Bilkent University
Department of Computer Engineering

IE400-Principles of Engineering Management

Term Project

Project Report

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Introduction

Santa wants to send presents to well behaved children and starts his journey where there are 30 villages in it. Distances between villages and probability of roads being out of use due to the snow are provided.

In this report, several different questions to help Santa deliver the presents are being discussed. For each problem, firstly, the problem is defined, then, a mathematical model is provided with explanation, and lastly, results given by a solver is shown.

Part A:

1. Problem Definition

In this part, Santa decides to choose 4 center villages to leave the gifts so that the parents from other villages can walk to the center villages. For parents to walk to the center villages, the longest distance that a parent should walk should be minimized. 4 center villages should be chosen considering the road parents should walk.

2. Mathematical Model

Parameters:

$$d_{ij} = \text{distance between villages } i \text{ and } j$$

Decision Variables:

$$x_{ij} = \begin{cases} 1 & \text{if village } i \text{ takes presents from village } j \\ 0 & \text{otherwise} \end{cases}$$

$$y_j = \begin{cases} 1 & \text{if village } j \text{ is a center village} \\ 0 & \text{otherwise} \end{cases}$$

$$f = \text{the longest distance that a parent should walk} \qquad f \in Z^+$$

Objective Function:

$$min\, f$$

Constraints:
s.t

The constraint given below indicates that f, which is in the objective function, should be greater or equal than all distances that each parent walks.

$$f \geq x_{ij} d_{ij} \qquad \forall i,j = 1, \dots, 30$$

The constraint given below shows that there are 4 villages that are considered as center villages.

$$\sum_{j=1}^{30} y_j = 4$$

The below constraint is to indicate that parents should not walk to a second center village, but they should also walk to one center village.

$$\sum_{j=1}^{30} x_{ij} = 1 \qquad \forall i = 1, \dots, 30$$

The last constraint is to ensure that the road to a center village from another village is valid.

$$y_j \geq x_{ij} \qquad \forall i,j = 1, \dots, 30$$

3. Results

```
Center Villages: 15 19 24 30

Villages that goes to  15 : 4 6 7 8 9 10 14

Villages that goes to  19 : 5 11 12 16 20 23 25 26 28

Villages that goes to  24 : 13 29

Villages that goes to  30 : 1 2 3 17 18 21 22 27
```

Figure 1: Screenshot of model results for part A

Our algorithm chooses villages 15, 19, 24 and 30 as the centers for this part. Also, maximum distance between a village and its center is calculated as 50.5.

Part B:

1. Problem Definition

In this part of the problem, an additional problem is added to the definition in Part A. This time, parents can walk the roads only if the probability that the road is out of use is less than 0.60. In addition to the distance parameter, we also have a probability parameter which indicates the probability that the road between village *i* and *j* is closed.

2. Mathematical Model

Parameters:

$d_{ij} =$ *distance between villages i and j*

$p_{ij} =$ *probability that the road is out of use due to snow*

Decision Variables:

$$x_{ij} = \begin{cases} 1 & if\ village\ i\ takes\ presents\ from\ village\ j \\ 0 & otherwise \end{cases}$$

$$y_j = \begin{cases} 1 & if\ village\ j\ is\ a\ center\ village \\ 0 & otherwise \end{cases}$$

$f =$ *the longest distance that a parent should walk* $\qquad f \in Z^+$

Objective Function:

$$min\ f$$

Constraints:

s.t

The constraint given below indicates that f, which is in the objective function, should be greater or equal than all distances that each parent walks.

$$f \geq x_{ij} d \qquad \forall i,j = 1, \dots, 30$$

The constraint given below shows that there are only 4 villages that are considered as center villages.

$$\sum_{j=1}^{30} y_j = 4$$

The below constraint is to indicate that parents should not walk to a second center village, but they should also walk to a center village.

$$\sum_{j=1}^{30} x_{ij} = 1 \qquad \forall i = 1, \dots, 30$$

The below constraint is to ensure that the road to a center village from another village is valid.

$$y_j \geq x_{ij} \qquad \forall i,j = 1, \dots, 30$$

Below constraint indicates that if the probability that a road is out of use is higher than 0.6, then a parent should not choose that road to walk.

$$p_{ij} x_{ij} \leq 0.6 \qquad \forall i,j = 1, \dots, 30$$

(Probability constraint)

3. Results

```
Center Villages: 9 13 20 24

Villages that goes to  9 : 1 8 12 15 21 25 27 30

Villages that goes to  13 : 3 7 10 16 18 26 28

Villages that goes to  20 : 4 5 6 11 14 19 23

Villages that goes to  24 : 2 17 22 29
```

