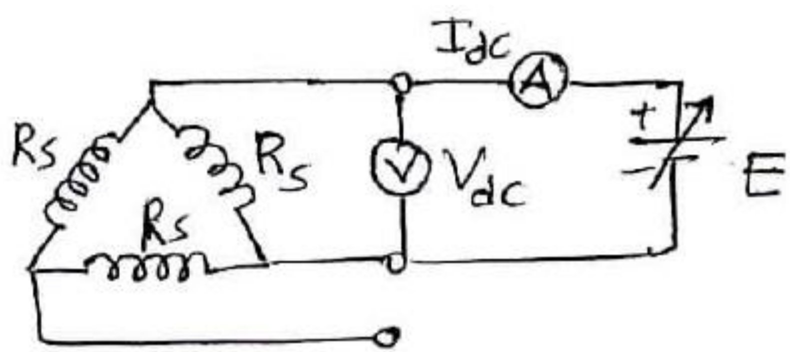
Determination of The Equivalent Circuit Parameters

- Equivalent circuit of the induction motor is used in performance calculations. Therefore, equivalent circuit parameters must be determined. Some motor tests are performed for this purpose. These tests are similar to open-circuit and short-circuit tests of the transformers and called as no-load test and locked-rotor test.

Determination of the stator resistance with DC current:

- In this experiment, a DC voltage is applied to the stator windings and then the DC current in the windings is measured. It must not be allowed current to flow in the windings more than the rated current. Stator windings could be either star or delta connected.

Delta connected stator



$$\frac{V_{dc}}{I_{dc}} = R_{sdc} \,//\, 2.R_{sdc}$$

$$\frac{V_{dc}}{I_{dc}} = \frac{R_{sdc} * 2R_{sdc}}{R_{sdc} + 2R_{sdc}}$$

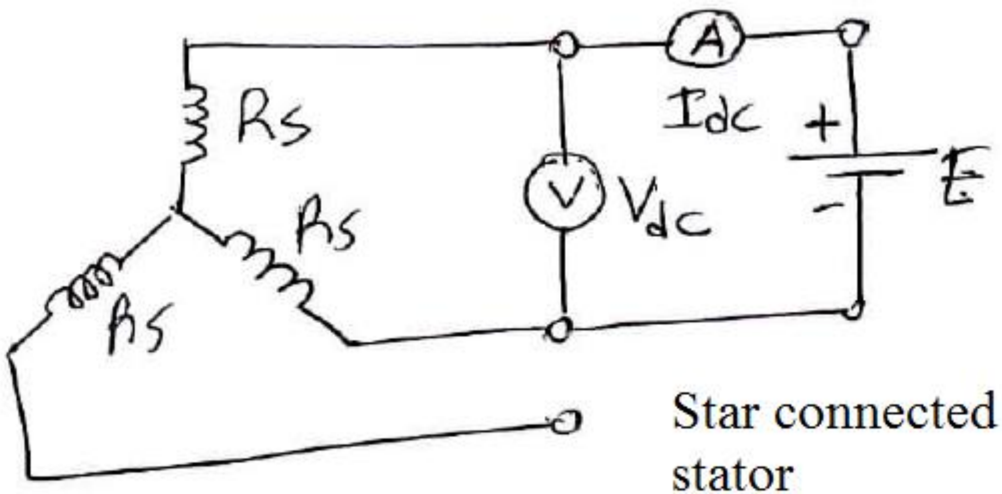
$$R_{sdc} = \frac{3}{2}.\frac{V_{dc}}{I_{dc}}$$

$$R_{sdc} * 1{,}25 = R_{sac}$$

- The reason for multiplying DC resistance by “1,25” coefficient is the skin effect. Resistance of the conductors against DC current is smaller than that of the conductors against AC current. The coefficient used here varies between 1,2 and 2,5.

