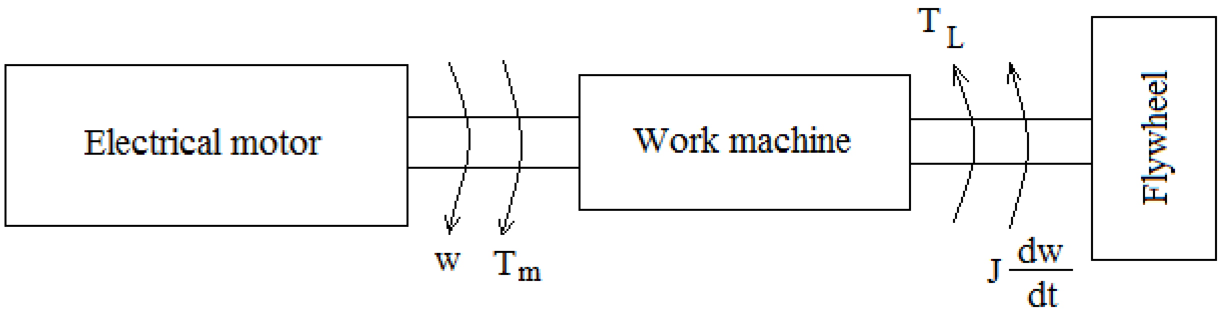
Electrical Machines

Motion Equation of A Drive System



Flywheel: It is used for obtaining smooth torque in case of cogging operations.

The torque, T_m obtained from the shaft of the electrical motor must meet the load torque of the work machine and the total moment of inertia ($jd\omega/dt$)referred to the shaft.

Motion equation of the system;

$$T_m = T_L + j\frac{d\omega}{dt}$$

If we consider the frictions in the rotating parts, the complete motion equation of the system becomes;

$$T_m = T_L + j\frac{d\omega}{dt} + B\omega$$

Where;

J: The total torque of the rotating mass referred to the shaft (kg. m^2).

It is also called moment of inertia.

B: Friction coefficient (Nm.s/rad)

t: Time (s)

ω: Angular velocity (rad/s)

- If we assume that the friction torque is small and can be neglected; the equation

$$j\frac{d\omega}{dt} = T_m - T_L$$

can be examined as;

a) If $T_m > T_L$, right hand side of $j\frac{d\omega}{dt} = T_m - T_L$ equation becomes positive. That is; $j\frac{d\omega}{dt} > 0$. The system is accelerating.

b) If $T_m = T_L$, right hand side of $j\frac{d\omega}{dt} = T_m - T_L$ equation becomes zero. That is; $j\frac{d\omega}{dt} = 0$. The system operates at constant speed.

c) If $T_m < T_L$, right hand side of $j\frac{d\omega}{dt} = T_m - T_L$ equation becomes negative. That is; $j\frac{d\omega}{dt} < 0$. The system is decelerating.

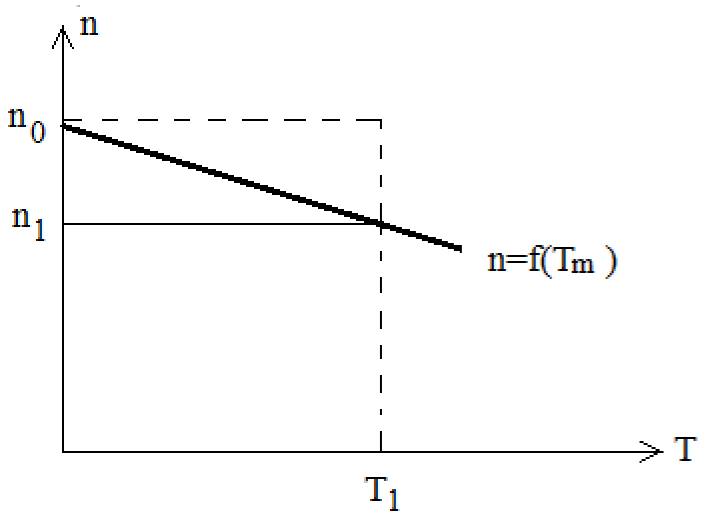
- As can be seen from these three operating conditions, to obtain continuous and stable operation it is necessary to know torque characteristic of electrical motor ($T_m = f(\omega)$) and torque characteristic of work machine ($T_L = f(\omega)$) . In practice, instead of using angular velocity, ω;

$$n = \frac{60.\omega}{2.\pi}\ (rpm)$$

n is used and the torque characteristic is characterized as a function of n.

