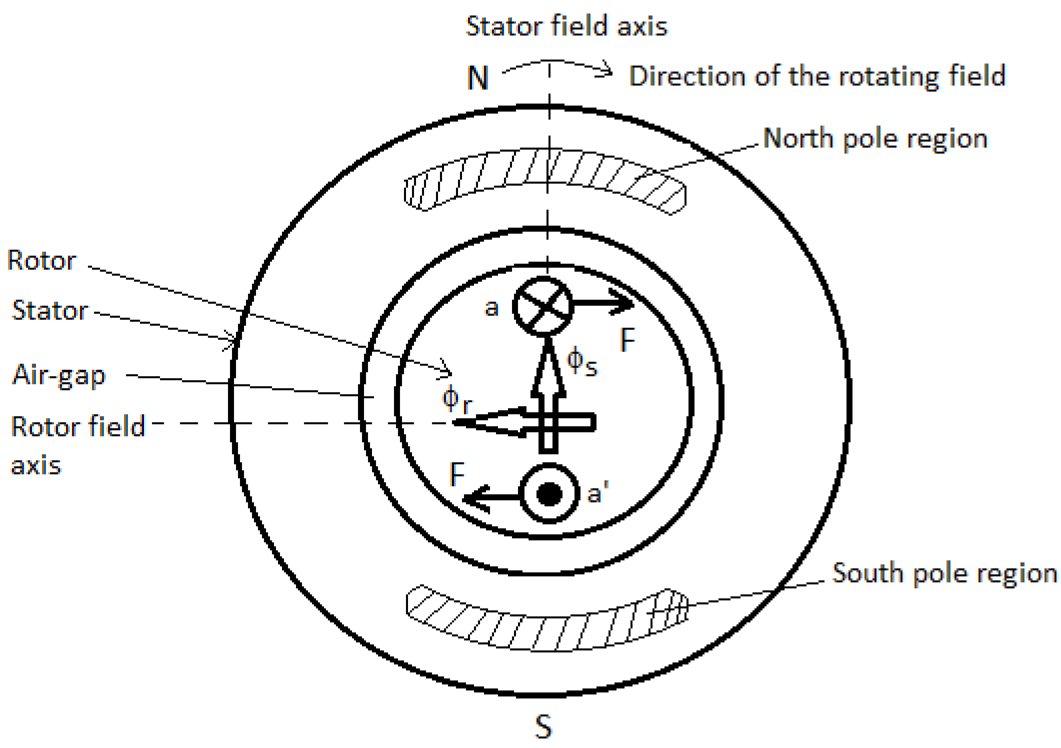
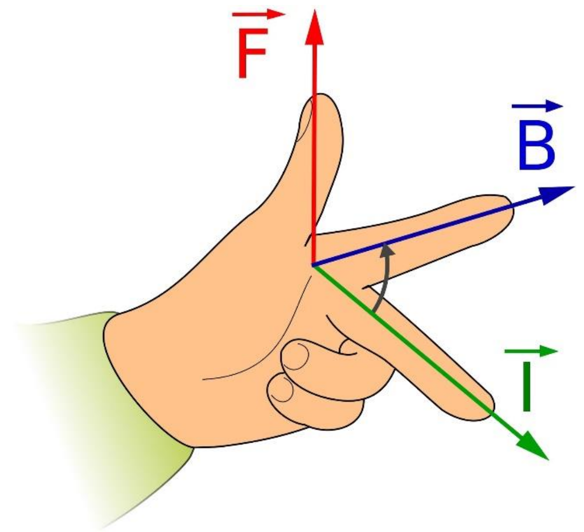
Induction Motors

Slip and Rotor Speed

- The rotor of the induction motor turns in the same direction with the stator rotating field. However, if the rotor was turning at synchronous speed, there would be no field cutting the rotor bars.





Left hand rule

- By considering the figure given above lets assume that the magnetic field rotates at clockwise and it has instantaneous position as shown with ϕ_s. When the field cuts the a-a' coil, it will result in a induced voltage on a-a' coil in the rotor. Since the rotor bars are short-circuited, current will flow in the direction as shown in the figure. Therefore, the produced force can be found. If left hand rule is applied to the "a" edge, a force shown with F direction is obtained. If the left hand rule is also applied to the " a' " edge, it will be seen that the produced torque rotates the rotor in the direction same with the rotating field. The rotor tries to catch the main field. In order to get currents in the rotor windings and hence to produce torque, it must be a relative movement between the rotor wires and the rotating field. Slip is defined as the difference between the synchronous speed, n_s and rotor speed, n_r. It can be expressed as percentage of the synchronous speed;

$$Slip\% = s = \frac{n_s - n_r}{n_s}.100$$

- Then, the rotor speed;
  - $n_r = (1-s).n_s$ (r/min)

