Linear Model Selection

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pdf_document

```
library(leaps)
```

Best Subset Selection

```
library(ISLR)
# fix(Hitters)
names(Hitters)
```

```
##  [1] "AtBat"     "Hits"      "HmRun"     "Runs"      "RBI"       "Walks"
##  [7] "Years"     "CAtBat"    "CHits"     "CHmRun"    "CRuns"     "CRBI"
## [13] "CWalks"    "League"    "Division"  "PutOuts"   "Assists"   "Errors"
## [19] "Salary"    "NewLeague"
```

```
dim(Hitters)
```

```
## [1] 322  20
```

```
sum(is.na(Hitters$Salary ))
```

```
## [1] 59
```

Hence we have salary is missing for 59 players.

```
Hitters = na.omit(Hitters)
dim(Hitters)
```

```
## [1] 263  20
```

```
sum(is.na(Hitters))
```

```
## [1] 0
```

Hence after removing no more na.

regsubsets() function performs best subset selection by identifying the best model that contains a given number of predictors where best is quantified using RSS.

```
library(leaps)
regfit.full = regsubsets(Salary ~.,Hitters)
summary(regfit.full)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., Hitters)
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: exhaustive
##          AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1  ( 1 ) " "   " "  " "   " "  " " " " " "   " "   " "   " "    " "   " "  "*"
## 2  ( 1 ) " "   "*"  " "   " "  " " " " " "   " "   " "   " "    " "   " "  "*"
## 3  ( 1 ) " "   "*"  " "   " "  " " " " " "   " "   " "   " "    " "   " "  "*"
## 4  ( 1 ) " "   "*"  " "   " "  " " " " " "   " "   " "   " "    " "   " "  "*"
## 5  ( 1 ) "*"   "*"  " "   " "  " " " " " "   " "   " "   " "    " "   " "  "*"
## 6  ( 1 ) "*"   "*"  " "   " "  " " " " "*"   " "   " "   " "    " "   " "  "*"
## 7  ( 1 ) " "   "*"  " "   " "  " " " " "*"   " "   "*"   "*"    "*"   " "  " "
## 8  ( 1 ) "*"   "*"  " "   " "  " " " " "*"   " "   " "   " "    "*"   "*"  " "
##          CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 ) " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 ) " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 ) " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 5  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 6  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 ) "*"    " "     "*"       "*"     " "     " "    " "
```

A * indicates that the given variable is included in the corresponding model. By default , regsubsets() only reports results up to best 8 variable model. But the nvmax option can be used in order to return as many variables as are desired.

```
regfit.full = regsubsets(Salary ~., data = Hitters, nvmax = 19)
reg.summary = summary(regfit.full)
```

The summary function returns R^2, RSS, and adjusted R2 Lets examine these to get the best overall model.

```
names(reg.summary)
```

```
## [1] "which"  "rsq"    "rss"    "adjr2"  "cp"     "bic"    "outmat" "obj"
```

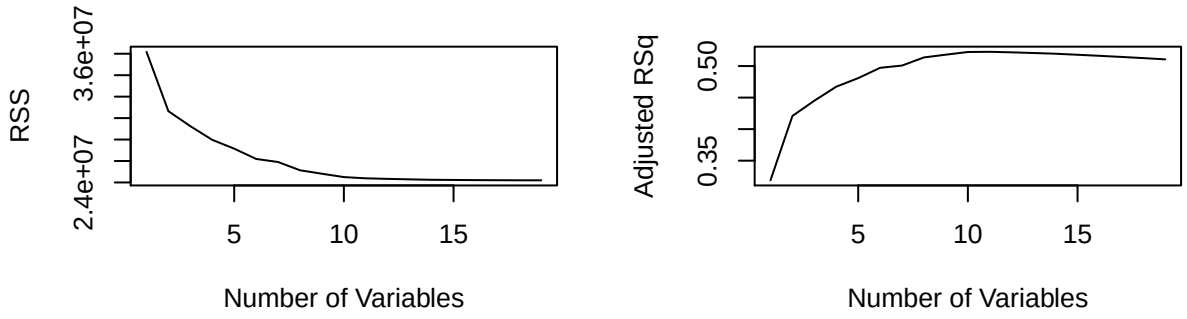
```
reg.summary$rsq
```

```
##  [1] 0.3214501 0.4252237 0.4514294 0.4754067 0.4908036 0.5087146 0.5141227
##  [8] 0.5285569 0.5346124 0.5404950 0.5426153 0.5436302 0.5444570 0.5452164
## [15] 0.5454692 0.5457656 0.5459518 0.5460945 0.5461159
```

thus R2 statistics increases from 32% when only one variable is included in the model to almost 55% when all variable are included. Hence R2 increases monotonically as more variables are included.

```
library(ggplot2)
par(mfrow = c(2,2))

plot(reg.summary$rss,xlab="Number of Variables", ylab="RSS", type = "l")
plot(reg.summary$adjr2,xlab="Number of Variables", ylab="Adjusted RSq", type = "l")
```

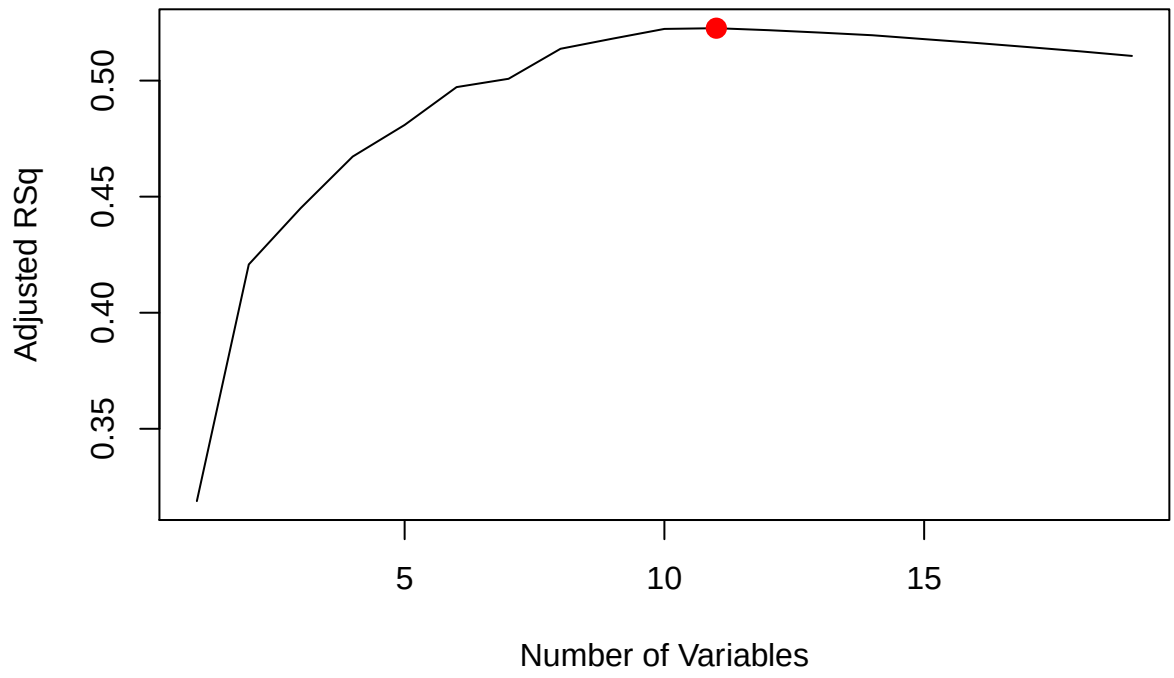


points() works like **plot()** except that it puts points on a plot that has already been created, instead of creating a new plot. the which.max() fucntiin xan be used to identofy the location of the maxima

```
which.max(reg.summary$adjr2)
```

```
## [1] 11
```

```
plot(reg.summary$adjr2,xlab="Number of Variables", ylab="Adjusted RSq", type = "l")
points(11, reg.summary$adjr2[11], col="red", cex=2, pch = 20)
```

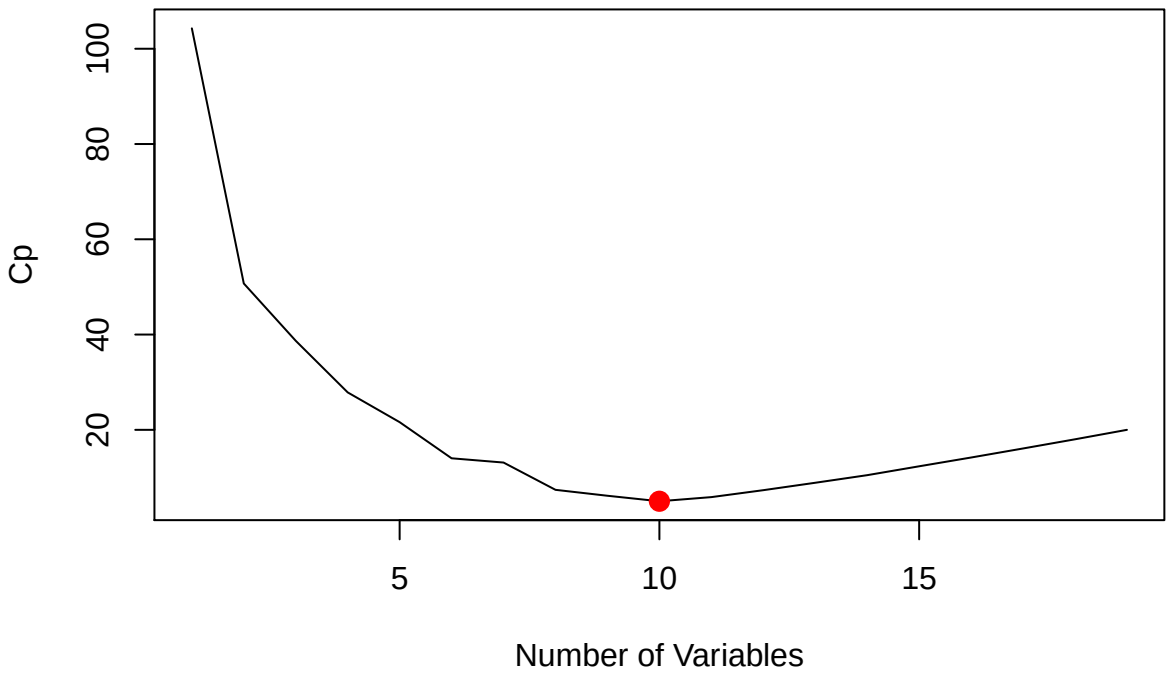



```
plot(reg.summary$cp,xlab="Number of Variables", ylab="Cp", type = "l")

which.max(reg.summary$cp)
```

```
## [1] 1
```

