A small trip into homogenization and echo imaging

Quentin Goepfert

Abstract

Homogenization

Homogenization is a powerful tool to study and simulate PDEs in which a small parameter ε (small in front of every other physical ones) is involved.
The idea of homogenization is to find a macroscopic equivalent model (independent of ε) for the microscopic ε-dependent model whilst controlling the error with regards to ε.
This allows to simulate the initial problem in a reasonable time, which was not possible with the first model.

Such problems often appear in the study of composite material for example, to find macroscopic properties such as density.

Two model can be considered :
Either, the structure is periodic and periodic homogenization tools are involved [1] (and were first introduced in the 1970's).
Or, stochastic models [2] can be considered. In that case, one can imagine ε being the characteristic size of small particles randomly disposed inside a domain.

Most of the structures of materials are periodic and is the most known (and studied) field in homogenization.
Still, this requires some hypotheses on our coefficients (periodic or stationary and ergodic coefficients).

Application echo imaging

Echo imaging is a common medical imaging method that presents several advantages. Indeed, it a cheap, portable method and has no side-effect (though, it is used only for soft tissues).
A lot of technical progress has been made since the 1970's where it was first introduced but the two main hypotheses made on the wave propagation remain unchanged - the wave propagates at constant speed and the echos correspond to simple diffusion.
Physical experiments have proven these two hypotheses wrong. We aim at finding

another model which does not require any of these hypotheses.
We remarked that the problem is naturally a multi-scale one and the inhomogeneities generating the echos are small (in front of the wavelength).
That being said, we try to use a stochastic homogenization model to find an asymptotic simpler equivalent macroscopic model which represents at best the propagation of the ultrasound wave inside a biologic tissue, which is too complex of a medium to be studied directly as such.

References

[1] S. Fliss, Cours d'Homogénéisation périodique (2020).

[2] L. Giovangigli, Cours d'Homogénéisation stochastique (2021).