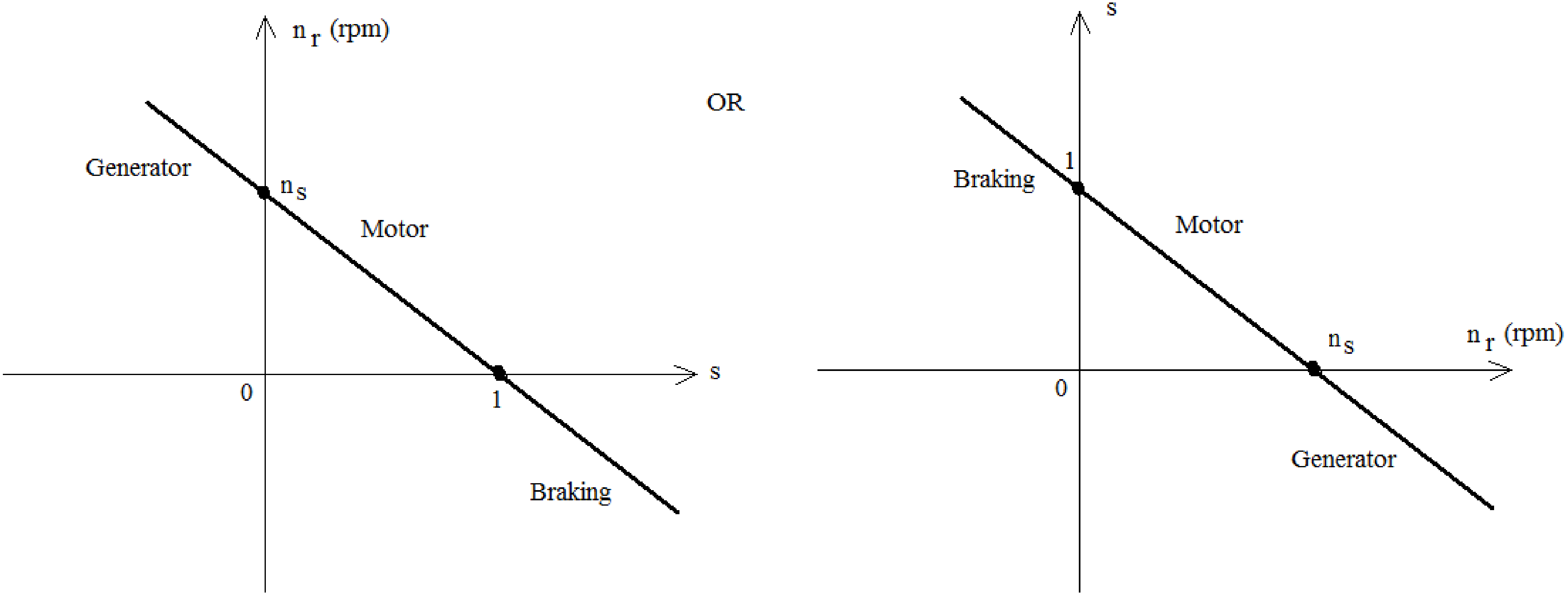
  Where s is not represented as percentage.

- **Example:**
- An induction motor having 6 poles is fed from 3-phase, 60 Hz supply. If the rotor speed at full load is 1140 rpm. Then calculate:
- a) The synchronous speed of the rotating magnetic field
- b) The slip
- c) The rotor speed for s=0.02.

- **Solution:**

- a) $$n_s = \frac{120.f}{P} = \frac{120.60}{6} = 1200\,rpm$$
- b) $$s = \frac{n_s - n_r}{n_s} = \frac{1200 - 1140}{1200}.100 = 5\%$$
- c) $$n_r = n_s(1 - s) = 1200(1 - 0.02) = 1176\,rpm$$

- Speed-slip characteristic of the induction motor having a certain pole number and supplied by source with a certain frequency can be drawn as follow:



The above figure has been drawn by considering the equation of;

$$n_r = (1\text{-}s).\ n_s \quad (r/min)$$

As can be seen there are three slip regions and therefore the machine operates at three different modes.

