Linear Regression

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Libraries

The **library()** function is used to load libraries or group of functions and datasets that are not included in the base R distribution.

```
library(ISLR )
library(MASS)
```

Simple Linear Regression

```
# fix(Boston)
```

```
names(Boston)
```

```
##  [1] "crim"    "zn"      "indus"   "chas"    "nox"     "rm"      "age"
##  [8] "dis"     "rad"     "tax"     "ptratio" "black"   "lstat"   "medv"
```

lm() function to fit a simple linear regression model with mdev as the response and lstat as the predictor

```
lm.fit = lm(Boston$medv ~ Boston$lstat, data = Boston)
# attach(Boston)
lm.fit = lm(Boston$medv~Boston$lstat)
lm.fit
```

```
##
## Call:
## lm(formula = Boston$medv ~ Boston$lstat)
##
## Coefficients:
##  (Intercept)  Boston$lstat
##        34.55         -0.95
```

```
summary(lm.fit)
```

```
##
## Call:
## lm(formula = Boston$medv ~ Boston$lstat)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.168  -3.990  -1.318   2.034  24.500
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  34.55384    0.56263   61.41   <2e-16 ***
```

```
## Boston$lstat -0.95005    0.03873  -24.53   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.216 on 504 degrees of freedom
## Multiple R-squared:  0.5441, Adjusted R-squared:  0.5432
## F-statistic: 601.6 on 1 and 504 DF,  p-value: < 2.2e-16
```

```
names(lm.fit)
```

```
##  [1] "coefficients"  "residuals"     "effects"       "rank"
##  [5] "fitted.values" "assign"        "qr"            "df.residual"
##  [9] "xlevels"       "call"          "terms"         "model"
```

```
coef(lm.fit)
```

```
##  (Intercept) Boston$lstat
##   34.5538409   -0.9500494
```

```
confint(lm.fit)
```

```
##                  2.5 %     97.5 %
## (Intercept)  33.448457 35.6592247
## Boston$lstat -1.026148 -0.8739505
```

the predict() function can be used to produce confidence intervals and prediction intervals for the prediction of medv for a given value of lstat

```
predict(lm.fit, data.frame(lstat = c(5,10,15)),interval ="confidence")
```

```
## Warning: 'newdata' had 3 rows but variables found have 506 rows
```

```
##            fit        lwr        upr
## 1   29.8225951 29.0252990 30.6198912
## 2   25.8703898 25.2652456 26.4755340
## 3   30.7251420 29.8734766 31.5768074
## 4   31.7606958 30.8435939 32.6777976
## 5   29.4900778 28.7120765 30.2680791
## 6   29.6040837 28.8195155 30.3886520
## 7   22.7447274 22.2015728 23.2878820
## 8   16.3603958 15.6261142 17.0946773
## 9    6.1188637  4.6964329  7.5412945
## 10  18.3079969 17.6682721 18.9477218
## 11  15.1253316 14.3211061 15.9295571
## 12  21.9466860 21.4017704 22.4916015
## 13  19.6285655 19.0379343 20.2191967
## 14  26.7064332 26.0688676 27.3439989
## 15  24.8063345 24.2337154 25.3789536
## 16  26.5069229 25.8775896 27.1362562
## 17  28.3025161 27.5895545 29.0154778
## 18  20.6166169 20.0524476 21.1807861
## 19  23.4477639 22.8999502 23.9955777
## 20  23.8372842 23.2844310 24.3901373
## 21  14.5838035 13.7470635 15.4205434
## 22  21.4146583 20.8644307 21.9648859
## 23  16.7689170 16.0562575 17.4815765
## 24  15.6668597 14.8940813 16.4396381
## 25  19.0680364 18.4583230 19.6777498
```

```
## 26   18.8685260 18.2513745 19.4856776
## 27   20.4836100 19.9164496 21.0507703
## 28   18.1369880 17.4899128 18.7840633
## 29   22.3932092 21.8502047 22.9362136
## 30   23.1722496 22.6269495 23.7175497
## 31   13.0827255 12.1512199 14.0142311
## 32   22.1651973 21.6215101 22.7088845
## 33    8.2279733  6.9600530  9.4958936
## 34   17.1204352 16.4256854 17.8151851
## 35   15.2298370 14.4317674 16.0279067
## 36   25.3573631 24.7692166 25.9455097
## 37   23.7137778 23.1627087 24.2648468
## 38   26.2219080 25.6038085 26.8400076
## 39   24.9298409 24.3539993 25.5056826
## 40   30.4496277 29.6148475 31.2844078
## 41   32.6727432 31.6958039 33.6496824
## 42   29.9556020 29.1504707 30.7607333
## 43   29.0340541 28.2817852 29.7863231
## 44   27.4854737 26.8130853 28.1578620
## 45   25.4808696 24.8888470 26.0728921
## 46   24.8538370 24.2799965 25.4276775
## 47   21.1106425 20.5559305 21.6653546
## 48   16.6929130 15.9762946 17.4095315
## 49    5.2828203  3.7982718  6.7673688
## 50   19.1630413 18.5567540 19.7693287
## 51   21.7756771 21.2294109 22.3219432
## 52   25.5948755 24.9991517 26.1905993
## 53   29.5375803 28.7568490 30.3183116
## 54   26.5449248 25.9140464 27.1758033
## 55   20.4931104 19.9261699 21.0600510
## 56   29.9841035 29.1772848 30.7909222
## 57   29.0720561 28.3176769 29.8264354
## 58   30.8011459 29.9447810 31.6575109
## 59   28.0365023 27.3371644 28.7358402
## 60   25.7943858 25.1919064 26.3968653
## 61   22.0606919 21.5164871 22.6048967
## 62   20.8351282 20.2754673 21.3947892
## 63   28.1600087 27.4543928 28.8656246
## 64   25.5283720 24.9348216 26.1219225
## 65   26.9059436 26.2598551 27.5520321
## 66   30.1171104 29.3023789 30.9318418
## 67   24.8253355 24.2522306 25.3984404
## 68   26.8584411 26.2144076 27.5024746
## 69   22.1176948 21.5737883 22.6616014
## 70   26.2029071 25.5855337 26.8202804
## 71   28.1695092 27.4634070 28.8756115
## 72   25.1673533 24.5848920 25.7498145
## 73   29.3095684 28.5418575 30.0772794
## 74   27.3904688 26.7225420 28.0583955
## 75   28.1125063 27.4093148 28.8156977
## 76   26.0603997 25.4483794 26.6724199
## 77   23.1817501 22.6363780 23.7271222
## 78   24.7968340 24.2244565 25.3692115
## 79   22.8302319 22.2868201 23.3736436
```

```
## 80  25.9083918 25.3018967 26.5148869
## 81  29.5280798 28.7478952 30.3082644
## 82  27.6944845 27.0120847 28.3768844
## 83  28.1695092 27.4634070 28.8756115
## 84  27.4189702 26.7497109 28.0882295
## 85  25.4143661 24.8244481 26.0042841
## 86  28.3500186 27.6345847 29.0654525
## 87  22.3362062 21.7930886 22.8793238
## 88  26.5354243 25.9049332 27.1659155
## 89  29.3285694 28.5597816 30.0973572
## 90  29.1385596 28.3804719 29.8966472
## 91  26.1839061 25.5672560 26.8005562
## 92  26.7634362 26.1234646 27.4034078
## 93  26.8014382 26.1598495 27.4430268
## 94  28.6540344 27.9225070 29.3855618
## 95  24.4928182 23.9276839 25.0579525
## 96  28.2360127 27.5264925 28.9455329
## 97  23.7802812 23.2282728 24.3322896
## 98  30.5541331 29.7129765 31.3952897
## 99  31.1621647 30.2832444 32.0410850
## 100 28.6730354 27.9404870 29.4055838
## 101 25.6043760 25.0083385 26.2004135
## 102 27.2669623 26.6047498 27.9291749
## 103 24.4548162 23.8905200 25.0191125
## 104 21.7851776 21.2389954 22.3313598
## 105 22.8397323 22.2962867 23.3831780
## 106 18.9065280 18.2908184 19.5222377
## 107 16.8259199 16.1162099 17.5356300
## 108 21.1676455 20.6138531 21.7214379
## 109 22.8967353 22.3530640 23.4404067
## 110 19.7805734 19.1946310 20.3665159
## 111 22.2031993 21.6596684 22.7467302
## 112 24.9013394 24.3262550 25.4764239
## 113 19.1535409 18.5469143 19.7601674
## 114 18.3174974 17.6781748 18.9568201
## 115 24.6258251 24.0576390 25.1940113
## 116 19.5810631 18.9889229 20.1732032
## 117 23.1152467 22.5703565 23.6601369
## 118 24.7683325 24.1966742 25.3399908
## 119 19.9515823 19.3706548 20.5325098
## 120 21.6236692 21.0759176 22.1714208
## 121 20.9016317 20.3432411 21.4600222
## 122 20.9966366 20.4399775 21.5532957
## 123 17.5194560 16.8441884 18.1947235
## 124 10.4130868  9.3008106 11.5253630
## 125 17.8519732 17.1921972 18.5117493
## 126 20.4836100 19.9164496 21.0507703
## 127  8.6554955  7.4184349  9.8925561
## 128 18.2224925 17.5791184 18.8658666
## 129 19.9325813 19.3511104 20.5140522
## 130 17.1299357 16.4356605 17.8242109
## 131 22.5832190 22.0403147 23.1261233
## 132 22.9062358 22.3625231 23.4499485
## 133 23.9892921 23.4340089 24.5445752
```

