№11

$H_0: \rho_0(x) = \begin{cases} 1; & x\in(0,1) \\ 0; & x\notin(0,1)\end{cases}$ $\quad H_1: \rho_1(x) = \begin{cases} \frac{e^{1-x}}{e-1}; & x\in(0,1)\\ 0; & x\notin(0,1)\end{cases}$

а) $n=1$; $l(x) = \frac{L_1}{L_0} = \frac{e^{1-x}}{e-1} \geq C$; $e^{-x} \geq \tilde{C}$

$G: x \leq B$ – крит. область

$P(x\in G|H_0)=\alpha$

$\int_0^B 1\cdot dx = \alpha$; $B=\alpha$; $G: x\le\alpha$

$W = P(x\in G|H_1) = \int_0^\alpha \frac{e^{1-x}}{e-1}dx = \frac{e}{e-1}(1-e^{-\alpha})$

$\alpha_2 = 1-W$; $\alpha_1=\alpha$

б) $n=2$; $l = \frac{L_1}{L_0} = \frac{e^{-x_1+1}}{(e-1)}\cdot\frac{e^{-x_2+1}}{(e-1)} \ge C$; $e^{-x_1-x_2}\ge\tilde C$

$G: x_1+x_2\le B$; $P(\vec{x}; G|H_0)=\alpha$

$\iint_G 1\cdot 1\cdot dx_1\cdot dx_2 = \alpha$; $\frac{B^2}{2}=\alpha$; $B=\sqrt{2\alpha}$

$G: x_1+x_2\le\sqrt{2\alpha}$ крит. область.

$W = P(\vec x\in G|H_1) = \frac{e^2}{(e-1)}\iint_G e^{-x_1-x_2}dx_1dx_2 =$

$= \frac{e^2}{(e-1)^2}\int_0^B dx_1\int_0^{B-x_1} e^{-x_1}\cdot e^{-x_2}dx_2 = \frac{e^2}{(e-1)}\int_0^B e^{-x_1}(1-e^{-(B-x_1)})dx_1$

$= \frac{e^2}{(e-1)^2}(1-e^{-B}-Be^{-B})$; $\alpha_2 = 1-W$

$\alpha_1=\alpha$

c) Асс. по n $\quad l = \frac{L_1}{L_0} = \prod_{i=1}^{n} p_i(x)$; $\ln l = \sum_{i=1}^{n} \ln p_i(x_i)$

$G: l \geq C \Rightarrow \eta = \ln p_i(\xi) = \ln \frac{e^{1-\xi}}{e-1} = \ln \frac{e}{e-1} - \xi$

$P(\ln l \geq \ln C \mid H_0) = \alpha$

$\frac{\sum \eta_i - n M[\eta]}{\sqrt{n D[\eta]}} \to N(0,1) \Rightarrow$ найти шкал X-ки

H_0: $M[\eta] = M[\ln \frac{e}{e-1} - \xi] = \ln \frac{e}{e-1} - \frac{1}{2}$

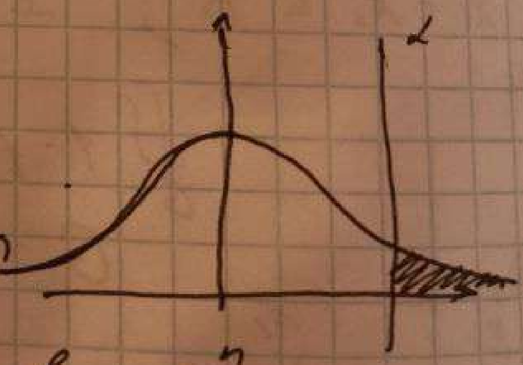
$M[\xi] = \int_0^1 x p_0(x) dx = \frac{1}{2} \to (p_0(x) = 1 \quad (0,1))$