- When the motor operates under the rated load, the heat of the stator windings is high. Since the experiment is performed under the lab. conditions, R_{sac} resistance given above must be also compensated against temperature changes.

$$R_s = R_{sac} \cdot \frac{234,5 + 75}{234,5 + 25}$$



Star connected stator

$$\frac{V_{dc}}{I_{dc}} = 2 * R_{sdc}$$

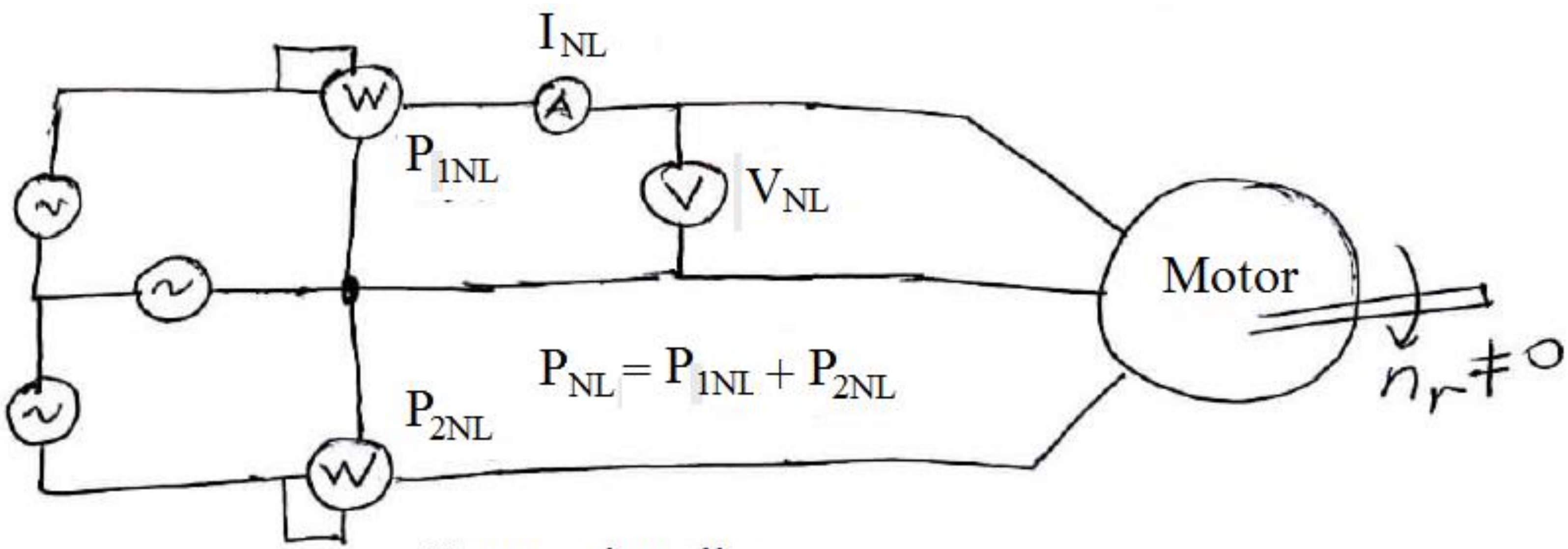
$$R_{sdc} = \frac{1}{2} \cdot \frac{V_{dc}}{I_{dc}}$$

