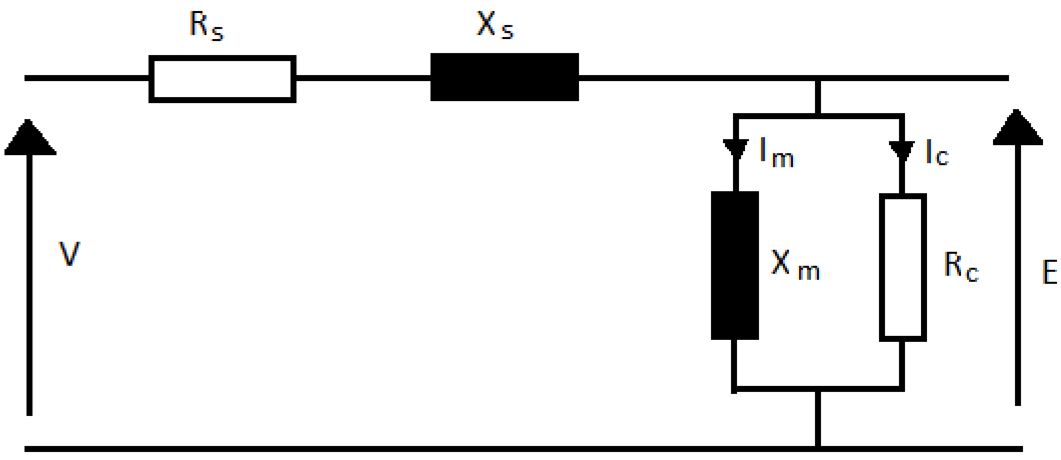
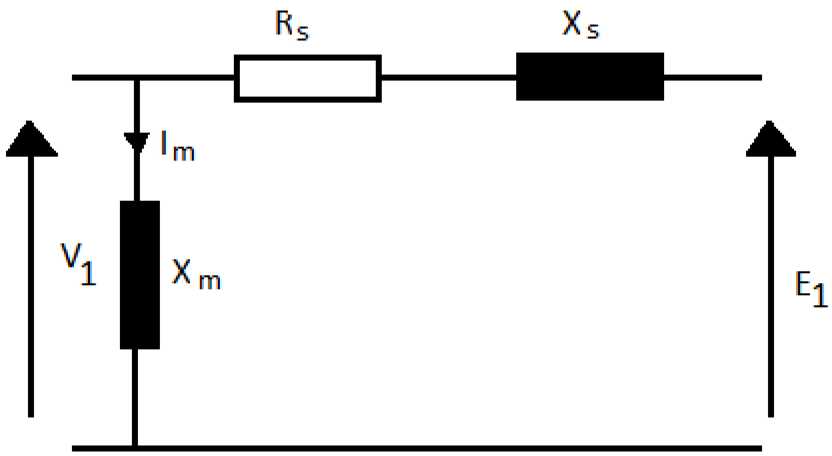
Complete Equivalent Circuit of The Induction Motor

Until now we saw the rotor power and the equivalent circuit representing the rotor power with two components (rotor copper loss and mechanical power). To obtain the complete equivalent circuit the stator circuit must be also considered. In addition to the winding resistance, R_S and leakage reactance, X_S the stator has also a component representing the magnetization. The magnetization reactance cannot be neglected in the motor unlike transformers because of the airgap. The stator equivalent circuit becomes as:



The following assumptions are made:

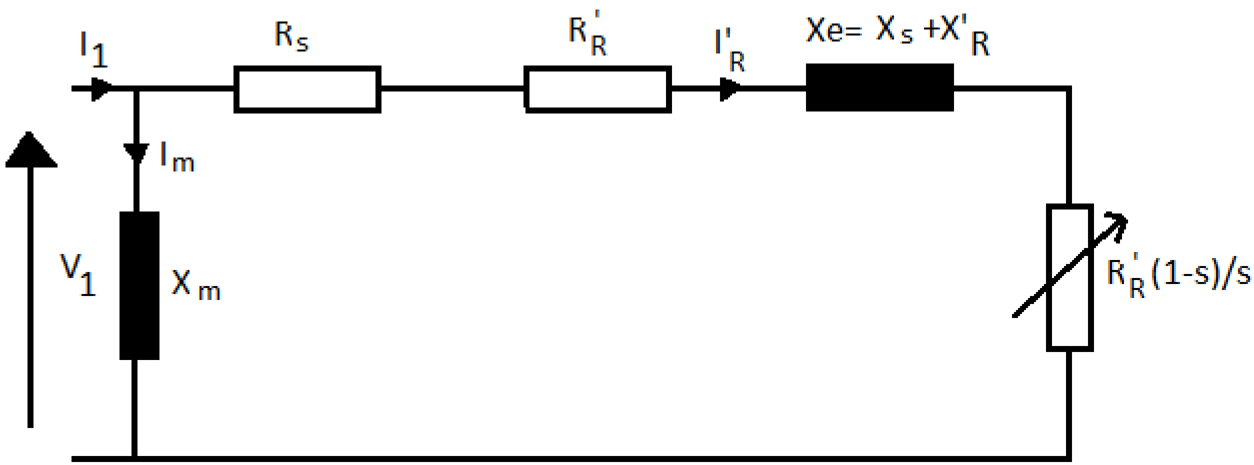
1. The core losses are assumed as constant and found by no-load test. It means that R_c component can be extracted from the equivalent circuit. Actually, the core losses are unneglectable. They are just constant. These losses must be considered in the performance calculation of the motor.
2. The magnetization current in transformers is small comparing the rated current and can be neglected. However, this current is between 30% - 50% in the motors depending on the size of the motor. Therefore, the magnetization reactance is basic component of the equivalent circuit. For simplicity in the calculations, the magnetization reactance can be taken to the input terminals. This will not make big difference in the calculations.



- Now, we can obtain the complete equivalent circuit of the motor for one phase by combining the rotor and stator circuits. To do this the rotor side must be referred to the stator side. For this reason E_{BR} must be made equal to E_1. The ratio between the stator and rotor phase windings is considered as:

$$E'_{BR} = \alpha E_{BR} = E_1$$

- α : The ratio between the stator and rotor windings.
- E'_{BR} : Rotor voltage referred to the stator.
- The final equivalent circuit is drawn as:



- By examining the equivalent circuit it can be seen why the induction motor operates with low power factor at light loads. At light loads (small slip) the resistance, $R'_R\left(\frac{1-s}{s}\right)$ representing the mechanical power will be big which means that I'R current is smaller than I_m. That's why the circuit mostly shows inductive characteristic. In this case, X_m behaves effective component in the circuit. Therefore, the power factor (cosφ) will be small. If the load increases, $R'_R\left(\frac{1-s}{s}\right)$ resistance value drops quickly since s slip value increases. As a result, the power factor gets bigger.

- From the equivalent circuit the rotor current can be calculated as;

$$I'_R = \frac{\bar{V}_1}{(R_s + R'_R/s) \quad + j(X_s + X'_R)}$$

- RPI, RCL and RPD powers can be calculated by using this current. The magnetization current is;

$$\bar{I}_m = \frac{V_1}{jX_m}$$

- And motor line current;

$$\bar{I}_1 = \bar{I}_m + \bar{I}'_R$$

- By replacing the magnetization reactance to the stator terminals, the stator values seem to be rotor values. This will result $E_1 = V_1$ error which can be neglected.

In practice, the complete equivalent circuit completely represents the motor except for the some neglectable errors. But for efficiency calculation, stator copper losses must be found as;