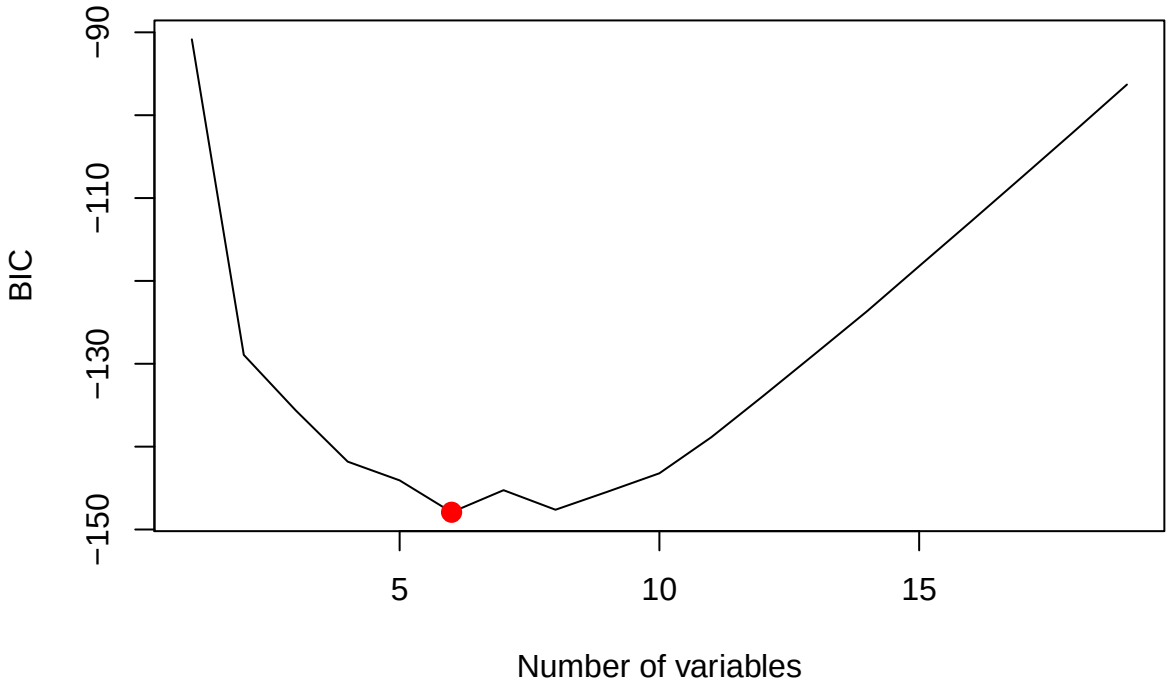
```
points(10, reg.summary$cp[10],col="red",cex=2,pch=20)
```



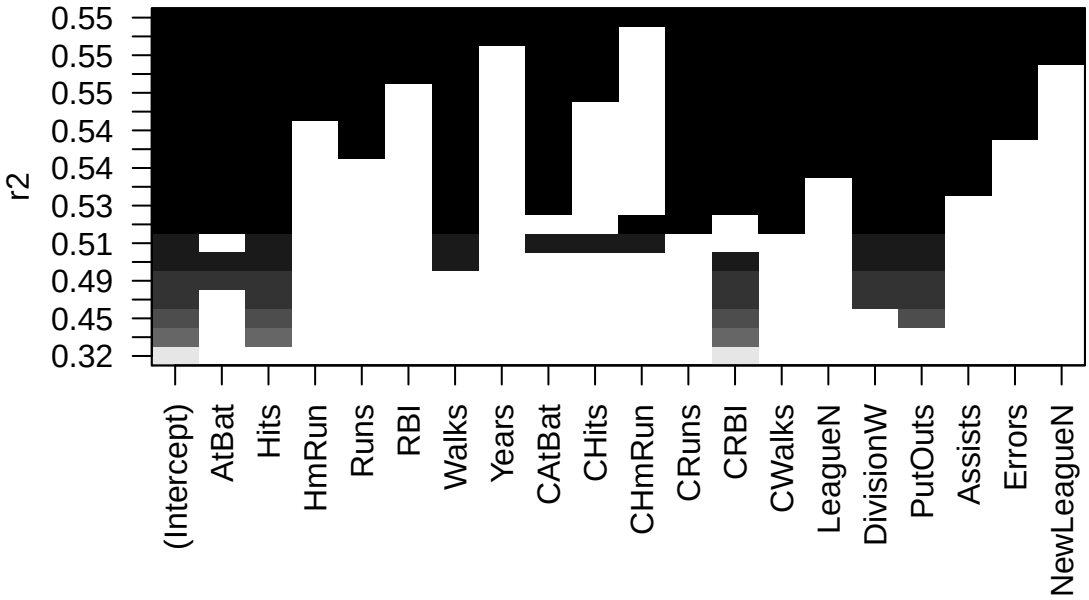

```
which.min(reg.summary$bic)
```

```
## [1] 6
```

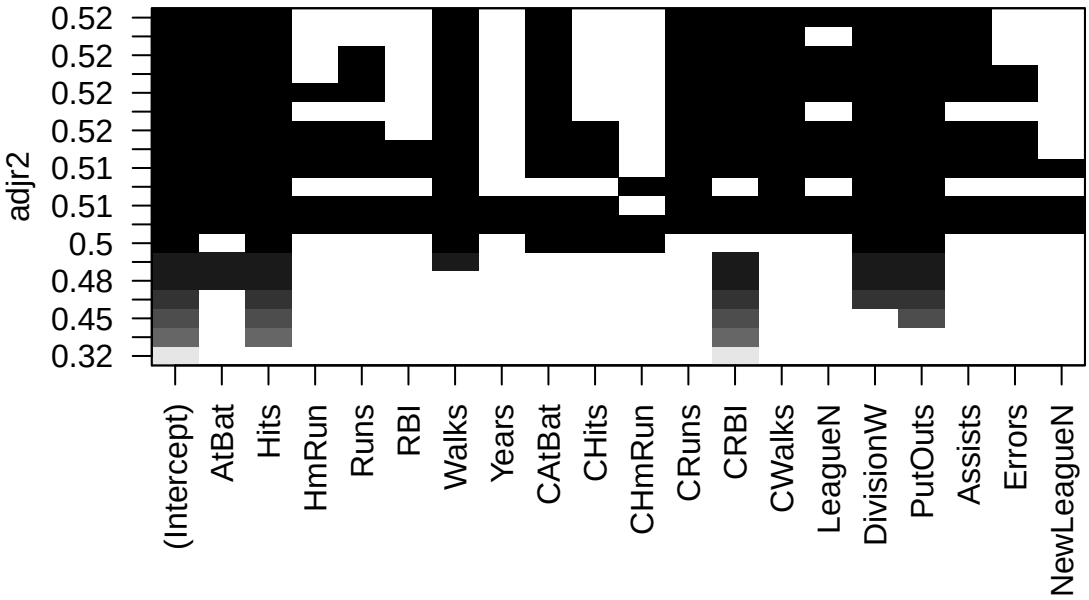
```
plot(reg.summary$bic,xlab ="Number of variables", ylab = "BIC", type="l")
points(6,reg.summary$bic[6],col="red",cex=2,pch=20)
```



