Первое задание:

№1

X_n - выборка:

$\xi \sim R(0,\theta)$

1) $\tilde{\theta}_1 = 2\bar{X} = \frac{2}{n}\sum_{i=1}^{n} x_i$

2) $\tilde{\theta}_2 = X_{min}$

3) $\tilde{\theta}_3 = X_{max}$

4) $\tilde{\theta}_4 = X_{min} + X_{max}$

5) $\tilde{\theta}_5 = X_1 + \frac{\sum_{k=2}^{n} X_k}{(n-1)}$

1) $M\tilde{\theta}_1 = M\left[\frac{2}{n}\sum_{i=1}^{n} x_i\right] = \frac{2}{n}\sum M x_i =$

$x_i \sim R(0,\theta) \; : \; M\xi = \frac{\theta}{2}$ $\quad = \frac{2}{n} n \frac{\theta}{2} =$

$= \theta$ - ~~несов~~ несмещенная

$D[\tilde{\theta}_1] = D\left[\frac{2}{n}\sum x_i\right] = \frac{4}{n^2}\sum D x_i = \frac{4}{n^2} n D\xi = \frac{\theta^2}{3n} \xrightarrow[n\to\infty]{} 0$

$D\xi = \frac{(b-a)^2}{12}$ - состоятельна

2) $M\tilde{\theta}_2 = M[X_{min}]$:

$\min(\xi_1 \ldots \xi_n) \sim 1-(1-F(y))^n$

$\varphi(y) = n(1-F(y))^{n-1} p(y) =$

$= n\left(1-\frac{y}{\theta}\right)^{n-1}\frac{1}{\theta}(0,\theta)$

$M[X_{min}] = \int_0^{\theta} y n \frac{1}{\theta}\left(1-\frac{y}{\theta}\right)^{n-1} dy =$

$= \left| t = \frac{y}{\theta} \right| = \int_0^1 t\,\theta n (1-t)^{n-1} dt = n\theta B(2,n) =$

$= n\theta \frac{\Gamma(2)\Gamma(n)}{\Gamma(n+2)} = \frac{\theta}{n+1}$ - смещена, не асс. несмещ.

Исправляем $\rightarrow$ $\theta_2' = (n+1) X_{min}$ - несмещ.

$M[X_{min}^2] = \int_0^{\theta} y^2 n\left(1-\frac{y}{\theta}\right)^{n-1}\frac{1}{\theta} dy = \int_0^1 \theta^2 t^2 n(1-t)^{n-1} dt$

$= \theta^2 n B(3,n) = \frac{2\theta^2}{(n+2)(n+1)}$

$D[X_{min}] = \frac{2\theta^2}{(n+2)(n+1)} - \frac{\theta^2}{(n+1)^2} = \frac{\theta^2[(n+1)2 - n - 2]}{(n+1)^2(n+2)} =$

$$= \frac{\Theta^2 n}{(n+1)^2(n+2)}$$

$$\mathcal{D}[\Theta_2'] = \mathcal{D}[(n+1)X_{min}] = \frac{\Theta^2 n}{n+2} \underset{n\to\infty}{\not\to} 0$$ ~~состоятельн~~

Проверка: Определено: $\tilde{\Theta}_2' \xrightarrow{P} \Theta$

$$\Theta_2' = (n+1)X_{min}\ ;\ \forall \varepsilon > 0\ P(|\Theta_2' - \Theta| \geq \varepsilon) \underset{n\to\infty}{\to} 0$$

$$P(|\Theta_2' - \Theta| \geq \varepsilon) \geq P(\Theta_2' \geq \Theta + \varepsilon) = P\left(X_{min} \geq \frac{\Theta+\varepsilon}{n+1}\right)$$

$$= P(X_1 \geq \_,\ X_2 \geq \_,\ X_n \geq \ldots) = \prod_{i=1}^{n} P\left(X_i \geq \frac{\Theta+\varepsilon}{n+1}\right) =$$

$$= \left(1 - F\left(\frac{\Theta+\varepsilon}{n+1}\right)\right)^n = \left(1 - \frac{\Theta+\varepsilon}{\Theta(n+1)}\right)^n \underset{n\to\infty}{\longrightarrow} e^{-\frac{\Theta+\varepsilon}{\Theta}} > 0$$

