

Unsupervised Learning

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R Markdown

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When you click the **Knit** button a document will be generated that includes both content as well as the output of any embedded R code chunks within the document. You can embed an R code chunk like this:

```
states <- row.names(USArrests)
states
```

```
## [1] "Alabama"      "Alaska"       "Arizona"      "Arkansas"
## [5] "California"   "Colorado"     "Connecticut"  "Delaware"
## [9] "Florida"      "Georgia"      "Hawaii"       "Idaho"
## [13] "Illinois"     "Indiana"      "Iowa"         "Kansas"
## [17] "Kentucky"     "Louisiana"    "Maine"        "Maryland"
## [21] "Massachusetts" "Michigan"     "Minnesota"    "Mississippi"
## [25] "Missouri"     "Montana"      "Nebraska"     "Nevada"
## [29] "New Hampshire" "New Jersey"   "New Mexico"   "New York"
## [33] "North Carolina" "North Dakota" "Ohio"         "Oklahoma"
## [37] "Oregon"       "Pennsylvania" "Rhode Island" "South Carolina"
## [41] "South Dakota" "Tennessee"    "Texas"        "Utah"
## [45] "Vermont"      "Virginia"     "Washington"   "West Virginia"
## [49] "Wisconsin"    "Wyoming"
```

```
names(USArrests)
```

```
## [1] "Murder" "Assault" "UrbanPop" "Rape"
```

```
apply(USArrests, 2, mean)
```

```
## Murder Assault UrbanPop Rape
## 7.788 170.760 65.540 21.232
```

```
apply(USArrests, 2, var)
```

```
## Murder Assault UrbanPop Rape
## 18.97047 6945.16571 209.51878 87.72916
```

We run PCA on our dataset using the `prcomp()` function.

```
pr.out <- prcomp(USArrests, scale = TRUE)
```

Now lets examing the results from The prcomp() function.

```
names(pr.out)
```

```
## [1] "sdev"      "rotation" "center"    "scale"     "x"
```

The center and scale components contain the mean and standard deviations prior to scaling.

```
pr.out$center
```

```
##      Murder  Assault UrbanPop      Rape
##      7.788  170.760   65.540   21.232
```

```
pr.out$scale
```

```
##      Murder  Assault UrbanPop      Rape
##  4.355510  83.337661 14.474763  9.366385
```

The rotation component correspondes to the rotation matrix whose columns contain the eigenvectors.

```
pr.out$rotation
```

```
##              PC1        PC2        PC3        PC4
## Murder   -0.5358995  0.4181809 -0.3412327  0.64922780
## Assault  -0.5831836  0.1879856 -0.2681484 -0.74340748
## UrbanPop -0.2781909 -0.8728062 -0.3780158  0.13387773
## Rape     -0.5434321 -0.1673186  0.8177779  0.08902432
```

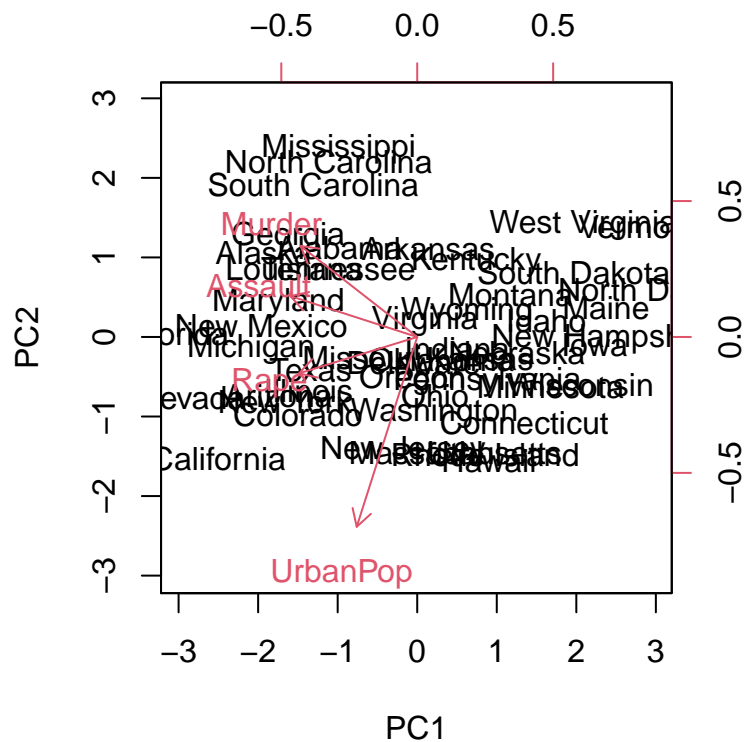
Let's check the dimentions of x component which returns the rotated data.

```
dim(pr.out$x)
```

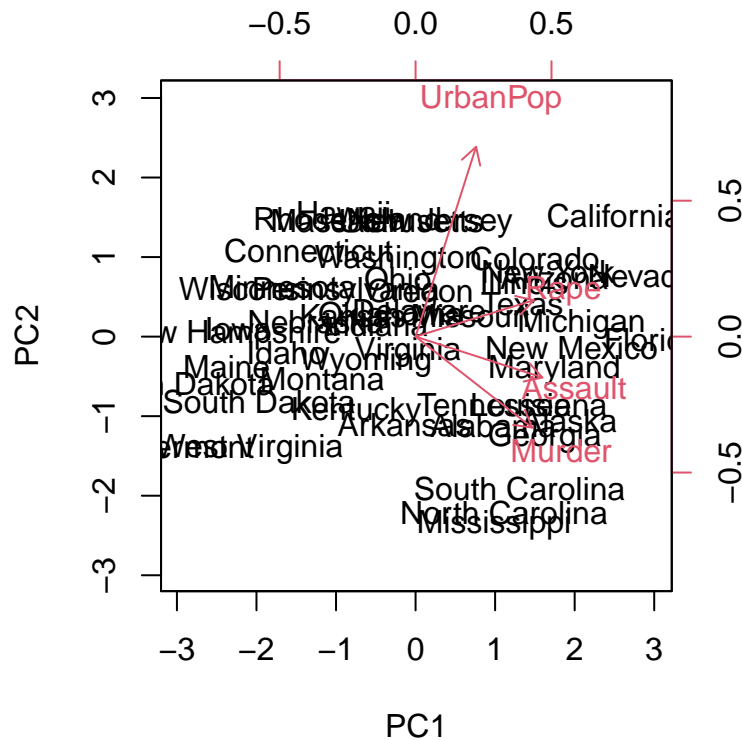
```
## [1] 50  4
```

We then plot the first two principal components using biplot().

```
biplot(pr.out, scale = 0)
```



```
pr.out$rotation <- -pr.out$rotation
pr.out$x <- -pr.out$x
biplot(pr.out, scale = 0)
```



