BILKENT UNIVERSITY

COMPUTER ENGINEERING

EEE391-BASICS OF SIGNALS AND SYSTEMS

MATLAB ASSIGNMENT 2

Sena Korkut

21703303

16/12/2020

Introduction

In this report, pole-zero diagrams and frequency responses of several different systems will be plotted along with the discussion.

To represent the infinite impulse response (IIR) filter, the following system function will be used:

$$H(z) = \frac{c_0+c_1 z+c_2 z^2+....+c_{M-1} z^{M-1}+c_M z^M}{d_0+d_1 z+d_2 z^2+....+d_{N-1} z^{N-1}+z^N}$$

(1)

Where $Y(z) = H(z)X(z)$, therefore, $H(z) = {}^{Y(z)}\!/_{X(z)}$

Part A:

In this part, a MATLAB function is written in order to plot the pole-zero diagram of an arbitrary IIR system. This function uses the Equation (1) as the system function. The function has the form pole_zero_plot(c, d) where c is a vector defined as: $c = [c_0, c_1, c_2, \dots, c_{M-1}, c_M]$ and d is a vector defined as: $d = [d_0, d_1, d_2, \dots, d_{N-1}]$.

For the representation of poles and zeros that make the $z = \infty$, upper right hand corner of the z-plane is used. When all the poles and zeros are counted, they should balance out in number, that is, the number of poles should be equal the number of zeros. Therefore, poles and zeros for $z = \infty$ are calculated with the following formulas:

Let $(\#of\ numerator\ roots) - (\#of\ denominator\ roots) = a$

Let $(\#of denominator\ roots) - (\#of\ numerator\ roots) = b$

$$at\ z = \infty, \#\ of\ Poles\ = \begin{cases} a, & a > 0 \\ 0, & a \leq 0 \end{cases} \tag{2}$$

$$at\ z = \infty, \#\ of\ Zeros\ = \begin{cases} b, & b > 0 \\ 0, & b \leq 0 \end{cases} \tag{3}$$

Part B:

In this part, a MATLAB function is written in order to plot the magnitude and phase responses of an arbitrary IIR system. This function uses the Equation (1) as the system function. The function has the form plot_freq_response(c, d) where c is a vector defined as: $c = [c_0, c_1, c_2, \ldots, c_{M-1}, c_M]$ and d is a vector defined as: $d = [d_0, d_1, d_2, \ldots, d_{N-1}]$.

For the magnitude response of the system, following equation is used by putting $z = e^{j\omega}$ into Equation (1) and then taking the magnitude of the function.

$$\left|H(e^{j\omega})\right| = \left|\frac{c_0 + c_1 e^{j\omega} + c_2 {e^{j\omega}}^2 + \ldots + c_{M-1} {e^{j\omega}}^{M-1} + c_M {e^{j\omega}}^M}{d_0 + d_1 e^{j\omega} + d_2 {e^{j\omega}}^2 + \ldots + d_{N-1} {e^{j\omega}}^{N-1} + {e^{j\omega}}^N}\right|$$

For the phase response of the system, following formulas are used by putting $z = e^{j\omega}$ into Equation (1) and then converting to polar form.

$$\angle H(e^{j\omega}) = \tan^{-1}\left(\frac{Im\{H(e^{j\omega})\}}{Re\{H(e^{j\omega})\}}\right)$$

Note: MATLAB codes for the functions in Part A and Part B can be found in Appendix A.

Part C:

In this part, several different system functions will be analyzed and pole-zero, magnitude and phase response graphs will be plotted by using the functions in Part A and Part B.

System A:

The function for system A is:

$$y[n] = y[n-1] + x[n] + x[n-1]$$

Taking the z transform of both sides,

$$Y(z) = (1)z^{-1}Y(z) + (1)X(z) + (1)z^{-1}X(z)$$
$$= \,(1 + \,(1)z^{-1})\,X(z)$$
$$\rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-1}}{1 - z^{-1}}$$