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## 134 20.2745991 19.7023691 20.8468291
## 135 18.1084866 17.4601662 18.7568069
## 136 18.4410038 17.8068503 19.0751574
## 137 18.4980068 17.8662010 19.1298126
## 138 20.6926208 20.1300773 21.2551643
## 139 14.2987887 13.4445481 15.1530292
## 140 17.0159298 16.3159259 17.7159337
## 141 11.6006485 10.5703489 12.6309481
## 142  1.8626426  0.1202312  3.6050541
## 143  9.0735172  7.8664540 10.2805804
## 144  9.4535370  8.2735814 10.6334925
## 145  6.7268953  5.3493556  8.1044350
## 146  8.1424688  6.8683558  9.4165819
## 147 18.7355191 18.1132302 19.3578080
## 148  6.4988835  5.1045396  7.8932273
## 149  7.6484432  6.3384222  8.9584642
## 150 14.1752822 13.3133807 15.0371838
## 151 21.1581450 20.6042018 21.7120882
## 152 21.9371855 21.3922039 22.4821671
## 153 23.0392427 22.4948400 23.5836454
## 154 19.5525616 18.9595062 20.1456170
## 155 20.1890947 19.6146630 20.7635263
## 156 20.2840996 19.7121097 20.8560895
## 157 19.2200443 18.6157758 19.8243128
## 158 30.1931143 29.3738334 31.0123953
## 159 28.4450235 27.7246102 29.1654369
## 160 27.5329762 26.8583359 28.2076164
## 161 29.3285694 28.5597816 30.0973572
## 162 32.9102555 31.9174432 33.9030678
## 163 32.7297461 31.7490076 33.7104847
## 164 31.3996770 30.5057178 32.2936362
## 165 23.4952664 22.9469306 24.0436022
## 166 25.2338567 24.6494444 25.8182690
## 167 31.0386583 30.1674966 31.9098200
## 168 23.0202417 22.4759503 23.5645332
## 169 24.0082931 23.4526882 24.5638979
## 170 23.7992822 23.2469962 24.3515682
## 171 20.8446287 20.2851522 21.4041052
## 172 23.1247472 22.5797913 23.6697030
## 173 20.5976159 20.0330307 21.1622011
## 174 25.9653947 25.3568503 26.5739391
## 175 25.3953651 24.8060409 25.9846893
## 176 29.4900778 28.7120765 30.2680791
## 177 24.9488419 24.3724910 25.5251928
## 178 28.5780304 27.8505694 29.3054914
## 179 27.9794994 27.2830307 28.6759680
## 180 29.7655921 28.9716338 30.5595504
## 181 27.3714678 26.7044264 28.0385091
## 182 25.5758745 24.9807757 26.1709733
## 183 29.9746030 29.1683471 30.7808589
## 184 29.1575606 28.3984099 29.9167112
## 185 21.2721509 20.7199504 21.8243514
## 186 22.0606919 21.5164871 22.6048967
## 187 30.3261213 29.4988312 31.1534114
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```
## 188 28.2075112 27.4994587 28.9155637
## 189 30.2216158 29.4006237 31.0426079
## 190 29.4330749 28.6583375 30.2078122
## 191 29.7085892 28.9179564 30.4992219
## 192 30.0981094 29.2845122 30.9117066
## 193 31.8271992 30.9057987 32.7485998
## 194 29.7750926 28.9805789 30.5696064
## 195 30.3926247 29.5613078 31.2239417
## 196 31.7321943 30.8169314 32.6474571
## 197 30.6776395 29.8289023 31.5263767
## 198 26.3739159 25.7499038 26.9979281
## 199 28.2645142 27.5535219 28.9755064
## 200 30.2216158 29.4006237 31.0426079
## 201 30.3261213 29.4988312 31.1534114
## 202 27.4949742 26.8221366 28.1678118
## 203 31.5991874 30.6924798 32.5058950
## 204 30.9341528 30.0695221 31.7987836
## 205 31.8176987 30.8969129 32.7384845
## 206 24.2268044 23.6672150 24.7863938
## 207 24.1317995 23.5740064 24.6895925
## 208 17.3959496 16.7147525 18.0771466
## 209 20.6261174 20.0621547 21.1900800
## 210 12.6172013 11.6551493 13.5792533
## 211 18.1464885 17.4998271 18.7931500
## 212 11.7716574 10.7529741 12.7903406
## 213 19.3245497 18.7239101 19.9251894
## 214 25.6423779 25.0450775 26.2396784
## 215  6.4798825  5.0841366  7.8756284
## 216 25.5568735 24.9623965 26.1513505
## 217 21.7186741 21.1718822 22.2654660
## 218 25.3478626 24.7600084 25.9357169
## 219 17.5289565 16.8541412 18.2037718
## 220 24.5783227 24.0112476 25.1453978
## 221 25.3288617 24.7415895 25.9161338
## 222 14.1657817 13.3032890 15.0282745
## 223 25.1198508 24.5387572 25.7009444
## 224 27.3334658 26.6681885 27.9987431
## 225 30.6206366 29.7754040 31.4658691
## 226 30.1551124 29.3381087 30.9721161
## 227 31.5801864 30.6746974 32.4856754
## 228 28.5115270 27.7876008 29.2354531
## 229 30.8296474 29.9715157 31.6877792
## 230 30.9816553 30.1140599 31.8492507
## 231 23.4857659 22.9375366 24.0339952
## 232 29.5660818 28.7837082 30.3484554
## 233 32.2072190 31.2610537 33.1533843
## 234 30.8011459 29.9447810 31.6575109
## 235 26.9059436 26.2598551 27.5520321
## 236 24.2173039 23.6578985 24.7767093
## 237 25.4903700 24.8980436 26.0826965
## 238 30.0601074 29.2487749 30.8714400
## 239 28.5115270 27.7876008 29.2354531
## 240 27.5519771 26.8764322 28.2275221
## 241 23.7422792 23.1908137 24.2937448
```

```
## 242 22.7732289 22.2299981 23.3164596
## 243 23.8942871 23.3405528 24.4480214
## 244 29.6230847 28.8374170 30.4087525
## 245 22.6782240 22.1352097 23.2212382
## 246 17.0159298 16.3159259 17.7159337
## 247 25.8513888 25.2469154 26.4558622
## 248 24.9108399 24.3355040 25.4861759
## 249 25.5093710 24.9164342 26.1023078
## 250 28.3215171 27.6075680 29.0354662
## 251 28.9485497 28.2010055 29.6960939
## 252 31.1431637 30.2654398 32.0208876
## 253 31.2001667 30.3188505 32.0814828
## 254 31.1906662 30.3099493 32.0713830
## 255 28.3120166 27.5985615 29.0254718
## 256 25.7658844 25.1643914 26.3673773
## 257 31.5991874 30.6924798 32.5058950
## 258 29.6895882 28.9000612 30.4791152
## 259 27.1529564 26.4959304 27.8099824
## 260 27.9985003 27.3010773 28.6959234
## 261 25.4428676 24.8520526 26.0336825
## 262 27.6564826 26.9759227 28.3370425
## 263 28.9390492 28.1920279 29.6860705
## 264 23.8657856 23.3124965 24.4190748
## 265 26.8584411 26.2144076 27.5024746
## 266 24.6258251 24.0576390 25.1940113
## 267 20.5026109 19.9358892 21.0693327
## 268 27.4854737 26.8130853 28.1578620
## 269 31.5516849 30.6480220 32.4553478
## 270 21.5856672 21.0375026 22.1338318
## 271 22.2031993 21.6596684 22.7467302
## 272 28.2930156 27.5805471 29.0054842
## 273 27.2099594 26.5503508 27.8695680
## 274 28.3025161 27.5895545 29.0154778
## 275 31.2001667 30.3188505 32.0814828
## 276 31.7226938 30.8080435 32.6373441
## 277 28.8060423 28.0662988 29.5457858
## 278 30.6016356 29.7575690 31.4457021
## 279 27.7229860 27.0392006 28.4067715
## 280 29.9461015 29.1415320 30.7506710
## 281 30.9816553 30.1140599 31.8492507
## 282 30.1931143 29.3738334 31.0123953
## 283 31.6941923 30.7813783 32.6070063
## 284 31.5516849 30.6480220 32.4553478
## 285 27.0959535 26.4414884 27.7504186
## 286 26.7349347 26.0961690 27.3737004
## 287 22.2697027 21.7264045 22.8130009
## 288 27.7704885 27.0843830 28.4565940
## 289 27.3334658 26.6681885 27.9987431
## 290 25.5188715 24.9256283 26.1121147
## 291 31.3901765 30.4968218 32.2835312
## 292 31.1716652 30.2921463 32.0511841
## 293 30.0886089 29.2755783 30.9016395
## 294 26.4024174 25.7772765 27.0275583
## 295 24.6733276 24.1040072 25.2426480
```

```
## 296 28.5970314 27.8685565 29.3255064
## 297 27.5329762 26.8583359 28.2076164
## 298 19.5050591 18.9104619 20.0996563
## 299 29.8320956 29.0342420 30.6299492
## 300 30.0506069 29.2398397 30.8613742
## 301 28.7870413 28.0483308 29.5257518
## 302 25.5283720 24.9348216 26.1219225
## 303 26.3169130 25.6951394 26.9386865
## 304 29.9366010 29.1325930 30.7406091
## 305 27.9699989 27.2740067 28.6659911
## 306 26.0699002 25.4575282 26.6822721
## 307 28.4070216 27.6886055 29.1254376
## 308 27.3999692 26.7315989 28.0683396
## 309 30.2406168 29.4184824 31.0627513
## 310 25.0818488 24.5018338 25.6618639
## 311 22.5452170 22.0023268 23.0881072
## 312 28.8725457 28.1291735 29.6159179
## 313 23.4192625 22.8717494 23.9667755
## 314 27.0484510 26.3961032 27.7007987
## 315 25.7373829 25.1368693 26.3378965
## 316 23.6282733 23.0783385 24.1782081
## 317 17.1394362 16.4456351 17.8332373
## 318 19.4100542 18.8123127 20.0077956
## 319 24.7113296 24.1410850 25.2815741
## 320 22.4597126 21.9167917 23.0026335
## 321 27.7134855 27.0301625 28.3968086
## 322 28.0270018 27.3281434 28.7258602
## 323 27.2384609 26.5775529 27.8993688
## 324 23.4002615 22.8529438 23.9475792
## 325 28.7395388 28.0034034 29.4756743
## 326 29.7275902 28.9358503 30.5193301
## 327 28.7110374 27.9764418 29.4456329
## 328 22.4027096 21.8597204 22.9456989
## 329 25.0818488 24.5018338 25.6618639
## 330 27.5804786 26.9035726 28.2573847
## 331 25.9178923 25.3110575 26.5247270
## 332 22.7447274 22.2015728 23.2878820
## 333 27.1149544 26.4596381 27.7702708
## 334 29.1575606 28.3984099 29.9167112
## 335 28.1410077 27.4363631 28.8456524
## 336 26.9439456 26.2962017 27.5916894
## 337 25.2433572 24.6586627 25.8280517
## 338 24.5213197 23.9555470 25.0870925
## 339 26.4689209 25.8411218 27.0967200
## 340 25.3003602 24.7139549 25.8867654
## 341 25.7278824 25.1276937 26.3280711
## 342 29.3380699 28.5687431 30.1073967
## 343 26.3359140 25.7133971 26.9584309
## 344 27.7324865 27.0482381 28.4167349
## 345 30.1741134 29.3559717 30.9922550
## 346 24.5498212 23.9834015 25.1162409
## 347 22.5167156 21.9738247 23.0596064
## 348 28.5115270 27.7876008 29.2354531
## 349 28.8630453 28.1201927 29.6058978
```

