed in theory.

iii. Find the probability that he failed in practical or theory.

iv. Find the probability that he failed in neither practical nor theory.

2. (a) Consider the relations R and S shown in Figure 2. Compute: $\overline{R}$, $R \cap S$, $R \cup S$, *and* S^{-1}.

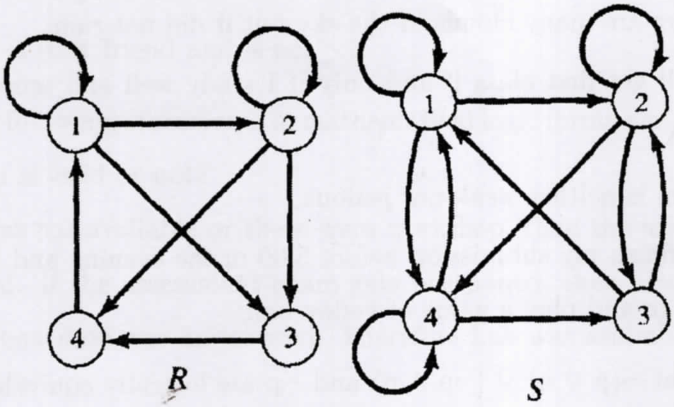

Figure 2

[20%]

(b) Figure 3 represents relations R and S respectively on Set A= {1, 2, 3, 4}. Determine whether the diagraph of relation R and S is *reflexive*, *symmetric* or *transitive*.

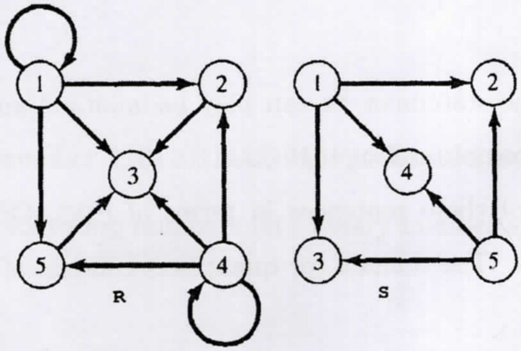

Figure 3

[30%]

(c) Let A={ 1, 2, 3, 4}. Let R={ (1,1),(1,2), (2,3), (2,4), (3,4), (4,1), (4,2)}, S={ (3,1), (4,4), (2,3), (2,4),(1,1), (1,4)}. Compute: $R \ o \ S$ and $S \ o \ R$. [10%]

(d) let f: $\mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f(x)= x^2\text{-}4x$ for all $x \in \mathbb{R}$. Find *Image(f)* and determine whether the function f *is Onto* or not. [20%]

(e) Let f: $\mathbb{R} \rightarrow \mathbb{R}$ and g: $\mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x)=4x\text{-}3$ and $g(x)=x^2+2$ for all $x \in \mathbb{R}$. Find $g \ o \ f$, $f \ o \ f$, $f \ o \ g$, and $g \ o \ g$. [20%]

3. (a) Express each of the following statements in propositional form:

   i. There are many clouds in the sky but it did not rain.

   ii. I will get first class if and only if I study well and score above 3.7 in final GPA.

   iii. Sham is neither weak nor jealous.

   iv. If I finish my submission before 5.00 in the evening and it is not very hot, I will go and play a game of volley ball.

[20

(b) Show that $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent by developing a series of logical equivalences.

[15

(c) State the *converse*, *contrapositive*, and *inverse* of the following conditional statement.

``A positive integer is a prime only if it has no divisors other than 1 and itself."

[15

(d) Let P(x) be the statement "x can play badminton" and Q(x) be the statement "x knows the computer language Java".

Express each of these sentences in terms of P(x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at Vavuniya Campus.

   i. There is a student at Vavuniya campus who can play badminton and who knows Java.

   ii. There is a student at Vavuniya campus who can play badminton but who doesn't know Java.

   iii. Every student at Vavuniya campus either can play badminton or knows Java.

[15%

(e) Let the domain (universe of discourse) be all people. Translate each of the following statements into logical expressions using predicates, quantifiers, and logical connectives.

   i. No one is perfect.

[This question is continued on the next page]

ii. All your friends are perfect.

iii. At least one of your friends is perfect.

iv. Everyone is your friend and is perfect. [20%]

(f) Represent the following statements in mathematical logic forms and prove whether the conclusion is valid or not?