Speed-Torque Characteristics of Electrical Motor

- a) Shunt Characteristic:



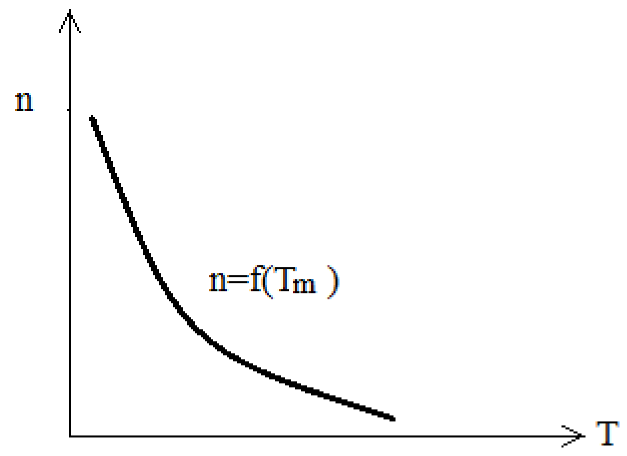
- Characteristics:
- i) When there is no load on the shaft, $n=n_o$ (no-load operation)
- ii) When there is a load of $T_L = T_1$ on the shaft, the speed of the motor drops to n_1 from n_o.

The motors giving these characteristics are:

1) Direct current (dc) shunt motor
2) Separately excited dc motor
3) Squirrel cage induction motor
4) Wound rotor induction motor

These motors are used to drive machine tools, paper and textile machines, lifts, ventilators, aspirators, fridge and washing machines.

b) Series Characteristic:



Characteristics:

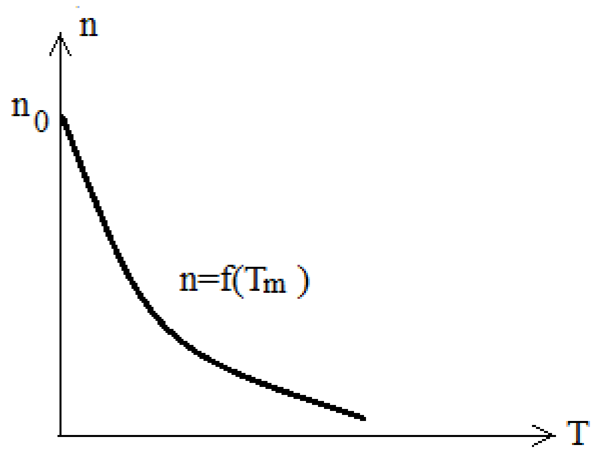
i) The motor must be never operated at no-load. There must be always T_L load on the shaft. If T_L =0, speed of the motor will increase severely and the motor will destroy itself.
ii) At high load torques the speed of the motor decreases rapidly.

The motor giving these characteristics is:

1) Direct current series motor

This motor is used to drive cranes, electrical trains, lifts in mining and trolleybus.

- c) Compound Characteristic:



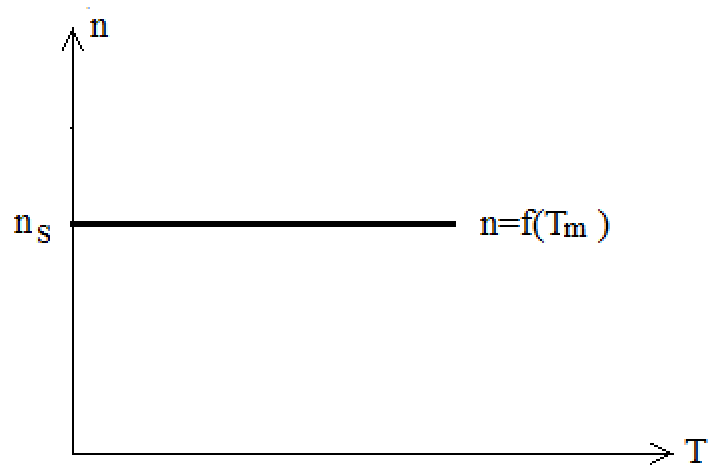
- Characteristics:
- i) At no-load, when T_L =0, the speed, n is as bigger as n_o.
- ii) When $T_L \neq 0$, the motor speed, n drops rapidly.