We can compute the variance associated with each principal component from the standard deviation returned by `prcomp()`.

```
pr.out$sdev
```

```
## [1] 1.5748783 0.9948694 0.5971291 0.4164494
```

```
pr.var <- pr.out$sdev^2
pr.var
```

```
## [1] 2.4802416 0.9897652 0.3565632 0.1734301
```

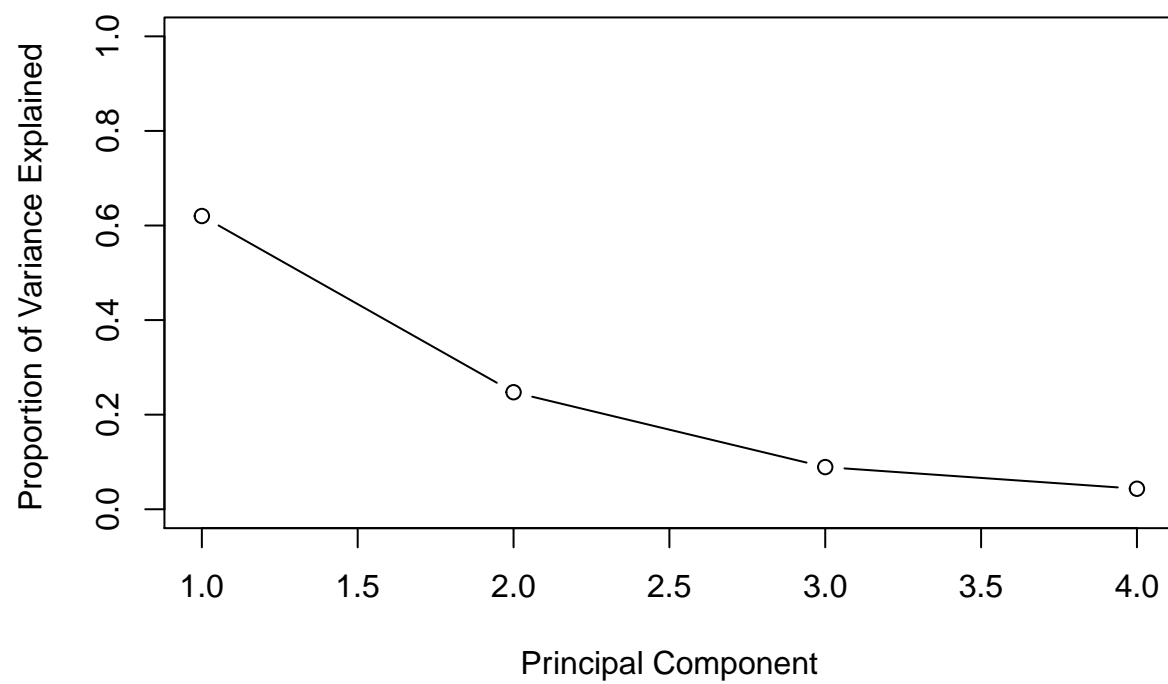
Let's compute the proportional variance as well.

```
pve <- pr.var/sum(pr.var)
pve
```

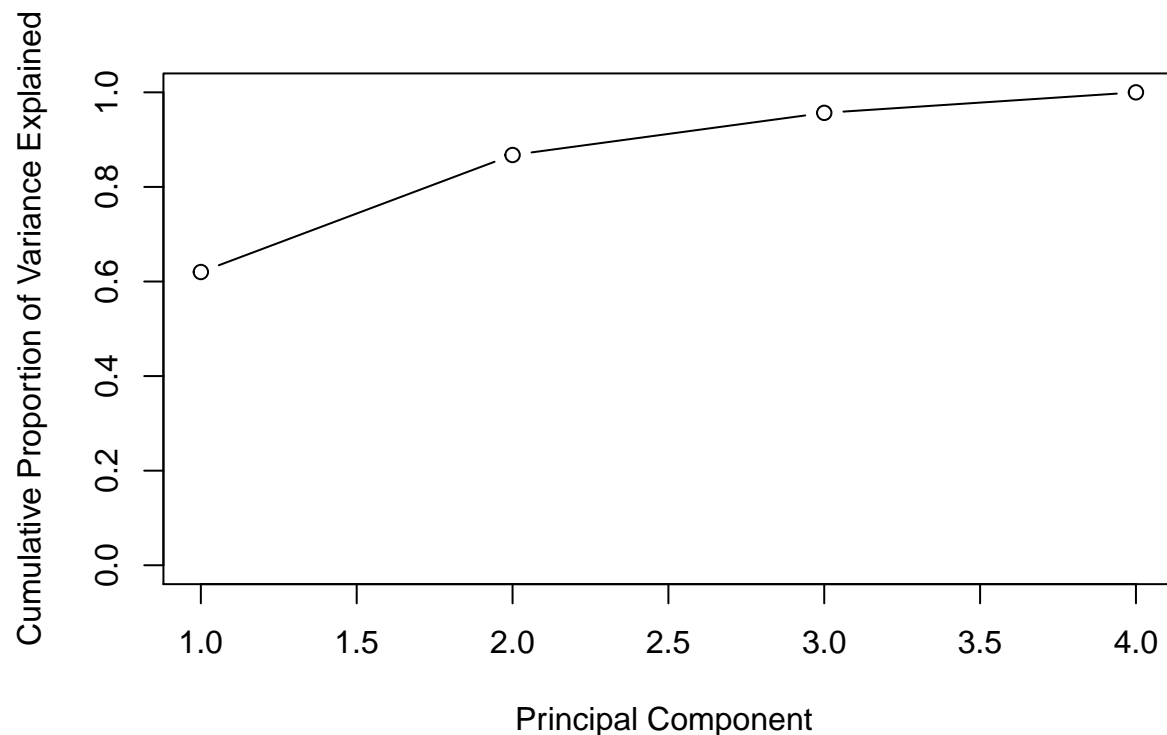
```
## [1] 0.62006039 0.24744129 0.08914080 0.04335752
```