$D[\eta] = D[\ln \frac{e}{e-1} - \xi] = D[\xi] = \frac{1}{12}$

$D[\xi] = \frac{(b-a)^2}{12} = \frac{1}{12} \to$

$P(\ln l \geq \ln C) = P\left(\frac{\sum \eta_i - n M[\eta]}{\sqrt{n D[\eta]}} \geq \frac{\ln C - n M[\eta]}{\sqrt{n D[\eta]}}\right) = \alpha$

$\frac{\ln C - n(\ln \frac{e}{e-1} - \frac{1}{2})}{\sqrt{n/12}} = u_{1-\alpha}$



$\ln C = n(\ln \frac{e}{e-1} - \frac{1}{2}) + u_{1-\alpha} \sqrt{\frac{n}{12}}$

$\ln l = \sum_{i=1}^{n} \ln (\frac{e}{e-1} \cdot e^{-x_i}) = n \ln \frac{e}{e-1} - \sum_{i=1}^{n} x_i.$

$G: \ln l \geq \ln C$; $\quad -\sum_{i=1}^{n} x_i \geq -\frac{n}{2} + u_{1-\alpha} \sqrt{\frac{n}{12}}$

$G: \bar{X} \leq \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}}$ — критическая область

$W = P(\overline{X_n} \in G | H_1) = P(\overline{X} \le \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}} | H_1)$; $\alpha_1 = \alpha$, $\alpha_2 = 1 - W$

$W = P(\overline{X} \le \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}} | H_1)$

$\frac{\overline{X} - M[\xi]}{\sqrt{D[\xi]}} \sqrt{n} \rightsquigarrow N(0,1)$

$H_1: M[\xi] = \int_0^1 x \frac{e}{e-1} e^{-x} dx = \frac{e}{e-1} [-xe^{-x}|_0^1 + \int_0^1 e^{-x} dx]$

$= \frac{e}{e-1} [-e^{-1} + 1 - e^{-1}] = \frac{e-2}{e-1}$

$M[\xi^2] = \int_0^1 x^2 \frac{e}{e-1} e^{-x} dx = \frac{2e-5}{e-1}$; $D[\xi] = \frac{e^2-3e+1}{(e-1)^2}$

$W = P(\overline{X} \le A | H_1) = P\left(\frac{\overline{X} - M[\xi]}{\sqrt{D[\xi]}} \sqrt{n} \le \frac{A - M[\xi]}{\sqrt{D[\xi]}} \sqrt{n}\right)$

$= \int_{-\infty}^{\frac{A - M[\xi]}{\sqrt{D[\xi]}} \sqrt{n}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$

d) $G: X_{min} < C$

$H_0: F_0(x) = \begin{cases} 0, & x < 0 \\ x, & x \in [0,1] \\ 1, & x > 1 \end{cases}$ $\quad F_{min}(x) = 1 - (1 - F(x))^n$

$P(X_{min} \le c | H_0) = F_{min}(c) = \alpha$

$1 - (1-c)^n = \alpha \Rightarrow c = 1 - \sqrt[n]{1-\alpha}$ $\quad \underline{\alpha_1 = \alpha}$

$W = P(X_{min} \le c | H_1)$

$H_1: F(x) = \int_0^x \frac{e}{e-1} e^{-t} dt = \frac{e}{e-1}(1 - e^{-x})$ $(0,1)$

$W = F_{min}(c) = 1 - (1 - \frac{e}{e-1}(1 - e^{-c}))^n = 1 - (1 - \frac{e}{e-1}(1 - e^{\sqrt[n]{1-\alpha} - 1}))^n$

$\alpha_2 = 1 - W = (1 - \frac{e}{e-1}(1 - e^{\sqrt[n]{1-\alpha} - 1}))^n$

№12

	1	2	3	4
H_0:	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{3}$
H_1:	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$$p_0(x) = \frac{1}{3}\delta(x-4) + \frac{1}{6}\delta(x-3) + \frac{1}{4}\delta(x-2) + \frac{1}{4}\delta(x-1)$$

$$p_1(x) = \frac{1}{4}\delta(x-4) + \frac{1}{4}\delta(x-3) + \frac{1}{4}\delta(x-2) + \frac{1}{4}\delta(x-1)$$

$$n=2: \quad l = \frac{L_1}{L_0} = \frac{\left[\frac{1}{4}\delta(x-4) + \frac{1}{4}\delta(x-3) + \frac{1}{4}\delta(x-2) + \frac{1}{4}\delta(x-1)\right]^2}{\left[\frac{1}{3}\delta(x-4) + \frac{1}{6}\delta(x-3) + \frac{1}{4}\delta(x-2) + \frac{1}{4}\delta(x-1)\right]^2}$$