```
## 350 28.9580502 28.2099826 29.7061177
## 351 28.8725457 28.1291735 29.6159179
## 352 29.3380699 28.5687431 30.1073967
## 353 27.1529564 26.4959304 27.8099824
## 354 30.2786188 29.4541960 31.1030416
## 355 26.9059436 26.2598551 27.5520321
## 356 29.2620660 28.4970407 30.0270913
## 357 17.8329723 17.1723301 18.4936144
## 358 21.9466860 21.4017704 22.4916015
## 359 23.6472743 23.0970946 24.1974540
## 360 22.5167156 21.9738247 23.0596064
## 361 27.1529564 26.4959304 27.8099824
## 362 21.0726405 20.5172954 21.6279857
## 363 24.8728380 24.2985026 25.4471734
## 364 20.6451183 20.0815659 21.2086707
## 365 29.5280798 28.7478952 30.3082644
## 366 27.7894895 27.1024523 28.4765267
## 367 21.2531499 20.7006691 21.8056308
## 368 21.8896830 21.3443551 22.4350109
## 369 31.4566800 30.5590890 32.3542710
## 370 31.0101568 30.1407794 31.8795342
## 371 31.7416948 30.8258192 32.6575704
## 372 25.4998705 24.9072393 26.0925018
## 373 26.1174026 25.5032610 26.7315443
## 374  1.5206248 -0.2478391  3.2890888
## 375 -1.5195331 -3.5211516  0.4820854
## 376 21.7851776 21.2389954 22.3313598
## 377 12.4746939 11.5031968 13.4461910
## 378 14.3747926 13.5252436 15.2243416
## 379 12.0471717 11.0470929 13.0472505
## 380 13.8617660 12.9802176 14.7433144
## 381 18.2034915 17.5592994 18.8476836
## 382 14.5268005 13.6865809 15.3670201
## 383 12.1326761 11.1383423 13.1270100
## 384 11.2206288 10.1643364 12.2769211
## 385  5.4538292  3.9820212  6.9256372
## 386  5.2828203  3.7982718  6.7673688
## 387  7.6864452  6.3791938  8.9936965
## 388  4.1617621  2.5932928  5.7302313
## 389  5.4633297  3.9922290  6.9344304
## 390 14.7453119 13.9183735 15.5722502
## 391 18.2984964 17.6583687 18.9386242
## 392 16.7309150 16.0162798 17.4455502
## 393 10.1565735  9.0263197 11.2868272
## 394 20.1415922 19.5659058 20.7172786
## 395 19.0205339 18.4090793 19.6319886
## 396 18.2889959 17.6484647 18.9295272
## 397 16.1513849 15.4057280 16.8970418
## 398 15.6288578 14.8539101 16.4038054
## 399  5.4918311  4.0228520  6.9608103
## 400  6.0808617  4.6556170  7.5061065
## 401  9.1210197  7.9173537 10.3246857
## 402 15.2488380 14.4518832 16.0457928
## 403 15.2583385 14.4619407 16.0547363
```

```
## 404 15.7713652 15.0045215 16.5382088
## 405  8.5414896  7.2962170  9.7867621
## 406 12.7217067 11.7665537 13.6768597
## 407 12.3796890 11.4018718 13.3575061
## 408 23.0297422 22.4853957 23.5740888
## 409  9.4725379  8.2939335 10.6511424
## 410 15.7618647 14.9944833 16.5292460
## 411 24.9488419 24.3724910 25.5251928
## 412 14.3937936 13.5454147 15.2421725
## 413  1.9006446  0.1611253  3.6401639
## 414 15.4768499 14.6931671 16.2605326
## 415 -0.5789842 -2.5081970  1.3502286
## 416  6.9549072  5.5941340  8.3156803
## 417 10.0520680  8.9144652 11.1896709
## 418  9.2445261  8.0496812 10.4393710
## 419 14.9638232 14.1500064 15.7776400
## 420 12.9497186 12.0095351 13.8899021
## 421 20.2840996 19.7121097 20.8560895
## 422 19.6380660 19.0477342 20.2283979
## 423 21.1581450 20.6042018 21.7120882
## 424 12.4271914 11.4525366 13.4018463
## 425 18.2509940 17.6088421 18.8931459
## 426 11.3821371 10.3369210 12.4273532
## 427 19.6475665 19.0575331 20.2375999
## 428 20.7591243 20.1979530 21.3202955
## 429 14.1087788 13.2427333 14.9748243
## 430 11.6766524 10.6515220 12.7017829
## 431 17.7949703 17.1325889 18.4573517
## 432 15.8473691 15.0848129 16.6099253
## 433 23.1247472 22.5797913 23.6697030
## 434 19.1440404 18.5370739 19.7510069
## 435 20.1415922 19.5659058 20.7172786
## 436 12.4461924 11.4728012 13.4195836
## 437 17.4054500 16.7247123 18.0861877
## 438  9.4250355  8.2430525 10.6070185
## 439  2.2331619  0.5189267  3.9473970
## 440 12.8167117 11.8678100 13.7656134
## 441 13.5482497 12.6467830 14.4497163
## 442 16.0088775 15.2553498 16.7624052
## 443 18.7925221 18.1724521 19.4125921
## 444 16.6454106 15.9263026 17.3645185
## 445 11.9521668 10.9456884 12.9586451
## 446 11.7716574 10.7529741 12.7903406
## 447 17.6524629 16.9834761 18.3214496
## 448 18.9350295 18.3203935 19.5496655
## 449 17.3294461 16.6450182 18.0138740
## 450 16.2083879 15.4658534 16.9509223
## 451 17.9849801 17.3311996 18.6387607
## 452 17.7094658 17.0431368 18.3757948
## 453 18.1464885 17.4998271 18.7931500
## 454 18.6500147 18.0243497 19.2756797
## 455 16.7784175 16.0662507 17.4905842
## 456 17.3294461 16.6450182 18.0138740
## 457 16.4934027 15.7662519 17.2205534
```

```
## 458 18.4600048 17.8266365 19.0933731
## 459 19.1345399 18.5272327 19.7418471
## 460 20.5881154 20.0233208 21.1529100
## 461 18.9540305 18.3401067 19.5679543
## 462 20.6356178 20.0718608 21.1993749
## 463 21.2626504 20.7103102 21.8149906
## 464 24.7778330 24.2059359 25.3497301
## 465 21.9941884 21.4495875 22.5387893
## 466 21.1296435 20.5752420 21.6840450
## 467 18.2604945 17.6187487 18.9022402
## 468 14.2987887 13.4445481 15.1530292
## 469 17.3294461 16.6450182 18.0138740
## 470 20.5311124 19.9650415 21.0971833
## 471 19.0775369 18.4681695 19.6869043
## 472 22.3267057 21.7835655 22.8698459
## 473 20.9111322 20.3529192 21.4693451
## 474 23.4762654 22.9281416 24.0243893
## 475 17.3199456 16.6350541 18.0048371
## 476 11.6576515 10.6312297 12.6840732
## 477 16.8069190 16.0962277 17.5176102
## 478 10.8881115  9.8088847 11.9673382
## 479 17.4244510 16.7446305 18.1042716
## 480 22.0986939 21.5546921 22.6426956
## 481 24.3503108 23.7882407 24.9123810
## 482 27.2004589 26.5412822 27.8596355
## 483 27.8939949 27.2017959 28.5861939
## 484 24.6543266 24.0854627 25.2231906
## 485 21.8801825 21.3347822 22.4255828
## 486 24.5023187 23.9369726 25.0676649
## 487 20.3221016 19.7510629 20.8931402
## 488 23.6757758 23.1252211 24.2263305
## 489 17.3959496 16.7147525 18.0771466
## 490 11.7811579 10.7631185 12.7991973
## 491  6.3563761  4.9515111  7.7612410
## 492 17.3864491 16.7047920 18.0681061
## 493 21.8706820 21.3252083 22.4161558
## 494 23.1437481 22.5986577 23.6888385
## 495 21.6426702 21.0951188 22.1902215
## 496 17.8329723 17.1723301 18.4936144
## 497 14.4697975 13.6260879 15.3135072
## 498 21.1581450 20.6042018 21.7120882
## 499 22.2792032 21.7359340 22.8224724
## 500 20.2080956 19.6341596 20.7820317
## 501 20.9396336 20.3819475 21.4973198
## 502 25.3668636 24.7784239 25.9553033
## 503 25.9273927 25.3202176 26.5345679
## 504 29.1955625 28.4342810 29.9568440
## 505 28.3975211 27.6796032 29.1154389
## 506 27.0674520 26.4142591 27.7206448
```

```r
predict(lm.fit, data.frame(lstat = c(5, 10, 15)), interval = "prediction")
```

```
## Warning: 'newdata' had 3 rows but variables found have 506 rows
```

```
##            fit         lwr      upr
```

```
## 1   29.8225951  17.5846032 42.06059
## 2   25.8703898  13.6434129 38.09737
## 3   30.7251420  18.4834878 42.96680
## 4   31.7606958  19.5143151 44.00708
## 5   29.4900778  17.2533279 41.72683
## 6   29.6040837  17.3669145 41.84125
## 7   22.7447274  10.5206618 34.96879
## 8   16.3603958   4.1263477 28.59444
## 9    6.1188637  -6.1756909 18.41342
## 10  18.3079969   6.0792598 30.53673
## 11  15.1253316   2.8868863 27.36378
## 12  21.9466860   9.7225420 34.17083
## 13  19.6285655   7.4022984 31.85483
## 14  26.7064332  14.4778089 38.93506
## 15  24.8063345  12.5809243 37.03174
## 16  26.5069229  14.2787250 38.73512
## 17  28.3025161  16.0697291 40.53530
## 18  20.6166169   8.3915995 32.84163
## 19  23.4477639  11.2234904 35.67204
## 20  23.8372842  11.6127838 36.06178
## 21  14.5838035   2.3431786 26.82443
## 22  21.4146583   9.1902764 33.63904
## 23  16.7689170   4.5361476 29.00169
## 24  15.6668597   3.4304407 27.90328
## 25  19.0680364   6.8408326 31.29524
## 26  18.8685260   6.6409490 31.09610
## 27  20.4836100   8.2584542 32.70877
## 28  18.1369880   5.9078642 30.36611
## 29  22.3932092  10.1691502 34.61727
## 30  23.1722496  10.9480885 35.39641
## 31  13.0827255   0.8352577 25.33019
## 32  22.1651973   9.9411080 34.38929
## 33   8.2279733  -4.0496643 20.50561
## 34  17.1204352   4.8886962 29.35217
## 35  15.2298370   2.9917948 27.46788
## 36  25.3573631  13.1312158 37.58351
## 37  23.7137778  11.4893579 35.93820
## 38  26.2219080  13.9942832 38.44953
## 39  24.9298409  12.7042793 37.15540
## 40  30.4496277  18.2091366 42.69012
## 41  32.6727432  20.4217361 44.92375
## 42  29.9556020  17.7170972 42.19411
## 43  29.0340541  16.7989133 41.26919
## 44  27.4854737  15.2549843 39.71596
## 45  25.4808696  13.2545351 37.70720
## 46  24.8538370  12.6283694 37.07930
## 47  21.1106425   8.8860579 33.33523
## 48  16.6929130   4.4599124 28.92591
## 49   5.2828203  -7.0190758 17.58472
## 50  19.1630413   6.9360079 31.39007
## 51  21.7756771   9.5514728 33.99988
## 52  25.5948755  13.3683612 37.82139
## 53  29.5375803  17.3006565 41.77450
## 54  26.5449248  14.3166473 38.77320
```