$0 \leq s \leq 1$

$0 \leq n_r < 1$ Motor operation mode

$s < 0$

$n_r > n_s$ Generator operation mode

$s > 1$

$n_r < n_s$ Breaking mode

- The relationship between the electrical and mechanical frequencies related to the pole number is given as;

$$\omega_m = \frac{2}{P}\omega_e$$

- Where; ω_e is electrical frequency and ω_m is mechanical frequency .

$$\omega_m = \frac{2\pi n}{60} \;\; and \;\; \omega_e = 2\pi f$$

$$s = \frac{n_s - n_r}{n_s} = \frac{\frac{2\pi n_s}{60} - \frac{2\pi n_r}{60}}{\frac{2\pi n_s}{60}} = \frac{\omega_s - \omega_r}{\omega_s}$$

- Where; ω_s is mechanical frequency of the rotating field (r/s) and ω_r is mechanical frequency of the rotor speed (r/s)

$$\omega_{se} = \frac{P}{2}\omega_s \quad and\ \omega_{re} = \frac{P}{2}\omega_r$$

$$s = \frac{\omega_s - \omega_r}{\omega_s} = \frac{\frac{P}{2}\omega_s - \frac{P}{2}\omega_r}{\frac{P}{2}\omega_s} = \frac{\omega_{se} - \omega_{re}}{\omega_{se}}$$

- Since $\omega_{se} = 2.\pi.f$ and $\omega_{re} = 2.\pi.f_r$

$$s = \frac{\omega_{se} - \omega_{re}}{\omega_{se}} = \frac{2.\pi.f - 2.\pi.f_r}{2.\pi.f} = \frac{f - f_r}{f}$$

- So, there are many alternative ways to express the slip;

$$\boxed{s = \frac{n_s - n_r}{n_s} = \frac{\omega_s - \omega_r}{\omega_s} = \frac{\omega_{se} - \omega_{re}}{\omega_{se}} = \frac{f - f_r}{f}}$$

Voltage and Frequency Induced In The Rotor

- At starting (when the rotor speed is zero) the rotating field cuts the rotor bars at maximum rate. In this case the voltage produced in the rotor circuit will be maximum and its value is determined by the turn number of the rotor winding. In the stator windings an opposite emf which is approximately equal to the applied voltage is produced with the rotating field. For this reason, the flux cutting the stator windings will be equal to the flux cutting the rotor windings at start-up. Therefore, the induced voltage per phase in the stator and rotor will depend on the turn ratio as in transformers. In addition, the frequency of the induced rotor voltage will be same as supply frequency when the rotor speed is zero. In this case, s=1 or 100%. As the slip becomes smaller, the flux cuts the windings less. The induced rotor voltage and frequency are;

$$E_R = s * E_{BR}$$

$$f_R = s * f$$

$$\frac{E_s}{E_{BR}} = \frac{N_s}{N_R}$$

E_R : Induced rotor voltage at slip, s.
E_{BR} : Induced rotor voltage per phase with the locked-rotor.
f_R : Rotor frequency.
E_S : Rms value of the induced voltage in the stator per phase.
N_s : Turn number of one phase stator winding.
N_R : Turn number of one phase rotor winding.

Example:

Stator windings and rotor windings of 3-phase, 60 Hz, 4 poles, 220 V wound rotor induction motor are connected in delta and star, respectively. The ratio of rotor and stator windings is 40%. For the speed of 1710 rpm calculate;

a) Slip
b) E_{BR} voltage
c) E_R voltage
d) Voltage between the rotor terminals
e) Rotor frequency

Solution:

a) $n_s = \frac{120 * f}{p} = \frac{120 * 60}{4} = 1800\ rpm$ $\quad s = \frac{n_s - n_r}{n_s} = \frac{1800 - 1710}{1800} = 0.05$

b) E_{BR} = 0.4*220 = 88 V/phase

c) E_R = s*E_{BR} = 0.05*88 = 4.4 V

d) $V_{L-L}(rotor) = \sqrt{3} * 4.4 = 7.62\ V$

e) f_R = s*f = 0.05*60 = 3 Hz

As can be seen at normal operation condition, the induced rotor voltage and frequency are so small.

The Rotor Circuit

Basically, an induction motor is similar to the transformer with the secondary side short-circuited and with moving capability regarding to the primary side. Therefore, its equivalent circuit is similar to that of the single-phase transformer. The induced rotor voltage per phase is s* E_{BR} which causes a current flow in the short-circuited rotor windings. This rotor current is restricted only by the rotor impedance which has two components:

1) Rotor resistance, R_R
2) Leakage reactance, s* X_{BR} where X_{BR} is the rotor leakage reactance at start-up.