Now we can plot the proportional variance for each principal component.

```
plot(pve, xlab = "Principal Component", ylab = "Proportion of Variance Explained ", ylim = c(0, 1), type = "n")
```



```
plot(cumsum(pve), xlab = "Principal Component ", ylab = " Cumulative Proportion of Variance Explained ")
```



```
a <- c(1, 2, 8, -3)
cumsum(a)
```

```
## [1] 1 3 11 8
```

```
##Clustering##
```

```
##K-Means Clustering##
```

In this exercise we use K-Means clustering on randomly generated data using the `kmeans()` function.

```
set.seed(2)
x <- matrix(rnorm(50 * 2), ncol = 2)
x[1:25, 1] <- x[1:25, 1] + 3
x[1:25, 2] <- x[1:25, 2] - 4
```

Let's start by clustering the data into two clusters with $K = 2$.

```
km.out <- kmeans(x, 2, nstart = 20)
```

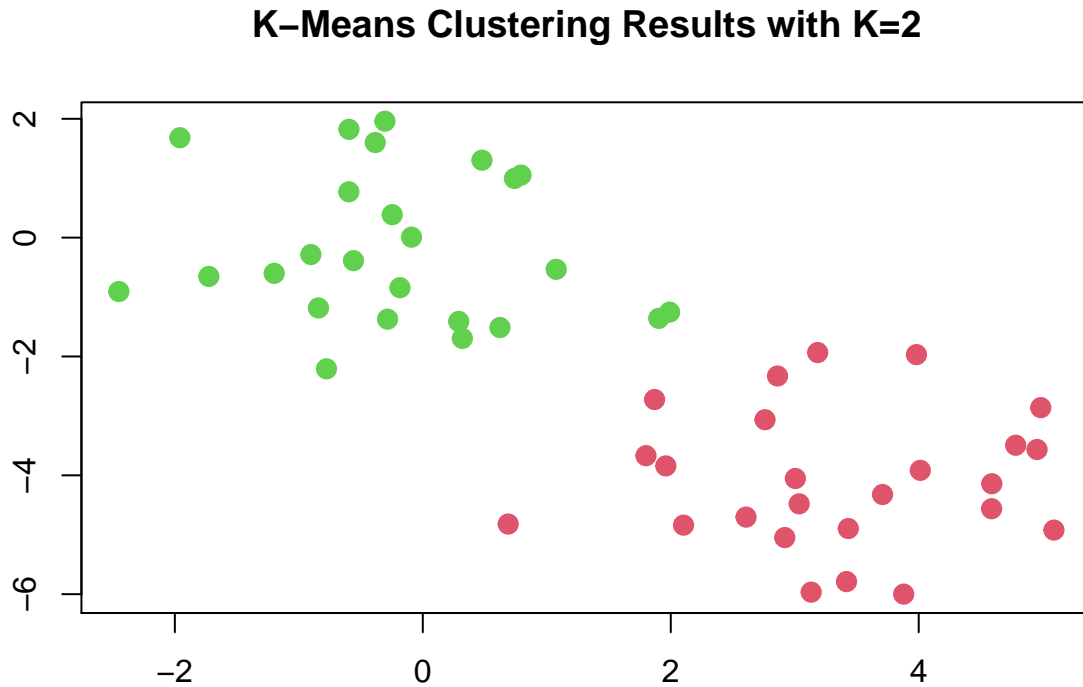
The `kmeans()` function returns the cluster assignments in the `cluster` component.

```
km.out$cluster
```

```
## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2
## [39] 2 2 2 2 2 2 2 2 2 2 2 2
```

Now let's plot the clusters.

```
plot(x, col = (km.out$cluster + 1), main = "K-Means Clustering Results with K=2", xlab = "", ylab = "",
```



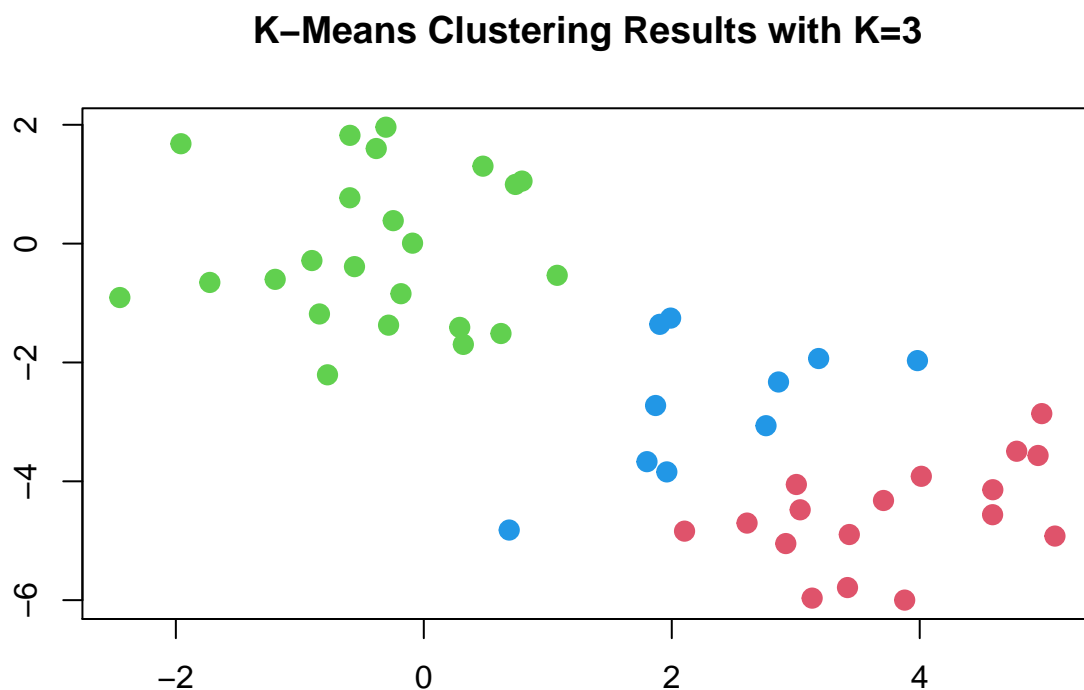
We can run K-means with different values for the number of clusters such as $K = 3$ and plot the results.

```
set.seed(4)
km.out <- kmeans(x, 3, nstart = 20)
km.out

## K-means clustering with 3 clusters of sizes 17, 23, 10
##
## Cluster means:
##      [,1]      [,2]
## 1  3.7789567 -4.56200798
## 2 -0.3820397 -0.08740753
## 3  2.3001545 -2.69622023
##
## Clustering vector:
## [1] 1 3 1 3 1 1 1 3 1 3 1 3 1 3 1 1 1 1 1 3 1 1 1 2 2 2 2 2 2 2 2 2 2 2
## [39] 2 2 2 2 2 3 2 3 2 2 2 2
##
## Within cluster sum of squares by cluster:
## [1] 25.74089 52.67700 19.56137
## (between_SS / total_SS = 79.3 %)
##
```

```
## Available components:
##
## [1] "cluster"      "centers"      "totss"        "withinss"     "tot.withinss"
## [6] "betweenss"    "size"         "iter"         "ifault"
```

```
plot(x, col = (km.out$cluster + 1), main = "K-Means Clustering Results with K=3", xlab = "", ylab = "",
```