```
## 55  20.4931104   8.2679648 32.71826
## 56  29.9841035  17.7454876 42.22272
## 57  29.0720561  16.8367853 41.30733
## 58  30.8011459  18.5591639 43.04313
## 59  28.0365023  15.8045018 40.26850
## 60  25.7943858  13.5675406 38.02123
## 61  22.0606919   9.8365795 34.28480
## 62  20.8351282   8.6103180 33.05994
## 63  28.1600087  15.9276477 40.39237
## 64  25.5283720  13.3019635 37.75478
## 65  26.9059436  14.6768719 39.13502
## 66  30.1171104  17.8779703 42.35625
## 67  24.8253355  12.5999025 37.05077
## 68  26.8584411  14.6294778 39.08740
## 69  22.1176948   9.8935958 34.34179
## 70  26.2029071  13.9753189 38.43050
## 71  28.1695092  15.9371201 40.40190
## 72  25.1673533  12.9414781 37.39323
## 73  29.3095684  17.0734684 41.54567
## 74  27.3904688  15.1602239 39.62071
## 75  28.1125063  15.8802848 40.34473
## 76  26.0603997  13.8330806 38.28772
## 77  23.1817501  10.9575858 35.40591
## 78  24.7968340  12.5714351 37.02223
## 79  22.8302319  10.6061548 35.05431
## 80  25.9083918  13.6813480 38.13544
## 81  29.5280798  17.2911908 41.76497
## 82  27.6944845  15.4634407 39.92553
## 83  28.1695092  15.9371201 40.40190
## 84  27.4189702  15.1886525 39.64929
## 85  25.4143661  13.1881334 37.64060
## 86  28.3500186  16.1170873 40.58295
## 87  22.3362062  10.1121422 34.56027
## 88  26.5354243  14.3071668 38.76368
## 89  29.3285694  17.0924018 41.56474
## 90  29.1385596  16.9030596 41.37406
## 91  26.1839061  13.9563544 38.41146
## 92  26.7634362  14.5346862 38.99219
## 93  26.8014382  14.5726034 39.03027
## 94  28.6540344  16.4201513 40.88792
## 95  24.4928182  12.2677563 36.71788
## 96  28.2360127  16.0034258 40.46860
## 97  23.7802812  11.5558190 36.00474
## 98  30.5541331  18.3132055 42.79506
## 99  31.1621647  18.9185842 43.40575
## 100 28.6730354  16.4390912 40.90698
## 101 25.6043760  13.3778464 37.83091
## 102 27.2669623  15.0370282 39.49690
## 103 24.4548162  12.2297930 36.67984
## 104 21.7851776   9.5609770 34.00938
## 105 22.8397323  10.6156538 35.06381
## 106 18.9065280   6.6790237 31.13403
## 107 16.8259199   4.5933220 29.05852
## 108 21.1676455   8.9431026 33.39219
```

```
## 109 22.8967353  10.6726467 35.12082
## 110 19.7805734   7.5545319 32.00661
## 111 22.2031993   9.9791169 34.42728
## 112 24.9013394  12.6758135 37.12687
## 113 19.1535409   6.9264905 31.38059
## 114 18.3174974   6.0887813 30.54621
## 115 24.6258251  12.4006217 36.85103
## 116 19.5810631   7.3547229 31.80740
## 117 23.1152467  10.8911038 35.33939
## 118 24.7683325  12.5429672 36.99370
## 119 19.9515823   7.7257801 32.17738
## 120 21.6236692   9.3993984 33.84794
## 121 20.9016317   8.6768796 33.12638
## 122 20.9966366   8.7719635 33.22131
## 123 17.5194560   5.2888080 29.75010
## 124 10.4130868  -1.8494546 22.67563
## 125 17.8519732   5.6221708 30.08178
## 126 20.4836100   8.2584542 32.70877
## 127  8.6554955  -3.6189936 20.92998
## 128 18.2224925   5.9935639 30.45142
## 129 19.9325813   7.7067533 32.15841
## 130 17.1299357   4.8982236 29.36165
## 131 22.5832190  10.3591645 34.80727
## 132 22.9062358  10.6821454 35.13033
## 133 23.9892921  11.7646815 36.21390
## 134 20.2745991   8.0492071 32.49999
## 135 18.1084866   5.8792968 30.33768
## 136 18.4410038   6.2125569 30.66945
## 137 18.4980068   6.2696814 30.72633
## 138 20.6926208   8.4676783 32.91756
## 139 14.2987887   2.0569550 26.54062
## 140 17.0159298   4.7838912 29.24797
## 141 11.6006485  -0.6547291 23.85603
## 142  1.8626426 -10.4730277 14.19831
## 143  9.0735172  -3.1979849 21.34502
## 144  9.4535370  -2.8153285 21.72240
## 145  6.7268953  -5.5625465 19.01634
## 146  8.1424688  -4.1358098 20.42075
## 147 18.7355191   6.5076818 30.96336
## 148  6.4988835  -5.7924533 18.79022
## 149  7.6484432  -4.6336136 19.93050
## 150 14.1752822   1.9329116 26.41765
## 151 21.1581450   8.9335952 33.38269
## 152 21.9371855   9.7130385 34.16133
## 153 23.0392427  10.8151216 35.26336
## 154 19.5525616   7.3261771 31.77895
## 155 20.1890947   7.9635994 32.41459
## 156 20.2840996   8.0587188 32.50948
## 157 19.2200443   6.9931108 31.44698
## 158 30.1931143  17.9536705 42.43256
## 159 28.4450235  16.2118000 40.67825
## 160 27.5329762  15.3023628 39.76359
## 161 29.3285694  17.0924018 41.56474
## 162 32.9102555  20.6579725 45.16254
```

```
## 163 32.7297461  20.4784355 44.98106
## 164 31.3996770  19.1550077 43.64435
## 165 23.4952664  11.2709695 35.71956
## 166 25.2338567  13.0078884 37.45983
## 167 31.0386583  18.7956323 43.28168
## 168 23.0202417  10.7961255 35.24436
## 169 24.0082931  11.7836679 36.23292
## 170 23.7992822  11.5748074 36.02376
## 171 20.8446287   8.6198270 33.06943
## 172 23.1247472  10.9006014 35.34889
## 173 20.5976159   8.3725793 32.82265
## 174 25.9653947  13.7382491 38.19254
## 175 25.3953651  13.1691610 37.62157
## 176 29.4900778  17.2533279 41.72683
## 177 24.9488419  12.7232563 37.17443
## 178 28.5780304  16.3443898 40.81167
## 179 27.9794994  15.7476625 40.21134
## 180 29.7655921  17.5278173 42.00337
## 181 27.3714678  15.1412712 39.60166
## 182 25.5758745  13.3493907 37.80236
## 183 29.9746030  17.7360242 42.21318
## 184 29.1575606  16.9219946 41.39313
## 185 21.2721509   9.0476800 33.49662
## 186 22.0606919   9.8365795 34.28480
## 187 30.3261213  18.0861387 42.56610
## 188 28.2075112  15.9750093 40.44001
## 189 30.2216158  17.9820574 42.46117
## 190 29.4330749  17.1965320 41.66962
## 191 29.7085892  17.4710296 41.94615
## 192 30.0981094  17.8590447 42.33717
## 193 31.8271992  19.5804959 44.07390
## 194 29.7750926  17.5372817 42.01290
## 195 30.3926247  18.1523693 42.63288
## 196 31.7321943  19.4859512 43.97844
## 197 30.6776395  18.4361887 42.91909
## 198 26.3739159  14.1459908 38.60184
## 199 28.2645142  16.0318418 40.49719
## 200 30.2216158  17.9820574 42.46117
## 201 30.3261213  18.0861387 42.56610
## 202 27.4949742  15.2644601 39.72549
## 203 31.5991874  19.3535807 43.84479
## 204 30.9341528  18.6915898 43.17672
## 205 31.8176987  19.5710417 44.06436
## 206 24.2268044  12.0019975 36.45161
## 207 24.1317995  11.9070747 36.35652
## 208 17.3959496   5.1649728 29.62693
## 209 20.6261174   8.4011095 32.85113
## 210 12.6172013   0.3673724 24.86703
## 211 18.1464885   5.9173866 30.37559
## 212 11.7716574  -0.4827491 24.02606
## 213 19.3245497   7.0977950 31.55130
## 214 25.6423779  13.4157868 37.86897
## 215  6.4798825  -5.8116134 18.77138
## 216 25.5568735  13.3304200 37.78333
```

```
## 217 21.7186741   9.4944463 33.94290
## 218 25.3478626  13.1217293 37.57400
## 219 17.5289565   5.2983335 29.75958
## 220 24.5783227  12.3531708 36.80347
## 221 25.3288617  13.1027563 37.55497
## 222 14.1657817   1.9233695 26.40819
## 223 25.1198508  12.8940407 37.34566
## 224 27.3334658  15.1033653 39.56357
## 225 30.6206366  18.3794282 42.86184
## 226 30.1551124  17.9158208 42.39440
## 227 31.5801864  19.3346699 43.82570
## 228 28.5115270  16.2780961 40.74496
## 229 30.8296474  18.5875417 43.07175
## 230 30.9816553  18.7388825 43.22443
## 231 23.4857659  11.2614738 35.71006
## 232 29.5660818  17.3290531 41.80311
## 233 32.2072190  19.9586275 44.45581
## 234 30.8011459  18.5591639 43.04313
## 235 26.9059436  14.6768719 39.13502
## 236 24.2173039  11.9925054 36.44210
## 237 25.4903700  13.2640209 37.71672
## 238 30.0601074  17.8211931 42.29902
## 239 28.5115270  16.2780961 40.74496
## 240 27.5519771  15.3213139 39.78264
## 241 23.7422792  11.5178415 35.96672
## 242 22.7732289  10.5491599 34.99730
## 243 23.8942871  11.6697469 36.11883
## 244 29.6230847  17.3858450 41.86032
## 245 22.6782240  10.4541646 34.90228
## 246 17.0159298   4.7838912 29.24797
## 247 25.8513888  13.6244451 38.07833
## 248 24.9108399  12.6853021 37.13638
## 249 25.5093710  13.2829923 37.73575
## 250 28.3215171  16.0886725 40.55436
## 251 28.9485497  16.7136984 41.18340
## 252 31.1431637  18.8996690 43.38666
## 253 31.2001667  18.9564139 43.44392
## 254 31.1906662  18.9469565 43.43438
## 255 28.3120166  16.0792009 40.54483
## 256 25.7658844  13.5390877 37.99268
## 257 31.5991874  19.3535807 43.84479
## 258 29.6895882  17.4521000 41.92708
## 259 27.1529564  14.9233020 39.38261
## 260 27.9985003  15.7666091 40.23039
## 261 25.4428676  13.2165915 37.66914
## 262 27.6564826  15.4255413 39.88742
## 263 28.9390492  16.7042299 41.17387
## 264 23.8657856  11.6412655 36.09031
## 265 26.8584411  14.6294778 39.08740
## 266 24.6258251  12.4006217 36.85103
## 267 20.5026109   8.2774755 32.72775
## 268 27.4854737  15.2549843 39.71596
## 269 31.5516849  19.3063033 43.79707
## 270 21.5856672   9.3613779 33.80996
```

