Some Derivations for DVSO

Sen Zhang
The University of Sydney

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Abstract

I found that the derivation of the Jacobian of the virtual stereo objective is missing in the DVSO paper. Since this part is essential for the C++ implementation, I have done the derivation and give the details in this material.

1 Jacobian of the Virtual Stereo Objective

The virtual stereo objective presents:

$$E_i^{\dagger p} = w_p ||I_i[p^\dagger + [D^R(p^\dagger) \;\; 0]^T] - I_i[p]||_r \tag{1}$$

$$p^\dagger = K(IK^{-1}(p, d_p) + t_b). \tag{2}$$

By using the Gauss-Newton optimization method, we want to derive the derivative of $I_i[p^\dagger + [D^R(p^\dagger) \;\; 0]^T] - I_i[p]$ w.r.t. d_p. Since $\partial I_i[p]/\partial d_p = 0$, we only need to consider the derivative of the former term. Let $p^* = p^\dagger + [D^R(p^\dagger) \;\; 0]^T$, then we have:

$$\frac{\partial I_i[p^*]}{\partial d_p} = \frac{I_i[p^*]}{\partial p^*} \cdot \frac{\partial [p^\dagger + [D^R(p^\dagger) \;\; 0]^T]}{\partial d_p} \tag{3}$$

$$\frac{I_i[p^*]}{\partial p^*} = \begin{bmatrix} \frac{\partial I_i}{\partial p_x^*} & \frac{\partial I_i}{\partial p_y^*} \end{bmatrix}, \tag{4}$$

where we use local image gradients to approximate Eq. 4. The second derivative is derived as:

$$\frac{\partial [p^\dagger + [D^R(p^\dagger) \;\; 0]^T]}{\partial d_p} = \frac{\partial p^\dagger}{\partial d_p} + [\frac{\partial D^R(p^\dagger)}{\partial d_p} \;\; 0]^T \tag{5}$$

$$= \begin{bmatrix} \frac{\partial p_x^\dagger}{\partial d_p} \\ \frac{\partial p_y^\dagger}{\partial d_p} \end{bmatrix} + \begin{bmatrix} \frac{\partial D^R(p^\dagger)}{\partial d_p} \\ 0 \end{bmatrix}, \tag{6}$$

$$\frac{\partial D^R(p^\dagger)}{\partial d_p} = \begin{bmatrix} \frac{\partial D^R(p^\dagger)}{\partial p_x^\dagger} & \frac{\partial D^R(p^\dagger)}{\partial p_y^\dagger} \end{bmatrix} \begin{bmatrix} \frac{\partial p_x^\dagger}{\partial d_p} \\ \frac{\partial p_y^\dagger}{\partial d_p} \end{bmatrix}. \tag{7}$$

By inserting Eq. 7 into Eq. 6, we have:

$$\frac{\partial [p^\dagger + [D^R(p^\dagger) \;\; 0]^T]}{\partial d_p} = \begin{bmatrix} \frac{\partial p_x^\dagger}{\partial d_p} \\ \frac{\partial p_y^\dagger}{\partial d_p} \end{bmatrix} + \begin{bmatrix} \frac{\partial D^R(p^\dagger)}{\partial p_x^\dagger} \frac{\partial p_x^\dagger}{\partial d_p} + \frac{\partial D^R(p^\dagger)}{\partial p_y^\dagger} \frac{\partial p_y^\dagger}{\partial d_p} \\ 0 \end{bmatrix} \tag{8}$$

$$= \begin{bmatrix} (1 + \frac{\partial D^R(p^\dagger)}{\partial p_x^\dagger}) \frac{\partial p_x^\dagger}{\partial d_p} + \frac{D^R(p^\dagger)}{\partial p_y^\dagger} \frac{\partial p_y^\dagger}{\partial d_p} \\ \frac{\partial p_y^\dagger}{\partial d_p} \end{bmatrix} \tag{9}$$

$$= \begin{bmatrix} 1 + \frac{\partial D^R(p^\dagger)}{\partial p_x^\dagger} & \frac{\partial D^R(p^\dagger)}{\partial p_y^\dagger} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial p_x^\dagger}{\partial d_p} \\ \frac{\partial p_y^\dagger}{\partial d_p} \end{bmatrix}. \tag{10}$$

Therefore, by inserting Eq. 10 and Eq. 4 into Eq. 3, we have:

$$\frac{\partial I_i[p^*]}{\partial d_p} = \begin{bmatrix} \frac{\partial I_i}{\partial p^*_x} & \frac{\partial I_i}{\partial p^*_y} \end{bmatrix} \begin{bmatrix} 1 + \frac{\partial D^R(p^\dagger)}{\partial p^\dagger_x} & \frac{\partial D^R(p^\dagger)}{\partial p^\dagger_y} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial p^\dagger_x}{\partial d_p} \\ \frac{\partial p^\dagger_y}{\partial d_p} \end{bmatrix} \tag{11}$$

$$= \begin{bmatrix} (1 + \frac{\partial D^R(p^\dagger)}{\partial p^\dagger_x}) \frac{\partial I_i}{\partial p^*_x} & \frac{\partial D^R(p^\dagger)}{\partial p^\dagger_y} \frac{\partial I_i}{\partial p^*_x} + \frac{\partial I_i}{\partial p^*_y} \end{bmatrix} \begin{bmatrix} \frac{\partial p^\dagger_x}{\partial d_p} \\ \frac{\partial p^\dagger_y}{\partial d_p} \end{bmatrix}, \tag{12}$$

$$p^* = p^\dagger + [D^R(p^\dagger) \;\; 0]^T. \tag{13}$$

In the C++ implementation, we re-use the codes in DSO to compute $\frac{\partial p^\dagger_x}{\partial d_p}$ and $\frac{\partial p^\dagger_y}{\partial d_p}$.