$$R_{sdc} * 1{,}25 = R_{sac}$$

$$R_s = R_{sac} \cdot \frac{234{,}5 + 75}{234{,}5 + 25}$$

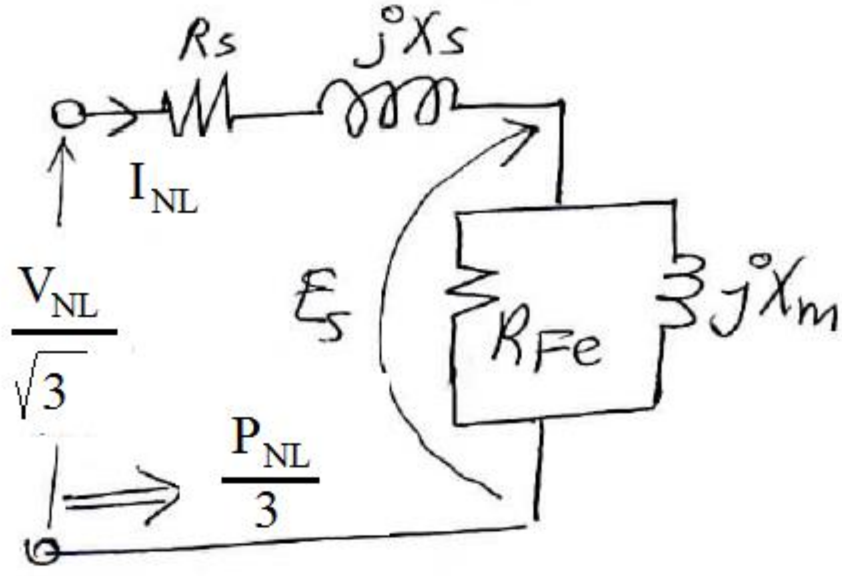
<u>No-Load Test:</u>

- Nominal voltage is applied to the stator windings and the motor is operated on no-load. If the motor is wound rotor type, the rotor windings are made short circuit. From the test, the electrical values given below are measured:
- V_{NL} = Stator line voltage (line-to-line)
- I_{NL} = Stator line current
- P_{NL} = 3-phase input power



Connection diagram

- After measuring P_{NL}, V_{NL} and I_{NL} by experiment, R_{Fe} and X_m parameters of the motor can be calculated as;



No-load equivalent circuit of the star connected motor.

$$S_{NL} = \frac{V_{NL}}{\sqrt{3}} . I_{NL}$$

$$Z_{NL} = \frac{V_{NL}}{\sqrt{3} * I_{NL}}$$

$$\theta = arccos\left(\frac{P_{NL}/3}{S_{NL}}\right)$$

$$Z_{NL} = |Z_{NL}|.(cos\theta + jsin\theta)$$

$$Y = \frac{1}{Z_{NL} - (R_s + jX_s)}$$

$$R_{Fe} = \frac{1}{Real(Y)}$$

$$X_M = \frac{-1}{Imaginary(Y)}$$

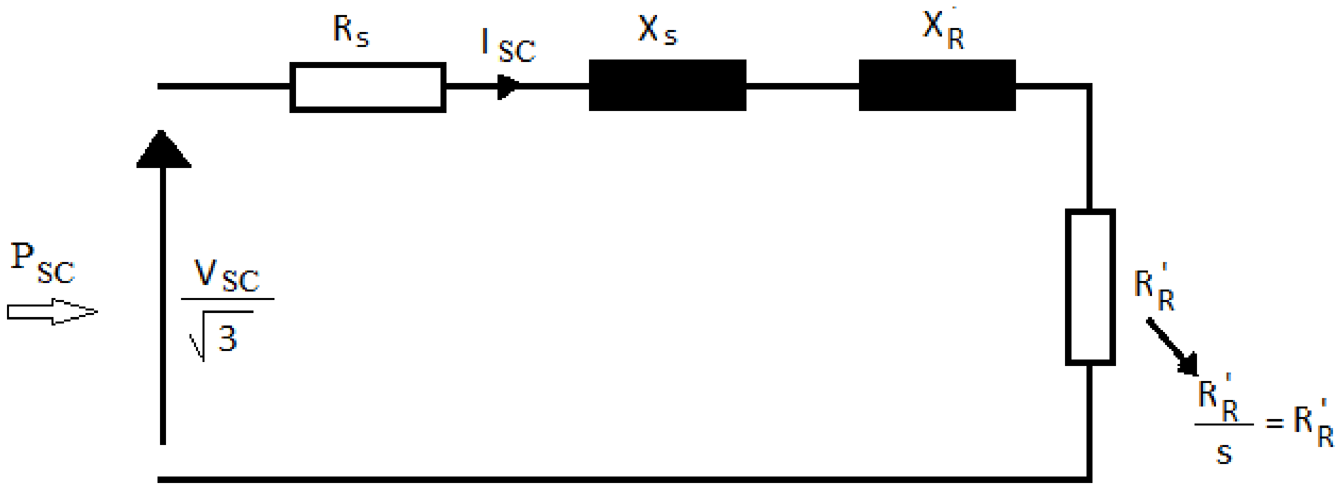
- Since the motor operates with very small slip value at no-load, the rotor copper losses are neglected. For this reason, the input power consists of the core losses, friction and ventilation losses and stator copper losses.