```
## 271 22.2031993  9.9791169 34.42728
## 272 28.2930156 16.0602574 40.52577
## 273 27.2099594 14.9801660 39.43975
## 274 28.3025161 16.0697291 40.53530
## 275 31.2001667 18.9564139 43.44392
## 276 31.7226938 19.4764965 43.96889
## 277 28.8060423 16.5716652 41.04042
## 278 30.6016356 18.3605077 42.84276
## 279 27.7229860 15.4918648 39.95411
## 280 29.9461015 17.7076337 42.18457
## 281 30.9816553 18.7388825 43.22443
## 282 30.1931143 17.9536705 42.43256
## 283 31.6941923 19.4481320 43.94025
## 284 31.5516849 19.3063033 43.79707
## 285 27.0959535 14.8664364 39.32547
## 286 26.7349347 14.5062477 38.96362
## 287 22.2697027 10.0456307 34.49377
## 288 27.7704885 15.5392374 40.00174
## 289 27.3334658 15.1033653 39.56357
## 290 25.5188715 13.2924779 37.74527
## 291 31.3901765 19.1455513 43.63480
## 292 31.1716652 18.9280417 43.41529
## 293 30.0886089 17.8495819 42.32764
## 294 26.4024174 14.1744346 38.63040
## 295 24.6733276 12.4480714 36.89858
## 296 28.5970314 16.3633305 40.83073
## 297 27.5329762 15.3023628 39.76359
## 298 19.5050591  7.2785997 31.73152
## 299 29.8320956 17.5940674 42.07012
## 300 30.0506069 17.8117301 42.28948
## 301 28.7870413 16.5527266 41.02136
## 302 25.5283720 13.3019635 37.75478
## 303 26.3169130 14.0891018 38.54472
## 304 29.9366010 17.6981701 42.17503
## 305 27.9699989 15.7381892 40.20181
## 306 26.0699002 13.8425634 38.29724
## 307 28.4070216 16.1739155 40.64013
## 308 27.3999692 15.1697001 39.63024
## 309 30.2406168 18.0009817 42.48025
## 310 25.0818488 12.8560899 37.30761
## 311 22.5452170 10.3211632 34.76927
## 312 28.8725457 16.6379487 41.10714
## 313 23.4192625 11.1950024 35.64352
## 314 27.0484510 14.8190470 39.27785
## 315 25.7373829 13.5106343 37.96413
## 316 23.6282733 11.4039045 35.85264
## 317 17.1394362  4.9077510 29.37112
## 318 19.4100542  7.1834415 31.63667
## 319 24.7113296 12.4860303 36.93663
## 320 22.4597126 10.2356574 34.68377
## 321 27.7134855 15.4823902 39.94458
## 322 28.0270018 15.7950287 40.25897
## 323 27.2384609 15.0085973 39.46832
## 324 23.4002615 11.1760102 35.62451
```

```
## 325 28.7395388  16.5053793 40.97370
## 326 29.7275902  17.4899590 41.96522
## 327 28.7110374  16.4769704 40.94510
## 328 22.4027096  10.1786514 34.62677
## 329 25.0818488  12.8560899 37.30761
## 330 27.5804786  15.3497401 39.81122
## 331 25.9178923  13.6908316 38.14495
## 332 22.7447274  10.5206618 34.96879
## 333 27.1149544  14.8853918 39.34452
## 334 29.1575606  16.9219946 41.39313
## 335 28.1410077  15.9087027 40.37331
## 336 26.9439456  14.7147863 39.17310
## 337 25.2433572  13.0173754 37.46934
## 338 24.5213197  12.2962282 36.74641
## 339 26.4689209  14.2408018 38.69704
## 340 25.3003602  13.0742964 37.52642
## 341 25.7278824  13.5011498 37.95461
## 342 29.3380699  17.1018684 41.57427
## 343 26.3359140  14.1080650 38.56376
## 344 27.7324865  15.5013394 39.96363
## 345 30.1741134  17.9347458 42.41348
## 346 24.5498212  12.3246997 36.77494
## 347 22.5167156  10.2926617 34.74077
## 348 28.5115270  16.2780961 40.74496
## 349 28.8630453  16.6284797 41.09761
## 350 28.9580502  16.7231669 41.19293
## 351 28.8725457  16.6379487 41.10714
## 352 29.3380699  17.1018684 41.57427
## 353 27.1529564  14.9233020 39.38261
## 354 30.2786188  18.0388297 42.51841
## 355 26.9059436  14.6768719 39.13502
## 356 29.2620660  17.0261342 41.49800
## 357 17.8329723   5.6031231 30.06282
## 358 21.9466860   9.7225420 34.17083
## 359 23.6472743  11.4228945 35.87165
## 360 22.5167156  10.2926617 34.74077
## 361 27.1529564  14.9233020 39.38261
## 362 21.0726405   8.8480272 33.29725
## 363 24.8728380  12.6473472 37.09833
## 364 20.6451183   8.4201294 32.87011
## 365 29.5280798  17.2911908 41.76497
## 366 27.7894895  15.5581861 40.02079
## 367 21.2531499   9.0286664 33.47763
## 368 21.8896830   9.6655206 34.11385
## 369 31.4566800  19.2117450 43.70161
## 370 31.0101568  18.7672576 43.25306
## 371 31.7416948  19.4954059 43.98798
## 372 25.4998705  13.2735066 37.72623
## 373 26.1174026  13.8899772 38.34483
## 374  1.5206248 -10.8187523 13.86000
## 375 -1.5195331 -13.8944771 10.85541
## 376 21.7851776   9.5609770 34.00938
## 377 12.4746939   0.2241196 24.72527
## 378 14.3747926   2.1332855 26.61630
```

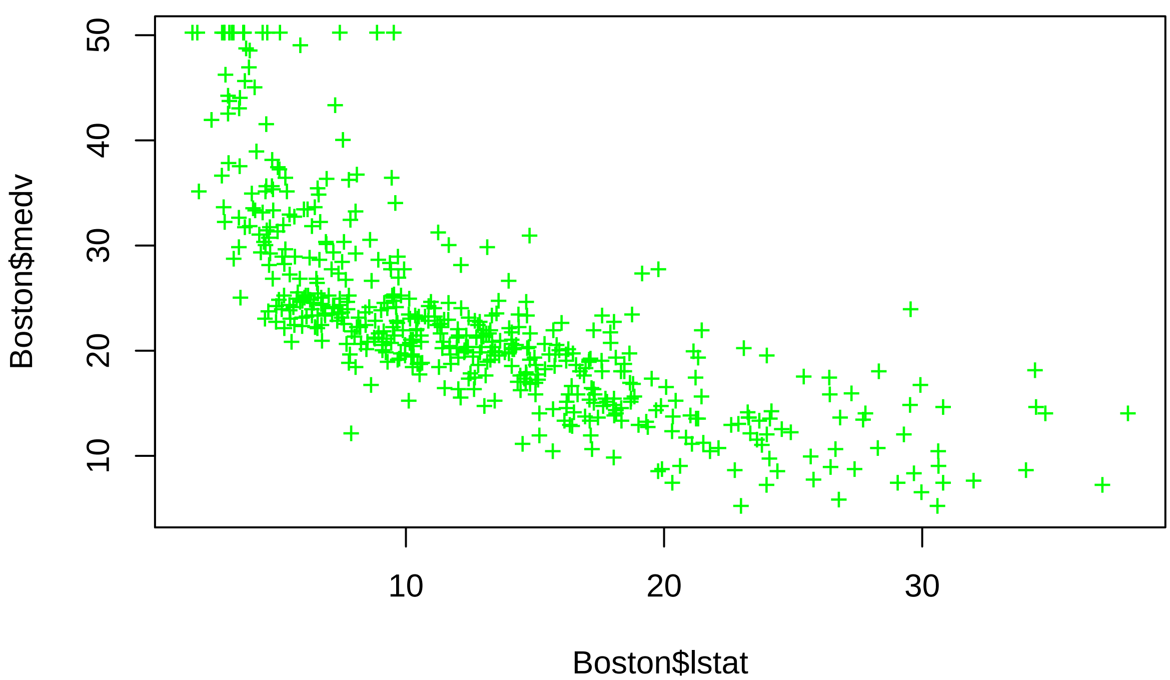
```
## 379 12.0471717  -0.2057023 24.30005
## 380 13.8617660   1.6179965 26.10554
## 381 18.2034915   5.9745199 30.43246
## 382 14.5268005   2.2859373 26.76766
## 383 12.1326761  -0.1197303 24.38508
## 384 11.2206288  -1.0369614 23.47822
## 385  5.4538292  -6.8465360 17.75419
## 386  5.2828203  -7.0190758 17.58472
## 387  7.6864452  -4.5953165 19.96821
## 388  4.1617621  -8.1505431 16.47407
## 389  5.4633297  -6.8369508 17.76361
## 390 14.7453119   2.5053531 26.98527
## 391 18.2984964   6.0697382 30.52725
## 392 16.7309150   4.4980303 28.96380
## 393 10.1565735  -2.1076117 22.42076
## 394 20.1415922   7.9160379 32.36715
## 395 19.0205339   6.7932431 31.24782
## 396 18.2889959   6.0602166 30.51778
## 397 16.1513849   3.9166488 28.38612
## 398 15.6288578   3.3923016 27.86541
## 399  5.4918311  -6.8081958 17.79186
## 400  6.0808617  -6.2140187 18.37574
## 401  9.1210197  -3.1501488 21.39219
## 402 15.2488380   3.0108684 27.48681
## 403 15.2583385   3.0204051 27.49627
## 404 15.7713652   3.5353195 28.00741
## 405  8.5414896  -3.7338299 20.81681
## 406 12.7217067   0.4724177 24.97100
## 407 12.3796890   0.1286119 24.63077
## 408 23.0297422  10.8056236 35.25386
## 409  9.4725379  -2.7961976 21.74127
## 410 15.7618647   3.5257853 27.99794
## 411 24.9488419  12.7232563 37.17443
## 412 14.3937936   2.1523676 26.63522
## 413  1.9006446 -10.4346175 14.23591
## 414 15.4768499   3.2397374 27.71396
## 415 -0.5789842 -12.9424233 11.78445
## 416  6.9549072  -5.3326665 19.24248
## 417 10.0520680  -2.2127966 22.31693
## 418  9.2445261  -3.0257802 21.51483
## 419 14.9638232   2.7247439 27.20290
## 420 12.9497186   0.7015878 25.19785
## 421 20.2840996   8.0587188 32.50948
## 422 19.6380660   7.4118133 31.86432
## 423 21.1581450   8.9335952 33.38269
## 424 12.4271914   0.1763663 24.67802
## 425 18.2509940   6.0221297 30.47986
## 426 11.3821371  -0.8745035 23.63878
## 427 19.6475665   7.4213282 31.87380
## 428 20.7591243   8.5342449 32.98400
## 429 14.1087788   1.8661158 26.35144
## 430 11.6766524  -0.5782917 23.93160
## 431 17.7949703   5.5650270 30.02491
## 432 15.8473691   3.6115914 28.08315
```