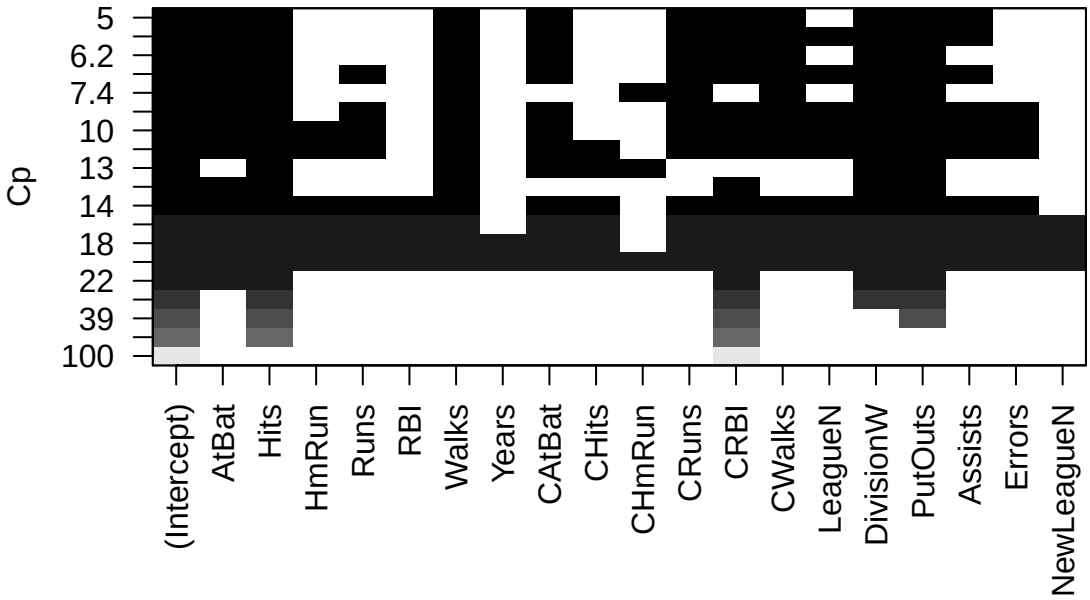

```
plot(regfit.full, scale="r2")
```



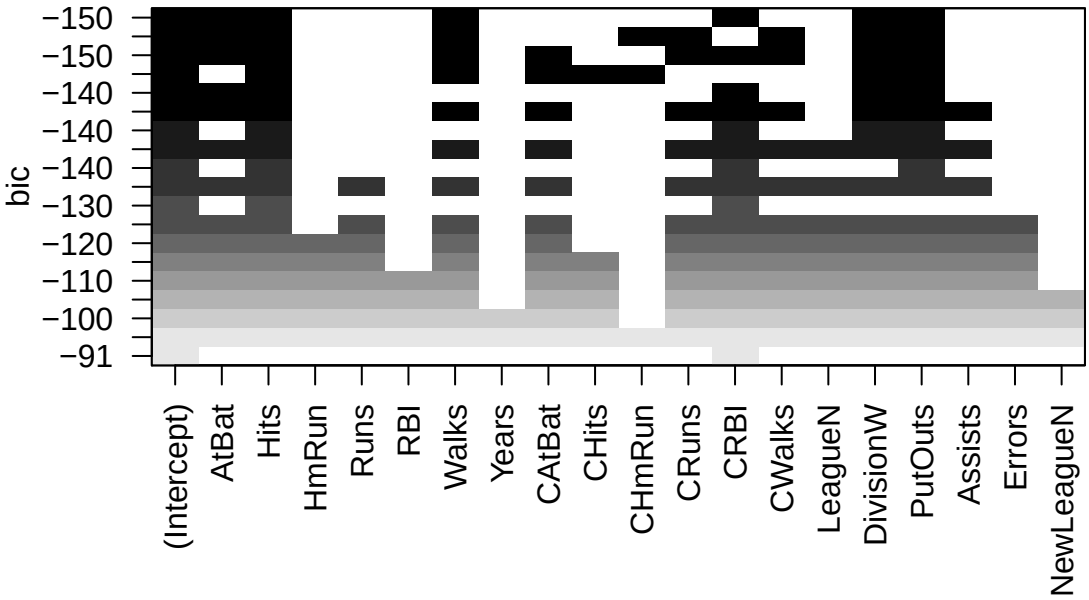

```
plot(regfit.full, scale="adjr2")
```




```
plot(regfit.full, scale="Cp")
```




```
plot(regfit.full, scale="bic")
```



The top row of each plot contains BLACK SQUARE for each variable selected according to the optimal model associated with the statistics,For instance we see several model share a BRIC close to -150. However the model with the lowest BRIC is the 6 variable model that contain only AtBat, Hits, CRBI, DivisionW and PutOuts.

```
coef(regfit.full, 6)
```

```
##  (Intercept)        AtBat         Hits        Walks         CRBI    DivisionW
##   91.5117981   -1.8685892    7.6043976    3.6976468    0.6430169 -122.9515338
##      PutOuts
##    0.2643076
```

##Forward and Backward Stepwise Selection

```
regfit.fwd = regsubsets(Salary ~., data = Hitters, nvmax = 19, method ="forward")
regfit.bwd = regsubsets(Salary ~., data = Hitters, nvmax = 19, method ="backward")
```

```
coef(regfit.full , 7)
```

```
##  (Intercept)         Hits        Walks       CAtBat        CHits       CHmRun
##   79.4509472    1.2833513    3.2274264   -0.3752350    1.4957073    1.4420538
##    DivisionW      PutOuts
## -129.9866432    0.2366813
```

```
coef(regfit.fwd , 7)
```

```
##  (Intercept)        AtBat         Hits        Walks         CRBI       CWalks
##  109.7873062   -1.9588851    7.4498772    4.9131401    0.8537622   -0.3053070
##    DivisionW      PutOuts
## -127.1223928    0.2533404
```

```
coef(regfit.bwd , 7)
```

```
##  (Intercept)        AtBat         Hits        Walks        CRuns       CWalks
##  105.6487488   -1.9762838    6.7574914    6.0558691    1.1293095   -0.7163346
##    DivisionW      PutOuts
## -116.1692169    0.3028847
```