The above H(z) is the system function of A. To make it similar to the Equation (1), it is multiplied by $^{z}/_{z}$, therefore, the resulting function is,

$$H(z)\frac{z}{z} = \frac{z+1}{z-1} \tag{4}$$

The vectors that are entered to the function parameters in Part A and Part B can be concluded as:

$$c = [1\,, 1]$$
$$d = [-1\,, 1]$$

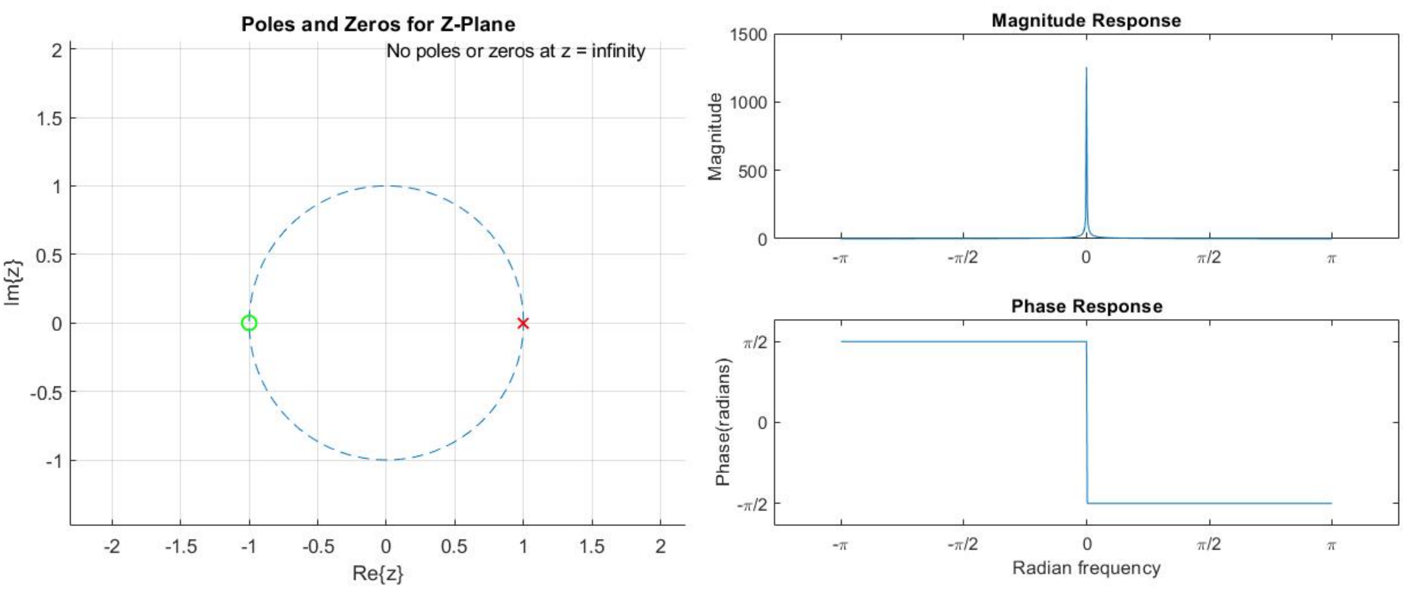

Figure 1: Poles-Zeros and Frequency Response Plots of System A

Poles and zeros are scattered at poles and zeros graph of z-plane in *Figure 1*. The zeros, marked by ‘o’ represents the root of numerator and the poles, marked by ‘x’ represents the root of denominator. The values and count of the roots of numerator and denominator parts of Equation(4) can be seen in the below table.

NUMERATOR	ROOT VALUES	ROOT COUNT
	-1	1

DENOMINATOR	ROOT VALUES	ROOT COUNT
	1	1

Table 1:Numerator and Denominator Root Values and Counts of System A

Moreover, because there is a balance between poles and zeros, calculated by using Equation (2) and (3), there is no zero or pole at $z = \infty$.

System B:

The function for system B is:

$$H(z) = (10z^2 - 1)z$$

Can be written as,

$$H(z) = 10z^3 + 0z^2 - z + 0 \tag{5}$$

The vectors that are entered to the function parameters in Part A and Part B can be concluded as:

$$c = [\,0\,, -1, 0\,, 10\,]$$

$$d = [1]$$

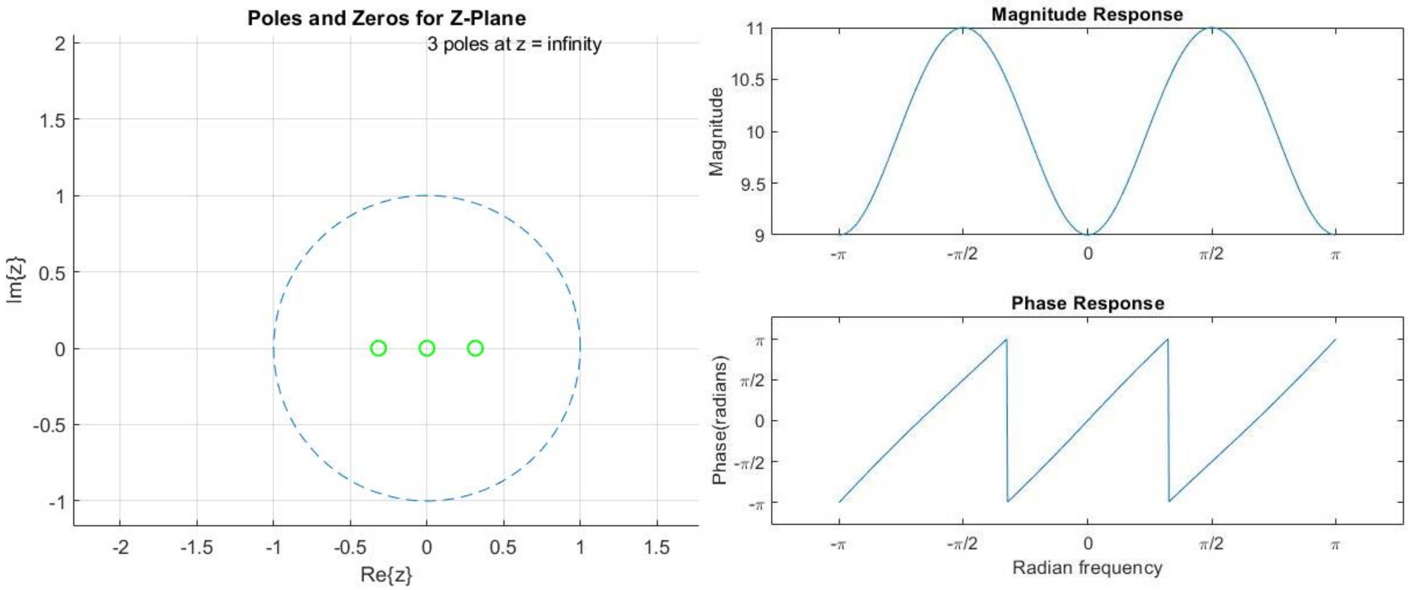

Figure 2: Poles-Zeros and Frequency Response Plots of System B

Zeros are scattered at poles and zeros graph of z-plane in *Figure 2*. The zeros, marked by 'o', represents the roots of numerator. The values and count of the roots of numerator part of Equation(5) can be seen in the below table.

NUMERATOR	ROOT VALUES	ROOT COUNT
	-0.316	1
	0	1
	0.316	1

Table 2:Numerator and Denominator Root Values and Counts of System B

Moreover, because there is no balance between poles and zeros, calculated by using Equation (2) and (3), there are three poles at $z = \infty$.

System C:

The function for system C is:

$$H(z) = \left(1 - \frac{\sqrt{2}}{2}(1+j)z^{-1}\right)\left(1 - \frac{\sqrt{2}}{2}(1-j)z^{-1}\right)$$

Can be written as,

$$H(z) = 1 - \frac{\sqrt{2}}{2}(1+j)z^{-1} - \frac{\sqrt{2}}{2}(1-j)z^{-1} + z^{-2}$$

$$= 1 - \frac{\sqrt{2}}{2}z^{-1}(1 + j + 1 - j) + z^{-2}$$

$$= 1 - \sqrt{2}z^{-1} + z^{-2}$$

The above H(z) is the system function of A. To make it similar to the Equation (1), it is multiplied by ${z^2}/{z^2}$, therefore, the resulting function is,

$$H(z)\frac{z^2}{z^2} = \frac{z^2 - \sqrt{2}\,z + 1}{z^2} \tag{6}$$

The vectors that are entered to the function parameters in Part A and Part B can be concluded as:

$$c = [\,1, -\sqrt{2}\,, 1]$$

$$d = [0\,, 0\,, 1]$$

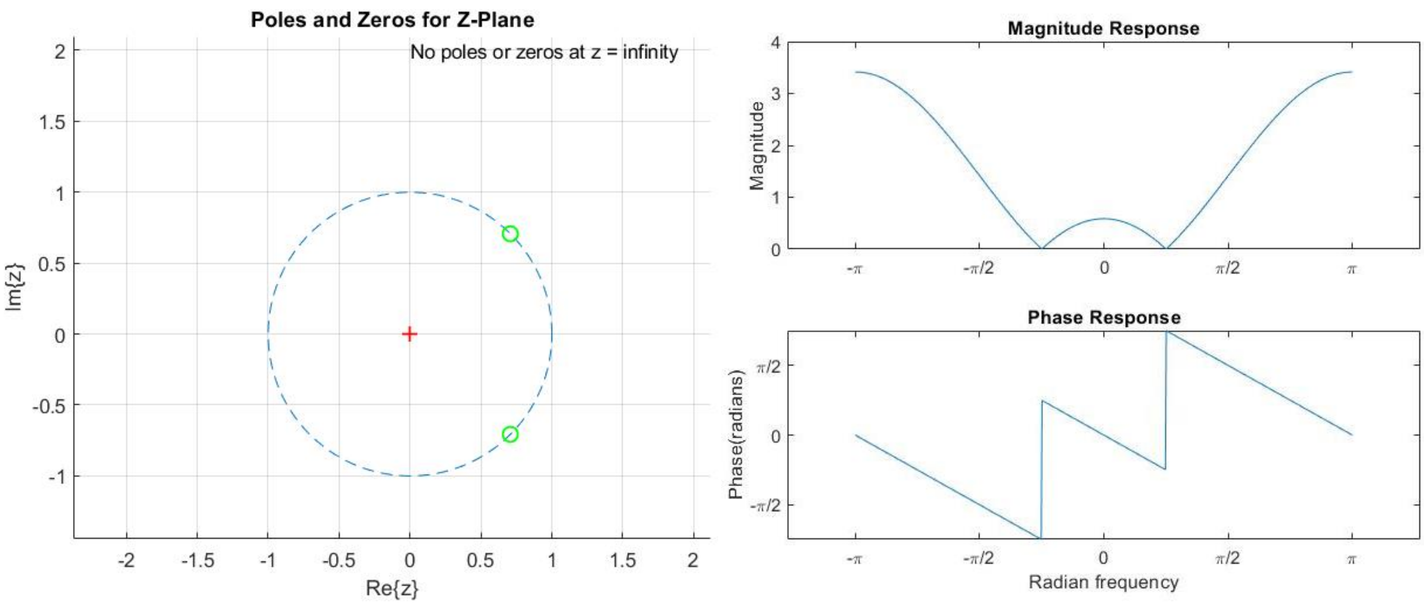

Figure 3: Poles-Zeros and Frequency Response Plots of System C

Poles and zeros are scattered at poles and zeros graph of z-plane in *Figure 1*. The zeros, marked by ‘o’ represents the roots of numerator and the poles, marked by ‘+’ represents the double root of denominator. The values and count of the roots of numerator and denominator parts of Equation(6) can be seen in the below table.

NUMERATOR	ROOT VALUES	ROOT COUNT
	0.707-0.707i	1
	0.707+0.707i	1

DENOMINATOR	ROOT VALUES	ROOT COUNT
	0	2

Table 3:Numerator and Denominator Root Values and Counts of System C

Moreover, because there is a balance between poles and zeros, calculated by using Equation (2) and (3), there is no zero or pole at $z = \infty$.

System D:

The function for system D is:

$$H(z) = \frac{z^3 + 7z^2 - 5z - 75}{2z^2 - 7z + 3}$$

After the normalization of the denominator, the final function for the system is

$$H(z) = \frac{{}^{1}/_{2}\, z^3 + {}^{7}/_{2}\, z^2 - {}^{5}/_{2}\, z - {}^{75}/_{2}}{z^2 - {}^{7}/_{2}\, z + {}^{3}/_{2}} \tag{7}$$

The vectors that are entered to the function parameters in Part A and Part B can be concluded as:

$$c = [-{}^{75}/_{2}, -{}^{5}/_{2}, {}^{7}/_{2}, {}^{1}/_{2}]$$
$$d = [1, -{}^{7}/_{2}, {}^{3}/_{2}]$$