```
## 433 23.1247472  10.9006014 35.34889
## 434 19.1440404   6.9169732 31.37111
## 435 20.1415922   7.9160379 32.36715
## 436 12.4461924   0.1954678 24.69692
## 437 17.4054500   5.1744988 29.63640
## 438  9.4250355  -2.8440251 21.69410
## 439  2.2331619 -10.0985601 14.56488
## 440 12.8167117   0.5679085 25.06551
## 441 13.5482497   1.3030300 25.79347
## 442 16.0088775   3.7736592 28.24410
## 443 18.7925221   6.5647974 31.02025
## 444 16.6454106   4.4122638 28.87856
## 445 11.9521668  -0.3012312 24.20556
## 446 11.7716574  -0.4827491 24.02606
## 447 17.6524629   5.4221601 29.88277
## 448 18.9350295   6.7075792 31.16248
## 449 17.3294461   5.0982890 29.56060
## 450 16.2083879   3.9738417 28.44293
## 451 17.9849801   5.7554997 30.21446
## 452 17.7094658   5.4793081 29.93962
## 453 18.1464885   5.9173866 30.37559
## 454 18.6500147   6.4220050 30.87802
## 455 16.7784175   4.5456768 29.01116
## 456 17.3294461   5.0982890 29.56060
## 457 16.4934027   4.2597805 28.72702
## 458 18.4600048   6.2315986 30.68841
## 459 19.1345399   6.9074558 31.36162
## 460 20.5881154   8.3630691 32.81316
## 461 18.9540305   6.7266160 31.18145
## 462 20.6356178   8.4106195 32.86062
## 463 21.2626504   9.0381732 33.48713
## 464 24.7778330  12.5524566 37.00321
## 465 21.9941884   9.7700585 34.21832
## 466 21.1296435   8.9050730 33.35421
## 467 18.2604945   6.0316515 30.48934
## 468 14.2987887   2.0569550 26.54062
## 469 17.3294461   5.0982890 29.56060
## 470 20.5311124   8.3060071 32.75622
## 471 19.0775369   6.8503503 31.30472
## 472 22.3267057  10.1026407 34.55077
## 473 20.9111322   8.6863882 33.13588
## 474 23.4762654  11.2519780 35.70055
## 475 17.3199456   5.0887625 29.55113
## 476 11.6576515  -0.5974008 23.91270
## 477 16.8069190   4.5742641 29.03957
## 478 10.8881115  -1.3714764 23.14770
## 479 17.4244510   5.1935508 29.65535
## 480 22.0986939   9.8745906 34.32280
## 481 24.3503108  12.1253901 36.57523
## 482 27.2004589  14.9706888 39.43023
## 483 27.8939949  15.6624005 40.12559
## 484 24.6543266  12.4290917 36.87956
## 485 21.8801825   9.6560169 34.10435
## 486 24.5023187  12.2772470 36.72739
```

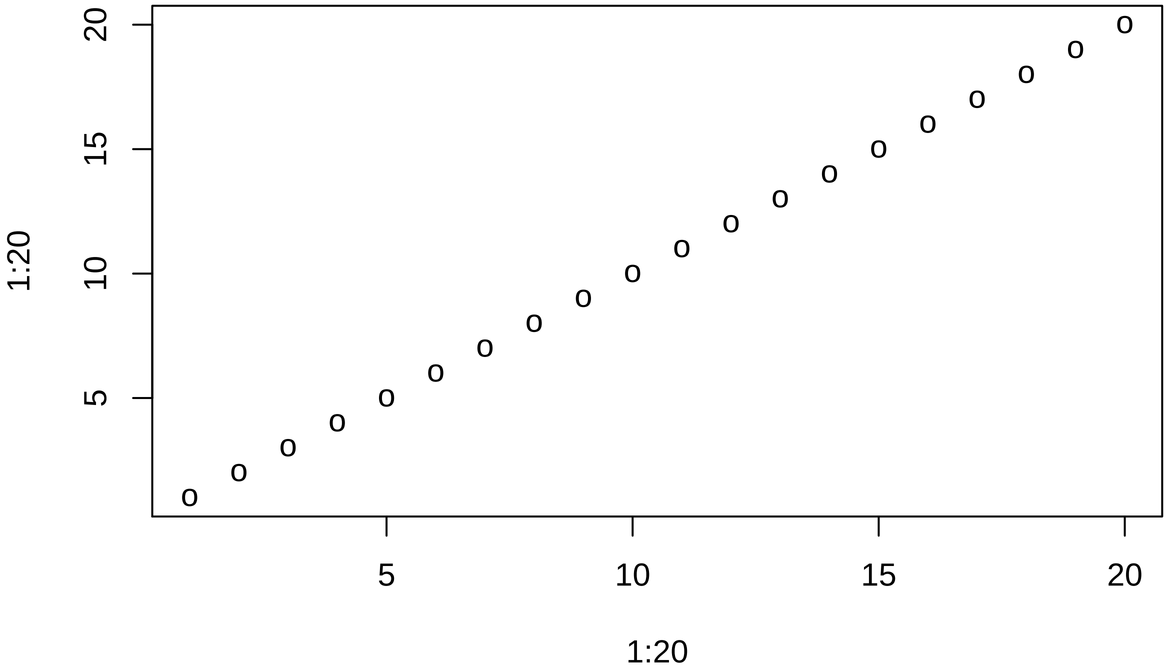
```
## 487 20.3221016   8.0967652 32.54744
## 488 23.6757758  11.4513791 35.90017
## 489 17.3959496   5.1649728 29.62693
## 490 11.7811579  -0.4731951 24.03551
## 491  6.3563761  -5.9361586 18.64891
## 492 17.3864491   5.1554467 29.61745
## 493 21.8706820   9.6465131 34.09485
## 494 23.1437481  10.9195964 35.36790
## 495 21.6426702   9.4184084 33.86693
## 496 17.8329723   5.6031231 30.06282
## 497 14.4697975   2.2286943 26.71090
## 498 21.1581450   8.9335952 33.38269
## 499 22.2792032  10.0551325 34.50327
## 500 20.2080956   7.9826236 32.43357
## 501 20.9396336   8.7149137 33.16435
## 502 25.3668636  13.1407021 37.59303
## 503 25.9273927  13.7003152 38.15447
## 504 29.1955625  16.9598642 41.43126
## 505 28.3975211  16.1644442 40.63060
## 506 27.0674520  14.8380029 39.29690
```

The **abline()** function can be used to draw any line, not just the least square regression lines. To draw a line with with intercept a and slope b - we write abline(a,b). lwd == the width of the line. pch == for plotting symbols.

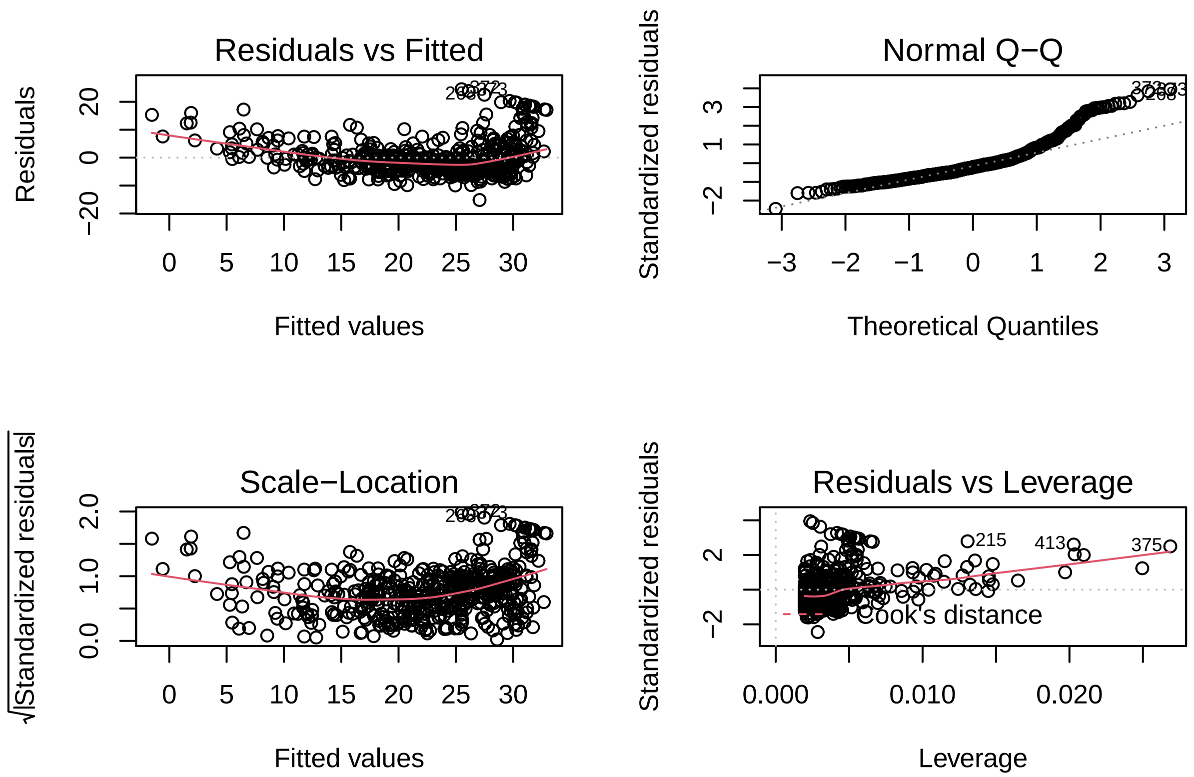
```
# plot(Boston$lstat,Boston$medv)
# abline(lm.fit)
# abline(lm.fit , lwd = 3)
# abline(lm.fit , lwd = 3, col ="red")
# abline(lm.fit , Boston$medv, col = "blue")
# abline(lm.fit , Boston$medv, pch = 20, col = "yellow")
plot(Boston$lstat , Boston$medv, pch = "+", col = "green")
```



