Indian Institute of Information Technology Sri City, Chittoor

Theory of Computation – Spring 2023

End Examination

Duration: 90 Minutes | Maximum Marks : 60

- It is a closed book exam.
- No electronic devices, books, any kind of material is allowed.

1. (a) Show that $A_{TM} \leq_m HALT_{TM}$ where $A_{TM} = \{< M, w > \mid M \text{ accepts } w\}$ and $HALT_{TM} = \{< M, w > \mid M \text{ halts on } w\}$. You need to give a clear and precise mapping reducibility from A_{TM} to $HALT_{TM}$ [8 Mark]

   (b) Design Turing Machine for the following language L = $\{a^n b^n \mid n \geq 1\}$. [6 Mark]

   (c) Which of the following problems are decidable or undecidable? [6 Mark]

   (i) Given NFAs N1 and N2, is $L(N1) \cap L(N2) = \varnothing$?

   (ii) If L is a recursive language, is the complement of L is recursive?

   (iii) Given a Turing Machine M, does $L(M) = \varnothing$?

   (iv) Given a CFG G= (V, T, P, S) and a string $x \in \Sigma^*$, does $x \in L(G)$?

   (v) Does a Turing Machine halt on taking particular input?

   (vi) Given a Turing machine M, decide if M takes more than 90 steps on every input string w?

2. (a) Disprove $(A \leq B) \Rightarrow (A \leq_m B)$. You need to give a counter-example such that LHS is true but RHS is false. [7 Marks]

   (b) Prove or disprove $(A \leq_m B) \Rightarrow (A \leq B)$. Your answer should be sound and precise. Unnecessary writings may attract negative marks. [7 Marks]

   (c) Prove or disprove $((A \leq_m B) \text{ and } B \text{ is Regular}) \Rightarrow (A \text{ is Regular})$. Your answer should be sound and precise. Unnecessary writings may attract negative marks. [6 Marks]

3. (a). Prove that the set B of all infinite binary sequences is uncountable. [10 Marks]

   (b). Prove that the set L of all languages over the alphabet Σ is uncountable. [10 Marks]