Choosing among the models using validation set approach and cross validation

```
set.seed(1)
train = sample(c(TRUE, FALSE), nrow(Hitters), rep = TRUE)
test =(!train)
```

```
regfit.best = regsubsets(Salary ~., data = Hitters[test,],nvmax=19)
regfit.best
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters[test, ], nvmax = 19)
## 19 Variables  (and intercept)
##             Forced in Forced out
## AtBat           FALSE      FALSE
## Hits            FALSE      FALSE
## HmRun           FALSE      FALSE
## Runs            FALSE      FALSE
## RBI             FALSE      FALSE
## Walks           FALSE      FALSE
## Years           FALSE      FALSE
## CAtBat          FALSE      FALSE
## CHits           FALSE      FALSE
## CHmRun          FALSE      FALSE
## CRuns           FALSE      FALSE
## CRBI            FALSE      FALSE
## CWalks          FALSE      FALSE
## LeagueN         FALSE      FALSE
## DivisionW       FALSE      FALSE
## PutOuts         FALSE      FALSE
## Assists         FALSE      FALSE
## Errors          FALSE      FALSE
## NewLeagueN      FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: exhaustive
```

```
test.mat = model.matrix(Salary~.,data=Hitters[test,])
test.mat
```

```
##                     (Intercept) AtBat Hits HmRun Runs RBI Walks Years CAtBat
## -Alvin Davis                  1   479  130    18   66  72    76     3   1624
## -Alfredo Griffin              1   594  169     4   74  51    35    11   4408
## -Andre Thornton               1   401   92    17   49  66    65    13   5206
## -Alan Trammell                1   574  159    21  107  75    59    10   4631
## -Buddy Biancalana             1   190   46     2   24   8    15     5    479
## -Bruce Bochy                  1   127   32     8   16  22    14     8    727
## -Barry Bonds                  1   413   92    16   72  48    65     1    413
## -Bobby Bonilla                1   426  109     3   55  43    62     1    426
## -Billy Hatcher                1   419  108     6   55  36    22     3    591
## -Bill Madlock                 1   379  106    10   38  60    30    14   6207
## -Bob Melvin                   1   268   60     5   24  25    15     2    350
## -BillyJo Robidoux             1   181   41     1   15  21    33     2    232
## -Chris Bando                  1   254   68     2   28  26    22     6    999
## -Chili Davis                  1   526  146    13   71  70    84     6   2648
## -Curt Ford                    1   214   53     2   30  29    23     2    226
## -Carney Lansford              1   591  168    19   80  72    39     9   4478
## -Chet Lemon                   1   403  101    12   45  53    39    12   5150
## -Candy Maldonado              1   405  102    18   49  85    20     6    950
## -Craig Reynolds               1   313   78     6   32  41    12    12   3742
## -Cal Ripken                   1   627  177    25   98  81    70     6   3210
## -Cory Snyder                  1   416  113    24   58  69    16     1    416
## -Chris Speier                 1   155   44     6   21  23    15    16   6631
## -Curt Wilkerson               1   236   56     0   27  15    11     4   1115
## -Daryl Boston                 1   199   53     5   29  22    21     3    514
## -Dave Concepcion              1   311   81     3   42  30    26    17   8247
## -Doug DeCinces                1   512  131    26   69  96    52    14   5347
## -Damaso Garcia                1   424  119     6   57  46    13     9   3651
## -Dan Gladden                  1   351   97     4   55  29    39     4   1258
## -Dave Henderson               1   388  103    15   59  47    39     6   2174
## -Don Mattingly                1   677  238    31  117 113    53     5   2223
## -Dale Murphy                  1   614  163    29   89  83    75    11   5017
## -Dwayne Murphy                1   329   83     9   50  39    56     9   3828
## -Dave Parker                  1   637  174    31   89 116    56    14   6727
## -Dan Pasqua                   1   280   82    16   44  45    47     2    428
## -Darrell Porter               1   155   41    12   21  29    22    16   5409
## -Don Slaught                  1   314   83    13   39  46    16     5   1457
## -Darryl Strawberry            1   475  123    27   76  93    72     4   1810
## -Dale Sveum                   1   317   78     7   35  35    32     1    317
## -Danny Tartabull              1   511  138    25   76  96    61     3    592
## -Eddie Milner                 1   424  110    15   70  47    36     7   2130
## -Eddie Murray                 1   495  151    17   61  84    78    10   5624
## -George Bell                  1   641  198    31  101 108    41     5   2129
## -Glenn Braggs                 1   215   51     4   19  18    11     1    215
## -George Brett                 1   441  128    16   70  73    80    14   6675
## -Gary Gaetti                  1   596  171    34   91 108    52     6   2862
## -George Hendrick              1   283   77    14   45  47    26    16   6840
## -Garth Iorg                   1   327   85     3   30  44    20     8   2140
## -Gary Pettis                  1   539  139     5   93  58    69     5   1469
## -Gary Redus                   1   340   84    11   62  33    47     5   1516
```

```
## -Glenn Wilson          1  584  158   15   70  84   42    5  2358
## -Harold Baines         1  570  169   21   72  88   38    7  3754
## -Howard Johnson        1  220   54   10   30  39   31    5  1185
## -Hal McRae             1  278   70    7   22  37   18   18  7186
## -Harold Reynolds       1  445   99    1   46  24   29    4   618
## -John Cangelosi        1  438  103    2   65  32   71    2   440
## -Jim Dwyer             1  160   39    8   18  31   22   14  2128
## -Julio Franco          1  599  183   10   80  74   32    5  2482
## -Jim Gantner           1  497  136    7   58  38   26   11  3871
## -Johnny Grubb          1  210   70   13   32  51   28   15  4040
## -John Kruk             1  278   86    4   33  38   45    1   278
## -Jeffrey Leonard       1  341   95    6   48  42   20   10  2964
## -Jim Morrison          1  537  147   23   58  88   47   10  2744
## -John Moses            1  399  102    3   56  34   34    5   670
## -Johnny Ray            1  579  174    7   67  78   58    6  3053
## -Jerry Royster         1  257   66    5   31  26   32   14  3910
## -John Russell          1  315   76   13   35  60   25    3   630
## -Jim Sundberg          1  429   91   12   41  42   57   13  5590
## -Joel Youngblood       1  184   47    5   20  28   18   11  3327
## -Kal Daniels           1  181   58    6   34  23   22    1   181
## -Kirk Gibson           1  441  118   28   84  86   68    8  2723
## -Ken Griffey           1  490  150   21   69  58   35   14  6126
## -Kent Hrbek            1  550  147   29   85  91   71    6  2816
## -Kevin Mitchell        1  328   91   12   51  43   33    2   342
## -Ken Oberkfell         1  503  136    5   62  48   83   10  3423
## -Ken Phelps            1  344   85   24   69  64   88    7   911
## -Kirby Puckett         1  680  223   31  119  96   34    3  1928
## -Len Dykstra           1  431  127    8   77  45   58    2   667
## -Lance Parrish         1  327   84   22   53  62   38   10  4273
## -Larry Parrish         1  464  128   28   67  94   52   13  5829
## -Larry Sheets          1  338   92   18   42  60   21    3   682
## -Lou Whitaker          1  584  157   20   95  73   63   10  4704
## -Mike Aldrete          1  216   54    2   27  25   33    1   216
## -Mariano Duncan        1  407   93    8   47  30   30    2   969
## -Mike Heath            1  288   65    8   30  36   27    9  2815
## -Mike Marshall         1  330   77   19   47  53   27    6  1928
## -Mike Schmidt          1   20    1    0    0   0    0    2    41
## -Milt Thompson         1  299   75    6   38  23   26    3   580
## -Mitch Webster         1  576  167    8   89  49   57    4   822
## -Mookie Wilson         1  381  110    9   61  45   32    7  3015
## -Marvell Wynne         1  288   76    7   34  37   15    4  1644
## -Ozzie Guillen         1  547  137    2   58  47   12    2  1038
## -Pete Incaviglia       1  540  135   30   82  88   55    1   540
## -Pete Rose             1  237   52    0   15  25   30   24 14053
## -Rafael Belliard       1  309   72    0   33  31   26    5   354
## -Randy Bush            1  357   96    7   50  45   39    5  1394
## -Rick Cerone           1  216   56    4   22  18   15   12  2796
## -Ron Cey               1  256   70   13   42  36   44   16  7058
## -Rick Dempsey          1  327   68   13   42  29   45   18  3949
## -Reggie Jackson        1  419  101   18   65  58   92   20  9528
## -Ron Kittle            1  376   82   21   42  60   35    5  1770
## -Rick Leach            1  246   76    5   35  39   13    6   912
## -Rick Manning          1  205   52    8   31  27   17   12  5134
## -Rance Mulliniks       1  348   90   11   50  45   43   10  2288
```

```
## -Rafael Ramirez              1   496  119     8   57  33    21     7   3358
## -Rick Schu                   1   208   57     8   32  25    18     3    653
## -Rob Wilfong                 1   288   63     3   25  33    16    10   2682
## -Robin Yount                 1   522  163     9   82  46    62    13   7037
## -Scott Bradley               1   220   66     5   20  28    13     3    290
## -Sid Bream                   1   522  140    16   73  77    60     4    730
## -Steve Buechele              1   461  112    18   54  54    35     2    680
## -Shawon Dunston              1   581  145    17   66  68    21     2    831
## -Scott Fletcher              1   530  159     3   82  50    47     6   1619
## -Steve Jeltz                 1   439   96     0   44  36    65     4    711
## -Steve Sax                   1   633  210     6   91  56    59     6   3070
## -Tom Foley                   1   263   70     1   26  23    30     4    888
## -Tony Gwynn                  1   642  211    14  107  59    52     5   2364
## -Terry Harper                1   265   68     8   26  30    29     7   1337
## -Tommy Herr                  1   559  141     2   48  61    73     8   3162
## -Tim Hulett                  1   520  120    17   53  44    21     4    927
## -Terry Kennedy               1    19    4     1    2   3     1     1     19
## -Tito Landrum                1   205   43     2   24  17    20     7    854
## -Tom Paciorek                1   213   61     4   17  22     3    17   4061
## -Terry Pendleton             1   578  138     1   56  59    34     3   1399
## -Terry Puhl                  1   172   42     3   17  14    15    10   4086
## -Tim Teufel                  1   279   69     4   35  31    32     4   1359
## -Vince Coleman               1   600  139     0   94  29    60     2   1236
## -Will Clark                  1   408  117    11   66  41    34     1    408
## -Willie Randolph             1   492  136     5   76  50    94    12   5511
## -Wayne Tolleson              1   475  126     3   61  43    52     6   1700
##                    CHits CHmRun CRuns CRBI CWalks LeagueN DivisionW PutOuts
## -Alvin Davis         457     63   224  266    263       0         1     880
## -Alfredo Griffin    1133     19   501  336    194       0         1     282
## -Andre Thornton     1332    253   784  890    866       0         0       0
## -Alan Trammell      1300     90   702  504    488       0         0     238
## -Buddy Biancalana    102      5    65   23     39       0         1     102
## -Bruce Bochy         180     24    67   82     56       1         1     202
## -Barry Bonds          92     16    72   48     65       1         0     280
## -Bobby Bonilla       109      3    55   43     62       0         1     361
## -Billy Hatcher       149      8    80   46     31       1         1     226
## -Bill Madlock       1906    146   859  803    571       1         1      72
## -Bob Melvin           78      5    34   29     18       1         1     442
## -BillyJo Robidoux     50      4    20   29     45       0         0     326
## -Chris Bando         236     21   108  117    118       0         0     359
## -Chili Davis         715     77   352  342    289       1         1     303
## -Curt Ford            59      2    32   32     27       1         0     109
## -Carney Lansford    1307    113   634  563    319       0         1      67
## -Chet Lemon         1429    166   747  666    526       0         0     316
## -Candy Maldonado     231     29    99  138     64       1         1     161
## -Craig Reynolds      968     35   409  321    170       1         1     106
## -Cal Ripken          927    133   529  472    313       0         0     240
## -Cory Snyder         113     24    58   69     16       0         0     203
## -Chris Speier       1634     98   698  661    777       1         0      53
## -Curt Wilkerson      270      1   116   64     57       0         1     125
## -Daryl Boston        120      8    57   40     39       0         1     152
## -Dave Concepcion    2198    100   950  909    690       1         1     153
## -Doug DeCinces      1397    221   712  815    548       0         1     119
## -Damaso Garcia      1046     32   461  301    112       0         0     224
```

```
## -Dan Gladden          353     16   196  110    117        1          1     226
## -Dave Henderson       555     80   285  274    186        0          1     182
## -Don Mattingly        737     93   349  401    171        0          0    1377
## -Dale Murphy         1388    266   813  822    617        1          1     303
## -Dwayne Murphy        948    145   575  528    635        0          1     276
## -Dave Parker         2024    247   978 1093    495        1          1     278
## -Dan Pasqua           113     25    61   70     63        0          0     148
## -Darrell Porter      1338    181   746  805    875        0          1     165
## -Don Slaught          405     28   156  159     76        0          1     533
## -Darryl Strawberry    471    108   292  343    267        1          0     226
## -Dale Sveum            78      7    35   35     32        0          0      45
## -Danny Tartabull      164     28    87  110     71        0          1     157
## -Eddie Milner         544     38   335  174    258        1          1     292
## -Eddie Murray        1679    275   884 1015    709        0          0    1045
## -George Bell          610     92   297  319    117        0          0     269
## -Glenn Braggs          51      4    19   18     11        0          0     116
## -George Brett        2095    209  1072 1050    695        0          1      97
## -Gary Gaetti          728    107   361  401    224        0          1     118
## -George Hendrick     1910    259   915 1067    546        0          1     144
## -Garth Iorg           568     16   216  208     93        0          0      91
## -Gary Pettis          369     12   247  126    198        0          1     462
## -Gary Redus           376     42   284  141    219        1          0     185
## -Glenn Wilson         636     58   265  316    134        1          0     331
## -Harold Baines       1077    140   492  589    263        0          1     295
## -Howard Johnson       299     40   145  154    128        1          0      50
## -Hal McRae           2081    190   935 1088    643        0          1       0
## -Harold Reynolds      129      1    72   31     48        0          1     278
## -John Cangelosi       103      2    67   32     71        0          1     276
## -Jim Dwyer            543     56   304  268    298        0          0      33
## -Julio Franco         715     27   330  326    158        0          0     231
## -Jim Gantner         1066     40   450  367    241        0          0     304
## -Johnny Grubb        1130     97   544  462    551        0          0       0
## -John Kruk             86      4    33   38     45        1          1     102
## -Jeffrey Leonard      808     81   379  428    221        1          1     158
## -Jim Morrison         730     97   302  351    174        1          0      92
## -John Moses           167      4    89   48     54        0          1     211
## -Johnny Ray           880     32   366  337    218        1          0     280
## -Jerry Royster        979     33   518  324    382        1          1      87
## -John Russell         151     24    68   94     55        1          0     498
## -Jim Sundberg        1397     83   578  579    644        0          1     686
## -Joel Youngblood      890     74   419  382    304        1          1      49
## -Kal Daniels           58      6    34   23     22        1          1      88
## -Kirk Gibson          750    126   433  420    309        0          0     190
## -Ken Griffey         1839    121   983  707    600        0          0      96
## -Kent Hrbek           815    117   405  474    319        0          1    1218
## -Kevin Mitchell        94     12    51   44     33        1          0     145
## -Ken Oberkfell        970     20   408  303    414        1          1      65
## -Ken Phelps           214     64   150  156    187        0          1       0
## -Kirby Puckett        587     35   262  201     91        0          1     429
## -Len Dykstra          187      9   117   64     88        1          0     283
## -Lance Parrish       1123    212   577  700    334        0          0     483
## -Larry Parrish       1552    210   740  840    452        0          1       0
## -Larry Sheets         185     36    88  112     50        0          0       0
## -Lou Whitaker        1320     93   724  522    576        0          0     276
```

```
## -Mike Aldrete         54      2    27   25     33        1          1     317
## -Mariano Duncan      230     14   121   69     68        1          1     172
## -Mike Heath          698     55   315  325    189        1          0     259
## -Mike Marshall       516     90   247  288    161        1          1     149
## -Mike Schmidt          9      2     6    7      4        1          0      78
## -Milt Thompson       160      8    71   33     44        1          0     212
## -Mitch Webster       232     19   132   83     79        1          0     325
## -Mookie Wilson       834     40   451  249    168        1          0     228
## -Marvell Wynne       408     16   198  120    113        1          1     203
## -Ozzie Guillen       271      3   129   80     24        0          1     261
## -Pete Incaviglia     135     30    82   88     55        0          1     157
## -Pete Rose          4256    160  2165 1314   1566        1          1     523
## -Rafael Belliard      82      0    41   32     26        1          0     117
## -Randy Bush          344     43   178  192    136        0          1     167
## -Rick Cerone         665     43   266  304    198        0          0     391
## -Ron Cey            1845    312   965 1128    990        1          0      41
## -Rick Dempsey        939     78   438  380    466        0          0     659
## -Reggie Jackson     2510    548  1509 1659   1342        0          1       0
## -Ron Kittle          408    115   238  299    157        0          1       0
## -Rick Leach          234     12   102   96     80        0          0      44
## -Rick Manning       1323     56   643  445    459        0          0     155
## -Rance Mulliniks     614     43   295  273    269        0          0      60
## -Rafael Ramirez      882     36   365  280    165        1          1     155
## -Rick Schu           170     17    98   54     62        1          0      42
## -Rob Wilfong         667     38   315  259    204        0          1     135
## -Robin Yount        2019    153  1043  827    535        0          0     352
## -Scott Bradley        80      5    27   31     15        0          1     281
## -Sid Bream           185     22    93  106     86        1          0    1320
## -Steve Buechele      160     24    76   75     49        0          1     111
## -Shawon Dunston      210     21   106   86     40        1          0     320
## -Scott Fletcher      426     11   218  149    163        0          1     196
## -Steve Jeltz         148      1    68   56     99        1          0     229
## -Steve Sax           872     19   420  230    274        1          1     367
## -Tom Foley           220      9    83   82     86        1          0      81
## -Tony Gwynn          770     27   352  230    193        1          1     337
## -Terry Harper        339     32   135  163    128        1          1      92
## -Tommy Herr          874     16   421  349    359        1          0     352
## -Tim Hulett          227     22   106   80     52        0          1      70
## -Terry Kennedy         4      1     2    3      1        1          1     692
## -Tito Landrum        219     12   105   99     71        1          0     131
## -Tom Paciorek       1145     83   488  491    244        0          1     178
## -Terry Pendleton     357      7   149  161     87        1          0     133
## -Terry Puhl         1150     57   579  363    406        1          1      65
## -Tim Teufel          355     31   180  148    158        1          0     133
## -Vince Coleman       309      1   201   69    110        1          0     300
## -Will Clark          117     11    66   41     34        1          1     942
## -Willie Randolph    1511     39   897  451    875        0          0     313
## -Wayne Tolleson      433      7   217   93    146        0          1      37
##                    Assists Errors NewLeagueN
## -Alvin Davis            82     14          0
## -Alfredo Griffin       421     25          0
## -Andre Thornton          0      0          0
## -Alan Trammell         445     22          0
## -Buddy Biancalana      177     16          0
```

```
## -Bruce Bochy            22      2          1
## -Barry Bonds             9      5          1
## -Bobby Bonilla          22      2          1
## -Billy Hatcher           7      4          1
## -Bill Madlock          170     24          1
## -Bob Melvin             59      6          1
## -BillyJo Robidoux       29      5          0
## -Chris Bando            30      4          0
## -Chili Davis             9      9          1
## -Curt Ford               7      3          1
## -Carney Lansford       147      4          0
## -Chet Lemon              6      5          0
## -Candy Maldonado        10      3          1
## -Craig Reynolds        206      7          1
## -Cal Ripken            482     13          0
## -Cory Snyder            70     10          0
## -Chris Speier           88      3          1
## -Curt Wilkerson        199     13          0
## -Daryl Boston            3      5          0
## -Dave Concepcion       223     10          1
## -Doug DeCinces         216     12          0
## -Damaso Garcia         286      8          1
## -Dan Gladden             7      3          0
## -Dave Henderson          9      4          0
## -Don Mattingly         100      6          0
## -Dale Murphy             6      6          1
## -Dwayne Murphy           6      2          0
## -Dave Parker             9      9          1
## -Dan Pasqua              4      2          0
## -Darrell Porter          9      1          0
## -Don Slaught            40      4          0
## -Darryl Strawberry      10      6          1
## -Dale Sveum            122     26          0
## -Danny Tartabull         7      8          0
## -Eddie Milner            6      3          1
## -Eddie Murray           88     13          0
## -George Bell            17     10          0
## -Glenn Braggs            5     12          0
## -George Brett          218     16          0
## -Gary Gaetti           334     21          0
## -George Hendrick         6      5          0
## -Garth Iorg            185     12          0
## -Gary Pettis             9      7          0
## -Gary Redus              8      4          0
## -Glenn Wilson           20      4          1
## -Harold Baines          15      5          0
## -Howard Johnson        136     20          1
## -Hal McRae               0      0          0
## -Harold Reynolds       415     16          0
## -John Cangelosi          7      9          1
## -Jim Dwyer               3      0          0
## -Julio Franco          374     18          0
## -Jim Gantner           347     10          0
## -Johnny Grubb            0      0          0
```

```
## -John Kruk              4      2          1
## -Jeffrey Leonard        4      5          1
## -Jim Morrison         257     20          1
## -John Moses             9      3          0
## -Johnny Ray           479      5          1
## -Jerry Royster        166     14          0
## -John Russell          39     13          1
## -Jim Sundberg          46      4          1
## -Joel Youngblood        2      0          1
## -Kal Daniels            0      3          1
## -Kirk Gibson            2      2          0
## -Ken Griffey            5      3          1
## -Kent Hrbek           104     10          0
## -Kevin Mitchell        59      8          1
## -Ken Oberkfell        258      8          1
## -Ken Phelps             0      0          0
## -Kirby Puckett          8      6          0
## -Len Dykstra            8      3          1
## -Lance Parrish         48      6          1
## -Larry Parrish          0      0          0
## -Larry Sheets           0      0          0
## -Lou Whitaker         421     11          0
## -Mike Aldrete          36      1          1
## -Mariano Duncan       317     25          1
## -Mike Heath            30     10          0
## -Mike Marshall          8      6          1
## -Mike Schmidt         220      6          1
## -Milt Thompson          1      2          1
## -Mitch Webster         12      8          1
## -Mookie Wilson          7      5          1
## -Marvell Wynne          3      3          1
## -Ozzie Guillen        459     22          0
## -Pete Incaviglia        6     14          0
## -Pete Rose             43      6          1
## -Rafael Belliard      269     12          1
## -Randy Bush             2      4          0
## -Rick Cerone           44      4          0
## -Ron Cey              118      8          0
## -Rick Dempsey          53      7          0
## -Reggie Jackson         0      0          0
## -Ron Kittle             0      0          0
## -Rick Leach             0      1          0
## -Rick Manning           3      2          0
## -Rance Mulliniks      176      6          0
## -Rafael Ramirez       371     29          1
## -Rick Schu             94     13          1
## -Rob Wilfong          257      7          0
## -Robin Yount            9      1          0
## -Scott Bradley         21      3          0
## -Sid Bream            166     17          1
## -Steve Buechele       226     11          0
## -Shawon Dunston       465     32          1
## -Scott Fletcher       354     15          0
## -Steve Jeltz          406     22          1
```

```
## -Steve Sax             432     16          1
## -Tom Foley             147      4          1
## -Tony Gwynn             19      4          1
## -Terry Harper            5      3          0
## -Tommy Herr            414      9          1
## -Tim Hulett            144     11          0
## -Terry Kennedy          70      8          0
## -Tito Landrum            6      1          1
## -Tom Paciorek           45      4          0
## -Terry Pendleton       371     20          1
## -Terry Puhl              0      0          1
## -Tim Teufel            173      9          1
## -Vince Coleman          12      9          1
## -Will Clark             72     11          1
## -Willie Randolph       381     20          0
## -Wayne Tolleson        113      7          0
## attr(,"assign")
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## attr(,"contrasts")
## attr(,"contrasts")$League
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##
## attr(,"contrasts")$Division
## [1] "contr.treatment"
##
## attr(,"contrasts")$NewLeague
## [1] "contr.treatment"
```

```
val.errors = rep(NA, 19)
for(i in 1:19){
        print(i)
        coefi = coef(regfit.best, id = i)
        pred = test.mat[,names(coefi)]%*%coefi
        s = paste(pred,mean((Hitters$Salary[test]-pred)^2), sep = ";")
        print(s)
        # pred = test.mat[,names(coefi)]%*%coefi
        val.errors[i] = mean((Hitters$Salary[test]-pred)^2)
}
```

