ASSIGNMENT-2

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1 Questions:-

Find the inverse and QR decomposition of the following.

1.1

$$\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \tag{1}$$

Solution:-

1.1.1 Inverse:-

Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

be a full-rank 2×2 matrix. Then $\det A \equiv |A| = 2 \times 1 - 1 \times 1 = 1$ and

$$A^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}^{-1} = \frac{1}{|A|} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}.$$

Therefore inverse of A=$\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

1.1.2 QR Decomposition:-

Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

The column vectors of the matrix is given by,

$$a = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{2}$$

Therefore,

$$\begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} q_1 & q_2 \end{bmatrix} \begin{bmatrix} u_1 & u_3 \\ 0 & u_2 \end{bmatrix} = QR \tag{3}$$

$$u_1 = || \ a \ || = \sqrt{2^2 + 1^2} = \sqrt{5} \tag{4}$$

$q_1 = \frac{a}{u_1} = \begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix}$
$u_3 = q_1^T \text{b} = \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{3}{\sqrt{5}}$
x=b-$u_3 q_1$
$q_2 = \frac{x}{||x||} = \begin{bmatrix} \frac{-1}{5\sqrt{5}} \\ \frac{2}{5\sqrt{5}} \end{bmatrix}$
$u_2 = q_2^T \text{b} = \begin{bmatrix} \frac{1}{5\sqrt{5}} \end{bmatrix}$
Therefore,

$$Q = \begin{bmatrix} \frac{2}{\sqrt{5}} & \frac{-1}{5\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{5\sqrt{5}} \end{bmatrix} \tag{5}$$

$$R = \begin{bmatrix} \sqrt{5} & \frac{3}{5\sqrt{5}} \\ 0 & \frac{1}{5\sqrt{5}} \end{bmatrix} \tag{6}$$

1.2

$$\begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix} \tag{7}$$

Solution:- Let

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

be a full-rank 2×2 matrix. Then $\det A \equiv |A| = 1 \times 7 - 3 \times 2 = 1$ and

$$A^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}^{-1} = \frac{1}{|A|} \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}.$$

Therefore inverse of A=$\begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$

1.2.1 QR Decomposition:-

Let

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

The column vectors of the matrix is given by,

$$a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, b = \begin{bmatrix} 3 \\ 7 \end{bmatrix} \tag{8}$$

Therefore,

$$\begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} q_1 & q_2 \end{bmatrix} \begin{bmatrix} u_1 & u_3 \\ 0 & u_2 \end{bmatrix} = QR \tag{9}$$

$$u_1 = \| a \| = \sqrt{2^2 + 1^2} = \sqrt{5} \tag{10}$$

$q_1 = \frac{a}{u_1} = \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix}$

$u_3 = q_1^T$b$= \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \frac{17}{\sqrt{5}}$

x=b-$u_3 q_1$

$q_2 = \frac{x}{||x||} = \begin{bmatrix} \frac{-2}{5\sqrt{5}} \\ \frac{1}{5\sqrt{5}} \end{bmatrix}$

$u_2 = q_2^T$b$= \begin{bmatrix} \frac{1}{5\sqrt{5}} \end{bmatrix}$

Therefore,

$$Q = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{-2}{5\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{5\sqrt{5}} \end{bmatrix} \tag{11}$$

$$R = \begin{bmatrix} \sqrt{5} & \frac{17}{5\sqrt{5}} \\ 0 & \frac{1}{5\sqrt{5}} \end{bmatrix} \tag{12}$$