несостоятельна

③ $\tilde{\Theta}_3 = X_{max}$: $X_i \sim F(x)$; $X_{max} \sim (F(y))^n$

$$\rho_{X_{max}}(y) = n(F(y))^{n-1}\rho(y)\quad (0, \Theta)$$

$$M[\tilde{\Theta}_3] = \int_0^\Theta y\, n\left(\frac{y}{\Theta}\right)^{n-1}\frac{1}{\Theta}\, dy = \frac{n}{\Theta^n}\int_0^\Theta y^n\, dy = \frac{n\Theta}{n+1}$$

– смещена, асс. несмещ

$$\Theta_3' = \frac{n+1}{n} X_{max} \text{ – несмещ.}$$

$$M[\tilde{\Theta}_3^2] = \int_0^\Theta y^2 n\left(\frac{y}{\Theta}\right)^{n-1}\frac{1}{\Theta}\, dy = \frac{n}{\Theta^n}\int_0^\Theta y^{n+1}\, dy = \frac{\Theta^2 n}{n+2}$$

$$\mathcal{D}[\tilde{\Theta}_3] = \frac{\Theta^2 n}{n+2} - \frac{n^2\Theta^2}{(n+1)^2} = \Theta^2\frac{n}{(n+2)(n+1)^2}$$

$$D[\Theta_3'] = \frac{(n+1)^2}{n^2}\,\frac{n\Theta^2}{(n+2)(n+1)^2} = \frac{\Theta^2}{n(n+2)} \xrightarrow[n\to\infty]{} 0$$

$\Leftarrow$ состоятельна

$$\tilde{\Theta}_4 = X_{min} + X_{max}$$

$$M[\tilde{\Theta}_4] = M[X_{min}] + M[X_{max}] = \frac{\Theta}{n+1} + \frac{n\Theta}{n+1} = \Theta$$

несмещ.

$$D[\tilde{\Theta}_4] = D[X_{min}] + D[X_{max}] + 2\,cov(X_{max}, X_{min})$$

$$cov(X_{max}, X_{min}) = M[X_{min} X_{max}] - M[X_{min}]\cdot M[X_{max}]$$

$$p(y, z) = n(n-1)\left(F(z) - F(y)\right)^{n-2} p(z)\,p(y)$$

$$M[X_{min} X_{max}] = \iint_D yz \cdot n(n-1)\left(\frac{z}{\Theta} - \frac{y}{\Theta}\right)^{n-2} \frac{1}{\Theta}\,\frac{1}{\Theta}\,dz\,dy$$

$$= \int_0^\Theta dz \int_0^z yz(n-1)n\left(\frac{z}{\Theta} - \frac{y}{\Theta}\right)^{n-2} \frac{1}{\Theta^2}\,dy = \left\{t = \frac{y}{z}\right\} =$$

$$= \frac{n(n-1)}{\Theta^n}\int_0^\Theta dz \int_0^1 t z^2 (z - tz)^{n-2}\,dt =$$

$$= \frac{n(n-1)}{\Theta^n}\int_0^\Theta z^{n+1}\,dz \int_0^1 t(1-t)^{n-2}\,dt =$$

$$= \frac{n(n-1)}{\Theta^n}\int_0^\Theta z^{n+1}\,\frac{1}{n(n-1)}\,dz = \frac{\Theta^{n+2}}{(n+2)\Theta^n} = \frac{\Theta^2}{n+2}$$

$$cov(X_{min}, X_{max}) = \frac{\Theta^2}{n+2} - \frac{\Theta}{n+1}\cdot\frac{n\Theta}{n+1} = \frac{\Theta^2}{(n+2)(n+1)^2}$$

$$D[\tilde{\Theta}_4] = \frac{\Theta^2 n}{(n+1)^2(n+2)} + \frac{\Theta^2 n}{(n+1)^2(n+2)} + \frac{2\Theta^2}{(n+2)(n+1)^2} =$$

$$= \frac{2\Theta^2}{(n+1)(n+2)} \xrightarrow[n\to\infty]{} 0$$ состоятельна.