The motor giving these is:

1) Direct current compound motor

- It is used in driving rolling mill.

- d) Synchronous Characteristic:

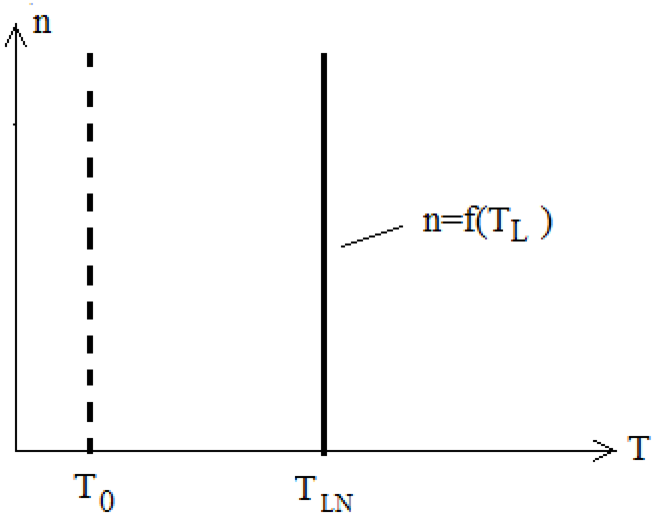


- Characteristic:
- It does not affected from the load torque. It operates at constant speed (n_s).
- The motor giving these characteristics is:

  Synchronous motor.
- It is used in the applications where constant speed is required. Such as, compressors.

Speed-Torque Characteristics of Work Machines (Heavy Machineries)

- **a) Constant Torque Characteristic:**

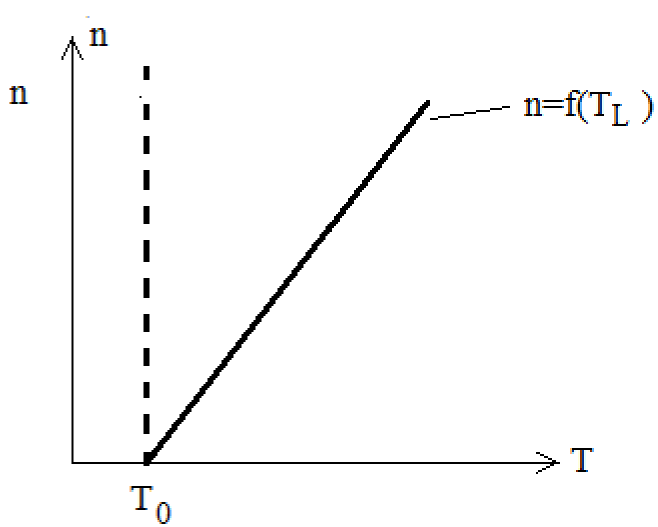


- Characteristics:
- i) It has minimum torque, T_o which corresponds to the friction torque.
- ii) It does not affected from the speed.

The work machines giving this characteristic are:

1) Machines which do friction work: Paper, textile, escalator.
2) Machines which do lifting work: Lifting machines, lifts.
3) Shaping machines: Machines tools like lathe tools, planers