$$P_{custator} = 3 * I_1^2 * R_s$$

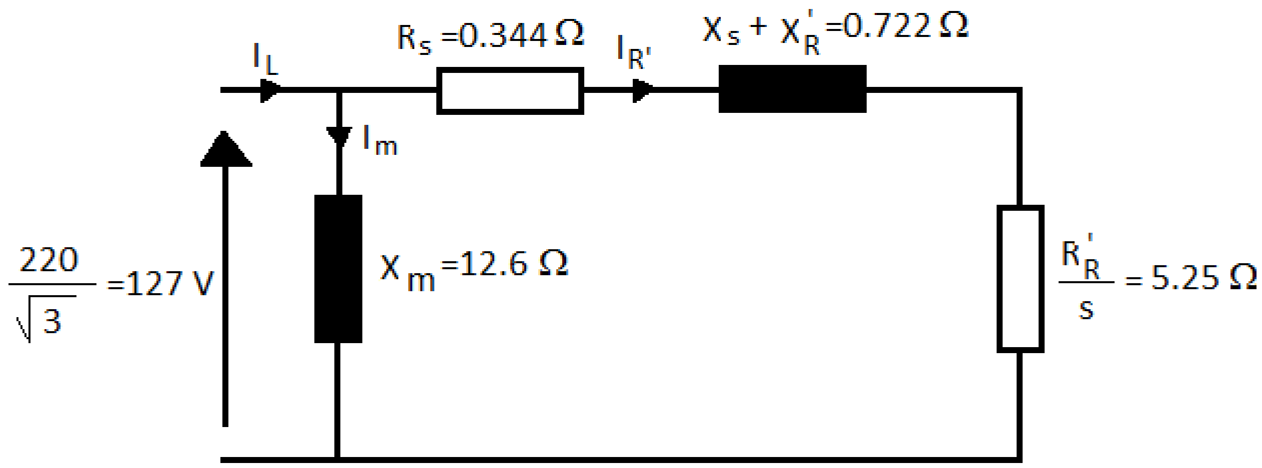
The net torque;

$$T = \frac{RPD - (Mechanical\ and\ core\ losses)}{\omega_R}$$

Mechanical losses are friction and ventilation losses. They can be determined by no-load test.

- **Example:**
- 3-phase, 220 V, 60 Hz, 6 poles, 10 HP induction motor has the equivalent parameters referred to the stator as:

  $R_s = 0.344\ \Omega, \quad R'_R = 0.147\ \Omega, \quad X_s = 0.498\ \Omega, \quad X'_R = 0.224\ \Omega, \quad X_m = 12.6\ \Omega$
- The total core and rotational losses are 262 W and they are constant. For s=2.8%, calculate;
- a) Line current and power factor
- b) Shaft torque and output power
- c) Efficiency
- **Solution:**
- Lets assume that the stator windings are connected in star. Hence, the phase voltage is; $\frac{220}{\sqrt{3}} = 127\ V$



- a)

$$I'_R = \frac{127}{0.344 + 5.25 + j0.722} = 22.52 \quad \angle -7.4^o\ A = 22.33 - j2.88\ A$$

$$I_m = \frac{127}{j12.6} = -j10.08\ A$$

$$\bar{I}_L = 22.33 - j(2.88 + 10.08) = 22.3 - j12.96\ A = 25.82\ \angle - 30.1^o\ A$$

- PF = cos(-30.1) = 0.865

- b)

$$n_s = \frac{120 * f}{p} = \frac{120 * 60}{6} = 1200\ rpm$$

$$n_R = n_s(1 - s) = 1200(1 - 0028) = 1166\ rpm$$

$$\omega_R = \frac{2\pi * 1166}{60} = 122.1\ rad/s$$

$$RPI = 3I'^2_R \frac{R'_R}{s} = 3(22.52)^2 \frac{0.147}{0.028} = 7988\ W$$

- RPD = RPI(1-s) = 7988(1-0028) = 7764 W
- P_{out} = RPD – P_{rot} = 7764 – 262 = 7502 W

$$T = \frac{P_{out}}{\omega_R} = \frac{7502}{122.1} = 61.4\ N.m$$

$$HP = \frac{7502}{746} = 10$$

c) Losses

Rotation and core losses = 262 W

RCC = s*RPI = 0.028*7988 = 224 W

Stator copper losses = $3 * (25.82)^2 * 0.344 = 688\ W$

Total = 1174 W

$$\eta = \frac{7502}{7502 + 1174} = 86.5\%$$

- The powers calculated from the equivalent circuit are for single-phase. When the power calculations for three-phase motor are performed it is necessary to multiply all powers by three.