$$P_{NL} = P_{f+v} + P_{Fe} + 3.I_{NL}^2.R_s$$
$$P_{f+v} + P_{Fe} = P_{NL} - 3.I_{NL}^2.R_s$$

- It is possible to calculate P_{f+v}+ P_{Fe} losses if the stator resistance, R_s is measured by ohmmeter across the stator ends. Be aware that if the stator windings are connected in star then the stator resistance measured from two ends will be twice of each phase resistance. Power factor is small at no-load operation therefore the circuit is basically a reactive circuit. In this case, the input current is at least 30% of the rated current depending on the motor size. R_s resistance is small comparing to X_m reactance and can be neglected.

The Locked-Rotor Test

- In this test the rotor is prevented to turn. That is; speed of the rotor is made zero (n_R =0). In this case, s = 1 or 100%. If the stator voltage is at rated value, the stator current will be 5-6 times bigger than the rated current (as the starting current). For this reason, the stator voltage is hold at a small value which allows the rated current to flow in the motor. This voltage is 10%-20% of the rated voltage and results in a small airgap field. Therefore, X_m reactance is much bigger than its normal value and can be neglected in the equivalent circuit as;



- **Figure:** One-phase equivalent circuit of star connected (stator side) three-phase induction motor for the locked-rotor test.