```r
plot(1:20, 1:20, pch = "o")
```

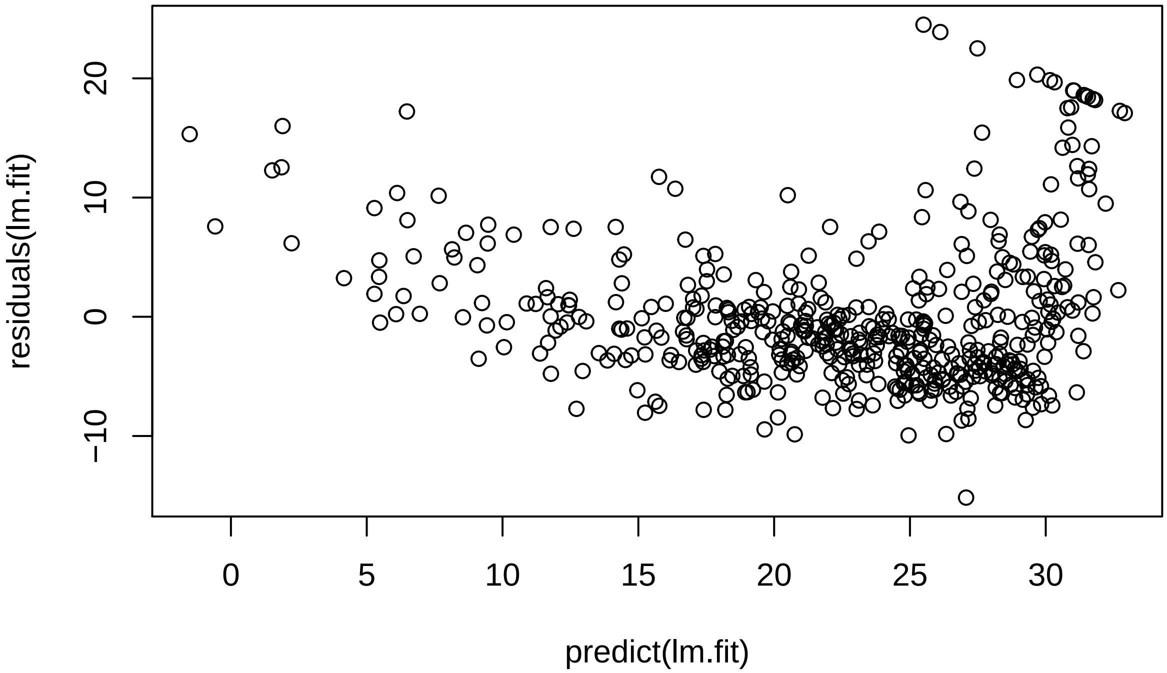



```r
par(mfrow=c(2,2))
plot(lm.fit)
```

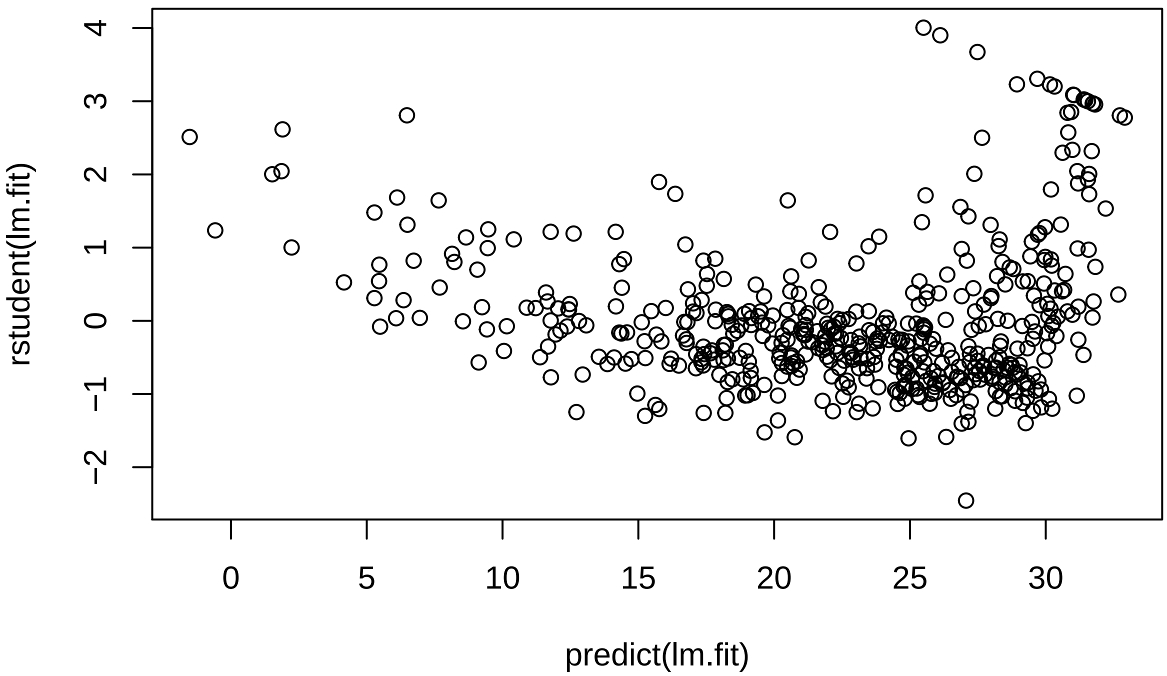


The **residual()** function will return thr studentized residuals. On the basis of residual plots, there is some evidence of non linearity.

```
plot(predict(lm.fit),residuals(lm.fit))
```



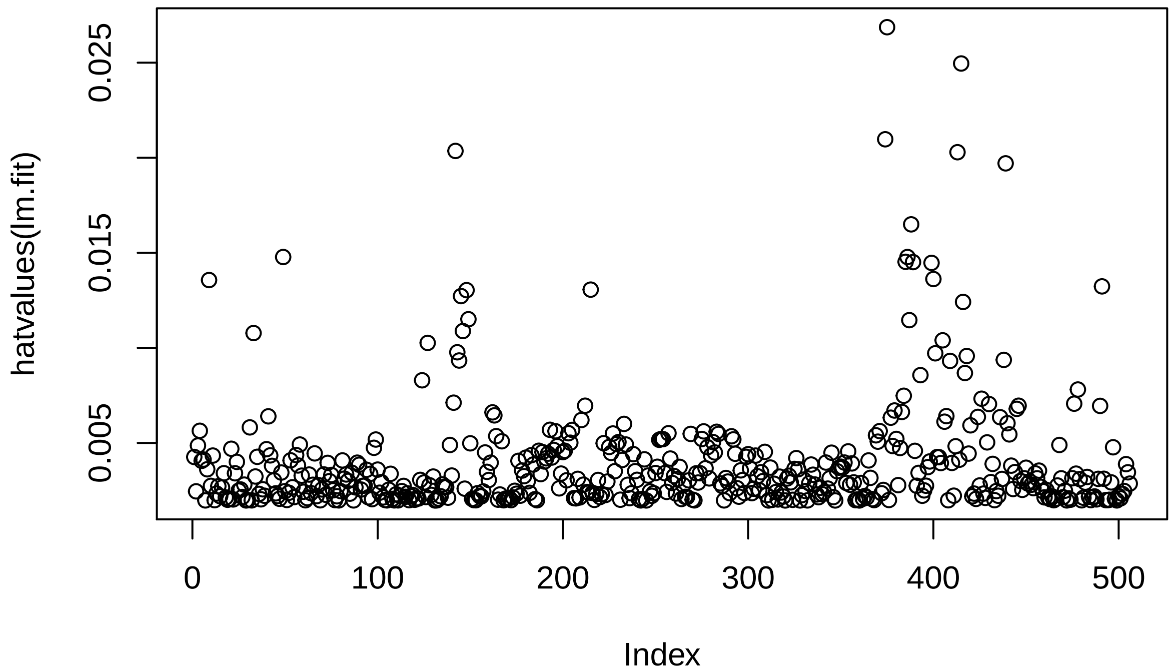

```
plot(predict(lm.fit),rstudent(lm.fit))
```



on the basis of the residual plots, there is some evidence of non-linearity. Leverage statistics can be computed for any number of predictirs using the hatvalues() function.

the **which.max()** function identifies the index of the largest element of a vector. In this case , it tells us which observation has the largest leverage statistic.

```
plot(hatvalues(lm.fit))
```




```
which.max(hatvalues(lm.fit))
```

```
## 375
## 375
```

Multiple Linear Regression

inorder to fit a multiple linear regression model using least squares, we again use the lm() function. the syntax lm(y ~ x1+x2+x3) is used to fit a modell 3 predictirs x1, x2, x3 the summary() fucniton now outputs the regression coefficients for all the predictors

```
lm.fit = lm(Boston$medv ~ Boston$lstat + Boston$age, data = Boston)
summary(lm.fit)
```

```
##
## Call:
## lm(formula = Boston$medv ~ Boston$lstat + Boston$age, data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.981  -3.978  -1.283   1.968  23.158
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  33.22276    0.73085  45.458  < 2e-16 ***
## Boston$lstat -1.03207    0.04819 -21.416  < 2e-16 ***
## Boston$age    0.03454    0.01223   2.826  0.00491 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.173 on 503 degrees of freedom
## Multiple R-squared:  0.5513, Adjusted R-squared:  0.5495
## F-statistic:   309 on 2 and 503 DF,  p-value: < 2.2e-16
```

the Boston dataset contins 13 variables and so it would be cumbersome to have to type all of these in order to perfrom a regresison using all of the predictors. Instead , we can use the following short - hand:

```
lm.fit = lm (Boston$medv ~ . , data = Boston)
summary(lm.fit)
```

```
##
## Call:
## lm(formula = Boston$medv ~ ., data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.595  -2.730  -0.518   1.777  26.199
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.646e+01  5.103e+00   7.144 3.28e-12 ***
## crim        -1.080e-01  3.286e-02  -3.287 0.001087 **
## zn           4.642e-02  1.373e-02   3.382 0.000778 ***
## indus        2.056e-02  6.150e-02   0.334 0.738288
## chas         2.687e+00  8.616e-01   3.118 0.001925 **
## nox         -1.777e+01  3.820e+00  -4.651 4.25e-06 ***
## rm           3.810e+00  4.179e-01   9.116  < 2e-16 ***
## age          6.922e-04  1.321e-02   0.052 0.958229
## dis         -1.476e+00  1.995e-01  -7.398 6.01e-13 ***
## rad          3.060e-01  6.635e-02   4.613 5.07e-06 ***
## tax         -1.233e-02  3.760e-03  -3.280 0.001112 **
## ptratio     -9.527e-01  1.308e-01  -7.283 1.31e-12 ***
## black        9.312e-03  2.686e-03   3.467 0.000573 ***
## lstat       -5.248e-01  5.072e-02 -10.347  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.745 on 492 degrees of freedom
## Multiple R-squared:  0.7406, Adjusted R-squared:  0.7338
## F-statistic: 108.1 on 13 and 492 DF,  p-value: < 2.2e-16
```

we can access the individual compnents of a sumary object by name . Hence **summary(lm.fit)**$r.sq * * * givesustheR^2 and * * summary(lm.fit)$**sigma** gives us ethe RSE. The **vif()** function, part of the car package, can be used to compute variance inflation factors. Most VIF's are low to moderate for this data. The car package is not part of the base R installation so it moust be downloaded the first time you use it via the install.packages option in R

```
library(car)
```

```
## Loading required package: carData
```

```
vif(lm.fit)
```

```
##     crim       zn    indus     chas      nox       rm      age      dis
## 1.792192 2.298758 3.991596 1.073995 4.393720 1.933744 3.100826 3.955945
##      rad      tax  ptratio    black    lstat
## 7.484496 9.008554 1.799084 1.348521 2.941491
```

What if we would like to perform a regression using all the varibales but one?

```
lm.fit1 = lm(Boston$medv ~. -age, data = Boston)
summary(lm.fit1)
```

```
##
## Call:
## lm(formula = Boston$medv ~ . - age, data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.6054  -2.7313  -0.5188   1.7601  26.2243
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  36.436927   5.080119   7.172 2.72e-12 ***
## crim         -0.108006   0.032832  -3.290 0.001075 **
## zn            0.046334   0.013613   3.404 0.000719 ***
## indus         0.020562   0.061433   0.335 0.737989
## chas          2.689026   0.859598   3.128 0.001863 **
## nox         -17.713540   3.679308  -4.814 1.97e-06 ***
## rm            3.814394   0.408480   9.338  < 2e-16 ***
## dis          -1.478612   0.190611  -7.757 5.03e-14 ***
## rad           0.305786   0.066089   4.627 4.75e-06 ***
## tax          -0.012329   0.003755  -3.283 0.001099 **
## ptratio      -0.952211   0.130294  -7.308 1.10e-12 ***
## black         0.009321   0.002678   3.481 0.000544 ***
## lstat        -0.523852   0.047625 -10.999  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.74 on 493 degrees of freedom
## Multiple R-squared:  0.7406, Adjusted R-squared:  0.7343
## F-statistic: 117.3 on 12 and 493 DF,  p-value: < 2.2e-16
```

Alternatively the update() function can be used

```
lm.fit1 = update(lm.fit, ~.-age)
summary(lm.fit1)
```

```
##
## Call:
## lm(formula = Boston$medv ~ crim + zn + indus + chas + nox + rm +
##     dis + rad + tax + ptratio + black + lstat, data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.6054  -2.7313  -0.5188   1.7601  26.2243
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  36.436927   5.080119   7.172 2.72e-12 ***
## crim         -0.108006   0.032832  -3.290 0.001075 **
## zn            0.046334   0.013613   3.404 0.000719 ***
## indus         0.020562   0.061433   0.335 0.737989
## chas          2.689026   0.859598   3.128 0.001863 **
## nox         -17.713540   3.679308  -4.814 1.97e-06 ***
## rm            3.814394   0.408480   9.338  < 2e-16 ***
## dis          -1.478612   0.190611  -7.757 5.03e-14 ***
```