⑤ $M[\tilde{\Theta}_5] = M[X_1] + \frac{n}{n-1} + \frac{n}{n-1} M[X_2] =$

$= \frac{\Theta}{2}\left(1 + \frac{n-1}{n-1}\right) = \Theta$; $\quad X_i \sim R(0,\Theta) : M[X_i] = \frac{\Theta}{2}$

несмещ.

$D[\tilde{\Theta}_5] = D[X_1] + \frac{(n-1)}{(n-1)^2} D[X_2] = \frac{\Theta^2}{12}\left(1 + \frac{1}{n-1}\right) \underset{n\to\infty}{\not\to} 0$

$X_1 + \frac{1}{n-1}\sum_{m=2}^{n} X_m \xrightarrow{P} X_1 + \frac{\Theta}{2}$ – не явл. состоят.

Эффективность: Θ_2 и Θ_5 – вычерк., Θ_3 тоже

Исследуем $\tilde{\Theta}_1$, $\tilde{\Theta}_3'$, $\tilde{\Theta}_4$: $D\tilde{\Theta}_1 = \frac{\Theta^2}{3n}$; $D\tilde{\Theta}_3' = \frac{\Theta^2}{n(n+2)}$

$D\tilde{\Theta}_4 = \frac{2\Theta^2}{(n+1)(n+2)}$

1–3: $\frac{\Theta^2}{3n} > \frac{\Theta^2}{n(n+2)}$ для $n > 1$

3–4: $\frac{\Theta^2}{n(n+2)} < \frac{2\Theta^2}{(n+1)(n+2)}$ для $n > 1$ $\Rightarrow \tilde{\Theta}_3'$ лучше

№2

а)

$p(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$; $\quad \gamma = \frac{\tilde{\mu}_3}{(\tilde{\mu}_2)^{\frac{3}{2}}}$

$\alpha_1 = \int_0^\infty x e^{-x} dx = -xe^{-x}\Big|_0^\infty + \int_0^\infty e^{-x} dx = -e^{-x}\Big|_0^\infty = 1$

~~$\mu_2 = \frac{1}{n}\sum(x_i - \bar{x})^2$~~

$\mu_2 = \int_0^\infty (x-1)^2 e^{-x} dx = \int_0^\infty x^2 e^{-x} dx - 2\int_0^\infty x e^{-x} dx$

$+ \int_0^\infty e^{-x} dx = 2 - 2 + 1 = 1$

$$\mu_3 = \int_0^\infty (x-1)^3 e^{-x}dx = \int_0^\infty x^3 e^{-x}dx - 3\int_0^\infty x^2 e^{-x}dx$$

$$+3\int_0^\infty x e^{-x}dx - \int_0^\infty e^{-x}dx = 6 - 3\cdot 2 + 3\cdot 1 - 1 = 2$$

$$\gamma = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}} = 2$$

c) Ядерная оценка плотности:

$$h = 2{,}344\,\frac{s}{\sqrt[5]{n}}; \quad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

$$\mu_2 = \frac{1}{n}\sum(x_i-\bar{x})^n; \quad \mu_2 n = \sum_{i=1}^{n}(x_i-\bar{x})^2$$

$$s^2 = \frac{\mu_2 n}{n-1}; \quad s = \sqrt[2]{\frac{\mu_2 n}{n-1}}$$

$$\tilde{p}(z) = \frac{1}{nh}\sum_{i=1}^{n} q\left(\frac{z-x_i}{h}\right) = \frac{1}{nh}\sum_{i=1}^{n}\left[\frac{3}{4}\left(1-\left(\frac{z-x_i}{h}\right)^2\right)\right]$$

$$\tilde{p}(z) = \frac{3}{4nh}\sum_{i=1}^{n}\left(1-\left(\frac{z-x_i}{h}\right)^2\right) = \cancel{\frac{3}{4nh}\sum_{i=1}^{n}\left(h-\frac{z-x_i}{h}\right)^2}$$