- For the above example, the input power could be also calculated as;

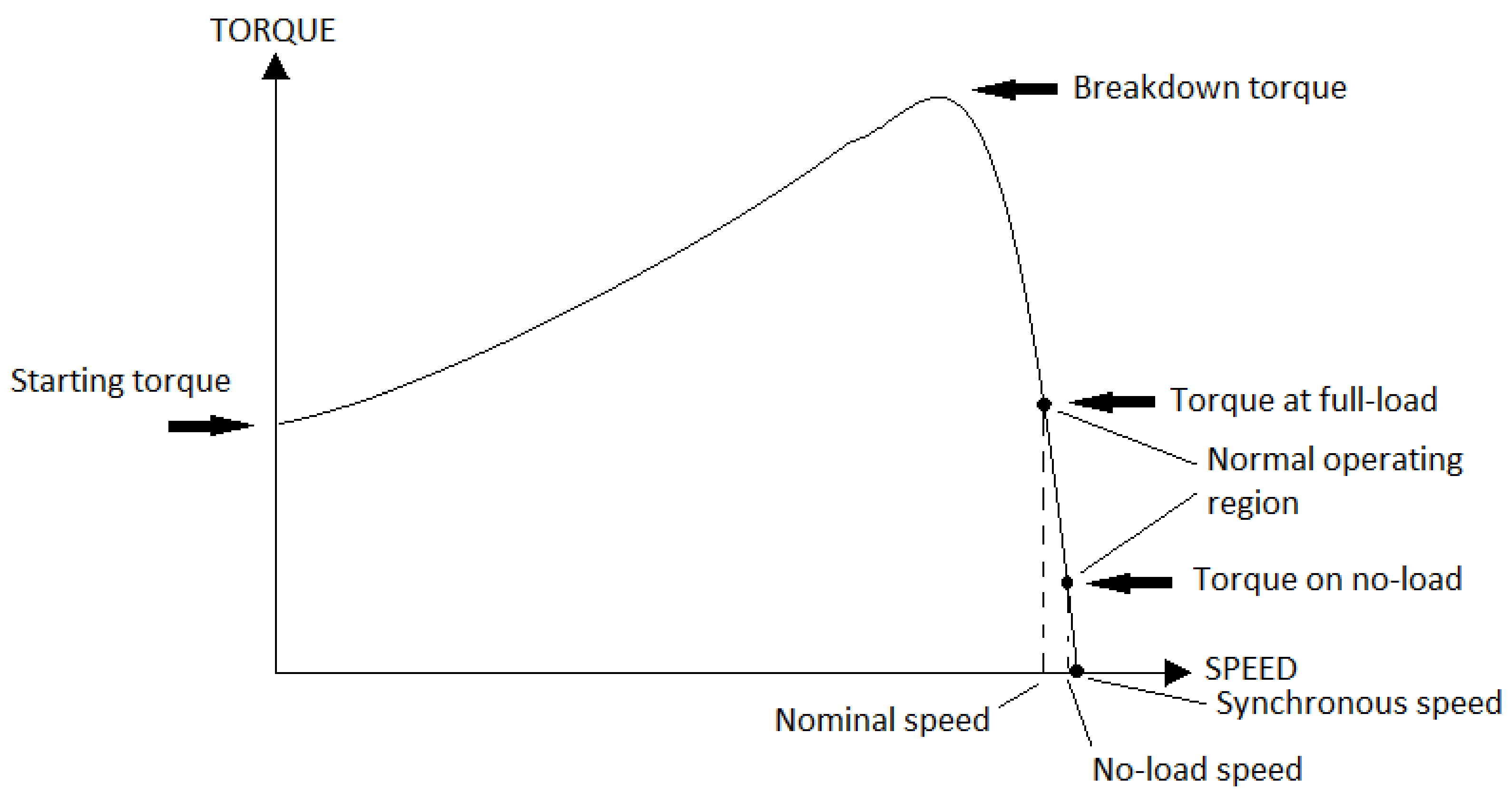
$$P_{in} = \sqrt{3} I_L V_L cos\varphi = \sqrt{3} * 25.82 * 220 * 0.865 = 8510.5 \ W$$

- If the efficiency is calculated by using the input power; $\eta = P_{out}/P_{in}$ = 88.1%.
- The difference in the efficiency is because of the magnetization reactance connected to the input terminals and the way of the power losses taken into consideration.
- In the above example, performance characteristics can be calculated as a function of the speed for each slip value.