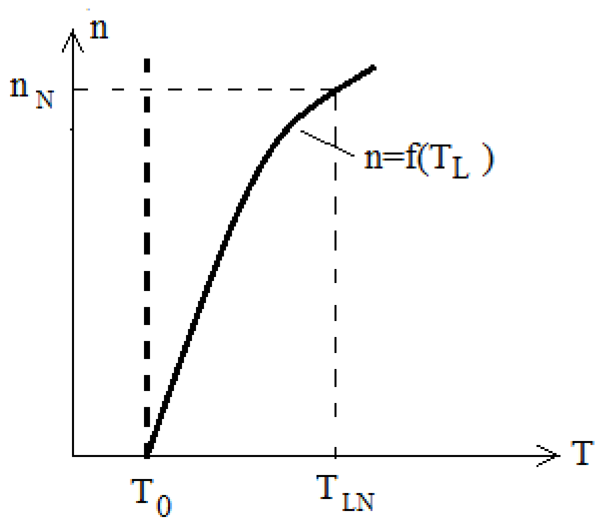
b) Proportional Torque Characteristic:



- Characteristics:
- i) It has minimum torque, T_o which corresponds to the friction torque.
- ii) The speed increases as the shaft torque increases.

- The work machines giving this characteristic are:
- Separately excited and shunt excited direct current generators loaded with a constant resistor.

- **c) Parabolic Characteristic:**

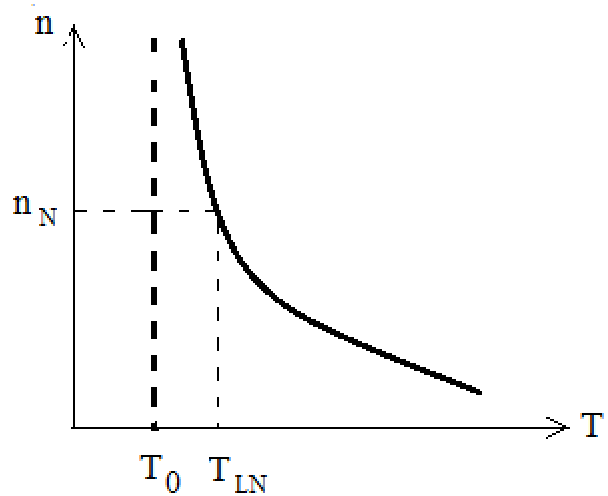


- Characteristics:
- i) It has a minimum torque, T_o which corresponds to the friction torque.
- ii) The load torque varies parabolically with the speed.

The work machines giving this characteristic are:

1) Ventilators, pumps, aspirators, ship propeller.

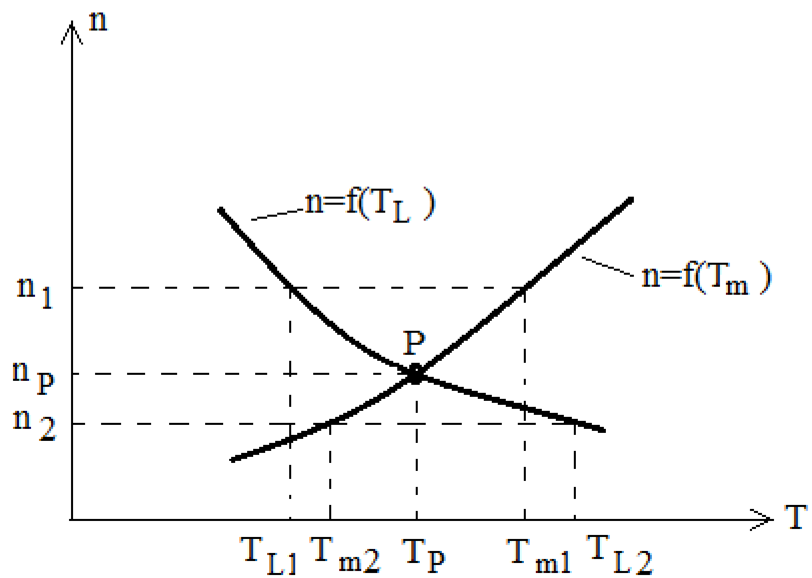
- **d) Hyperbolic Characteristic:**



- Characteristics:
- i) It has a minimum torque, which corresponds to the friction torque.
- ii) The load torque varies hyperbolically with the speed.
- The work machines giving this characteristic are:
- Paper winding and sheet winding machines.

Stable Operating Conditions of A System Consisting of An Electric Motor and A Work Machine

Aim: To determine what kind of motor characteristic can be used with the work machine which has a certain torque characteristic for stable operation.