Figure 2: Screenshot of model results for part B

Our algorithm chooses villages 9, 13, 20 and 24 as the centers for this part. Also, maximum distance between a village and its center is calculated as 54.5

Part C:

1. Problem Definition

In this part of the problem, Santa decides to visit all villages by himself. He travels with an average speed of 40 km/h. It is assumed that Santa is currently at village 1 and he does not use the roads where the probability that the road is out of use is greater than 0.60. The time it takes Santa to visit all villages are minimized so that he can return to village 1 at the end. This question is an example of a Travelling Salesman Problem with a minor tweak. In addition to the distance parameter, we also have a probability parameter which indicates the probability that the road between village *i* and *j* is closed.

2. Mathematical Model

Parameters:

$d_{ij} = $ *distance between villages i and j*

$p_{ij} = $ *probability that road between village i and j is closed*

Decision Variables:

$$x_{ij} = \begin{cases} 1 & if\ Santa\ goes\ from\ village\ i\ to\ j \\ 0 & otherwise \end{cases}$$

Objective Function:

$$min \sum_{i=1}^{30} \sum_{i \neq j, j=1}^{30} d_{ij} x_{ij}$$

Constraints:

$$\sum_{i=1, i \neq j}^{30} x_{ij} = 1$$ (Santa must enter each city once)

$$\sum_{j=1, j \neq i}^{30} x_{ij} = 1$$ (Santa must leave each city once)

Constraint below indicates that there is only a single complete tour which eliminates the subtouring problem.

$$y_i - (30 + 1) * x_{ij} \geq y_j - 30 \qquad 2 \leq i \neq j \leq 30$$

$$0 \leq y_i \leq 29 \qquad 2 \leq i \leq 30$$

(Eliminating subtours)

Constraint below indicates that Santa should not use the roads where the probability that the road is closed is greater than 0.6.

$$p_{ij} x_{ij} \leq 0.6 \qquad \forall i,j = 1, \ldots, 30$$

(Probability constraint)

3. Results

```
Route with total distance 1115 found: 1 -> 21 -> 11 -> 19 -> 2
6 -> 10 -> 2 -> 24 -> 29 -> 8 -> 7 -> 15 -> 3 -> 4 -> 23 -> 20
-> 5 -> 6 -> 28 -> 12 -> 16 -> 22 -> 18 -> 13 -> 9 -> 27 -> 25
-> 14 -> 17 -> 30 -> 1
```

Figure 3: Screenshot of model results for part C

Optimal route for Santa is as follows with respect to not using the roads that probability is higher than 0.6. It can be seen that Santa starts from village 1, travels to all the other villages and then returns back to village 1 again, his initial point.

Part D:

1. Problem Definition

In this part, while Santa visits other districts on the globe, volunteers can visit the villages and give gifts to the children with an average of 40km/h) . It is assumed that the volunteers are currently at village 1. There is a time limit (10 hours) for the volunteers to turn back to their initial positions. The number of volunteers should be found. This question is an example of a Distance Constrained Vehicle Routing Problem (DVRP). So, volunteers should not travel more than 400km which is equivalent to a 10 hour time limit for each traveller whose speeds are 40km/h.

2. Mathematical Model

Parameters:

$d_{ij} = $ *distance between villages i and j*

$s_{ij} = $ *spare distance capacity a volunteer has after covering the link to j*

Decision Variables:

$$x_{ij} = \begin{cases} 1 \; if \; volunteer \; uses \; the \; road \; between \; village \; i \; and \; j \\ 0 \quad\quad\quad\quad\quad\quad otherwise \end{cases}$$

$m = Number\ of\ volunteers$

$$m \in Z^{++}$$

Objective Function:

$$min\ m$$

Constraints:

$$\sum_{i=1}^{30} x_{ij} = 1;$$

$$\forall j = 1, \dots, 30$$

(Volunteers must enter each city once)

$$\sum_{j=1}^{n} x_{ij} = 1;$$

$$\forall i = 1, \dots, 30$$

(Volunteers must exit each city once)

$$\sum_{j=1}^{30} x_{1j} = m$$ (m volunteers leave from village 1)

$$\sum_{i=1}^{30} x_{i1} = m$$ (m volunteers return to the village 1)

Constraint below is the distance and subtour elimination constraint, it guarantees that the solution contains no illegal subtours, while also balancing inflow and outflow at each node.

$$\sum_{j=1, j \neq i}^{30} S_{ij} - \sum_{j=1, j \neq i}^{30} S_{ji} + \sum_{j=1}^{30} d_{ij} x_{ij} = 0$$

$$\forall i = 1, \dots, 30$$

Three constraints below together impose $0 \leq s_{ij} \leq 400$ and ensure that the distance traveled up to the village 1 does not exceed the predetermined value 400 which ensures that 10 hour time limit is not exceeded.

