Report: comparison of 9 global optimization methods on several test problems classes

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1 List of the algorithms

- Algorithm of global search (AGS) (`https://github.com/sovrasov/ags_nlp_solver`)
- Multi Level Single Linkage (MLSL) (`https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/#mlsl-multi-level-single-linkage`)
- DIRECT (`https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/#direct-and-direct-l`)
- Locally-based DIRECT (DIRECTl) (`https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/#direct-and-direct-l`)
- Dual Simulated Annealing (`https://github.com/sgubianpm/sdaopt`)
- Differential Evolution (`https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.differential_evolution.html#scipy.optimize.differential_evolution`)
- Controlled Random Search (`https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/#controlled-random-search-crs-with-local-mutation`)
- Simple (`https://github.com/chrisstroemel/Simple`)
- StoGO (`https://nlopt.readthedocs.io/en/latest/NLopt_Algorithms/#stogo`)

When conducting the comparison, the following parameters for the methods were employed:

- in the AGSl method, the parameter of alternation the global and local iterations was set to be equal to 5:1;
- in the DIRECT and DIRECTl methods, the parameter $\epsilon = 10^{-4}$;
- in the SDA method, the parameter $visit = 2.72$.

The rest parameters were varied subject to the problem class (see Table 1).

Since NLOpt hasn't an API to control parameters of the algorithms from Python, it was built with $\varepsilon = 10^{-4}$ for DIRECT and DIRECTl methods.

Table 1: Class-specific parameters of the optimization algorithms

	AGS, AGSl	CRS	DE
F_{GR}	$r = 3$	popsize=150	mutation=(1.1,1.9), popsize=60
GKLS 2d Simple	$r = 4.6$	popsize=200	mutation=(1.1,1.9), popsize=60
GKLS 2d Hard	$r = 6.5$	popsize=400	mutation=(1.1,1.9), popsize=60
GKLS 3d Simple	$r = 3.7$	popsize=1000	mutation=(1.1,1.9), popsize=70
GKLS 3d Hard	$r = 4.4$	popsize=2000	mutation=(1.1,1.9), popsize=80
GKLS 4d Simple	$r = 4.7$	popsize=8000	mutation=(1.1,1.9), popsize=90
GKLS 4d Hard	$r = 4.9$	popsize=16000	mutation=(1.1,1.9), popsize=100
GKLS 5d Simple	$r = 4$	popsize=25000	mutation=(1.1,1.9), popsize=120
GKLS 5d Hard	$r = 4$	popsize=30000	mutation=(1.1,1.9), popsize=140

Table 2: Trials limits for the test problem classes

Problems class	Trials limit
F_{GR}	5000
GKLS 2d Simple	8000
GKLS 2d Hard	9000
GKLS 3d Simple	15000
GKLS 3d Hard	25000
GKLS 4d Simple	150000
GKLS 4d Hard	250000
GKLS 5d Simple	350000
GKLS 5d Hard	600000

2 List of the test problems

- Functions from F_{GR} class. It consists of 100 multi-extremal problems of the same structure. The description can be found in `https://core.ac.uk/download/pdf/82313177.pdf`.

- Functions from classes generated by the GKLS generator (`http://wwwinfo.deis.unical.it/yaro/GKLS.html`).

Each class consists of 100 multi-extremal problems with 10 and more local minima. Problem is considered solved when optimization method placed a new trial point in the Δ-vicinity of the known global optima x^*: $\|x^* - \widetilde{x}\|_{\inf} \leq \Delta$.
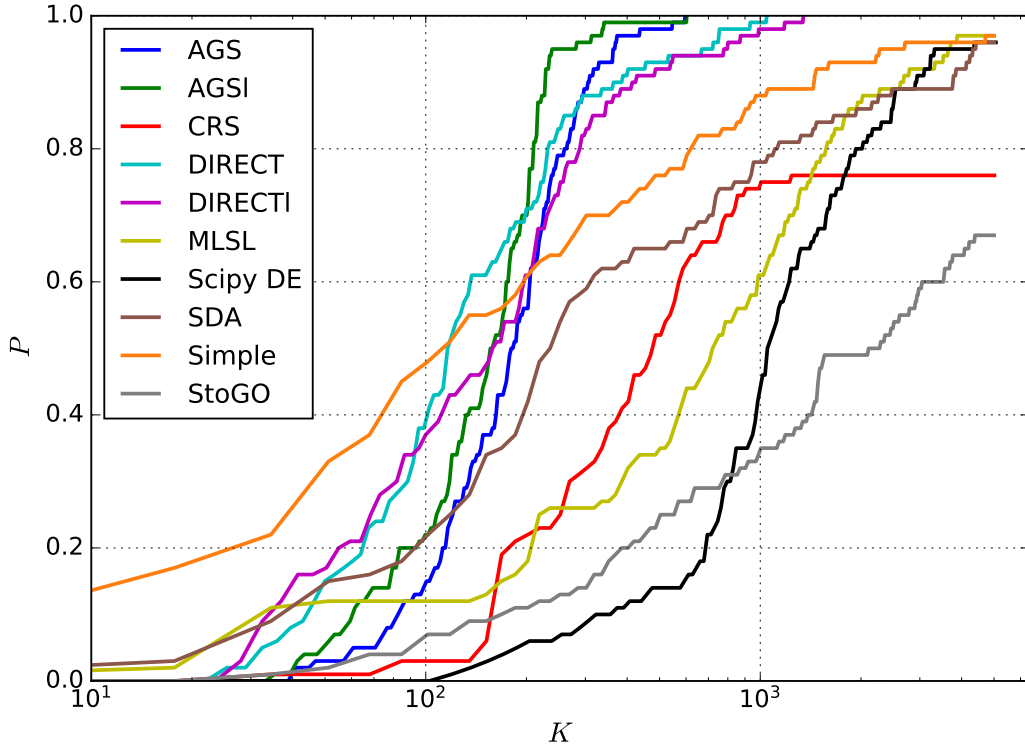
3 Results on the F_{GR} class