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## [129] "114.855732025029;99062.8177214346"
```

```
which.min(val.errors)
```

```
## [1] 19
```

```
coef(regfit.best,10)
```

```
## (Intercept)       AtBat        Hits       HmRun      CAtBat       CHits
## 228.1292887  -1.1904601   3.7961718   9.4079608  -0.3578420   1.3268139
##      CHmRun   DivisionW     PutOuts     Assists      Errors
##   2.2693220 -91.9852691   0.4211761   0.9119121  -9.4019561
```

this is little tedious as there is no predict() method for regsubsets(). we define our own function.

```
predict.regsubsets = function(object, newdata, id){
        form = as.formula(object$call[[2]])
        mat = model.matrix(form, newdata)
        coefi = coef(object,id = id)
        xbars = names(coefi)
        mat[, xbars]%*%coefi
}
```

```
regfit.best = regsubsets(Salary~.,data = Hitters, nvmax = 19)
coef(regfit.best , 10)
```

```
##  (Intercept)        AtBat         Hits        Walks       CAtBat        CRuns
##  162.5354420   -2.1686501    6.9180175    5.7732246   -0.1300798    1.4082490
##         CRBI       CWalks    DivisionW      PutOuts      Assists
##    0.7743122   -0.8308264 -112.3800575    0.2973726    0.2831680
```

Thus we see that the best 10 variable model on the full dataset has a different set of variables than the best ten variable model on the training set.