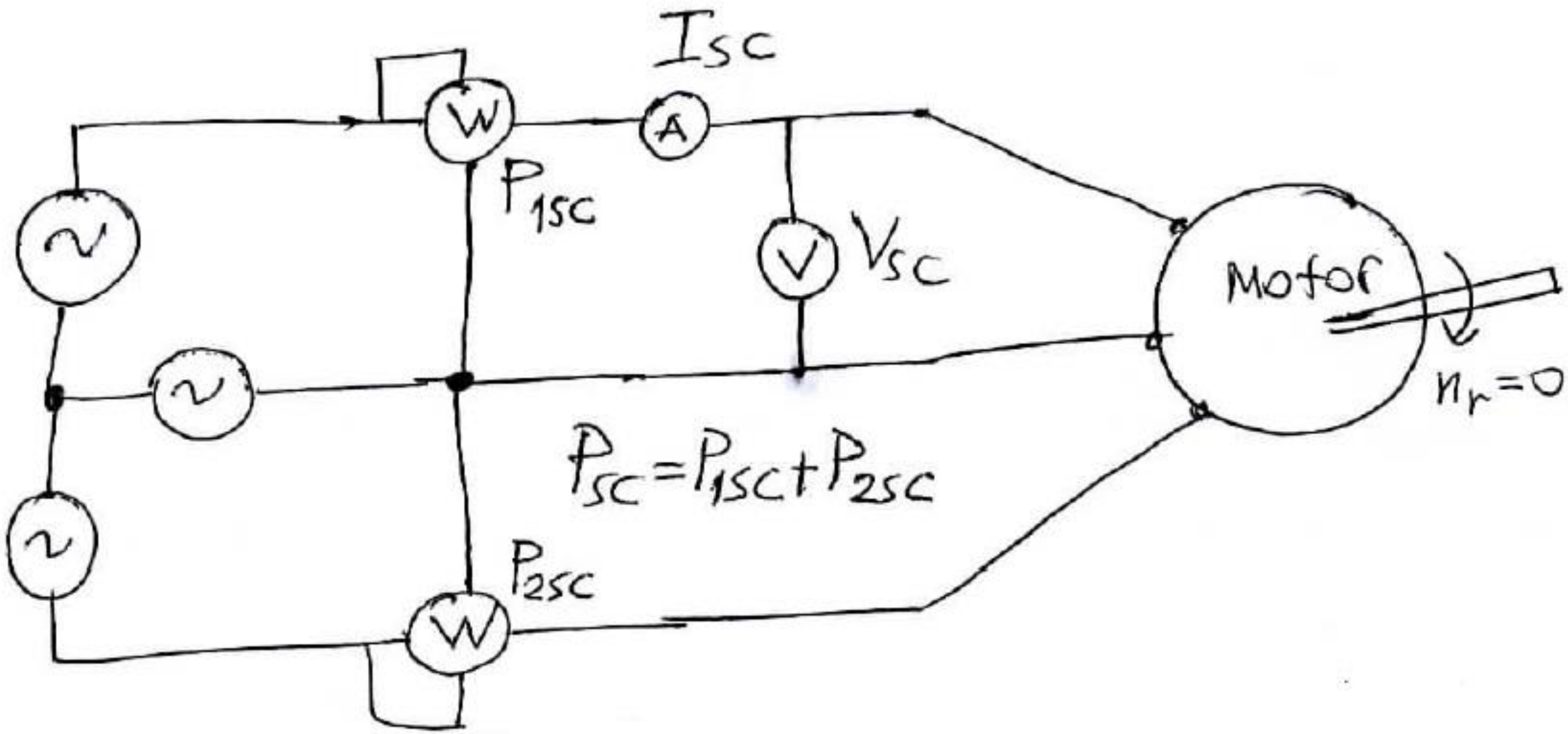


Figure: Experimental setup for the locked-rotor test.

- During the test the measurements given below are performed;
- V_{SC} = Stator line voltage
- I_{SC} = Line current
- P_{SC} = Three-phase input power
- By using the measurement results R'_R, X_S and X'_R parameters can be calculated. By assuming star connection;

$$\frac{P_{SC}}{3} = I_{SC}^2 * (R_S + R'_R)$$

$$R'_R = \frac{P_{SC}}{3.\, I_{SC}^2} - R_S$$

$$|Z_{SC}| = \sqrt{(R_S + R'_R)^2 + (X_S + X'_R)^2} = \frac{V_{SC}/\sqrt{3}}{I_{SC}}$$

Where; Z_{SC} is the equivalent impedance of the motor per phase referred to the stator.

$$X_S + X'_R = \sqrt{\left(\frac{V_{SC}/\sqrt{3}}{I_{SC}}\right)^2 - (R_S + R'_R)^2}$$

- Relationship between X_S and X'_R in the cage wound motor depends on the design class. These motors have A, B, C and D classes. According to NEMA (National Electrical Manufacturers Association) standards, the relationship between X_S and X'_R is given in the table;

Motor Class	$\mathbf{X_S}$	$\mathbf{X'_R}$
Wound Rotor Induction Motor	$\frac{X_S + X'_R}{2}$	$\frac{X_S + X'_R}{2}$
A	$\frac{X_S + X'_R}{2}$	$\frac{X_S + X'_R}{2}$
B	$\frac{2.(X_S + X'_R)}{5}$	$\frac{3.(X_S + X'_R)}{5}$
C	$\frac{3.(X_S + X'_R)}{10}$	$\frac{7.(X_S + X'_R)}{10}$
D	$\frac{X_S + X'_R}{2}$	$\frac{X_S + X'_R}{2}$

- External resistors are connected to the rotor circuit of the wound rotor induction motor for starting and speed control. The effective turn ratio between the stator and rotor should be determined to find the real rotor reactance. R_R resistance is determined in the same way as R_S.
- First, the rotor circuit is made open circuit and then the voltage is measured through the slip rings to determine the turn ratio. Meanwhile, the rotor will not turn because there will be no rotor current. In this case, the motor behaves as a transformer with the secondary side open.

- If the nominal voltage applied to the stator is V_{LL} and the voltage measured through the slip rings is V_{SR}, the turn ratio can be found as;

$$\alpha = \frac{V_{LL}}{V_{SR}} = \frac{N_S}{N_R}$$

- Where; N_S and N_R are the stator and rotor turn number per phase, respectively. The real rotor reactance is;

$$X_R = \frac{X'_R}{\alpha^2}$$

Similarly the real rotor resistance can be found as;

$$R_R = \frac{R'_R}{\propto^2}$$

R_R resistance found in the above equation must be same as the measured one.

- **Working Example:**
- Experimental results taken from 220 V, 50 Hz, 4 pole motor having star connected stator windings at 5 HP are (R_S=0,18 Ω);

No-load test	220 V	6,2 A	340 W
Locked-rotor test	54 V	15,2 A	430 W

- Calculate efficiency of the motor at 4% slip.