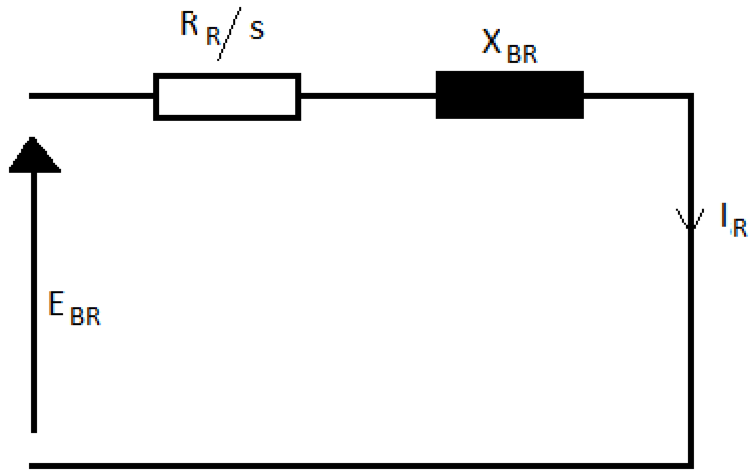
Since the reactance is a function of frequency, it will be proportional with the slip. The rotor current can be expressed as:

$$I_R = \frac{sE_{BR}}{\sqrt{R_R^2 + (sX_{BR})^2}}$$

If the nominator and denominator are both divided by s slip;

$$I_R = \frac{E_{BR}}{\sqrt{\left(\frac{R_R}{s}\right)^2 + X_{BR}{}^2}}$$

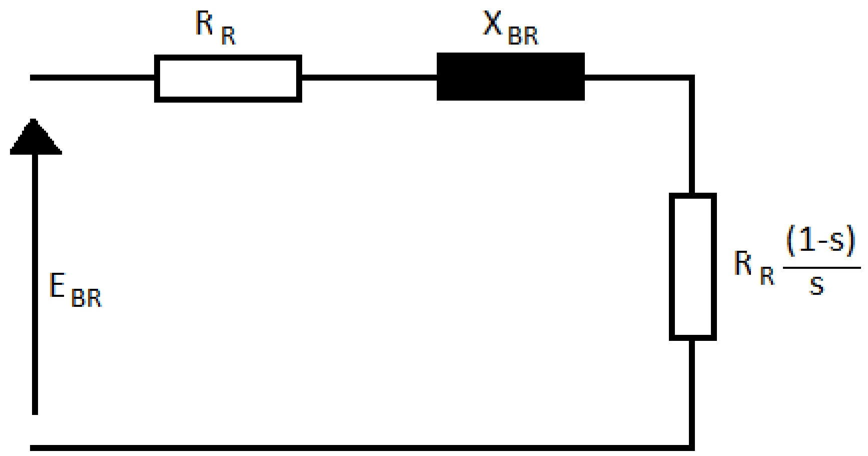
Although this mathematical operation (dividing by s) seems simple, it is very important process. The current, I_R was a function of the rotor voltage at slip frequency. Now it is defined by the supply frequency. In other words, process dividing by s changed the reference point from the rotor to the stator. We can draw an equivalent circuit related the above equation:



R_R/s term can be separated into two terms as;

$$\frac{R_R}{s} = \frac{R_R}{s} + R_R - R_R = R_R + R_R\left(\frac{1-s}{s}\right)$$

The equivalent circuit becomes as;



If both side of $\frac{R_R}{s} = \frac{R_R}{s} + R_R - R_R = R_R + R_R\left(\frac{1-s}{s}\right)$ equation is multiplied by I_R^2, an equation consisting of power terms is obtained:

$$I_R^2 \frac{R_R}{s} = I_R^2 R_R + I_R^2 R_R \left(\frac{1-s}{s}\right)$$

The term at the left represents the total power entering to the rotor circuit and it has two components:

1) Copper losses dissipated in the rotor winding ($I_R^2 R_R$)

2) Electrical power converted to the mechanical power ($I_R^2 R_R \left(\frac{1-s}{s}\right)$). Therefore, per phase:

Rotor input power (RPI) = Rotor copper loss (RCL) + Mechanical power developed in the rotor (RPD)

$$RPI = I_R^2 \frac{R_R}{s}$$

$$RCL = I_R^2 R_R$$

$$RPD = I_R^2 R_R \left(\frac{1-s}{s}\right) = RPI(1-s)$$

- In general, the developed power by the motor is equal to the multiplication of the developed torque and rotor angular velocity:

$$P_D = \omega_R * T_D$$

$$T_D = \frac{RPD}{\omega_R}$$

- Where; $\omega_R = \frac{2\pi n_R}{60}$ (rad/s) and n_R (rpm).

- In order to define the output torque (net torque) the losses must be taken into consideration.