~~d) ЦПТ:~~ $\cancel{\sqrt{n}\,\frac{\tilde{\alpha}_n-\alpha_n}{\sqrt{\tilde{\alpha}_{2n}-\tilde{\alpha}_n^2}} \to N(0,1)}$

$\cancel{\tilde{\alpha}_1 = \frac{1}{n}\sum_{i=1}^{n}x_i; \quad \tilde{\alpha}_2 = \frac{1}{n}\sum_{i=1}^{n}x_i^2;}$

$\cancel{\sqrt{\tilde{\alpha}_2-\tilde{\alpha}_1^2} \quad N(0,1)}$

$M[\tilde{\theta}_{(2)}] = \frac{n(n-1)}{\theta} \int\limits_0 x (1 - e \ldots) \ldots dx =$

$= \frac{n(n-1)}{\theta} \left(\int\limits_0^{\infty} x e^{-\frac{x}{\theta}(n-1)} dx + \int\limits_0^{\infty} x e^{-\frac{x}{\theta} n} dx \right) =$

$= \frac{n(n-1)}{\theta} \left(\frac{\theta^2}{(n-1)^2} - \frac{\theta^2}{n^2} \right) = \frac{n\theta}{n-1} - \frac{(n-1)\theta}{n} = \frac{2n-1}{n(n-1)} \theta$ // смещ.

$\tilde{\theta}_3 = \frac{n(n-1)}{2n-1} X_{(2)}$ – несмещ

б) Нер-во Крамера-Рао: $D[\tilde{\theta}] \geq \frac{1}{n J(\theta)}$

№2

д) $\bar{x} = \frac{1}{n} x_i$; $n = 25$

Характ. ф-я: $f(t; x_i) = \int\limits_0^a e^{ixt} e^{-x} dx = \frac{-1}{it-1}$

$f(t; \sum \frac{1}{n} x_i) = f^n(\frac{t}{n}; x_i) = \left[\frac{-1}{i\frac{t}{n} - 1}\right]^n = \left[\frac{n}{n-it}\right]^n$

$\xi \sim \Gamma(\lambda, a)$: $\left(\frac{\lambda}{\lambda - it}\right)^a \Rightarrow \Gamma(\lambda, a)$

$\bar{x} \sim \left(\frac{n}{n-it}\right)^n \sim \Gamma(n, n)$

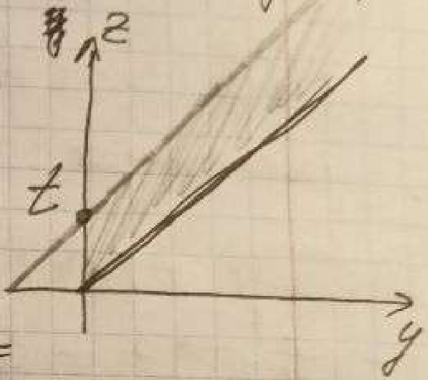
е) Найти плотность распред. размаха выборки

$l = X_{max} - X_{min}$; $\xi \sim e^{-x} (0, +\infty)$

$$\Phi(t) = P(\ell < t) = P(x_{max} - x_{min} < t) = \iint\limits_{z-y<t} \rho(y,z)\,dy\,dz$$