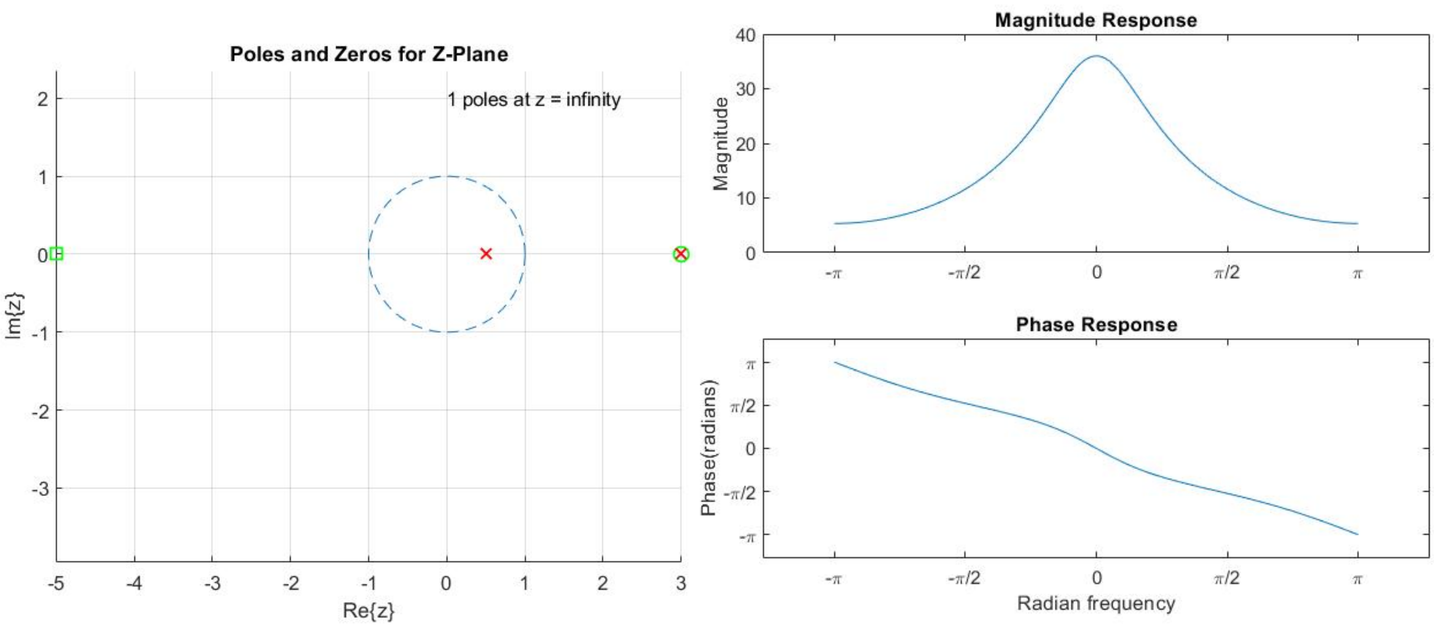

Figure 4: Poles-Zeros and Frequency Response Plots of System D

Poles and zeros are scattered at poles and zeros graph of z-plane in *Figure 1*. The zeros, marked by ‘o’ and ‘□’represents the single and double roots of numerator respectively and the poles, marked by ‘x’ represents the root of denominator. The values and count of the roots of numerator and denominator parts of Equation(7) can be seen in the below table.

NUMERATOR	ROOT VALUES	ROOT COUNT
	-5	2
	3	1

DENOMINATOR	ROOT VALUES	ROOT COUNT
	0.5	1
	3	1

Table 4:Numerator and Denominator Root Values and Counts of System D

Moreover, because there is no balance between poles and zeros, calculated by using Equation (2) and (3), there is one pole at $z = \infty$.

System E:

The function for system E is:

$$h[n] \; = \frac{1}{3}\, \delta[n-1]$$

Taking the z transform of both sides,

$$H(z) \; = \frac{1}{3}\, z^{-1}$$

The above H(z) is the system function of A. To make it similar to the Equation (1), it is multiplied by z/z, therefore, the resulting function is,

$$H(z)\frac{z}{z} \; = \frac{1}{3z}$$

After the normalization of the denominator, the final function for the system is

$$H(z) = \frac{1/3}{z} \tag{8}$$

The vectors that are entered to the function parameters in Part A and Part B can be concluded as:

$$c = [\ ^1/_3\ ]$$

$$d = [\ 0, 1\ ]$$

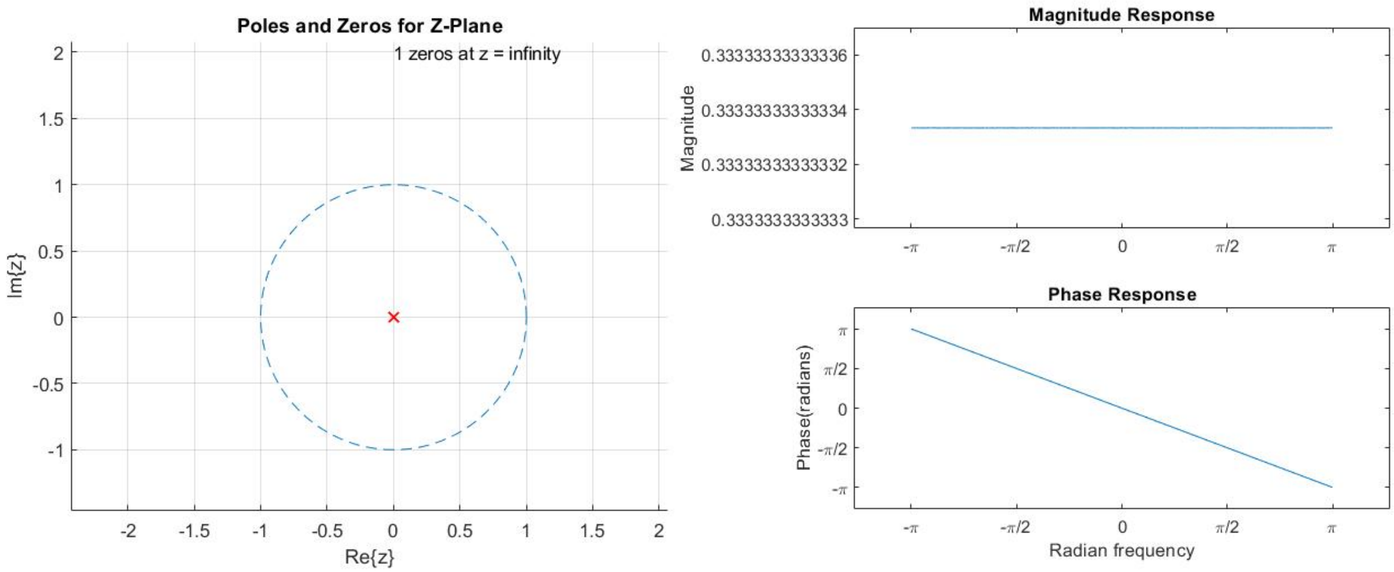

Figure 5: Poles-Zeros and Frequency Response Plots of System E

Poles are scattered at poles and zeros graph of z-plane in *Figure 2*. The pole, marked by ‘x’, represents the roots of denominator. The values and count of the roots of denominator part of Equation(8) can be seen in the below table.

DENOMINATOR	ROOT VALUES	ROOT COUNT
	1	1

Table 5: Denominator Root Values and Counts of System E

Moreover, because there is no balance between poles and zeros, calculated by using Equation (2) and (3), there is one zero at $z = \infty$.

Note: The screenshots of the results for the tables can be found in Appendix B.

