

2.2 Consider an outcome  $Y$  with 3 classes A, B, and C. Further, assume  $x \sim \text{Uniform}(0, 1)$ .  $Y$  is a function of  $x$  as follows:

$$\left. \begin{array}{l} P(Y=A) = 0.4 \\ P(Y=B) = 0.3 \\ P(Y=C) = 0.3 \end{array} \right\} \text{if } x > 0.6 \quad \left. \begin{array}{l} P(Y=A) = 0 \\ P(Y=B) = 0.2 \\ P(Y=C) = 0.8 \end{array} \right\} \text{if } x \leq 0.6$$

What is the Bayes classifier? What is the Bayes error?

Q.2.2. want to choose  $j$  s.t. we find  $\max_j P(Y=j|x)$ .  
i.e. given the data, want to maximize the probability.

we have, for  $x \leq 0.6$ , that  $\max_j P(Y=j|x) \Leftrightarrow j = C$ , as  $P(Y=C) = 0.8$  when  $x \leq 0.6$ .  
 $x > 0.6$ , that  $\max_j P(Y=j|x) \Leftrightarrow j = A$ , as  $P(Y=A) = 0.4$  when  $x > 0.6$ .

$$\therefore \text{BayesClassifier}(x) = \begin{cases} C, & x \leq 0.6 \\ A, & x > 0.6 \end{cases}$$

the Bayes error is thus given by  $1 - E(\max_j P(Y=j|x))$

$$\begin{aligned} \text{so, when } x \leq 0.6, \text{ Bayes Error} &= (P(Y=A) + P(Y=B)) \cdot 0.6 && \text{Since } x \text{ is uniform.} \\ &= 0.2 \cdot 0.6 = 0.12 \\ x > 0.6, \text{ Bayes Error} &= (P(Y=B) + P(Y=C)) \cdot (1 - 0.6) \\ &= (0.3 + 0.3) \cdot 0.4 = 0.24 \end{aligned}$$

$$\begin{aligned} \text{thus, the Bayes Error Rate} &= 0.12 + 0.24 \\ &= 0.36 \end{aligned}$$

$$\therefore \text{BayesClassifier}(x) = \begin{cases} C, & x \leq 0.6 \\ A, & x > 0.6 \end{cases}$$

with a Bayes Error Rate of 0.36. #