We can control the initial cluster assignments with the `nstart` argument to `kmeans()`.

```
set.seed(3)
km.out <- kmeans(x, 3, nstart = 1)
km.out$tot.withinss
```

```
## [1] 97.97927
```

```
km.out <- kmeans(x, 3, nstart = 20)
km.out$tot.withinss
```

```
## [1] 97.97927
```

##Hierarchical Clustering##

We can use hierarchical clustering on the dataset we generated in the previous exercise using the `hclust()` function.

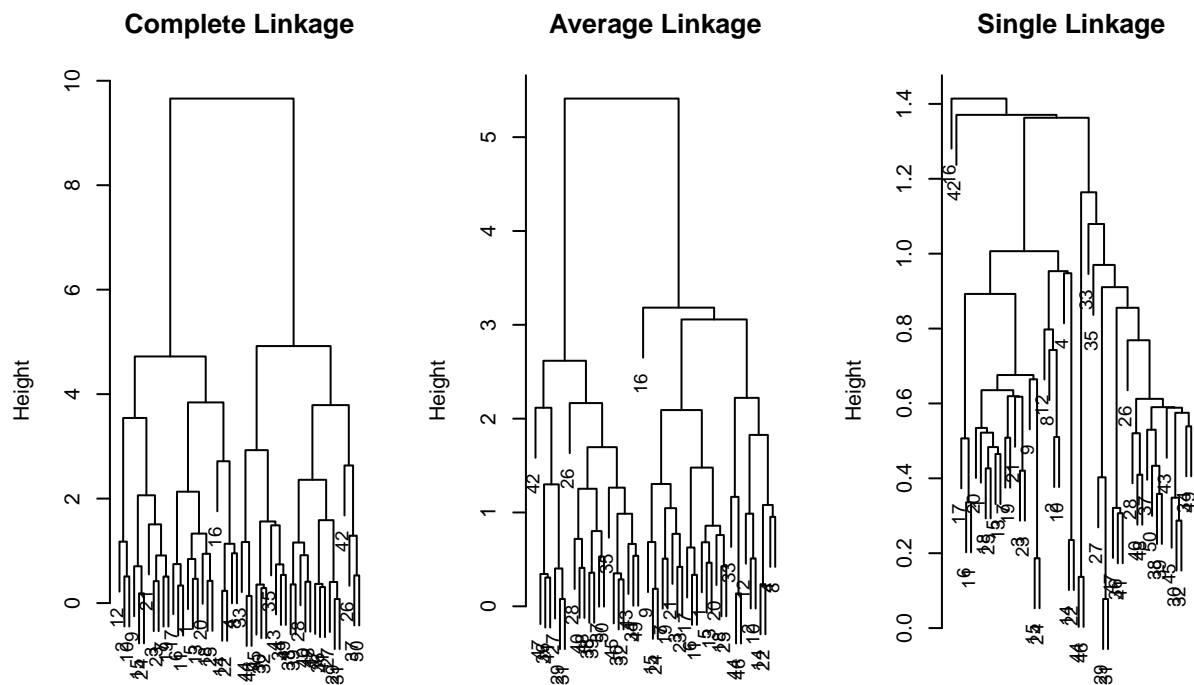

```
hc.complete <- hclust(dist(x), method = "complete")
```

The `hclust()` function supports various agglomeration methods including “single”, “complete”, and “average” linkages.

```
hc.average <- hclust(dist(x), method = "average")
hc.single <- hclust(dist(x), method = "single")
```

We can compare the different linkages by plotting the results obtained with different methods.

```
par(mfrow = c(1, 3))
plot(hc.complete, main = "Complete Linkage", xlab = "", sub = "", cex = 0.9)
plot(hc.average, main = "Average Linkage", xlab = "", sub = "", cex = 0.9)
plot(hc.single, main = "Single Linkage", xlab = "", sub = "", cex = 0.9)
```



We can cut the tree into different groups using the `cutree()` function.

```
cutree(hc.complete, 2)
```

```
## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2
## [39] 2 2 2 2 2 2 2 2 2 2 2 2 2
```

```
cutree(hc.average, 2)
```

```
## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 1 2 2 2 2 2
## [39] 2 2 2 2 2 1 2 1 2 2 2 2
```

```
cutree(hc.single, 2)
```

