

Probability

1. When Alice plays chess against a computer, she wins 75% of the time.
When Bob plays, he wins only 25% of the time.
If each of Alice and Bob plays exactly one chess game against the computer and exactly one of them wins, what is the probability that the winner is Alice?
2. Alice and Bob agree to meet at a public garden between 2 and 3 PM. Independently of one another, each will arrive at a random time during the hour. Each will wait up to 15 minutes for the other person and then leave the garden.
What is the probability that Alice and Bob meet?
3. X and Y are two independent uniform random variables on the interval $(0, 1)$. What is the probability that $\lfloor \frac{1}{X} \rfloor = \lfloor \frac{1}{Y} \rfloor$, where $\lfloor x \rfloor$ is the largest integer less than or equal to x ?
4. Six numbers are chosen from the set $\{1, 2, 3, \dots, 2021\}$.
What is the probability that the difference between two of those six numbers is a multiple of 5?
5. X and Y are two uncorrelated random variables.
 $E\{X\} = E\{Y\}$, $\text{Var}\{X\} = 4$, $\text{Var}\{Y\} = 8$.
Variance
Random variable $Z = cX + (1-c)Y$.
What is the value of c that minimizes $\text{Var}\{Z\}$?