```
k = 10
set.seed(1)

folds = sample(1:k, nrow(Hitters), replace = TRUE)
cv.errors= matrix(NA,k,19, dimnames = list(NULL, paste(1:19)))
folds
```

```
##   [1]  9  4  7  1  2  7  2  3  1  5  5 10  6 10  7  9  5  5  9  9  5  5  2 10  9
##  [26]  1  4  3  6 10 10  6  4  4 10  9  7  6  9  8  9  7  8  6 10  7  3 10  6  8
##  [51]  2  2  6  6  1  3  3  8  6  7  6  8  7  1  4  8  9  9  7  4  7  6  1  5  6
##  [76]  1  9  7  7  3  6  2 10 10  7  3  2 10  1 10 10  8 10  5  7  8  5  6  8  1
## [101]  3 10  3  1  6  6  4  9  5  1  3  6  3  7  3  3  1  9  2  8  6  1  2  7  7
## [126]  4  9  8  3  5  3  4  2  1  7  9 10 10  2  2  3  1  2  3  3  3  8  9  2 10
## [151]  8 10  4  5  9  5  7  5  6  4  2  1  3  8  9  6  1  4  5  9  5  8  4  1  9
## [176]  5  1  5  4 10 10  9  8  5  5  6  6  2  2  8  4 10  8  5  5  8  8  7  4  4
## [201]  1 10  4  9  9  9  9  6  6  4  3  3  9  9  7  9  5  7  4  4 10  8  1 10  2
## [226] 10  1  1  4  5  5  6  9  8  5  1  2  1  8  5  8 10  7  7  2  9  4  2  5  2
## [251]  4  3  6  9  7  5  5  1  1 10  1  3 10
```

```
cv.errors
```

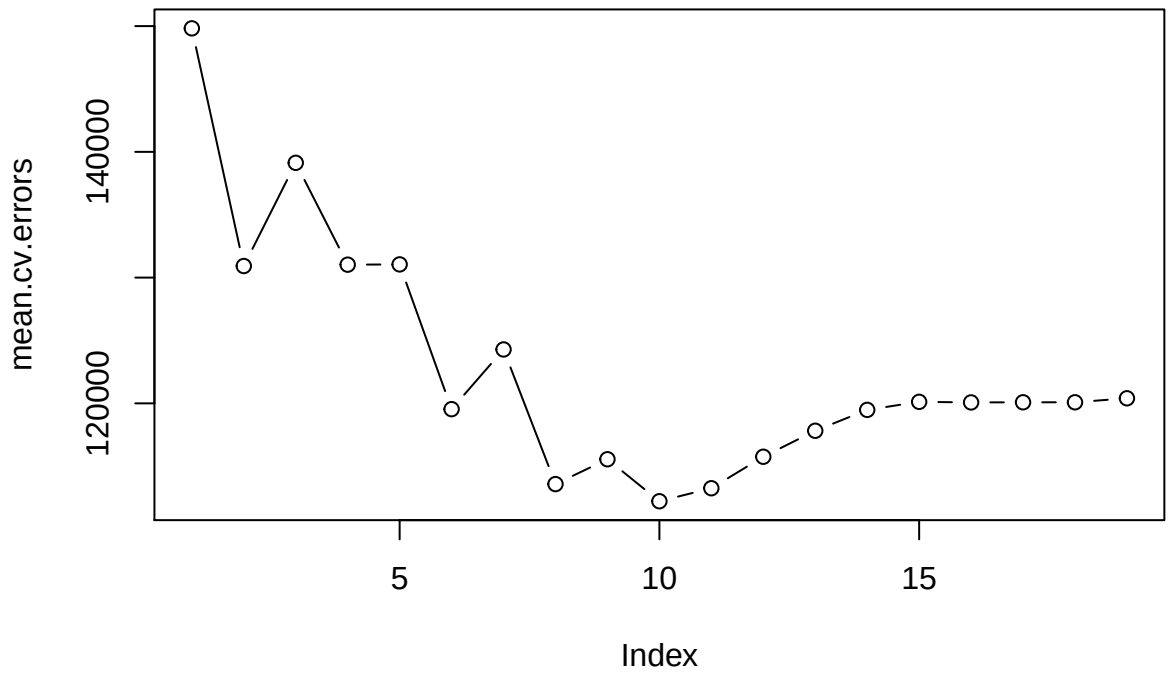
```
##        1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19
##  [1,] NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA
##  [2,] NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA
##  [3,] NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA
##  [4,] NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA
##  [5,] NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA
##  [6,] NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA
##  [7,] NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA
##  [8,] NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA
##  [9,] NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA
## [10,] NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA NA
```

```
for ( j in 1:k){
        best.fit=regsubsets(Salary ~., data=Hitters[folds!=j,],nvmax = 19)
        for (i in 1:19){
                pred = predict(best.fit, Hitters[folds ==j,],id = i)
                cv.errors[j,i] = mean((Hitters$Salary[folds == j]-pred)^2)
        }
}
```

```
mean.cv.errors = apply(cv.errors,2, mean)
mean.cv.errors
```

```
##        1        2        3        4        5        6        7        8
## 149821.1 130922.0 139127.0 131028.8 131050.2 119538.6 124286.1 113580.0
##        9       10       11       12       13       14       15       16
## 115556.5 112216.7 113251.2 115755.9 117820.8 119481.2 120121.6 120074.3
##       17       18       19
## 120084.8 120085.8 120403.5
```

```
par(mfrow=c(1,1))
plot(mean.cv.errors, type ='b')
```




```
reg.best=regsubsets(Salary~., data= Hitters, nvmax = 19)
coef(reg.best , 11)
```

```
##  (Intercept)        AtBat         Hits        Walks       CAtBat        CRuns
##  135.7512195   -2.1277482    6.9236994    5.6202755   -0.1389914    1.4553310
##         CRBI       CWalks      LeagueN    DivisionW      PutOuts      Assists
##    0.7852528   -0.8228559   43.1116152 -111.1460252    0.2894087    0.2688277
```

Ridge Regression and Lasso

We will perform ridge regression and lasso in order to predict Salary on the hitters data. Make sure that the missing data has been removed

```
x=model.matrix(Salary ~., Hitters)[,-1]
y = Hitters$Salary
```

the matrix() function automatically transforms any qualitative variable into dummy variables.

Ridge Regression

glmnet function with alpha = 1

```
library(glmnet)
```

```
## Loading required package: Matrix
```

```
## Loaded glmnet 4.1-1
```

```
grid = 10^seq(10, -2, length = 100)
ridge.mod = glmnet(x,y, alpha =0, lambda= grid)
```

```
dim(coef(ridge.mod))
```

```
## [1]  20 100
```

When large value of lambda is used the coef would be much smaller

```
ridge.mod$lambda[50]
```

```
## [1] 11497.57
```

```
sqrt(sum(coef(ridge.mod)[-1,50]^2))
```

```
## [1] 6.360612
```

```
ridge.mod$lambda[60]
```

```
## [1] 705.4802
```

```
sqrt(sum(coef(ridge.mod)[-1,60]^2))
```

```
## [1] 57.11001
```

```
ridge.mod$lambda[70]
```

```
## [1] 43.28761
```

```
sqrt(sum(coef(ridge.mod)[-1,70]^2))
```

```
## [1] 129.4454
```

```
ridge.mod$lambda[100]
```

```
## [1] 0.01
```

```
sqrt(sum(coef(ridge.mod)[-1,100]^2))
```

```
## [1] 136.2012
```

```
coef(ridge.mod)[,50]
```

```
##   (Intercept)         AtBat          Hits         HmRun          Runs
## 407.356050200   0.036957182   0.138180344   0.524629976   0.230701523
##           RBI         Walks         Years        CAtBat         CHits
##   0.239841459   0.289618741   1.107702929   0.003131815   0.011653637
##        CHmRun         CRuns          CRBI        CWalks       LeagueN
##   0.087545670   0.023379882   0.024138320   0.025015421   0.085028114
##     DivisionW       PutOuts       Assists        Errors    NewLeagueN
##  -6.215440973   0.016482577   0.002612988  -0.020502690   0.301433531
```

```
coef(ridge.mod)[,60]
```

```
##  (Intercept)         AtBat          Hits         HmRun          Runs           RBI
##  54.32519950   0.11211115   0.65622409   1.17980910   0.93769713   0.84718546
##        Walks         Years        CAtBat         CHits        CHmRun         CRuns
##   1.31987948   2.59640425   0.01083413   0.04674557   0.33777318   0.09355528
##         CRBI        CWalks       LeagueN    DivisionW       PutOuts       Assists
##   0.09780402   0.07189612  13.68370191 -54.65877750   0.11852289   0.01606037
##       Errors    NewLeagueN
##  -0.70358655   8.61181213
```

```
predict(ridge.mod, s=50, type ="coefficients")[1:20,]
```

```
##   (Intercept)         AtBat          Hits         HmRun          Runs
##  4.876610e+01 -3.580999e-01  1.969359e+00 -1.278248e+00  1.145892e+00
##           RBI         Walks         Years        CAtBat         CHits
##  8.038292e-01  2.716186e+00 -6.218319e+00  5.447837e-03  1.064895e-01
##        CHmRun         CRuns          CRBI        CWalks       LeagueN
##  6.244860e-01  2.214985e-01  2.186914e-01 -1.500245e-01  4.592589e+01
##     DivisionW       PutOuts       Assists        Errors    NewLeagueN
## -1.182011e+02  2.502322e-01  1.215665e-01 -3.278600e+00 -9.496680e+00
```

splitting data into train and test

```
set.seed(1)
train = sample(1:nrow(x), nrow(x)/2)
test = (-train)
y.test = y[test]
```

lets fit a ridge regression model on the training set and evaluate the MSE on the test set using lambda = 4

```
train<-train[!sapply(train,is.null)]
# x[train]
# y[train]
```

```
ridge.mod = glmnet(x[train,], y[train],alpha =0, lambda = grid, thresh = 1e-12)
ridge.pred = predict(ridge.mod, s=4, newx = x[test,])
mean((ridge.pred - y.test)^2)
```

```
## [1] 142199.2
```

so the test MSE = 142199.2

```
mean((mean(y[train])-y.test)^2)
```

```
## [1] 224669.9
```

Now with a very large lambda

```
ridge.pred=predict(ridge.mod, s = 1e10, newx = x[test,])
mean((ridge.pred - y.test)^2)
```

```
## [1] 224669.8
```

So fitting a ridge regression model with lambda = 4 leads to a much lower test MSE than fitting a model with just an intercept.