- **Solution:**

From the locked-rotor test;

$$R'_R = \frac{P_{SC}}{3.\,I_{SC}^2} - R_S = \frac{430}{3.\,(15{,}2)^2} - 0{,}18 = 0{,}44\ \Omega$$

$$X_S + X'_R = \sqrt{\left(\frac{V_{SC}/\sqrt{3}}{I_{SC}}\right)^2 - (R_S + R'_R)^2}$$

$$X_S + X'_R = \sqrt{\left(\frac{54/\sqrt{3}}{15{,}2}\right)^2 - (0{,}18 + 0{,}44)^2}$$

$$X_S + X'_R = 2{,}14\ \Omega$$

$$X_S = X'_R = \frac{2{,}14}{2} = 1{,}07\ \Omega$$

- From the no-load test;

$$S_{NL} = \frac{V_{NL}}{\sqrt{3}}.I_{NL} = \frac{220}{\sqrt{3}}.6,2 = 787,505\,VA$$

$$|Z_{NL}| = \frac{V_{NL}/\sqrt{3}}{I_{NL}} = \frac{220}{\sqrt{3}.6,2} = 20,486\,\Omega$$

$$\theta = arccos\left(\frac{P_{NL}/3}{S_{NL}}\right)$$

$$\theta = arccos\left(\frac{340/3}{787,505}\right) = 81,725^o$$

$$Z_{NL} = |Z_{NL}|.(cos\theta + jsin\theta)$$

$$Z_{NL} = 2,948 + j20,273\,\Omega$$

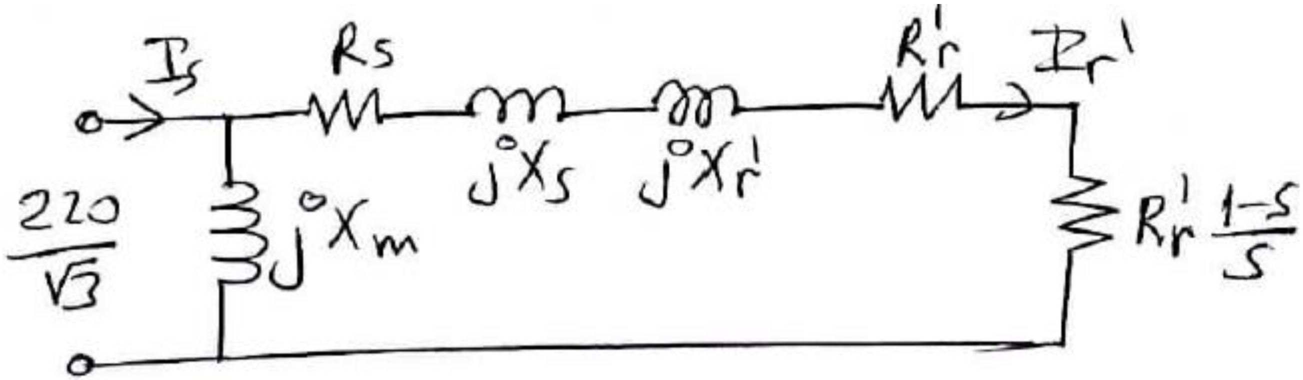
$$Y = \frac{1}{Z_{NL} - (R_s + jX_s)} = \frac{1}{2{,}948 + j20{,}273 - (0{,}18 + j1.07)}$$

$$Y = 735{,}5167 * 10^{-5} - j51{,}01{,}7742 * 10^{-5}\ Siemens$$

$$R_{Fe} = \frac{1}{Real(Y)} = \frac{1}{735{,}5167 * 10^{-5}} = 136\ \Omega$$

$$X_M = \frac{-1}{Imaginary(Y)} = \frac{-1}{-51{,}01{,}7742 * 10^{-5}} = 19{,}6\ \Omega$$

- Lets use the approximate equivalent circuit;