Lets assume that the work machine and electric motor have the characteristics as shown in the figure and operate at P point. Therefore; $T_L = T_m$

- Obviously this is the continuous operating point. Therefore;

$$J\frac{d\omega}{dt} = 0$$

- If we move from P point as much as $\Delta n = n_1 - n_p$ (that is; if we accelerate the motor); the moment of inertia will be,

$$J\frac{d\omega}{dt} > 0$$

- since $T_{m1} > T_{L1}$. The system will accelerate and $T_{m1} - T_{L1}$ difference will get bigger. Since the torque difference gets bigger and bigger the system will accelerate more and more. Therefore we will not have stable operation anymore.

Lets make the same analysis by decreasing the speed ($\Delta n = n_P - n_2$) while operating at P point. In this case, since $T_{m2} < T_{L2}$,

$$T_{m2} - T_{L2} < 0.$$

That is;

$$J\frac{d\omega}{dt} < 0$$

The physical meaning of this is that the system is decelerating. As it decelerates the speed will decrease more and more.

These two analysis show us that if at P point

$$\frac{dT_m}{dn} > \frac{dT_L}{dn}$$

stable operation cannot be obtained. We can proof this mathematically as;

Lets the speed at P point is ω. If there is $\Delta\omega$ increase in ω then; the load torque, motor torque and speed will change as;

$T_L - \Delta T_L$, $T_m + \Delta T_m$ and $\omega + \Delta\omega$

At P point, $T_L = T_m$

$$J\frac{d\omega}{dt} = T_m - T_L = 0 \rightarrow J\frac{d\omega}{dt} = 0$$

When there is an increase in speed;

$$J\frac{d(\omega + \Delta\omega)}{dt} = T_m + \Delta T_m - T_L + \Delta T_L$$

$$T_m + \Delta T_m - T_L + \Delta T_L - J\frac{d\omega}{dt} - J\frac{d\Delta\omega}{dt} = 0$$

Since $T_m = T_L$ at P point;

$$\Delta T_m - \Delta T_L - J\frac{d\Delta\omega}{dt} = 0$$

$$\frac{\Delta T_m}{\Delta\omega} \cong \frac{dT_m}{d\omega} \rightarrow \Delta T_m = \frac{dT_m}{d\omega}\Delta\omega$$

$$\frac{\Delta T_L}{\Delta\omega} \cong \frac{dT_L}{d\omega} \rightarrow \Delta T_L = \frac{dT_L}{d\omega}\Delta\omega$$

$$\frac{dT_m}{d\omega}\Delta\omega - \frac{dT_L}{d\omega}\Delta\omega - j\frac{d(\Delta\omega)}{dt} = 0$$

$$\Delta\omega\left(\frac{dT_m}{d\omega} - \frac{dT_L}{d\omega}\right) - j\frac{d(\Delta\omega)}{dt} = 0$$

Dividing both side by j;

$$\frac{d(\Delta\omega)}{dt} - \Delta\omega\frac{1}{j}\left(\frac{dT_m}{d\omega} - \frac{dT_L}{d\omega}\right) = 0$$

- This is a differential equation with constant coefficient.
- If $A_0 = \frac{1}{j}\left(\frac{dT_m}{d\omega} - \frac{dT_L}{d\omega}\right)$ then;

$$\frac{d(\Delta\omega)}{dt} - \Delta\omega A_0 = 0$$

- It has a solution of;

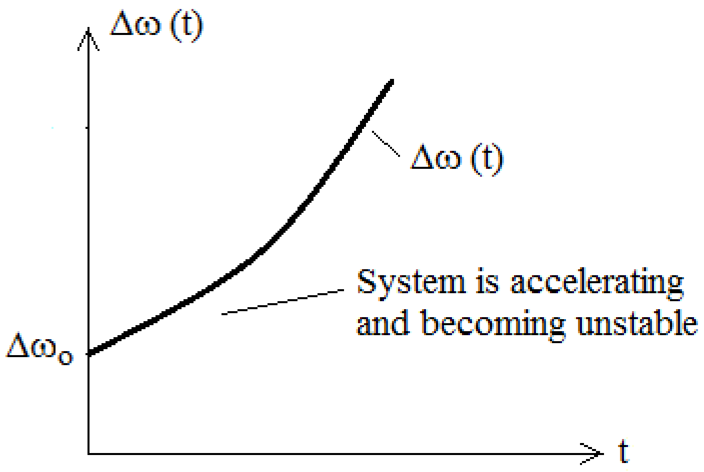
$$\Delta\omega(t) = \Delta\omega_0 e^{A_0 t}$$