```
# ridge.pred = predict(ridge.mod, s= 0, newx = x[test,], exact=T )
mean((ridge.pred-y.test)^2)
```

```
## [1] 224669.8
```

```
lm(y~x, subset = train)
```

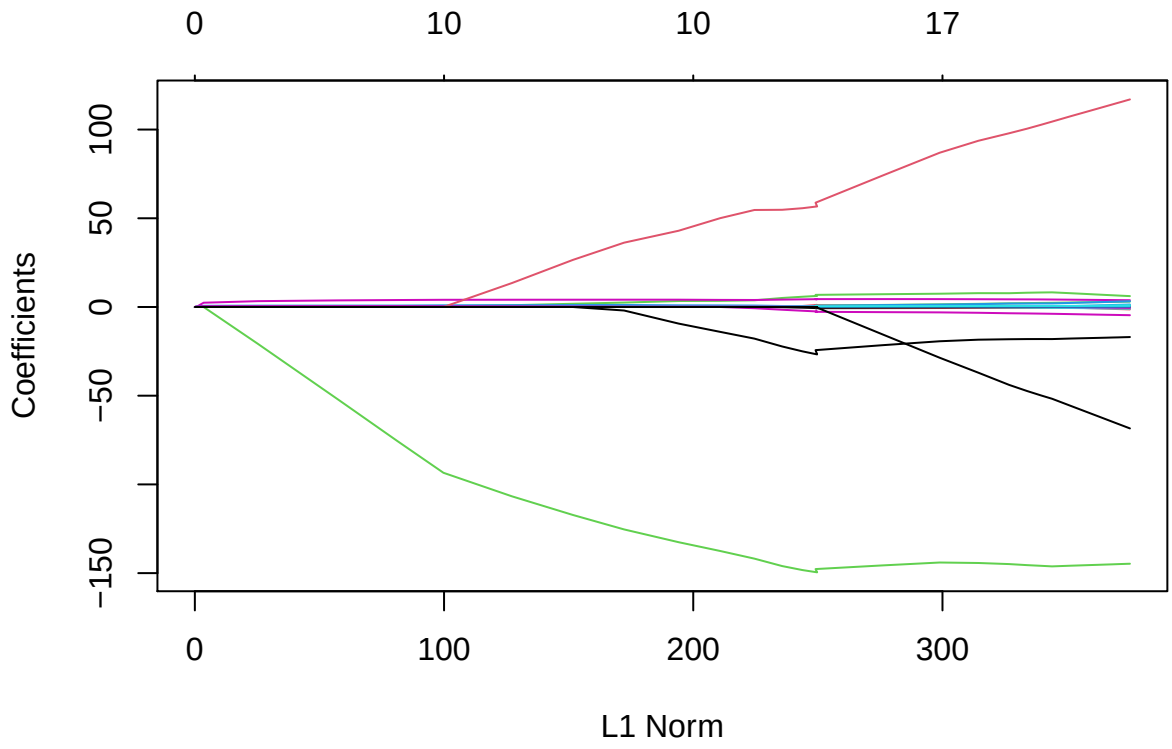
```
##
## Call:
## lm(formula = y ~ x, subset = train)
##
## Coefficients:
## (Intercept)       xAtBat        xHits       xHmRun        xRuns         xRBI
##    274.0145      -0.3521      -1.6377       5.8145       1.5424       1.1243
##      xWalks       xYears      xCAtBat       xCHits      xCHmRun       xCRuns
##      3.7287     -16.3773      -0.6412       3.1632       3.4008      -0.9739
##       xCRBI      xCWalks     xLeagueN   xDivisionW     xPutOuts     xAssists
##     -0.6005       0.3379     119.1486    -144.0831       0.1976       0.6804
##     xErrors  xNewLeagueN
##     -4.7128     -71.0951
```

```
# predict(ridge.mod, s=0, exact = T, type ="coefficients")[1:20]
```

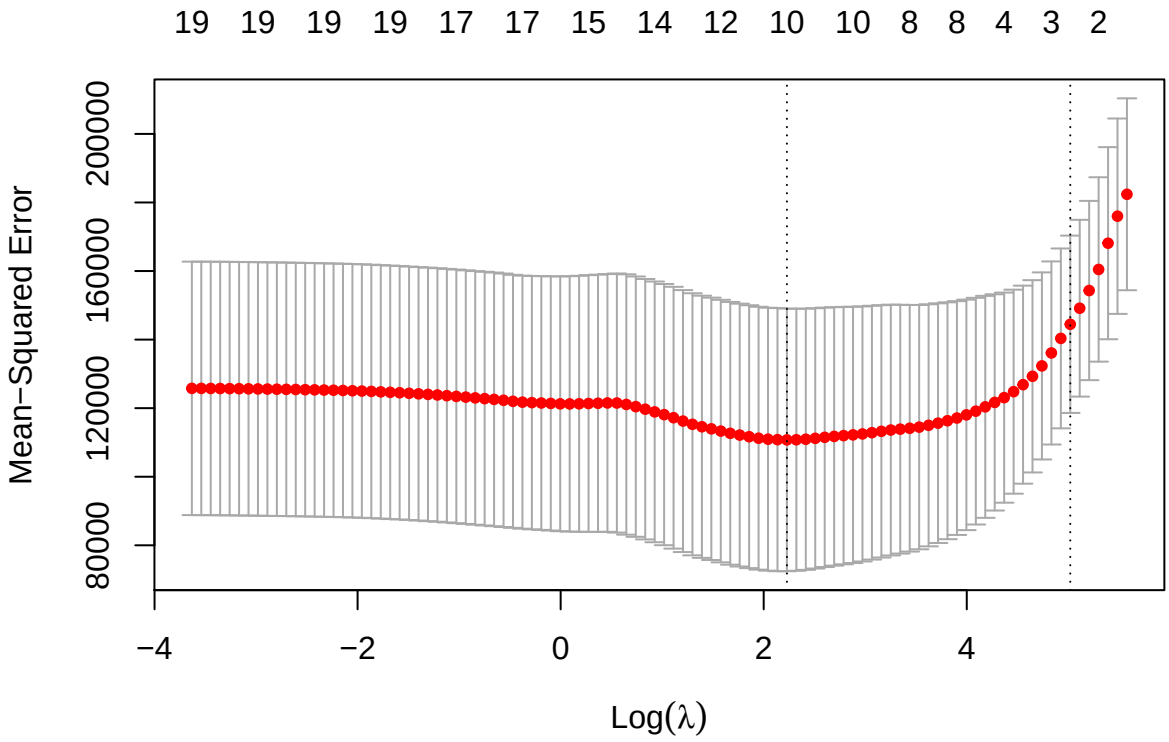
Lasso

```
lasso.mod = glmnet(x[train,], y[train],alpha =1, lambda = grid)
plot(lasso.mod)
```

```
## Warning in regularize.values(x, y, ties, missing(ties), na.rm = na.rm):
## collapsing to unique 'x' values
```




```
set.seed(1)
cv.out = cv.glmnet(x[train,],y[train], alpha = 1)
plot(cv.out)
```




```
bestlam = cv.out$lambda.min
lasso.pred = predict(lasso.mod, s= bestlam, newx=x[test,])
mean((lasso.pred -y.test)^2)
```

```
## [1] 143673.6
```

This is substantially lower than the test set MSE of the null model and of least squares and very similar to the test MSE of ridge regression with lambda chosen by cross validation

The lasso has a substantial advantage over ridge regression in that the resulting coefficient estimates are sparse. Hence we see that 12 of the 19 coeff estimates are exactly 0. So the lasso model with lambda chosen by cross validation contains only 7 variables.

```
out = glmnet(x,y, aplha = 1, lambda = grid)
lasso.coef=predict(out, type = "coefficients", s= bestlam)[1:20,]
lasso.coef
```

```
##   (Intercept)         AtBat          Hits         HmRun          Runs
##    1.27479059   -0.05497143    2.18034583    0.00000000    0.00000000
##           RBI         Walks         Years        CAtBat         CHits
##    0.00000000    2.29192406   -0.33806109    0.00000000    0.00000000
##        CHmRun         CRuns          CRBI        CWalks       LeagueN
##    0.02825013    0.21628385    0.41712537    0.00000000   20.28615023
##     DivisionW       PutOuts       Assists        Errors    NewLeagueN
## -116.16755870    0.23752385    0.00000000   -0.85629148    0.00000000
```

```
lasso.coef[lasso.coef!=0]
```

```
##   (Intercept)         AtBat          Hits         Walks         Years
##    1.27479059   -0.05497143    2.18034583    2.29192406   -0.33806109
##        CHmRun         CRuns          CRBI       LeagueN     DivisionW
##    0.02825013    0.21628385    0.41712537   20.28615023 -116.16755870
##       PutOuts        Errors
##    0.23752385   -0.85629148
```

PCR and PLS Regression