Appendix

Appendix A: Full MATLAB Code

```
clc
clear
close all
prompt ="Choose system to plot";
prompt = prompt+newline+"1-System A";
prompt = prompt+newline+"2-System B";
prompt = prompt+newline+"3-System C";
prompt = prompt+newline+"4-System D";
prompt = prompt+newline+"5-System E"+newline;
choice = input(prompt);
if (choice == 1)
    c = [1 1 0 0];
    d = [-1 1 0 0];
end
if (choice == 2)
    c = [0 -1 0 10];
    d = [1 0 0 0];
end
if (choice == 3)
    c = [1 -sqrt(2) 1 0];
    d = [0 0 1 0];
end
if (choice == 4)
    c = [-75/2 -5/2 7/2 1/2];
    d = [3/2 -7/2 1 0];
end
if (choice == 5)
    c = [1/3 0 0 0];
    d = [0 1 0 0];
end
%pole_zero_plot(c,d);
plot_freq_response(c,d);

function pole_zero_plot(c,d)
    %Draw unit circle
    angle = 0:2*pi/500:2*pi;
    R = 1;
    x = R*cos(angle);
    y = R*sin(angle);
    hold on
    plot(x,y, '--');
    axis equal;
    grid on;
    xlabel('Re\{z\}');
    ylabel('Im\{z\}');
    title('Poles and Zeros for Z-Plane');
```

```
    %Find the roots of the functions with coefficients c and d
    clear roots
    %Plot zeros
    coefs = flip(c);
    r_c = roots(coefs);
    r_c = round(r_c,3);
    [root_count,unique_roots] = groupcounts(r_c);
    for root = 1:length(unique_roots)
        if(root_count(root) == 1)
            scatter(real(unique_roots(root)),imag(unique_roots(root)),
60, 'g', 'o','LineWidth',1.1);
        elseif(root_count(root) == 2)
            scatter(real(unique_roots(root)),imag(unique_roots(root)),
60, 'g', 's','LineWidth',1.1);
        else
            scatter(real(unique_roots(root)),imag(unique_roots(root)),
60, 'g', 'filled','LineWidth',1.1);
        end
    end
    %Plot poles
    coefs = flip(d);
    r_d = roots(coefs);
    r_d = round(r_d,3);
    [root_count,unique_roots] = groupcounts(r_d);
    for root = 1:length(unique_roots)
        if(root_count(root) == 1)
            scatter(real(unique_roots(root)),imag(unique_roots(root)),
60, 'r', 'x','LineWidth',1.1);
        elseif(root_count(root) == 2)
            scatter(real(unique_roots(root)),imag(unique_roots(root)),
60, 'r', '+','LineWidth',1.1);
        else
            scatter(real(unique_roots(root)),imag(unique_roots(root)),
60, 'r', '*','LineWidth',1.1);
        end
    end
    if (length(r_d) == length(r_c))
       text(0,2,'No poles or zeros at z = infinity');
    end
    if (length(r_d) < length(r_c))
       text(0,2,sprintf('%d poles at z = infinity', length(r_c) -
length(r_d)));
    end
    if (length(r_c) < length(r_d))
       text(0,2,sprintf('%d zeros at z = infinity', length(r_d) -
length(r_c)));
    end

    hold off
end

function plot_freq_response(c,d)
```

```
    rad_freq = -pi:0.01:pi;
    H_func = @(x) (c(1) + c(2)*x + c(3)*x.^2 + c(4)*x.^3)./(d(1) +
d(2)*x + d(3)*x.^2 +d(4)*x.^3);
    %Plot frequency response
    subplot(2,1,1);
    %Plot magnitude
    plot(rad_freq,abs(H_func(exp(1i*rad_freq))));
    xticks([ -pi -pi/2 0 pi/2 pi ]);
    xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
    ylabel('Magnitude');
    title('Magnitude Response');
    subplot(2,1,2);
    %Plot phase
    plot(rad_freq,angle(H_func(exp(1i*rad_freq))));
    xlabel('Radian frequency');
    xticks([ -pi -pi/2 0 pi/2 pi ]);
    xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
    ylabel('Phase(radians)');
    yticks([ -pi -pi/2 0 pi/2 pi ]);
    yticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
    title('Phase Response');

end
```

Appendix B:Screenshots of Root Values and Counts:

```
root_count =                root_count =

     1                           1


unique_roots =              unique_roots =

    -1                           1
```

Screenshot 1: Root Values as unique_roots and Corresponding Counts as root_count for c and d coefficients of System A respectively

```
root_count =

     1
     1
     1


unique_roots =

   -0.3160
         0
    0.3160
```

```
root_count =

  0×1 empty double column vector


unique_roots =

  0×1 empty double column vector
```

Screenshot 2: Root Values as unique_roots and Corresponding Counts as root_count for c and d coefficients of System B respectively

```
root_count =

     1
     1


unique_roots =

   0.7070 - 0.7070i
   0.7070 + 0.7070i
```

```
root_count =

     2


unique_roots =

     0
```

Screenshot 3: Root Values as unique_roots and Corresponding Counts as root_count for c and d coefficients of System C respectively

```
root_count =

     2
     1


unique_roots =

    -5
     3
```

```
root_count =

     1
     1


unique_roots =

    0.5000
    3.0000
```

Screenshot 4: Root Values as unique_roots and Corresponding Counts as root_count for c and d coefficients of System D respectively

```
root_count =

  0×1 empty double column vector


unique_roots =

  0×1 empty double column vector
```

```
root_count =

     1


unique_roots =

     0
```

Screenshot 5: Root Values as unique_roots and Corresponding Counts as root_count for c and d coefficients of System E respectively