```
## rad           0.305786   0.066089   4.627 4.75e-06 ***
## tax          -0.012329   0.003755  -3.283 0.001099 **
## ptratio      -0.952211   0.130294  -7.308 1.10e-12 ***
## black         0.009321   0.002678   3.481 0.000544 ***
## lstat        -0.523852   0.047625 -10.999  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.74 on 493 degrees of freedom
## Multiple R-squared:  0.7406, Adjusted R-squared:  0.7343
## F-statistic: 117.3 on 12 and 493 DF,  p-value: < 2.2e-16
```

Interaction Terms

It is easy to include interaction terms in a linear model using the lm() function. The syntax lstat : black tells R to include an interaction term between lstat and black. The syntax lstat*age simultaneously includes lstat, age, and the interaction term lstat* age as predictors. It is a shorthand for lstat + age + lstat : age

```
summary(lm(Boston$medv~Boston$lstat*age, data = Boston))
```

```
##
## Call:
## lm(formula = Boston$medv ~ Boston$lstat * age, data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.806  -4.045  -1.333   2.085  27.552
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)       36.0885359  1.4698355  24.553  < 2e-16 ***
## Boston$lstat      -1.3921168  0.1674555  -8.313 8.78e-16 ***
## age               -0.0007209  0.0198792  -0.036   0.9711
## Boston$lstat:age   0.0041560  0.0018518   2.244   0.0252 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.149 on 502 degrees of freedom
## Multiple R-squared:  0.5557, Adjusted R-squared:  0.5531
## F-statistic: 209.3 on 3 and 502 DF,  p-value: < 2.2e-16
```

Non-linear Transformations of the Predictors

The lm() function can also accommodate non-linear transformations of the predictors. For instance, given a predictor X, we can create a predictor X^2 using I(X^2). The function I() is needed since the ^ has a special meaning in a formula; wrapping as we do allows the the standard usage in R, which is to raise X to the power 2. We now perform a regression of medv onto lstat and lstat^2

```
lm.fit2 = lm(Boston$medv ~ Boston$lstat + I (Boston$lstat^2))
summary(lm.fit2)
```

```
##
## Call:
## lm(formula = Boston$medv ~ Boston$lstat + I(Boston$lstat^2))
##
## Residuals:
```

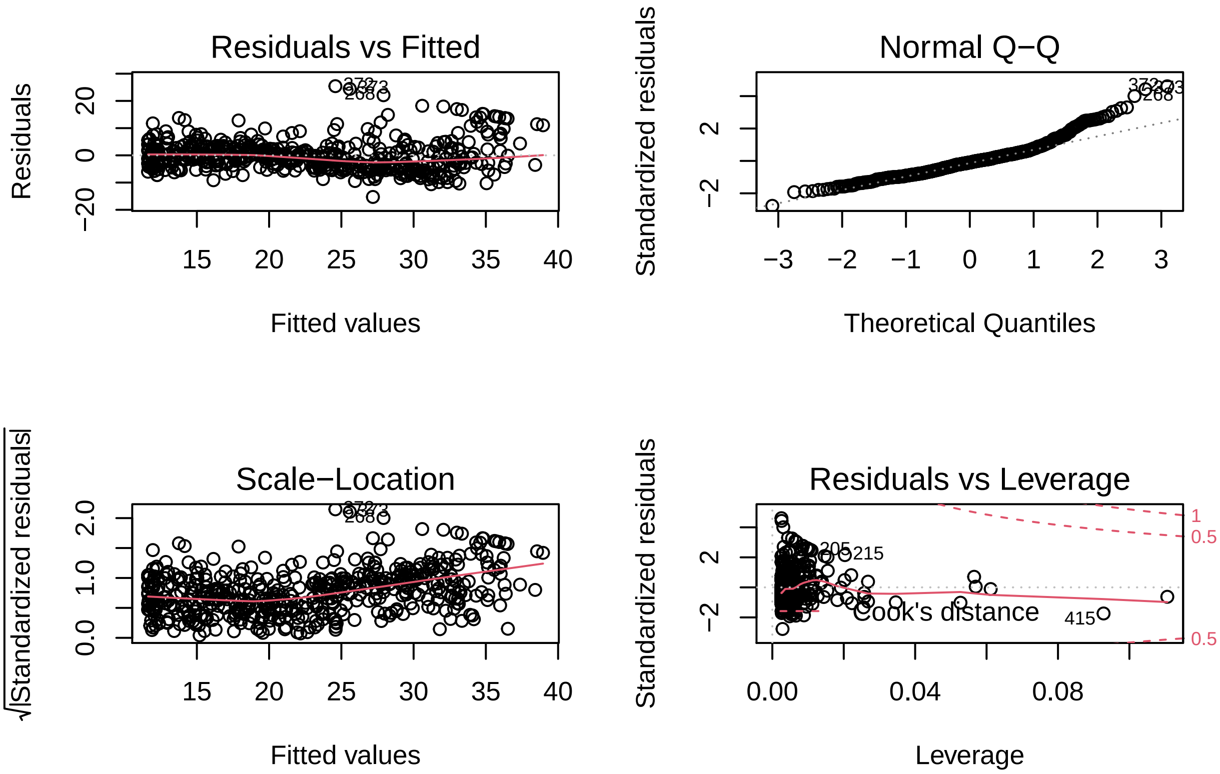
```
##      Min       1Q   Median       3Q      Max
## -15.2834  -3.8313  -0.5295   2.3095  25.4148
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)        42.862007   0.872084   49.15   <2e-16 ***
## Boston$lstat       -2.332821   0.123803  -18.84   <2e-16 ***
## I(Boston$lstat^2)   0.043547   0.003745   11.63   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.524 on 503 degrees of freedom
## Multiple R-squared:  0.6407, Adjusted R-squared:  0.6393
## F-statistic: 448.5 on 2 and 503 DF,  p-value: < 2.2e-16
```

The near -zero p-value associated with the quadratic term suggests that it leads to an improved model. We use the anova() function to further quantify the extent to which the quadratic fit is superior to linear fit

```
lm.fit = lm(Boston$medv~Boston$lstat)
anova(lm.fit, lm.fit2)
```

```
## Analysis of Variance Table
##
## Model 1: Boston$medv ~ Boston$lstat
## Model 2: Boston$medv ~ Boston$lstat + I(Boston$lstat^2)
##   Res.Df   RSS Df Sum of Sq     F    Pr(>F)
## 1    504 19472
## 2    503 15347  1    4125.1 135.2 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
par(mfrow=c(2,2))
plot(lm.fit2)
```




```r
lm.fit5 = lm(Boston$medv~poly(Boston$lstat,5))
summary(lm.fit5)
```

```
##
## Call:
## lm(formula = Boston$medv ~ poly(Boston$lstat, 5))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.5433  -3.1039  -0.7052   2.0844  27.1153
##
## Coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)               22.5328     0.2318  97.197  < 2e-16 ***
## poly(Boston$lstat, 5)1 -152.4595     5.2148 -29.236  < 2e-16 ***
## poly(Boston$lstat, 5)2   64.2272     5.2148  12.316  < 2e-16 ***
## poly(Boston$lstat, 5)3  -27.0511     5.2148  -5.187 3.10e-07 ***
## poly(Boston$lstat, 5)4   25.4517     5.2148   4.881 1.42e-06 ***
## poly(Boston$lstat, 5)5  -19.2524     5.2148  -3.692 0.000247 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 5.215 on 500 degrees of freedom
## Multiple R-squared:  0.6817, Adjusted R-squared:  0.6785
## F-statistic: 214.2 on 5 and 500 DF,  p-value: < 2.2e-16
```

This suggests that including additional polynomial terms, up to the fifth order, leads to the improvement in the model fit.

```r
summary(lm(Boston$medv~log(rm),data =Boston))
```

```
##
## Call:
## lm(formula = Boston$medv ~ log(rm), data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -19.487  -2.875  -0.104   2.837  39.816
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -76.488      5.028  -15.21   <2e-16 ***
## log(rm)       54.055      2.739   19.73   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.915 on 504 degrees of freedom
## Multiple R-squared:  0.4358, Adjusted R-squared:  0.4347
## F-statistic: 389.3 on 1 and 504 DF,  p-value: < 2.2e-16
```

Qualitative Predictors

```r
# fix(Carseats)
names(Carseats)
```

```
##  [1] "Sales"       "CompPrice"   "Income"      "Advertising" "Population"
##  [6] "Price"       "ShelveLoc"   "Age"         "Education"   "Urban"
## [11] "US"
```

Now we try to fit a multiple regression mode that includes some interaction terms

```r
lm.fit = lm(Carseats$Sales~.+Carseats$Income:Carseats$Advertising+ Carseats$Price:Carseats$Age, data = C
summary(lm.fit)
```

```
##
## Call:
## lm(formula = Carseats$Sales ~ . + Carseats$Income:Carseats$Advertising +
##     Carseats$Price:Carseats$Age, data = Carseats)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.9208 -0.7503  0.0177  0.6754  3.3413
##
## Coefficients:
##                                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)                          6.5755654  1.0087470   6.519 2.22e-10 ***
## CompPrice                            0.0929371  0.0041183  22.567  < 2e-16 ***
## Income                               0.0108940  0.0026044   4.183 3.57e-05 ***
## Advertising                          0.0702462  0.0226091   3.107 0.002030 **
## Population                           0.0001592  0.0003679   0.433 0.665330
## Price                               -0.1008064  0.0074399 -13.549  < 2e-16 ***
## ShelveLocGood                        4.8486762  0.1528378  31.724  < 2e-16 ***
## ShelveLocMedium                      1.9532620  0.1257682  15.531  < 2e-16 ***
## Age                                 -0.0579466  0.0159506  -3.633 0.000318 ***
```

```
## Education                              -0.0208525  0.0196131  -1.063 0.288361
## UrbanYes                                0.1401597  0.1124019   1.247 0.213171
## USYes                                  -0.1575571  0.1489234  -1.058 0.290729
## Carseats$Income:Carseats$Advertising   0.0007510  0.0002784   2.698 0.007290 **
## Carseats$Price:Carseats$Age            0.0001068  0.0001333   0.801 0.423812
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.011 on 386 degrees of freedom
## Multiple R-squared:  0.8761, Adjusted R-squared:  0.8719
## F-statistic:   210 on 13 and 386 DF,  p-value: < 2.2e-16
```

The **contrasts()** function returns the coding that R uses for the dummy variables

```
attach(Carseats)
contrasts(ShelveLoc)
```

```
##        Good Medium
## Bad       0      0
## Good      1      0
## Medium    0      1
```

```
?contrasts
```

Writing Function

```
Loadlibraries = function(){
  library(MASS)
  library(ISLR)
  print("The libraries have been loaded!!!")
}
```

```
Loadlibraries()
```

```
## [1] "The libraries have been loaded!!!"
```