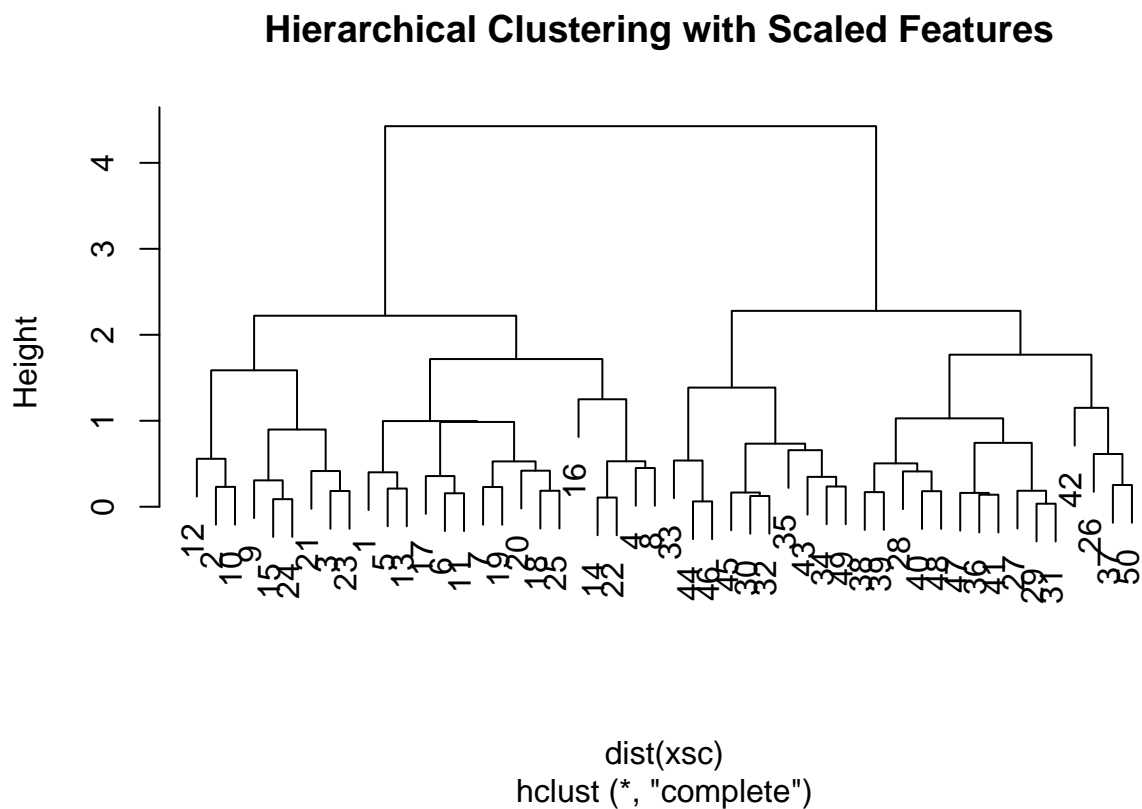
```
## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
## [39] 1 1 1 1 1 1 1 1 1 1 1 1 1
```

```
cutree(hc.single, 4)
```

```
## [1] 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 1 1 1 1 1 3 3 3 3 3 3 3 3 3 3 3
## [39] 3 3 3 4 3 3 3 3 3 3 3 3 3
```

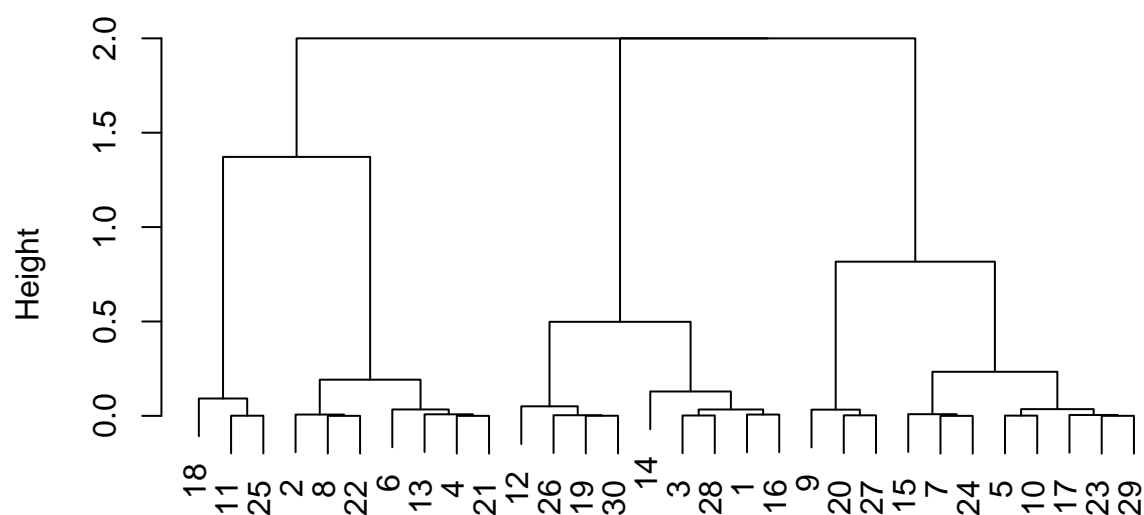
We can scale the dataset before it to the clustering algorithm by first calling `scale()`.

```
xsc <- scale(x)
plot(hclust(dist(xsc), method = "complete"), main = "Hierarchical Clustering with Scaled Features ")
```



```
x <- matrix(rnorm(30 * 3), ncol = 3)
dd <- as.dist(1 - cor(t(x)))
plot(hclust(dd, method = "complete"), main = "Complete Linkage with Correlation -Based Distance", xlab = "dist(xsc) hclust (*, "complete")")
```

Complete Linkage with Correlation –Based Distance



next example

```
library(ISLR)
nci.labs <- NCI60$labs
nci.data <- NCI60$data
```

```
dim(nci.data)
```

```
## [1] 64 6830
```

The `table()` function can be used to produce crosstabs from the dataset.

```
nci.labs[1:4]
```

```
## [1] "CNS" "CNS" "CNS" "RENAL"
```

```
table(nci.labs)
```

```
## nci.labs
## BREAST CNS COLON K562A-repro K562B-repro LEUKEMIA
## 7 5 7 1 1 6
## MCF7A-repro MCF7D-repro MELANOMA NSCLC OVARIAN PROSTATE
```

```
##          1          1          8          9          6          2
##      RENAL      UNKNOWN
##          9          1
```

PCA on the NCI60 Data

We use `prcomp()` to run principal component analysis as shown in the PCA exercise above.

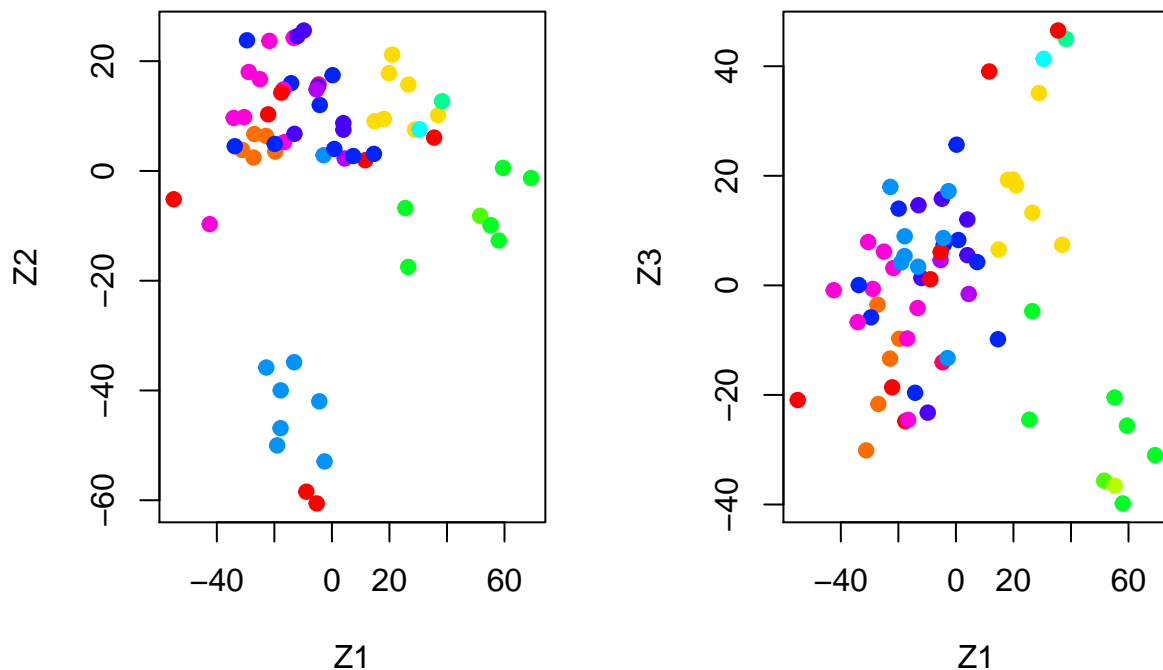
```
pr.out <- prcomp(nci.data, scale = TRUE)
```