"If the Lab was not available or there were workshop, then the assessment exam was postponed. If the assessment exam gets postponed, then new date was announced. No new date was announced. Therefore Lab was available." [15%]

4. (a) Convert each of the following numbers into decimal number system:

i. 1000110111110.001_2

ii. 100.01_8

iii. $A2DB.3_{16}$ [15%]

(b) Convert the binary number 1101101111110101 to hexadecimal number system. [10%]

(c) Convert each of the following numbers into binary number system:

i. 568.1875_{10}

ii. $ABC7.A7_{16}$

iii. 35.32_8 [15%]

(d) Covert the following binary IP address into a dotted-decimal notation:
00011110 11010010 11011101 01011010 [20%]

(e) Convert the following MAC (Media Access Control) address into a 48-bit binary string removing colons (:).

BD:32:FA:61:D0:40 [15%]

[This question is continued on the next page]

(f) Perform the binary addition/ subtraction using two's complement method in an 8-bit word size computer system:

i. (-3) + 11

ii. 75 + (-40)

iii. (-19) + (-7)

iv. 92 - 19

v. (-23) - (-18) [25

5. (a) Using the properties of Boolean algebra, simplify each of the following Boolean functions:

i. $F(x.y)=y+\overline{(xy)}$

ii. $F(x.y)=\overline{(xy)}(\overline{x}+y)(\overline{y}+y)$ [30

(b) Consider the combinational circuit shown in Figure 4:

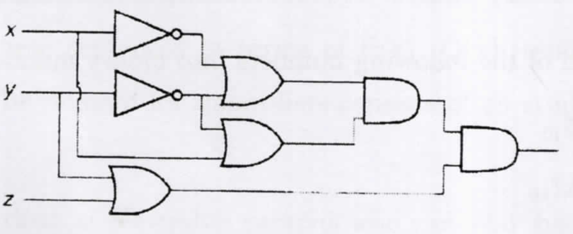

Figure 4

i. Write down the Boolean expression corresponding to the digital circuit shown in Figure 4. [05

ii. Using the laws of Boolean algebra, obtain a simpler equivalent expression, and draw the corresponding circuit. [20

(c) Design a combination logic circuit with three input variables that will produce logical value 1 output when more than one input variables are logical value 1. [25

[This question is continued on the next page]

(d) Use a Karnaugh map to simplify each of the following Boolean expressions:

i. $x\bar{y}z + xy\bar{z} + x\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}$

ii. $xyzw + xy\bar{z}w + xyz\bar{w} + \bar{x}yzw + \bar{x}yz\bar{w} + \bar{x}y\bar{z}\bar{w} + x\bar{y}zw + x\bar{y}\bar{z}w + x\bar{y}z\bar{w} +$

$\bar{x}\bar{y}zw + \bar{x}\bar{y}z\bar{w}$ [20%]

6. (a) Find the degree of each vertex of the graph given in Figure 5.

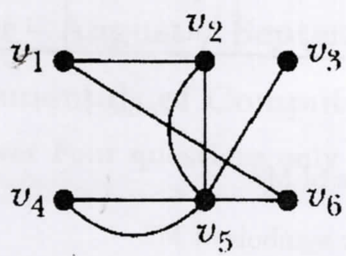

Figure 5

[20%]

(b) Let $A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 0 \end{pmatrix}$

Draw a directed graph that has A as its adjacency matrix.

[15%]

(c) Represent the following graph in an adjacency list.

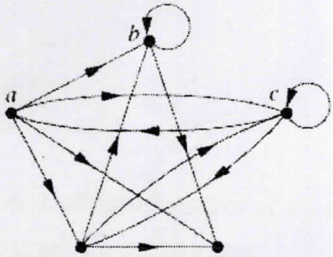

Figure 6

[15%]

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(d) Define a finite-state machine with the usual notation.

(e) Consider the finite-state machine M defined by the state table shown in Table 1.

	f		g	
	Input		*Input*	
State	0	1	0	1
s_0	s_1	s_0	0	0
s_1	s_2	s_0	1	1
s_2	s_0	s_3	0	1
s_3	s_1	s_2	1	0

Table 1

i. What are the states of M?

ii. What are the input symbols of M?

iii. What is the initial state of M?

iv. Draw the state diagrams for the finite-state machine M.