Figure 1: $\Delta = 10^{-2}$

2

Method	Average number of trials	Problems solved
AGS	193.11	100
AGSl	158.30	100
CRS	400.30	76
DIRECT	182.25	100
DIRECTl	214.92	100
MLSL	947.18	97
SDA	691.24	96
Scipy DE	1257.34	96
Simple	374.12	97
StoGO	1336.78	67

4   Results on the $GKLS$ problems
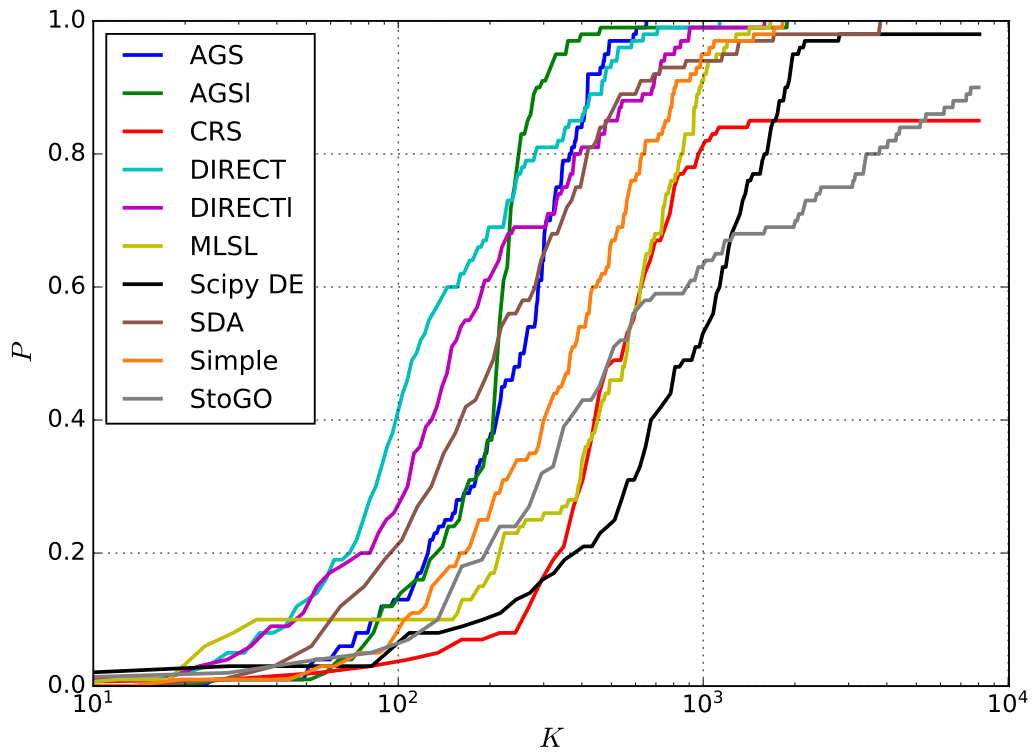
Figure 2: Class GKLS Simple 2d. $\Delta = 2 \cdot 10^{-2}$