$$S_{ij} \leq 400\, x_{ij}$$
$$\forall i,j = 1, \ldots, 30$$
$$S_{1j} = 400\, x_{ij}\, x_{1j} - d_{1j}\, x_{1j}$$
$$\forall j = 1, \ldots, 30$$

$$S_{ij} \geq 0$$
$$\forall i,j = 1, \ldots, 30$$

3. Results

The result which is shown below indicates that minimum 3 volunteers are needed to fulfill the task not exceeding 10 hours time limits. The 10 hours time limit means that all volunteers should travel less or equal 400 km. Our model can give the minimum number of volunteers, their distance of the route and maximum of the route distance among them.. We know that the question asks us the minimum number of volunteers; however, firstly we did understand the minimum number of volunteers and minimum number of distances. That's why we conduct this model and then we make some changes; however, we want to share our not necessarily results. Our model works with a time limit which is determined by us to find the optimal solution. It finds a lot of optimal solutions for maximum route distance in this determined time and compares them and give us the most optimal one end of the time-limit.

```
Fail! Number of Valunteers :  1
Fail! Number of Valunteers :  2
Sucsess! Number of Valunteers :  3
Route for volunteer 1:
 1 ->  9 ->  13 ->  12 ->  28 ->  25 ->  8 ->  7 ->  15 ->  11 -> 1
Distance of the route: 376km

Route for volunteer 2:
 1 ->  2 ->  16 ->  18 ->  22 ->  17 ->  14 ->  6 ->  27 ->  19 ->  26 ->  10 -> 1
Distance of the route: 389km

Route for volunteer 3:
 1 ->  30 ->  3 ->  24 ->  29 ->  5 ->  20 ->  23 ->  4 ->  21 -> 1
Distance of the route: 375km

Maximum of the route distances: 389km
```

Figure 4: Screenshot of model results for part D

The routes below are constructed when 3 volunteers are used. It can be seen that none of the volunteers exceed the 10 hour time limit because all their distances are less than 400km. Sec means how long our model works to find the most optimal solution to minimize the maximum of the route distance. Actually, we execute the model until 600 secs (10 minutes); however, they always give the same result after 100. For 20 seconds execution it gives 389km, For 100 seconds execution it gives 387km and for longer than 100 seconds it gives 383km as a maximum of the route distance. As a result, Santa should select 3 volunteers so that every village is visited without exceeding the time constraint.

```
Sec:  100
Route for volunteer 1:
 1 ->  11 ->  19 ->  26 ->  13 ->  12 ->  28 ->  25 ->  8 ->  7 ->  15 -> 1
Distance of the route: 387km

Route for volunteer 2:
 1 ->  2 ->  16 ->  18 ->  22 ->  17 ->  14 ->  6 ->  27 ->  9 ->  21 -> 1
Distance of the route: 364km

Route for volunteer 3:
 1 ->  30 ->  3 ->  4 ->  23 ->  20 ->  5 ->  29 ->  24 ->  10 -> 1
Distance of the route: 383km

Maximum of the route distances: 387km
```

```
Sec:  200
Route for volunteer 1:
 1 ->  2 ->  16 ->  22 ->  18 ->  13 ->  12 ->  26 ->  19 ->  11 ->  21 -> 1
Distance of the route: 382km

Route for volunteer 2:
 1 ->  15 ->  7 ->  8 ->  25 ->  28 ->  17 ->  14 ->  6 ->  27 ->  9 -> 1
Distance of the route: 379km

Route for volunteer 3:
 1 ->  30 ->  3 ->  4 ->  23 ->  20 ->  5 ->  29 ->  24 ->  10 -> 1
Distance of the route: 383km

Maximum of the route distances: 383km
```

```
Sec:  300
Route for volunteer 1:
 1 ->  2 ->  16 ->  22 ->  18 ->  13 ->  12 ->  26 ->  19 ->  11 ->  21 -> 1
Distance of the route: 382km

Route for volunteer 2:
 1 ->  15 ->  7 ->  8 ->  25 ->  28 ->  17 ->  14 ->  6 ->  27 ->  9 -> 1
Distance of the route: 379km

Route for volunteer 3:
 1 ->  30 ->  3 ->  4 ->  23 ->  20 ->  5 ->  29 ->  24 ->  10 -> 1
Distance of the route: 383km

Maximum of the route distances: 383km
```

Figure 5: Results for Different Seconds Values