$$\rho(y,z) = n(n-1)\,\rho(y)\,\rho(z)\,[F(y) - F(z)]^{n-2} \qquad (y \le z)$$

$$(z = y+t;\quad F(z) = 1 - e^{-z})$$



$$= \int_0^\infty dy \int_y^{y+t} \rho(y,z)\,dz$$

$$\psi(t) = \Phi'(t) = \int_0^\infty \rho(y, y+t)\,dy =$$

$$= \int_0^\infty n(n-1)\, e^{-y} e^{-z} (e^{-y} - e^{-z})^{n-2}\, dy =$$

$$= n(n-1)(1-e^{-t})^{n-2} e^{-t} \int_0^\infty e^{-2y} \cdot e^{-y(n-2)}\, dy = \quad \left(= \frac{1}{n}\right)$$

$$= (n-1)(1-e^{-t})^{n-2} e^{-t} \quad //$$

№3

$$\xi(x) \sim \rho(x) = \begin{cases} e^{-\frac{x}{\theta}}/\theta, & x \ge 0 \\ 0, & x < 0 \end{cases}, \quad \theta > 0 \qquad n = 3$$

1) $\tilde{\Theta}_1 = \bar{X}$ 2) $\tilde{\Theta}_2 = \dfrac{X_{min} + X_{max}}{2}$ 3) $\tilde{\Theta}_3 = X_{(2)}$

0) $\xi \sim \rho(x)$: $M[\xi] = \int_0^\infty x \cdot \dfrac{e^{-\frac{x}{\theta}}}{\theta}\, dx = \dfrac{1}{\theta}\int_0^\infty x \cdot e^{-\frac{x}{\theta}}\, dx = \dfrac{1}{\theta}\cdot \theta^2 = \theta$

$$M[\xi^2] = \frac{1}{\theta}\int_0^\infty x^2 e^{-\frac{x}{\theta}}\, dx = \frac{1}{\theta}\cdot 2 \cdot \theta^3 = 2\theta^2$$

$D[\xi] = M[\xi^2] - M^2[\xi] = 2\Theta^2 - \Theta^2 = \Theta^2$

1) $\tilde{\Theta}_1 = \bar{X}$

$M[\tilde{\Theta}_1] = M[\frac{1}{n}\sum X_i] = \frac{1}{n} M[\sum X_i] = \frac{1}{n} \cdot n \cdot M[\xi] = \Theta$

– несмещенность

$D[\tilde{\Theta}_1] = D[\frac{1}{n}\sum X_i] = \frac{1}{n^2}\sum DX_i = \frac{1}{n^2} \cdot \Theta^2 \cdot n = \frac{\Theta^2}{n} \xrightarrow[n\to\infty]{} 0$ – состоятельность

2) $\tilde{\Theta}_2 = \frac{X_{max} + X_{min}}{2}$

$M[\tilde{\Theta}_2] = M[\frac{X_{max} + X_{min}}{2}] = \frac{1}{2}[M[X_{max}] + M[X_{min}]] =$

$\xi \sim F(x)$

$\min(\xi_1 \ldots \xi_n) \sim 1 - (1 - F(\xi))^n$

$\varphi(y) = n(1 - F(y))^{n-1} p(y)$

$F(x) = \begin{cases} 1 - e^{-\frac{x}{\Theta}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$; $\varphi(y) = n \cdot (e^{-\frac{x}{\Theta}})^{n-1} \frac{e^{-\frac{x}{\Theta}}}{\Theta} = \frac{n}{\Theta} \cdot e^{-\frac{x(n-1)}{\Theta}} \cdot e^{-\frac{x}{\Theta}} = \frac{n}{\Theta} e^{-\frac{xn}{\Theta}}$