Method	Average number of trials	Problems solved
AGS	254.89	100
AGSl	217.60	100
CRS	510.61	85
DIRECT	189.03	100
DIRECTl	255.21	100
MLSL	556.83	100
SDA	356.30	100
Scipy DE	952.16	98
Simple	440.63	100
StoGO	1251.52	90
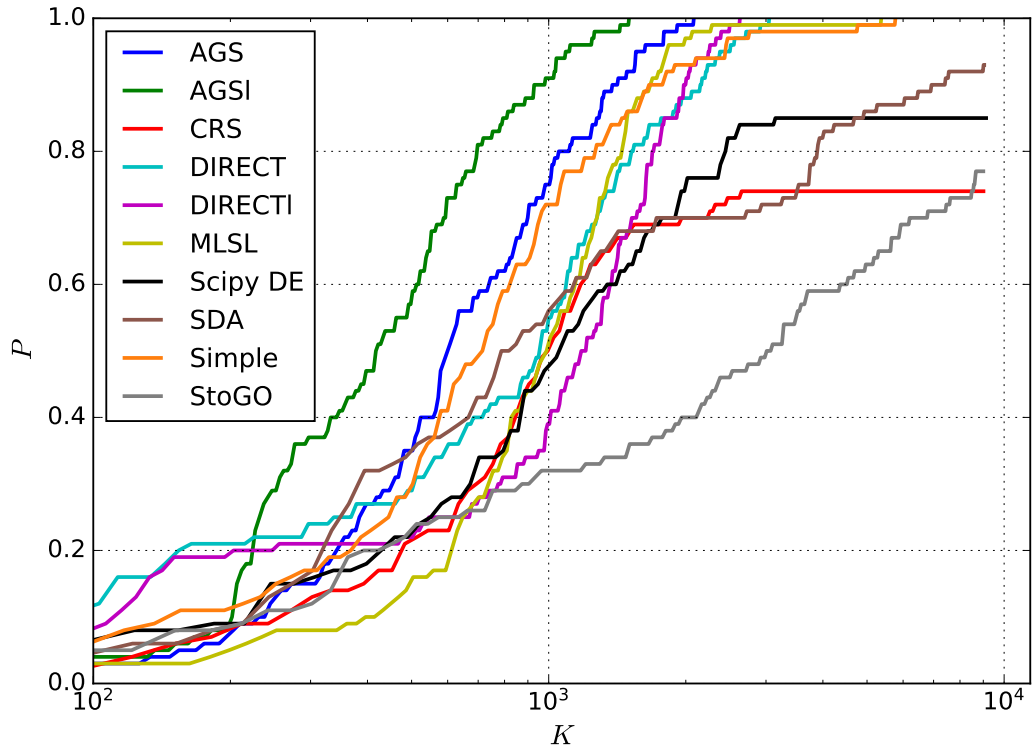
Figure 3: Class GKLS Hard 2d. $\Delta = 2 \cdot 10^{-2}$

Method	Average number of trials	Problems solved
AGS	728.71	100
AGSl	487.96	100
CRS	844.74	74
DIRECT	985.44	100
DIRECTl	1126.65	100
MLSL	1042.54	100
SDA	1637.92	93
Scipy DE	1041.12	85
Simple	898.19	100
StoGO	2532.23	77
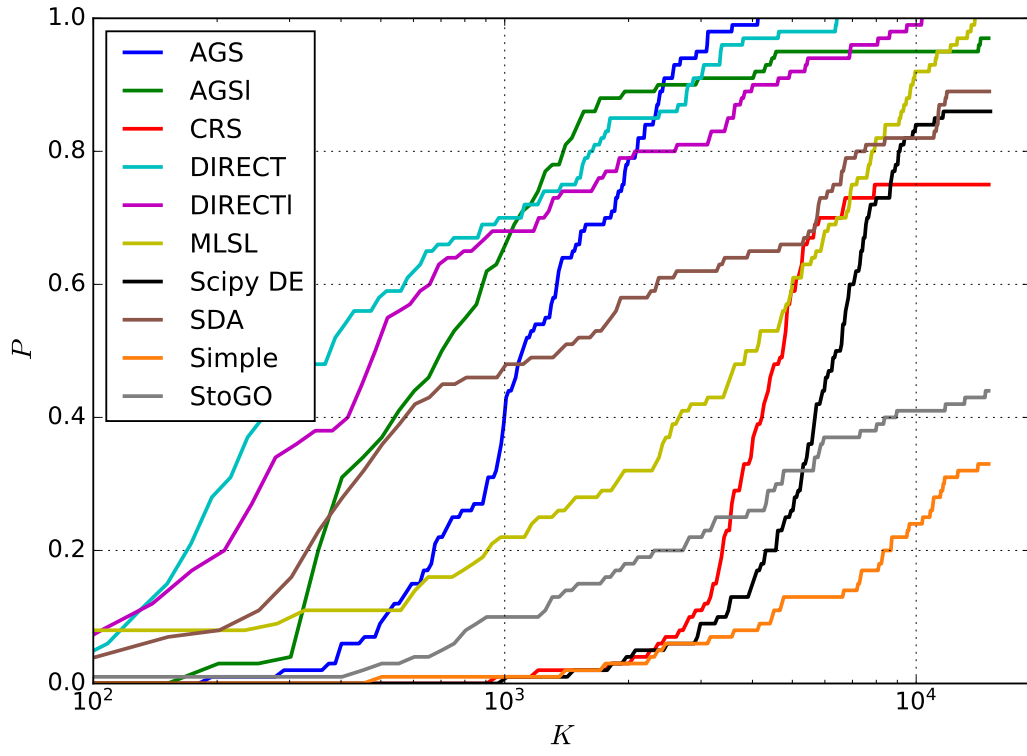
Figure 4: Class GKLS Simple 3d. $\Delta = 2 \cdot 10^{-2}$