Induction Motor Characteristics

- Variable speed induction motor has wound rotor with external resistance to increase starting torque and to control speed. This type of the motor is used in the load applications such as lifts and cranes where frequent starting-up is required. But, squirrel cage induction motors which are cheaper and simple are preferred in the applications like pumps and aspirators where constant power is required and frequent starting-up is not demanded. Typical speed-torque characteristic of a three-phase induction motor is shown in the figure given below.

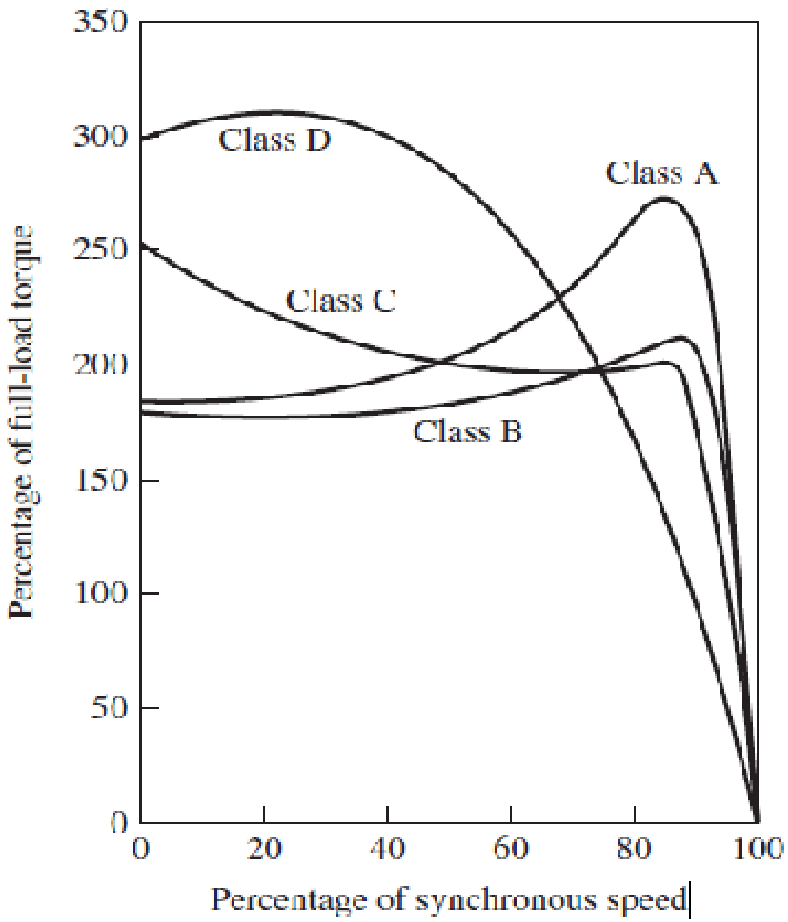
Torque-Speed Characteristic



Induction Motor Classes

- Squirrel cage induction motors are classified as A, B, C and D class motors.
- **1) <u>Class A Design Motors:</u>**
- This type of motors is characterized with operating of small slip values ($s<0.01$) at full-load because of its small rotor resistance. This motor can be used in the applications where small starting torque is required. Because of the small rotor resistance the starting current of Class A motor is high.
- **2) <u>Class B Design Motors:</u>**
- They are general purpose motors from the point of starting torque and starting current. Speed regulation of them at full load is small (usually less than 5%). Their starting torque is 150% of the rated torque. But, for bigger power and smaller speed of the motors the starting torque is a bit smaller than 150% of the rated torque. Although the starting current is normal it can be as big as 600% of the rated current.

- 3) **<u>Class C Design Motors:</u>**
- Class C motors have higher starting torques comparing to Class B motors. Starting currents are normal. The slip value at full load is smaller than 0.05. The starting torque of the Class C motor is 200% of the rated torque. This type of the motors are usually designed to start-up at full load. They are used in the applications of conveyors, reciprocating pumps, compressors.
- 4) **<u>Class D Design Motors:</u>**
- These motors have high slip values, high starting torques and low starting currents. Their efficiency comparing to the other class motors is low because of the high slip values at full load. The peak value of the torque characteristic is shifted to the origin of the torque-speed curve. As a results the starting torque of Class D motors which is also breakdown (pull-out) torque is around 300% of the rated torque.



- Typical torque-speed curves for different rotor designs