We create a function to assign unique colors to each cancer type.

```
Cols <- function(vec) {
  cols <- rainbow(length(unique(vec)))
  return(cols[as.numeric(as.factor(vec))])
}
```

We can now use our `Cols()` function to plot the PCA results.

```
par(mfrow = c(1, 2))
plot(pr.out$x[, 1:2], col = Cols(nci.labs), pch = 19, xlab = "Z1", ylab = "Z2")
plot(pr.out$x[, c(1, 3)], col = Cols(nci.labs), pch = 19, xlab = "Z1", ylab = "Z3")
```



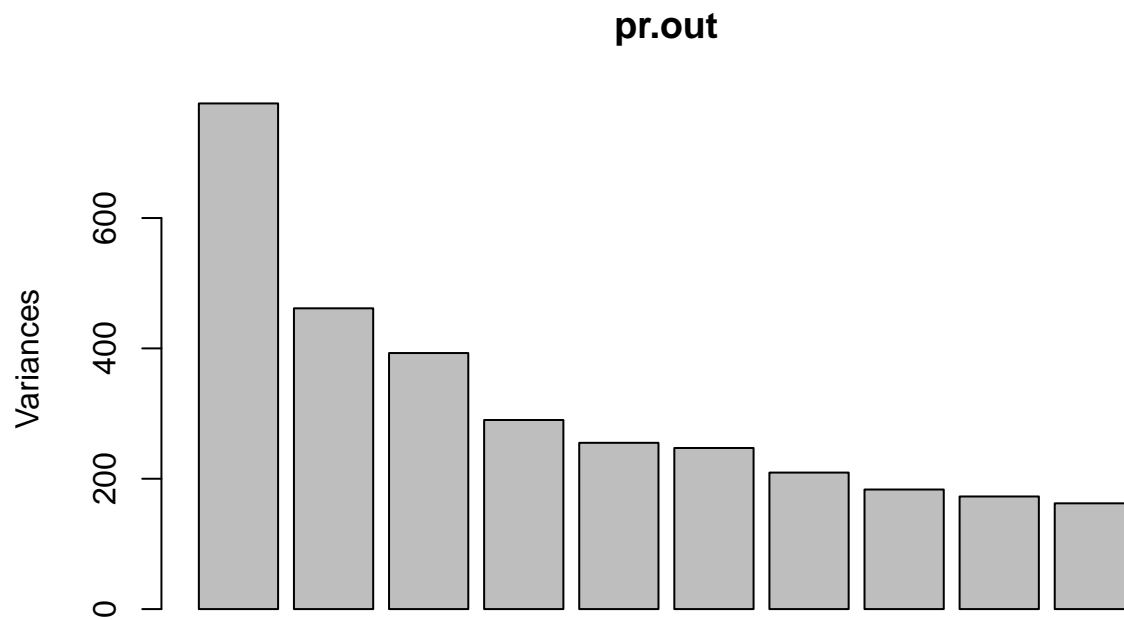
We can get a summary of the proportional variance and plot the variance explained by each principal component.

```
summary(pr.out)
```

```
## Importance of components:
```