Method	Average number of trials	Problems solved
AGS	1372.13	100
AGSl	1195.32	97
CRS	4145.81	75
DIRECT	973.64	100
DIRECTl	1477.79	100
MLSL	4609.17	100
SDA	2706.52	89
Scipy DE	5956.94	86
Simple	7098.45	33
StoGO	3856.11	44
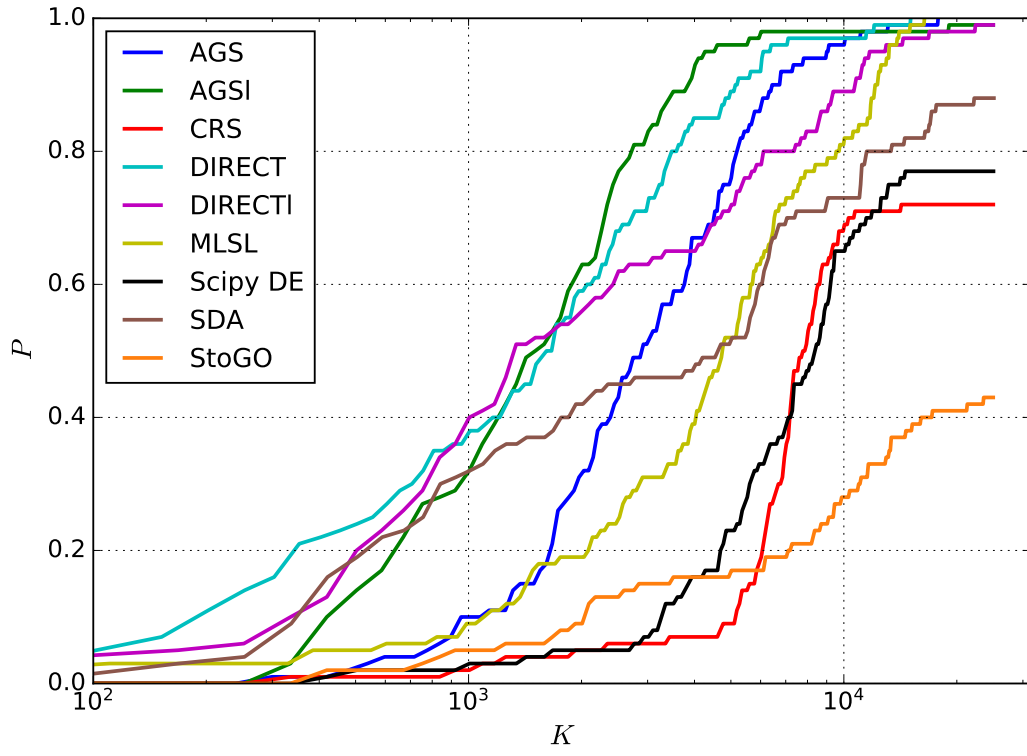
Figure 5: Class GKLS Hard 3d. $\Delta = 2 \cdot 10^{-2}$

Method	Average number of trials	Problems solved
AGS	3636.12	100
AGSl	1930.49	99
CRS	6786.96	72
DIRECT	2298.74	100
DIRECTl	3553.33	99
MLSL	5640.10	100
SDA	4708.43	88
Scipy DE	6914.34	77
StoGO	7843.23	43
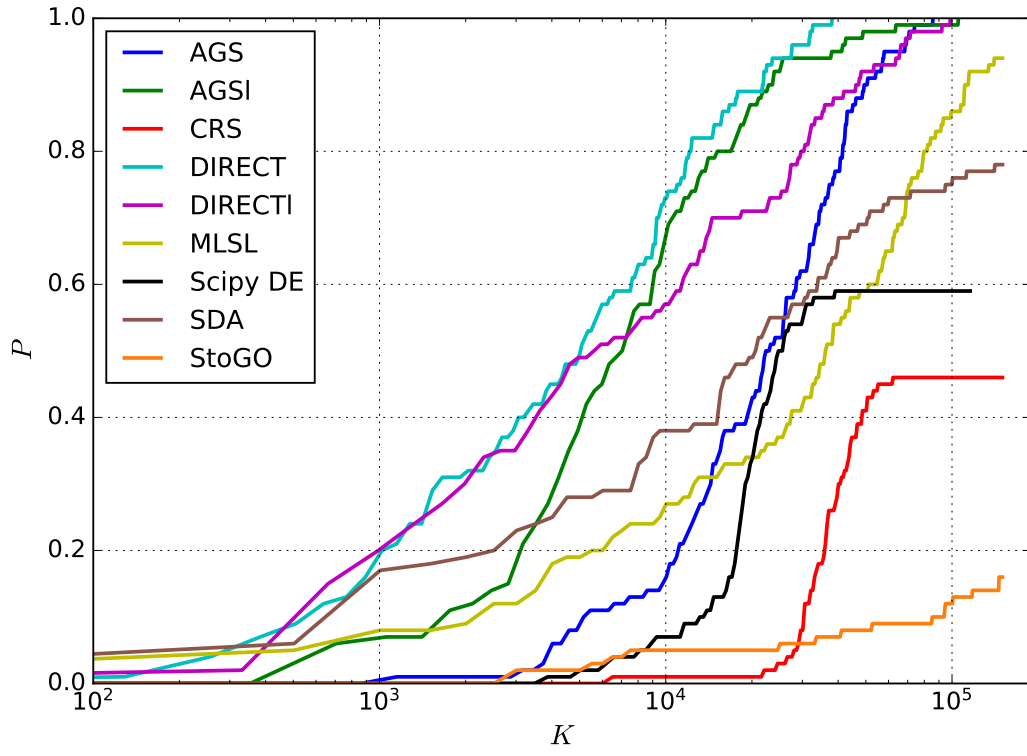
Figure 6: Class GKLS Simple 4d. $\Delta = 2 \cdot 10^{-2}$