$M[\xi^{X_{min}}] = \int_0^{\infty} x \cdot \frac{n}{\Theta} \cdot e^{-\frac{xn}{\Theta}} dx = \frac{\Theta}{n} = \frac{\Theta}{3}$

max: $(\xi_1 \dots \xi_n) = n\,(F(y))^{n-1};$

$$\varphi(y) = n\,(F(y))^{n-1}\,p(y)$$

$$\varphi(x) = n\cdot\left[1-e^{-\frac{x}{\theta}}\right]^{n-1}\cdot\frac{e^{-\frac{x}{\theta}}}{\theta}$$

$$M[X_{max}] = \int_0^\infty x\cdot\varphi(x)\,dx = \{n=3\} =$$

$$= \int_0^\infty x\cdot\frac{3}{\theta}\left(1-e^{-\frac{x}{\theta}}\right)^2 e^{-\frac{x}{\theta}} = \frac{3}{\theta}\left[\int_0^\infty x e^{-\frac{x}{\theta}}dx - 2\int_0^\infty x e^{-\frac{2x}{\theta}}dx + \int_0^\infty x e^{-\frac{3x}{\theta}}dx\right] = \frac{3}{\theta}\left[\theta^2 - 2\frac{\theta^2}{4} + \frac{\theta^2}{9}\right] = \frac{11}{6}\theta$$

$$M[\tilde{\theta}_2] = \frac{1}{2}\left[\frac{\theta}{3} + \frac{11}{6}\theta\right] = \frac{13}{12}\theta$$ — смещенная

$$\tilde{\theta}_2' = \frac{12}{13}\left(\frac{X_{min}+X_{max}}{2}\right)$$ — несмещ.

3) $\tilde{\theta}_3 = X_{(2)}$; $n=3$

$$P(x \le X_{(2)} < x+\Delta x) = C_3^1\cdot P(x \le \xi < x+\Delta x)\cdot C_2^1\cdot P(\xi < x)\,C_1^1\cdot P(\xi \ge x+\Delta x)$$

Переход к $\Delta x \to 0$: $\Phi(x+\Delta x) - \Phi(x) =$
$= 6\,(F(x+\Delta x) - F(x))\,F(x)\,(1-F(x+\Delta x))$; $/\Delta x$

$$\Phi'(x) = \varphi(x) = 6\,p(x)\,F(x)\,(1-F(x))$$

$$\varphi(x) = 6\cdot\frac{1}{\theta}\cdot e^{-\frac{x}{\theta}}\cdot\left[1-e^{-\frac{x}{\theta}}\right]\cdot\left(e^{-\frac{x}{\theta}}\right)$$

$$\varphi(x) = \frac{6}{\Theta} \cdot e^{-\frac{2x}{\Theta}} \left[1 - e^{-\frac{x}{\Theta}}\right] = \frac{6}{\Theta} e^{-\frac{2x}{\Theta}} - \frac{6}{\Theta} e^{-\frac{3x}{\Theta}}$$

$$M[\tilde{\Theta}_3] = \int_0^\infty x\left[\frac{6}{\Theta} e^{-\frac{2x}{\Theta}} - \frac{6}{\Theta} e^{-\frac{3x}{\Theta}}\right] dx = \frac{6}{\Theta}\left[\int_0^\infty x e^{-\frac{2x}{\Theta}} - \int_0^\infty x e^{-\frac{3x}{\Theta}} dx\right]$$

$$= \frac{6}{\Theta}\left[\frac{\Theta^2}{4} - \frac{\Theta^2}{9}\right] = \frac{3\Theta}{2} - \frac{2\Theta}{3} = \frac{5}{6}\Theta$$

смещенная

$$\tilde{\Theta}_3' = \frac{6}{5}\Theta$$

Сравнени эффективност: $D[\tilde{\Theta}_1] = \frac{\Theta^2}{n} = \frac{\Theta^2}{3}$

$D[\tilde{\Theta}_2']$ - как найти??