$l \geqslant C$

$P(l \geqslant C \mid H_0) = \alpha = 0{,}2$

H_0:

II \ I	1	2	3	4
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{24}$	$\frac{1}{12}$
2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{24}$	$\frac{1}{12}$
3	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{36}$	$\frac{1}{18}$
4	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{9}$

H_1: все значения $\frac{1}{16}$

l:

II \ I	1	2	3	4
1	1	1	$\frac{3}{2}$	$\frac{3}{4}$
2	1	1	$\frac{3}{2}$	$\frac{3}{4}$
3	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{9}{4}$	$\frac{9}{8}$
4	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{9}{8}$	$\frac{9}{16}$

увеличение

Выбираем разные l:

C	$\frac{9}{16}$	$\frac{3}{4}$	1	$\frac{9}{8}$	$\frac{3}{2}$	$\frac{9}{4}$
α	1	$\frac{8}{9}$	$\frac{5}{9}$	$\frac{11}{36}$	$\frac{7}{36}$	$\frac{1}{36}$
W	1	$\frac{15}{16}$	$\frac{11}{16}$	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{1}{16}$

$\alpha_1 = \sum l$; при $l \geqslant C$ в H_0

↳ смотрим в табл H_0 и $\sum$ всё у кого в $l \geqslant C$

W – аналог w

$W \to max$ при $\alpha_1 < 0{,}2 \Rightarrow W = \frac{5}{16}$; $\alpha = \frac{3}{2}$; G: ~~[illegible]~~ выпадет хотя бы один гербик

№1~~3~~4

$X_n \sim N(a, 2)$; $y_m \sim N(b, 1)$ $\alpha = 0{,}05$

$x = \{-1{,}11; -6{,}10; 2{,}42\}$ $H_0: a = b$

$y = \{-2{,}29; -2{,}91\}$ $H_1: a > b;\ a < b;\ a \neq b$

$$\Delta = \frac{(\bar{x} - \bar{y})\sqrt{\frac{n \cdot m}{n+m}}}{\sqrt{\frac{S_x^2(n-1) + S_y^2(m-1)}{n+m-2}}} \sim t(n+m-2)$$

$\Leftrightarrow$ или $\tilde{\Delta} = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}}$?

$$\tilde{\Delta} = \frac{(-1{,}596 + 2{,}6)\sqrt{\frac{6}{5}}}{\sqrt{\frac{S_x^2(2) + S_y^2}{3}}} \sim t(3)\ ;\quad \tilde{\Delta} = 0{,}313 \sim N(0,1)$$

1) $a > b$

$$p\text{-value} = P(\Delta \geq |\tilde{\Delta}|) = \int_{0{,}313}^{+\infty} \frac{2}{\sqrt{\pi}\sqrt{\pi}\sqrt{3}}\left(1 + \frac{y^2}{3}\right)^{-2} dy = 0{,}387 > 0{,}05$$

$\Rightarrow$ нет осн. отв.

2) $a < b$

$$p\text{-value} = P(\Delta \leq -|\tilde{\Delta}|) = \int_{-\infty}^{-0{,}313} t(3)\,dy = 0{,}387 > 0{,}05$$

$\Rightarrow$ нет осн. отвергнуть

3) $a \neq b$

$$p\text{-value} = P(|\Delta| \geq |\tilde{\Delta}|) = 2\int_{-\infty}^{-0{,}313} = 0{,}774 > 0{,}05$$

$\Rightarrow$ нет осн. отвергнуть H_0