Method	Average number of trials	Problems solved
AGS	26654.07	100
AGSl	11095.65	100
CRS	37436.76	46
DIRECT	7824.32	100
DIRECTl	15994.11	100
MLSL	41514.32	94
SDA	21417.90	78
Scipy DE	19157.73	59
StoGO	59895.44	16
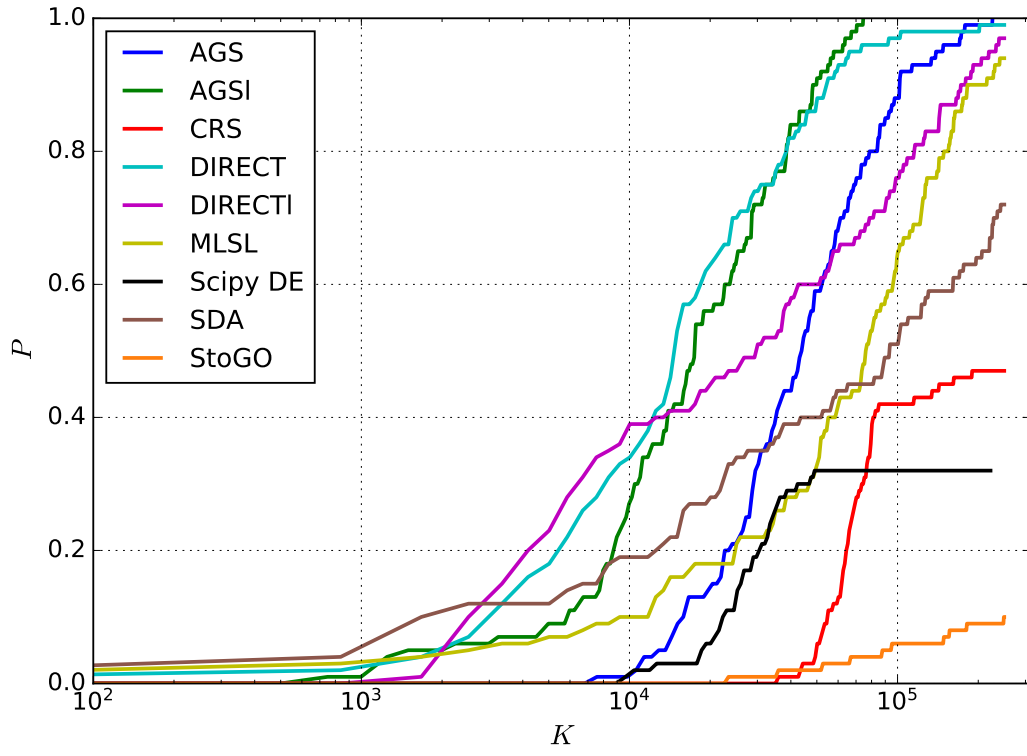
Figure 7: Class GKLS Hard 4d. $\Delta = 2 \cdot 10^{-2}$

Method	Average number of trials	Problems solved
AGS	54536.84	100
AGSl	23167.84	100
CRS	73779.32	47
DIRECT	23204.38	99
DIRECTl	54489.92	97
MLSL	80247.19	94
SDA	68815.53	72
Scipy DE	27466.06	32
StoGO	109328.10	10
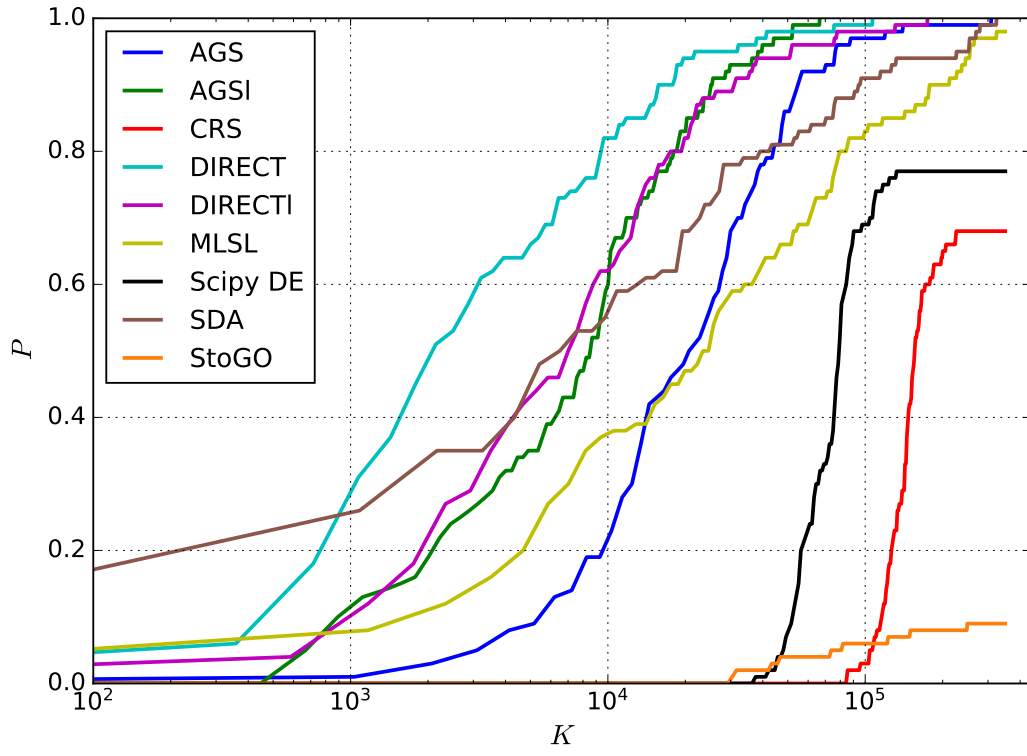
Figure 8: Class GKLS Simple 5d. $\Delta = 2 \cdot 10^{-2}$