- If;

$$A_0 = \frac{1}{j}\left(\frac{dT_m}{d\omega} - \frac{dT_L}{d\omega}\right) > 0$$

- Then;

$$\frac{dT_m}{d\omega} > \frac{dT_L}{d\omega}$$

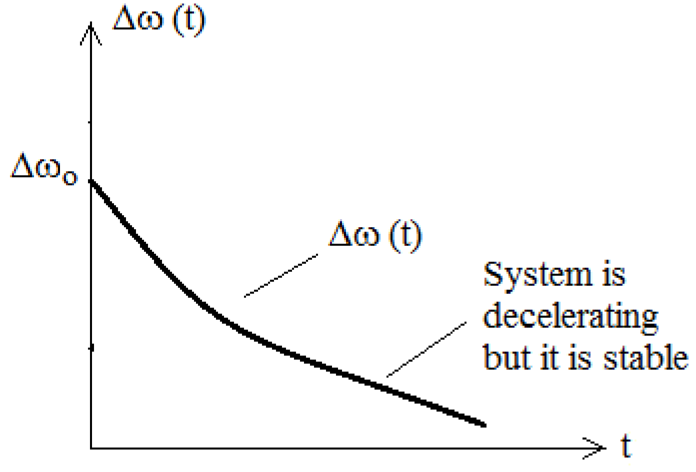


If;

$$A_0 = \frac{1}{j}\left(\frac{dT_m}{d\omega} - \frac{dT_L}{d\omega}\right) < 0$$

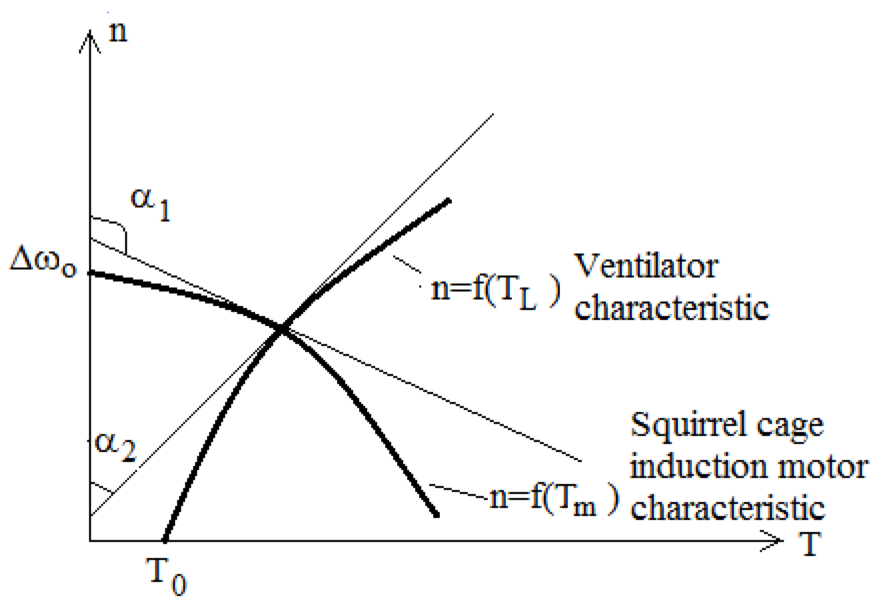
Then;

$$\frac{dT_m}{d\omega} < \frac{dT_L}{d\omega}$$



Therefore, if the criteria of $\frac{dT_m}{d\omega} < \frac{dT_L}{d\omega}$ is provided in addition to the condition of $T_m - T_L = 0$ at P point then the system will be stable.

Working Example: A ventilator is driven by a squirrel cage induction motor. Determine weather the system is stable or not.