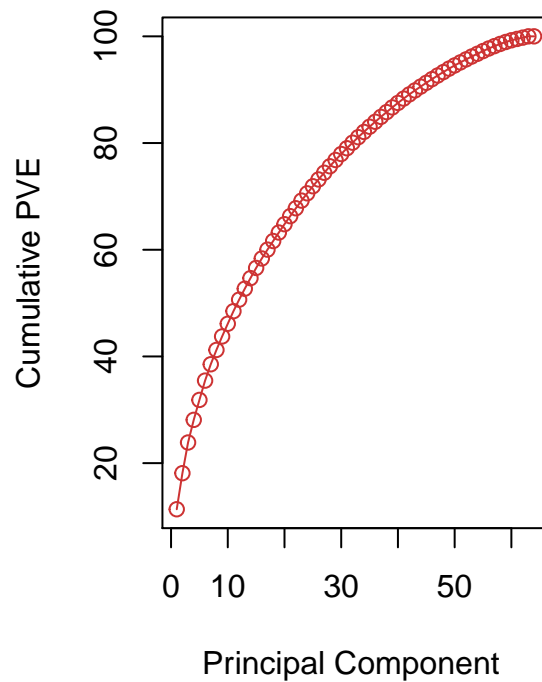
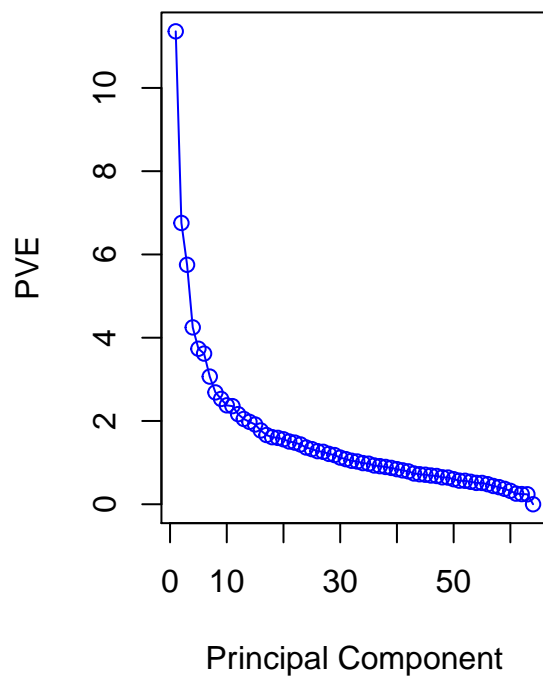
```
##          PC1      PC2      PC3      PC4      PC5      PC6
## Standard deviation 27.8535 21.48136 19.82046 17.03256 15.97181 15.72108
## Proportion of Variance 0.1136 0.06756 0.05752 0.04248 0.03735 0.03619
## Cumulative Proportion 0.1136 0.18115 0.23867 0.28115 0.31850 0.35468
##          PC7      PC8      PC9      PC10     PC11     PC12
## Standard deviation 14.47145 13.54427 13.14400 12.73860 12.68672 12.15769
## Proportion of Variance 0.03066 0.02686 0.02529 0.02376 0.02357 0.02164
## Cumulative Proportion 0.38534 0.41220 0.43750 0.46126 0.48482 0.50646
##          PC13     PC14     PC15     PC16     PC17     PC18
## Standard deviation 11.83019 11.62554 11.43779 11.00051 10.65666 10.48880
## Proportion of Variance 0.02049 0.01979 0.01915 0.01772 0.01663 0.01611
## Cumulative Proportion 0.52695 0.54674 0.56590 0.58361 0.60024 0.61635
##          PC19     PC20     PC21     PC22     PC23     PC24
## Standard deviation 10.43518 10.3219 10.14608 10.0544 9.90265 9.64766
## Proportion of Variance 0.01594 0.0156 0.01507 0.0148 0.01436 0.01363
## Cumulative Proportion 0.63229 0.6479 0.66296 0.6778 0.69212 0.70575
##          PC25     PC26     PC27     PC28     PC29     PC30     PC31
## Standard deviation 9.50764 9.33253 9.27320 9.0900 8.98117 8.75003 8.59962
## Proportion of Variance 0.01324 0.01275 0.01259 0.0121 0.01181 0.01121 0.01083
## Cumulative Proportion 0.71899 0.73174 0.74433 0.7564 0.76824 0.77945 0.79027
##          PC32     PC33     PC34     PC35     PC36     PC37     PC38
## Standard deviation 8.44738 8.37305 8.21579 8.15731 7.97465 7.90446 7.82127
## Proportion of Variance 0.01045 0.01026 0.00988 0.00974 0.00931 0.00915 0.00896
## Cumulative Proportion 0.80072 0.81099 0.82087 0.83061 0.83992 0.84907 0.85803
##          PC39     PC40     PC41     PC42     PC43     PC44     PC45
## Standard deviation 7.72156 7.58603 7.45619 7.3444 7.10449 7.0131 6.95839
## Proportion of Variance 0.00873 0.00843 0.00814 0.0079 0.00739 0.0072 0.00709
## Cumulative Proportion 0.86676 0.87518 0.88332 0.8912 0.89861 0.9058 0.91290
##          PC46     PC47     PC48     PC49     PC50     PC51     PC52
## Standard deviation 6.8663 6.80744 6.64763 6.61607 6.40793 6.21984 6.20326
## Proportion of Variance 0.0069 0.00678 0.00647 0.00641 0.00601 0.00566 0.00563
## Cumulative Proportion 0.9198 0.92659 0.93306 0.93947 0.94548 0.95114 0.95678
##          PC53     PC54     PC55     PC56     PC57     PC58     PC59
## Standard deviation 6.06706 5.91805 5.91233 5.73539 5.47261 5.2921 5.02117
## Proportion of Variance 0.00539 0.00513 0.00512 0.00482 0.00438 0.0041 0.00369
## Cumulative Proportion 0.96216 0.96729 0.97241 0.97723 0.98161 0.9857 0.98940
##          PC60     PC61     PC62     PC63     PC64
## Standard deviation 4.68398 4.17567 4.08212 4.04124 2.148e-14
## Proportion of Variance 0.00321 0.00255 0.00244 0.00239 0.000e+00
## Cumulative Proportion 0.99262 0.99517 0.99761 1.00000 1.000e+00
```

```
plot(pr.out)
```



We can also plot the proportional variance explained (PVE) and the cumulative PVE for each principal component.

```
pve <- 100 * pr.out$sdev^2/sum(pr.out$sdev^2)
par(mfrow = c(1, 2))
plot(pve, type = "o", ylab = "PVE", xlab = "Principal Component", col = "blue")
plot(cumsum(pve), type = "o", ylab = "Cumulative PVE", xlab = "Principal Component ", col = "brown3")
```



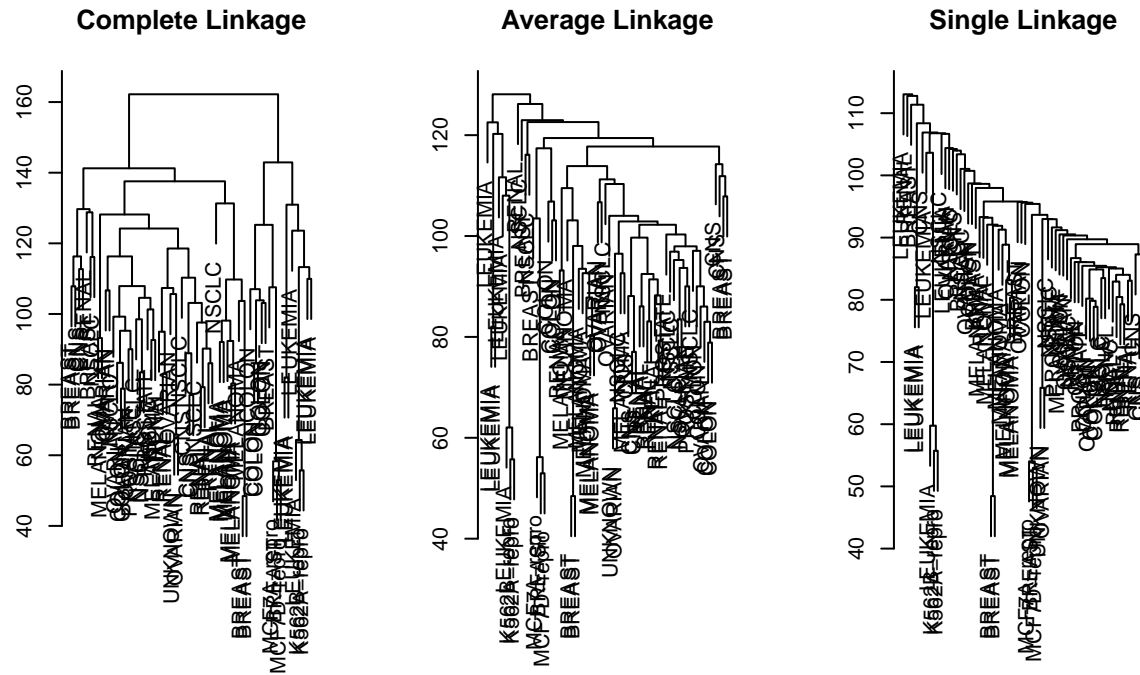
Clustering the Observations of the NCI60 Data