Method	Average number of trials	Problems solved
AGS	29809.99	100
AGSl	11529.03	100
CRS	143574.99	68
DIRECT	7166.49	100
DIRECTl	13970.53	100
MLSL	52647.63	98
SDA	34255.31	100
Scipy DE	73074.52	77
StoGO	91580.44	9
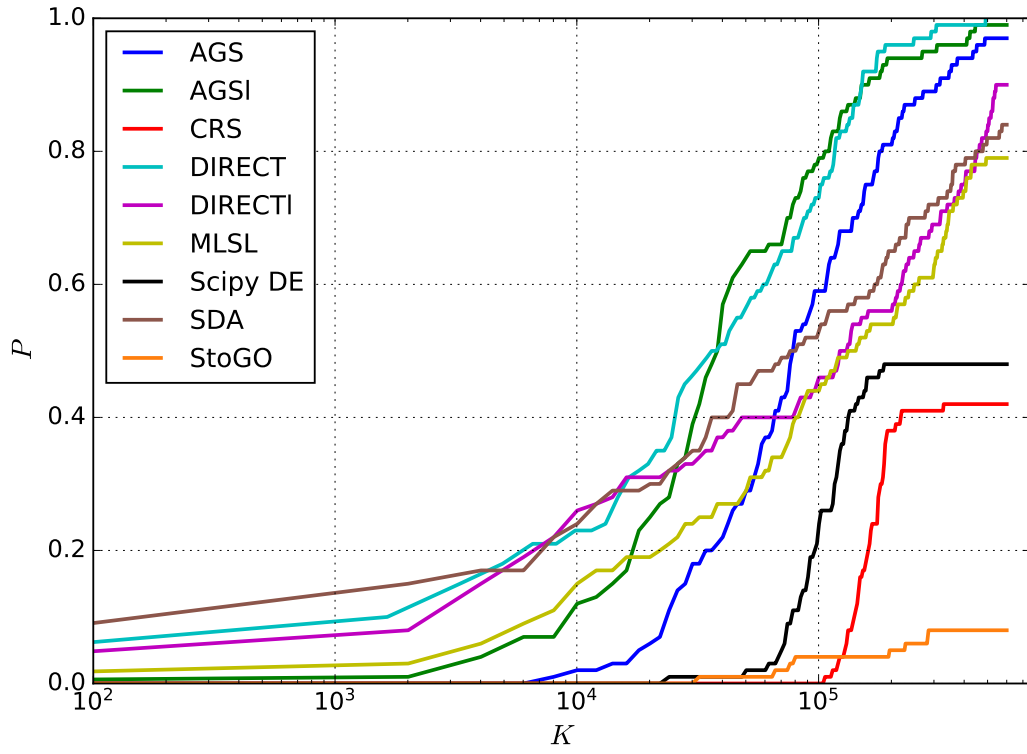
Figure 9: Class GKLS Hard 5d. $\Delta = 2 \cdot 10^{-2}$

Method	Average number of trials	Problems solved
AGS	113129.08	97
AGSl	67652.72	99
CRS	165192.76	42
DIRECT	66327.42	100
DIRECTl	164390.63	90
MLSL	138766.23	79
SDA	116973.10	84
Scipy DE	105496.88	48
StoGO	155123.75	8
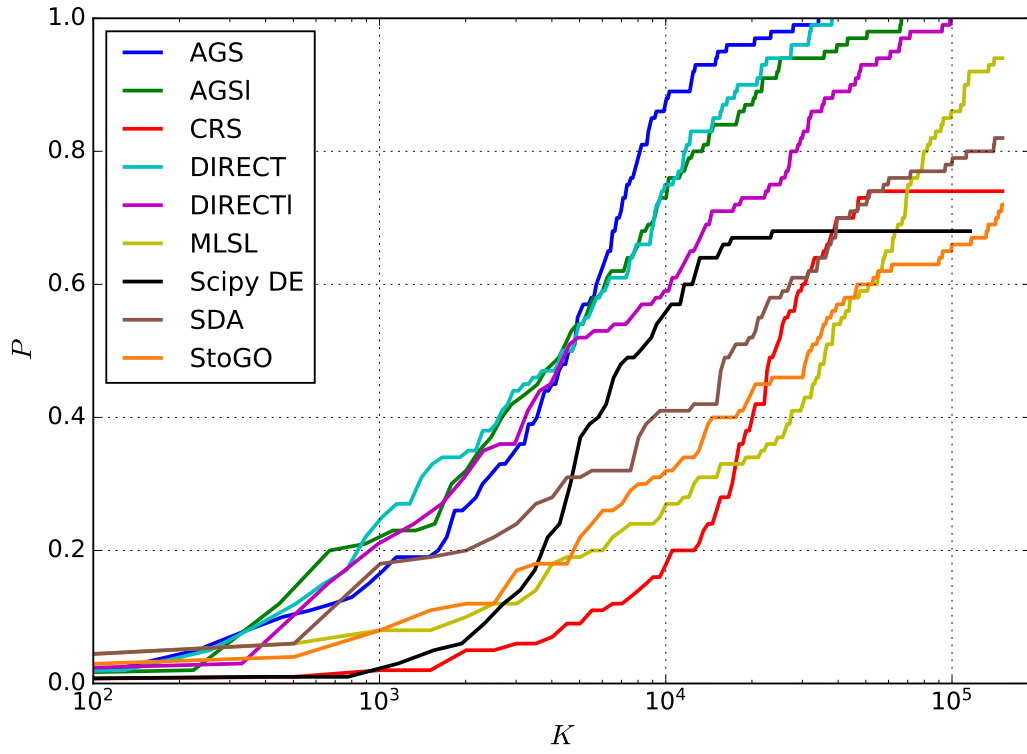
Figure 10: Class GKLS Simple 4d. $\Delta = 0.0632$