- **Solution:**
- α_1 and α_2 angles are found from;

$$\frac{dT_m}{dn} = tan\alpha_1 \ ; \ \frac{dT_L}{dn} = tan\alpha_2$$

- Since α_1 is obtuse angle, $tan\alpha_1 < 0$. Since α_2 is acute angle, $tan\alpha_2 > 0$. Then; the criteria

$$\frac{dT_m}{dn} < \frac{dT_L}{dn}$$

- is met and since $T_m - T_L = 0$ at P point, the system is stable.

Definitions on Electric Motors

- **Rated (Nominal) Value:** Electrical and mechanical values given on the motor plate.
- **Rated Power:** The mechanical power in Watt taken from the motor shaft and it is given on the motor plate. It is represented by P_N.
- **Rated Voltage:** Voltage between the terminals of the motor connected to the supply (line voltage) when the motor operates at rated power. It is represented by U_N. Its unit is volt and given on the motor plate.
- **Rated Current:** It is current drawn from each phase of the supply when the motor is connected to the supply and operates at rated power. Its unit is Ampere and given on the motor plate.

Rated Value of Power Factor: If the motor is AC motor, the motor power taken from the supply is;

$$P = \sqrt{3} U_N I_N cos\varphi_N$$

Where; $cos\varphi_N$ is known as power factor. It is given on the motor plate.

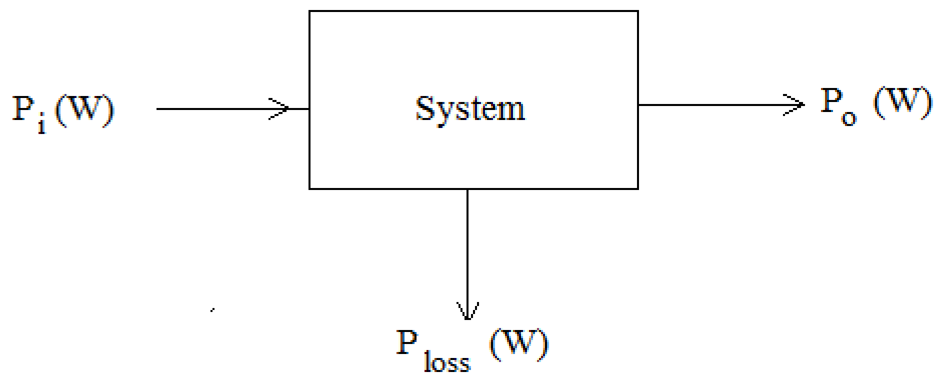
Load: It is the resistive torque of work machine mounted on motor shaft.

Operation on No-Load: When there is no load mounted on the shaft of motor.

Rated Speed: It is the shaft speed when the electric motor operates at the rated power, P_N. It is given on the motor plate and its unit is revolution per minute (rpm).

- **Motor Efficiency:** General definition is;

$$\eta = \frac{Output\ Power\ of\ System}{Input\ Power\ of\ System}$$



- If the power loss is represented by P_{loss}, output power of the system will be $P_o = P_i - P_{loss}$ and the efficiency;

$$\eta = \frac{P_o}{P_i} = \frac{P_i - P_{loss}}{P_i} = 1 - \frac{P_{loss}}{P_i}$$

Efficiency at Rated Value:

- $P_o = P_N$. If the motor is:
- a) DC motor;

$$\eta_N = \frac{P_o}{I_N U_N} = \frac{P_N}{I_N U_N}$$

$$P_{loss} = I_N U_N - P_N$$

- b) Single-Phase AC Motor;

$$P_i = I_N U_N cos\varphi_N$$

$$\eta_N = \frac{P_N}{I_N U_N cos\varphi_N} \quad and \ P_{loss} = I_N U_N cos\varphi_N - P_N$$

- c) Three-Phase AC Motor;

$$P_i = \sqrt{3} I_N U_N cos\varphi_N$$

$$\eta_N = \frac{P_N}{\sqrt{3} I_N U_N cos\varphi_N} \quad and \quad P_{loss} = \sqrt{3} I_N U_N cos\varphi_N - P_N$$