```
library(pls)
```

```
##
## Attaching package: 'pls'
```

```
## The following object is masked from 'package:stats':
##
##     loadings
```

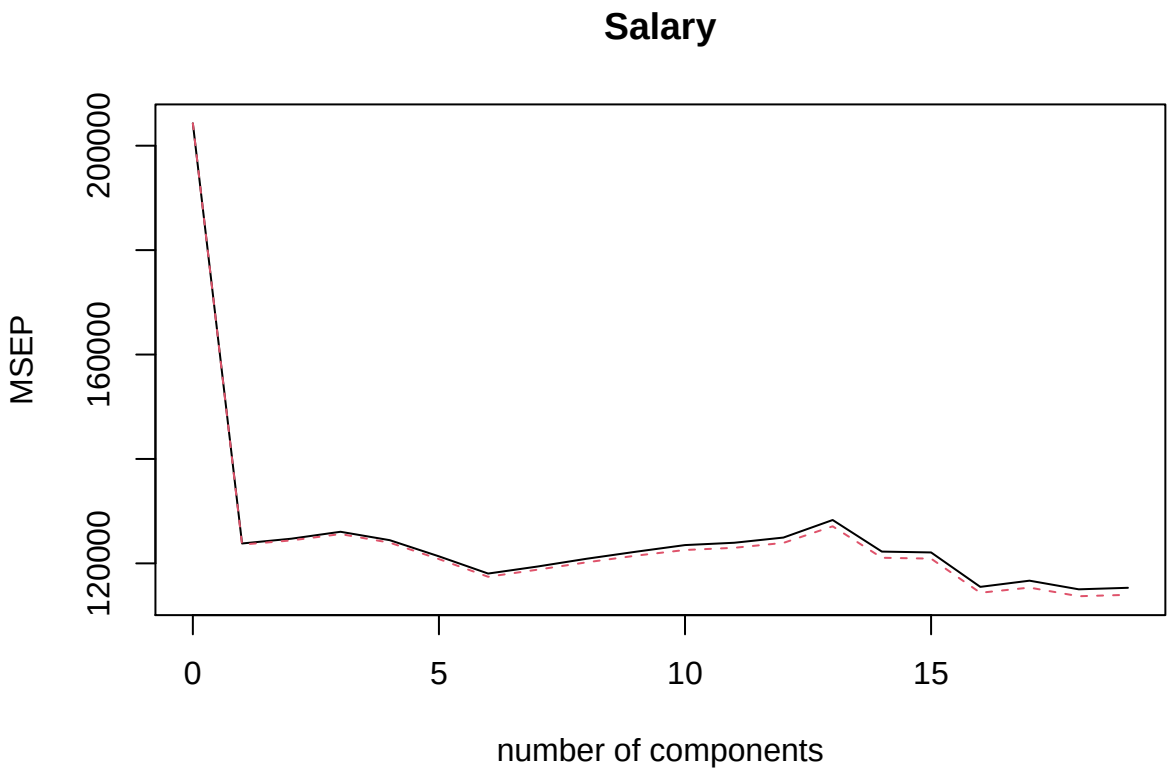
```
set.seed(2)
pcr.fit = pcr(Salary~., data=Hitters, scale =TRUE, validation ="CV")
```

```
summary(pcr.fit)
```

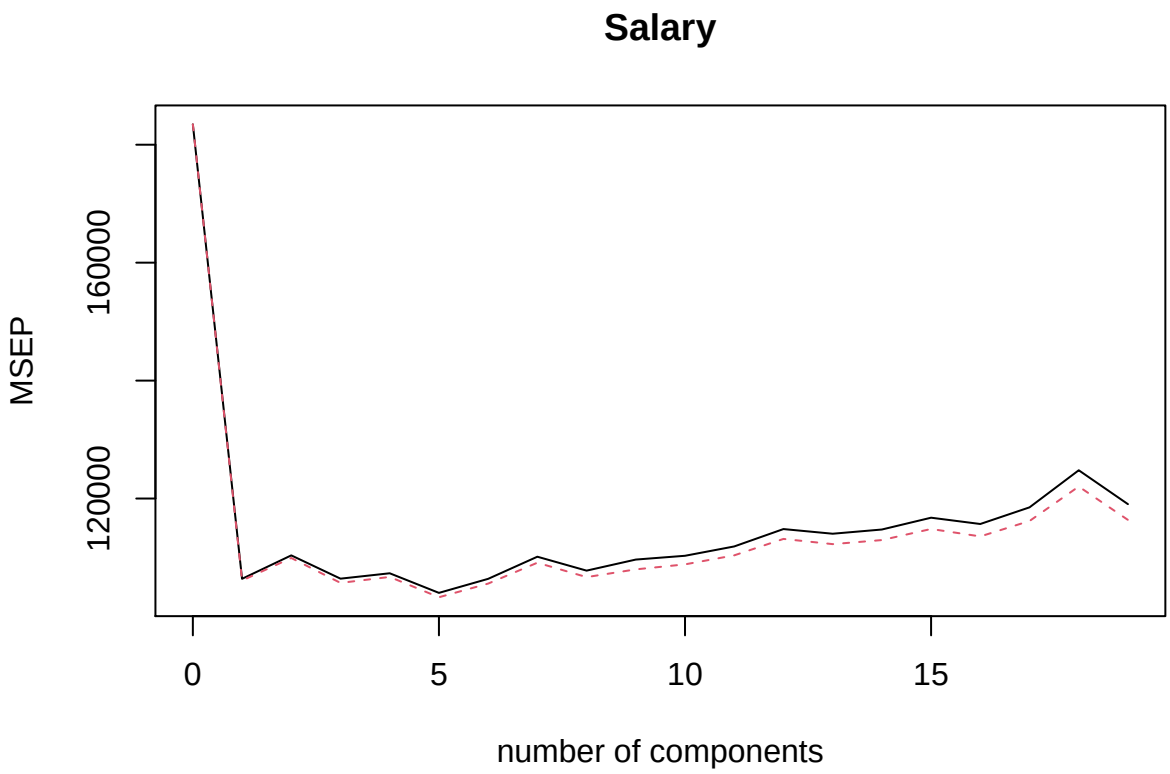
```
## Data:    X dimension: 263 19
##  Y dimension: 263 1
## Fit method: svdpc
## Number of components considered: 19
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##        (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV             452    351.9    353.2    355.0    352.8    348.4    343.6
## adjCV          452    351.6    352.7    354.4    352.1    347.6    342.7
##        7 comps  8 comps  9 comps  10 comps  11 comps  12 comps  13 comps
## CV       345.5    347.7    349.6     351.4     352.1     353.5     358.2
## adjCV    344.7    346.7    348.5     350.1     350.7     352.0     356.5
##        14 comps  15 comps  16 comps  17 comps  18 comps  19 comps
## CV        349.7     349.4     339.9     341.6     339.2     339.6
## adjCV     348.0     347.7     338.2     339.7     337.2     337.6
##
## TRAINING: % variance explained
##         1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps
## X         38.31    60.16    70.84    79.03    84.29    88.63    92.26    94.96
## Salary    40.63    41.58    42.17    43.22    44.90    46.48    46.69    46.75
##         9 comps  10 comps  11 comps  12 comps  13 comps  14 comps  15 comps
## X         96.28     97.26     97.98     98.65     99.15     99.47     99.75
## Salary    46.86     47.76     47.82     47.85     48.10     50.40     50.55
```

```
##         16 comps  17 comps  18 comps  19 comps
## X          99.89     99.97     99.99    100.00
## Salary     53.01     53.85     54.61     54.61
```

```
validationplot(pcr.fit, val.type = "MSEP")
```




```
set.seed(1)
pcr.fit=pcr(Salary~.,data = Hitters, subset = train, scale=TRUE, validation="CV")
validationplot(pcr.fit, val.type="MSEP")
```




```
pcr.pred=predict(pcr.fit, x[test,],ncomp=7)
mean((pcr.pred - y.test)^2)
```

```
## [1] 140751.3
```

```
pcr.fit = pcr(y~x, scale = TRUE, ncomp = 7)
summary(pcr.fit)
```

```
## Data:    X dimension: 263 19
##  Y dimension: 263 1
## Fit method: svdpc
## Number of components considered: 7
## TRAINING: % variance explained
##    1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps
## X    38.31    60.16    70.84    79.03    84.29    88.63    92.26
## y    40.63    41.58    42.17    43.22    44.90    46.48    46.69
```

##PLS: Partial Least Square

```
set.seed(1)
pls.fit = plsr(Salary~., data = Hitters, subset= train, scale = TRUE, validation = "CV")
summary(pls.fit)
```

```
## Data:    X dimension: 131 19
```

```
##  Y dimension: 131 1
## Fit method: kernelpls
## Number of components considered: 19
##
## VALIDATION: RMSEP
## Cross-validated using 10 random segments.
##        (Intercept)  1 comps  2 comps  3 comps  4 comps  5 comps  6 comps
## CV           428.3    325.5    329.9    328.8    339.0    338.9    340.1
## adjCV        428.3    325.0    328.2    327.2    336.6    336.1    336.6
##        7 comps  8 comps  9 comps  10 comps  11 comps  12 comps  13 comps
## CV       339.0    347.1    346.4     343.4     341.5     345.4     356.4
## adjCV    336.2    343.4    342.8     340.2     338.3     341.8     351.1
##        14 comps  15 comps  16 comps  17 comps  18 comps  19 comps
## CV        348.4     349.1     350.0     344.2     344.5     345.0
## adjCV     344.2     345.0     345.9     340.4     340.6     341.1
##
## TRAINING: % variance explained
##         1 comps  2 comps  3 comps  4 comps  5 comps  6 comps  7 comps  8 comps
## X         39.13    48.80    60.09    75.07    78.58    81.12    88.21    90.71
## Salary    46.36    50.72    52.23    53.03    54.07    54.77    55.05    55.66
##         9 comps  10 comps  11 comps  12 comps  13 comps  14 comps  15 comps
## X         93.17     96.05     97.08     97.61     97.97     98.70     99.12
## Salary    55.95     56.12     56.47     56.68     57.37     57.76     58.08
##         16 comps  17 comps  18 comps  19 comps
## X          99.61     99.70     99.95    100.00
## Salary     58.17     58.49     58.56     58.62
```

```
pls.pred = predict(pls.fit, x[test,], ncomp=2)
mean((pls.pred - y.test)^2)
```

```
## [1] 145367.7
```

```
pls.fit = plsr(Salary~.,data = Hitters, scale = TRUE, ncomp =2)
summary(pls.fit)
```

```
## Data:    X dimension: 263 19
##  Y dimension: 263 1
## Fit method: kernelpls
## Number of components considered: 2
## TRAINING: % variance explained
##         1 comps  2 comps
## X         38.08    51.03
## Salary    43.05    46.40
```