Method	Average number of trials	Problems solved
AGS	5729.82	100
AGSl	8847.40	100
CRS	19883.59	74
DIRECT	7328.78	100
DIRECTl	15010.01	100
MLSL	41484.80	94
SDA	22065.96	82
Scipy DE	6271.24	68
StoGO	29359.22	72
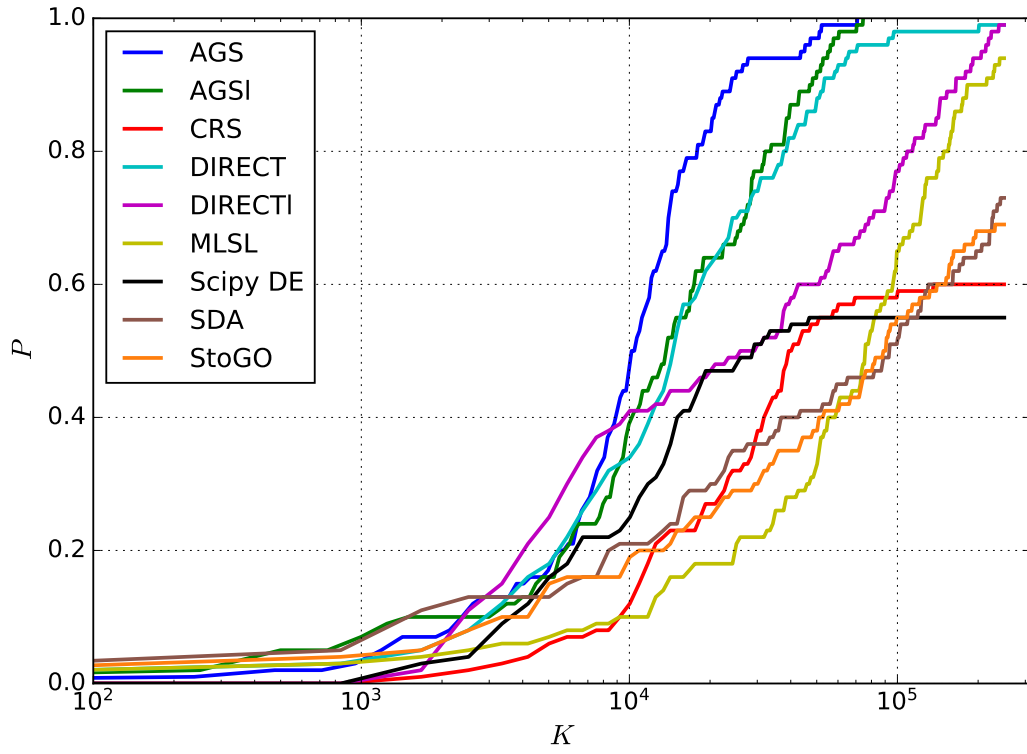
Figure 11: Class GKLS Hard 4d. $\Delta = 0.0632$