$S = 0{,}04$, $R_r' = 0{,}44\,\Omega$, $X_s + X_r' = 2{,}14\,\Omega$, $R_s = 0{,}18\,\Omega$
$X_m = 19{,}6\,\Omega$

$$\bar{I}_R' = \frac{(220/\sqrt{3}).e^{j0}}{\left(0{,}18 + 0.44 + 0{,}44.\left(\frac{1-0{,}04}{0{,}04}\right) + j2{,}14\right)}$$

$$\bar{I}_R' = 11{,}15.e^{-j10{,}836^{o}}\,A \quad \rightarrow I_R' = 11{,}15\,A$$

$$RPD = 3.I'^{2}_{R}.R'_{R}.\left(\frac{1-s}{s}\right) = 3 * (11{,}15)^{2} * 0{,}44 * \left(\frac{1-0{,}04}{0{,}04}\right)$$

$$RPD = 3944{,}55\ W$$

$$P_o = RPD - \left(P_{f+v} + P_{Fe}\right) = 3944{,}55 - \left(P_{f+v} + P_{Fe}\right)$$

$$P_{NL} = 340\ W = P_{f+v} + P_{Fe} + 3.I_{NL}^{2}.R_s$$

$$340 = P_{f+v} + P_{Fe} + 3.(6{,}2)^{2}.0{,}18$$

$$P_{f+v} + P_{Fe} = 319{,}242\ W$$

$$P_o = 3944{,}55 - 319{,}242 = 3625{,}3\ W$$

$$RCL = 3.\,I'^{2}_{R}.\,R'_{R}$$

$$RCL = 3.\,(11{,}15)^{2}.\,0{,}44 = 164{,}356\ W$$

$$\bar{I}_m = \frac{\left(\frac{220}{\sqrt{3}}\right) e^{j0}}{j19{,}6} = 6{,}48.\,e^{-j90^o}\ A$$

$$\bar{I}_S = \bar{I}_m + \bar{I}'_R$$

$$\bar{I}_S = \frac{\bar{V}_S}{jX_m} + \frac{\bar{V}_S}{(R_S + R'_R/s) + (X_S + X'_R)}$$

$$\bar{I}_S = 6{,}48.\,e^{-j90^o} + 11{,}15.\,e^{-j10{,}836^o}\,A$$

$$\bar{I}_S = 13{,}917.\,e^{-j38{,}05^o}\ A$$

$$SCL = 3.I_S^2.R_S = 3 * (13{,}917)^2 * 0{,}18$$

$$SCL = 104{,}597\ W$$

$$Losses = RCL + SCL + P_{f+v} + P_{Fe}$$

$$Losses = 164{,}356 + 104{,}597 + 319{,}249 = 588{,}195\ W$$

$$\eta = 100.\frac{P_o}{P_o + Losses} = 100.\frac{3625{,}3}{3625{,}3 + 588{,}195}$$

$$\boxed{\eta = 86\%}$$

- **Example:**

The no-load and locked-rotor test results of a three-phase 10 HP, 220 V, 60 Hz, 6 poles star

V_{NL}=220 V, P_{NL}=340 W, I_{NL}=6.2 A

The locked-rotor test:

V_{SC}=54 V, P_{SC} =430 W, I_{SC} =15.2 A

The voltage across the stator windings is 4 V when nominal DC current flows through it. Coefficient for DC resistance is 1,25.

a) Calculate the efficiency of the motor for s=0.06 by using the complete equivalent circuit.
b) Slip value at the maximum (breakdown) torque,
c) Maximum (breakdown) torque of the induction motor.
d) Speed of the motor at maximum torque point.

- **Solution:**
- For star connected stator windings;

$$R_{sdc} = \frac{1}{2} \cdot \frac{V_{dc}}{I_{dc}} \qquad R_{Sdc} = \frac{1}{2} \cdot \frac{4}{15{,}2} = 0{,}1316\ \Omega$$

- The effective AC resistance (because of skin effect);

$$R_{Sac} = 1{,}25 * R_{Sdc} = 1{,}25 * 0{,}1316 = 0{,}1645\ \Omega$$

When the motor operates under the rated load, the heat of the stator windings is high. Since the experiment is performed under the lab. conditions, R_{sac} resistance given above must be also compensated against temperature changes.

$$R_s = R_{sac} \cdot \frac{234{,}5 + 75}{234{,}5 + 25}$$

$$R_S = 0{,}1645 * \frac{234{,}5 + 75}{234{,}5 + 25} = 0{,}1962\ \Omega$$