ASSIGNMENT-2

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1 Questions:-

Find the inverse and QR decomposition of the following.

1.1

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \tag{1}$$

Solution:-

1.1.1 Inverse:-

Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

be a full-rank 2×2 matrix. Then $\det A \equiv |A| = 2 \times 1 - 1 \times 1 = 1$ and

$$A^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{-1} = \frac{1}{|A|} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}.$$

Therefore inverse of A=$\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

1.1.2 QR Decomposition:-

Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

with the vectors a1 = $(2,1)^T$, a2 = $(1,1)^T$
Note that all the vectors considered above and below are column vectors. From now on, I will drop T notation for simplicity, but we have to remember that all the vectors are column vectors. Performing the Gram-Schmidt procedure, we obtain:
$u_1 = a_1$
e_1=$\frac{u_1}{|u_1|}$
$u_2 = a_2$ - $(a_2 \cdot e_1) \cdot e_1$
e_2=$\frac{u_2}{|u_2|}$
Thus

$$Q = \begin{bmatrix} 0.894 & -0.447 \\ 0.447 & 0.894 \end{bmatrix} \tag{2}$$

$$R = \begin{bmatrix} 2.236 & 1.342 \\ 0.000 & 0.447 \end{bmatrix} \tag{3}$$

1.2

$$\begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix} \tag{4}$$

Solution:- Let

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

be a full-rank 2×2 matrix. Then $\det A \equiv |A| = 1 \times 7 - 3 \times 2 = 1$ and

$$A^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}^{-1} = \frac{1}{|A|} \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}.$$

Therefore inverse of A=$\begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$

1.2.1 QR Decomposition:-

Let

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

with the vectors a1 = $(1,2)^T$, a2 = $(3,7)^T$
Performing the Gram-Schmidt procedure, we obtain:

$u_1 = a_1$
e_1=$\frac{u_1}{|u_1|}$
u_2 = a_2 - $(a_2 \cdot e_1) \cdot e_1$
e_2=$\frac{u_2}{|u_2|}$
Thus

$$Q = \begin{bmatrix} 0.447 & -0.894 \\ 0.894 & 0.447 \end{bmatrix} \tag{5}$$

$$R = \begin{bmatrix} 2.236 & 7.603 \\ 0.000 & 0.447 \end{bmatrix} \tag{6}$$