Method	Average number of trials	Problems solved
AGS	13113.40	100
AGSl	19826.36	100
CRS	27137.40	60
DIRECT	22884.35	99
DIRECTl	55596.07	99
MLSL	80220.11	94
SDA	68048.01	73
Scipy DE	12487.64	55
StoGO	58925.54	69
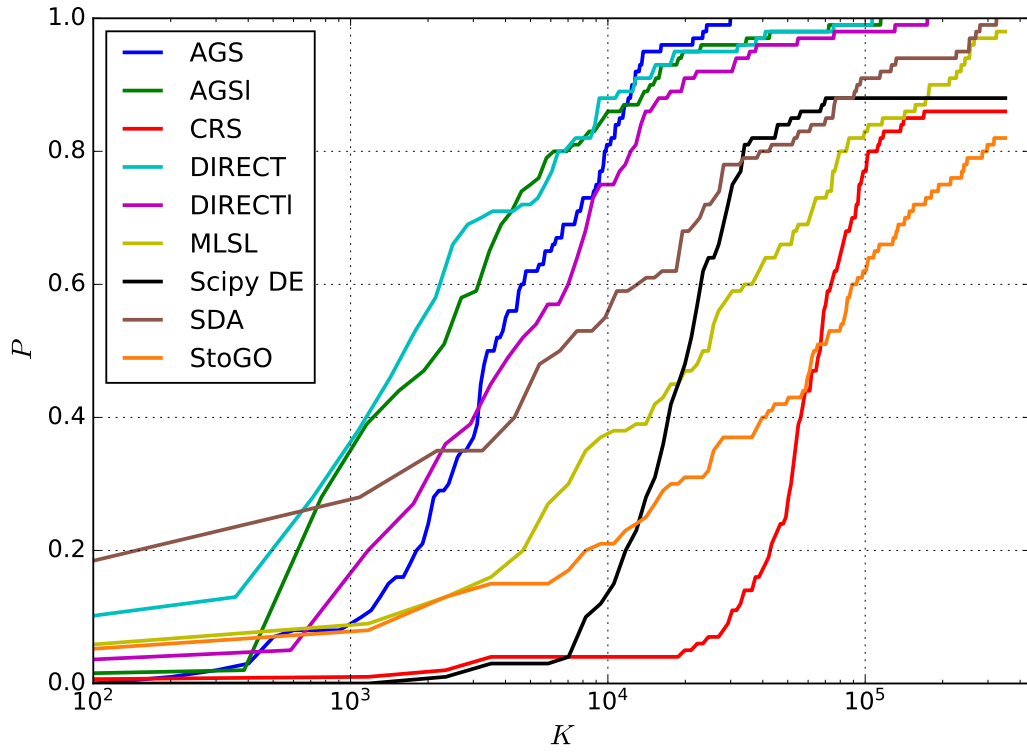
Figure 12: Class GKLS Simple 5d. $\Delta = 0.0796$

Method	Average number of trials	Problems solved
AGS	5821.47	100
AGSl	6314.25	100
CRS	62921.69	86
DIRECT	5966.13	100
DIRECTl	10795.46	100
MLSL	52609.18	98
SDA	34208.83	100
Scipy DE	20859.38	88
StoGO	69206.76	82
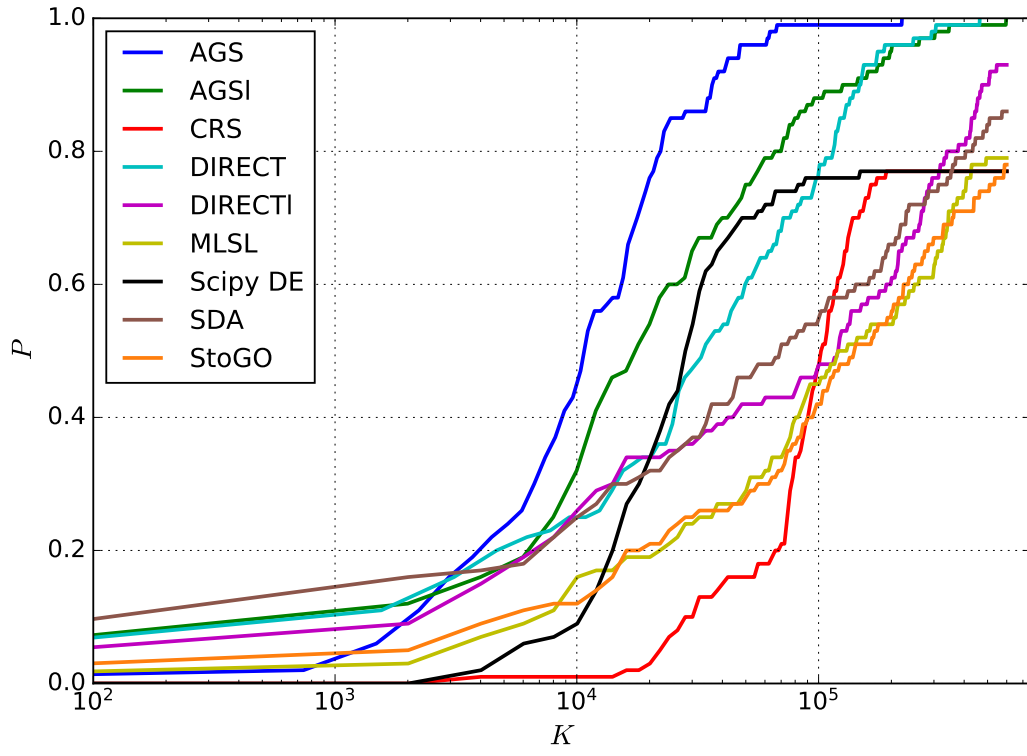
Figure 13: Class GKLS Hard 5d. $\Delta = 0.0796$

Method	Average number of trials	Problems solved
AGS	17008.61	100
AGSl	48514.29	100
CRS	87563.88	77
DIRECT	61657.32	100
DIRECTl	148637.82	93
MLSL	138011.78	79
SDA	115634.59	86
Scipy DE	26850.04	77
StoGO	141886.49	78