In this final exercise we use heirchical and K-means clustering on the NC160 dataset. We first scale the data to have a zero mean and standard deviation of one.

```
sd.data <- scale(nci.data)
```

We run heirchical clustering with different linakges and plot the results.

```
par(mfrow = c(1, 3))
data.dist <- dist(sd.data)
plot(hclust(data.dist), labels = nci.labs, main = "Complete Linkage", xlab = "", sub = "", ylab = "")
plot(hclust(data.dist, method = "average"), labels = nci.labs, main = "Average Linkage", xlab = "", sub = "", ylab = "")
plot(hclust(data.dist, method = "single"), labels = nci.labs, main = "Single Linkage", xlab = "", sub = "", ylab = "")
```



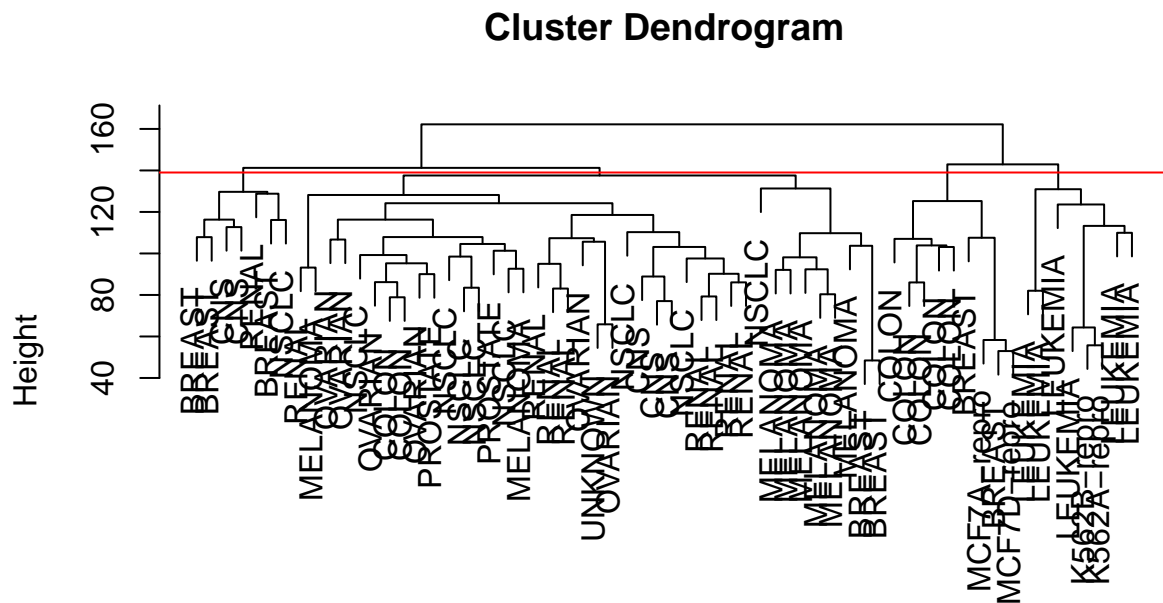
We cut the tree to give us four clusters using `cutree()`.

```
hc.out <- hclust(dist(sd.data))
hc.clusters <- cutree(hc.out, 4)
table(hc.clusters, nci.labs)
```

```
##           nci.labs
## hc.clusters BREAST CNS COLON K562A-repro K562B-repro LEUKEMIA MCF7A-repro
##           1      2   3     2           0           0           0           0
##           2      3   2     0           0           0           0           0
##           3      0   0     0           1           1           6           0
##           4      2   0     5           0           0           0           1
##
##           nci.labs
## hc.clusters MCF7D-repro MELANOMA NSCLC OVARIAN PROSTATE RENAL UNKNOWN
##           1           0           8     8           6           2     8           1
##           2           0           0     1           0           0     1           0
##           3           0           0     0           0           0     0           0
##           4           1           0     0           0           0     0           0
```

And plot the results with four clusters.

```
par(mfrow = c(1, 1))
plot(hc.out, labels = nci.labs)
abline(h = 139, col = "red")
```

```
dist(sd.data)
hclust(*, "complete")
```

We can get a summary of the result from the return value of `hclust()`.

```
hc.out
```

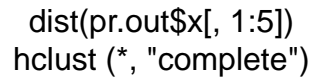
```
##
## Call:
## hclust(d = dist(sd.data))
##
## Cluster method   : complete
## Distance        : euclidean
## Number of objects: 64
```

For clustering the cancer types in four groups with K-means, we simply run `kmeans()` with `K = 4`.

```
set.seed(2)
km.out <- kmeans(sd.data, 4, nstart = 20)
km.clusters <- km.out$cluster
table(km.clusters, hc.clusters)
```

```
##           hc.clusters
## km.clusters  1  2  3  4
##           1 11  0  0  9
##           2 20  7  0  0
##           3  9  0  0  0
##           4  0  0  8  0
```

```
hc.out <- hclust(dist(pr.out$x[, 1:5]))
plot(hc.out, labels = nci.labs, main = "Hier. Clust. on First Five Score Vectors ")
```



```
##      nci.labs
##      BREAST  CNS  COLON  K562A-repro  K562B-repro  LEUKEMIA  MCF7A-repro  MCF7D-repro
##  1      0    2      7              0              0              2              0
##  2      5    3      0              0              0              0              0
##  3      0    0      0              1              1              4              0
##  4      2    0      0              0              0              0              1
##      nci.labs
##      MELANOMA  NSCLC  OVARIAN  PROSTATE  RENAL  UNKNOWN
##  1            1      8          5          2      7          0
##  2            7      1          1          0      2          1
##  3            0      0          0          0      0          0
##  4            0      0          0          0      0          0
```