$\tilde{\Theta}_3'$: ~~$\varphi(x)=$~~ $M[\tilde{\Theta}_3^2] = \frac{6}{\Theta}\left[\int_0^\infty x^2 e^{-\frac{2x}{\Theta}} - \int_0^\infty x^2 e^{-\frac{3x}{\Theta}} dx\right] =$

$$= \frac{6}{\Theta}\left[-\frac{2\Theta^3}{27} + \frac{\Theta^3}{4}\right] = \frac{19}{18}\Theta^2$$

Для $\tilde{\Theta}_3'$: $D[\tilde{\Theta}_3'] = D\left[\frac{6}{5}\tilde{\Theta}_3\right] = \frac{36}{25} \cdot \frac{19}{18}\Theta^2 = \frac{19}{50}\Theta^2$

$D[\tilde{\Theta}_1] = \frac{\Theta^2}{3}$; $D[\tilde{\Theta}_2]$ - ? ; $D[\tilde{\Theta}_3] = \frac{19}{50}\Theta^2$

$$D[\tilde{\Theta}_3'] > D[\tilde{\Theta}_1]$$

б) Неравенство Крамера - Рао: $D[\tilde{\Theta}] \geq \frac{1}{n\,J(\Theta)}$

$$J(\Theta) = M\left[\left(\frac{\partial \ln \varphi}{\partial \Theta}\right)^2\right]$$

Доказательство Регулярности:

$$\int_0^{\infty} \frac{d}{d\Theta} \rho(x,\Theta)\,dx = \int_0^{\infty} \frac{d \ln \rho(x,\Theta)}{d\Theta} \rho(x,\Theta)\,dx = 0$$

$\tilde{\Theta}_1$: $\rho(x,\Theta) = \frac{1}{\Theta} \exp\left(-\frac{x}{\Theta}\right)$

$\ln \rho(x,\Theta) = -\frac{x}{\Theta} - \ln\Theta$; $x \geq 0$; $x > 0$

$\frac{d\ln(x,\Theta)}{d\Theta} = \frac{x-\Theta}{\Theta^2}$; $\frac{d\rho(x,\Theta)}{d\Theta} = \frac{(x-\Theta)}{\Theta^3} \cdot e^{-\frac{x}{\Theta}}$

$\frac{d\ln(x,\Theta)}{d\Theta} \cdot \rho(x,\Theta) = \frac{(x-\Theta)}{\Theta^3} \cdot e^{-\frac{x}{\Theta}}$ => регулярна

$\tilde{\Theta}_2$: $\rho(x,\Theta) = 3\left[1 - e^{-\frac{x}{\Theta}}\right]^2 \frac{1}{\Theta} e^{-\frac{x}{\Theta}}$

$\ln \rho(x,\Theta) = \ln\left(\frac{e^{-\frac{3x}{\Theta}}\left(e^{\frac{x}{\Theta}} - 1\right)^2}{\Theta}\right) + \log 3$

=> не регуляр.

$\tilde{\Theta}_3$: $\rho(x,\Theta) = \frac{6}{\Theta} e^{-\frac{2x}{\Theta}} - \frac{6}{\Theta} e^{-\frac{3x}{\Theta}}$

$$\frac{d\ln(x,\Theta)}{d\Theta} = \frac{1}{\Theta^2} \cdot \frac{1}{\left(e^{-\frac{x}{\Theta}} - 1\right)} \left[\Theta\left(-e^{-\frac{x}{\Theta}}\right) + x\left(2e^{\frac{x}{\Theta}} - 3\right) + \Theta\right] =$$

$$\rho(x,\Theta) \frac{d\ln(x,\Theta)}{d\Theta} = \frac{6}{\Theta^3} \frac{\left[e^{-\frac{2x}{\Theta}} - e^{-\frac{3x}{\Theta}}\right]}{\left(e^{-\frac{x}{\Theta}} - 1\right)} \left[\Theta\left(1 - e^{-\frac{x}{\Theta}}\right) + x\left(2e^{\frac{x}{\Theta}} - 3\right)\right]$$

$$\frac{d\rho(x,\Theta)}{d\Theta} = -\frac{6}{\Theta} e^{-\frac{3x}{\Theta}} \left[\Theta\left(e^{\frac{x}{\Theta}} - 1\right) + x\left(3 - 2e^{\frac{x}{\Theta}}\right)\right]$$ => все регул.

$\tilde{\Theta}_1$: $\frac{d \ln(x,\Theta)}{d\Theta} = \frac{x-\Theta}{\Theta^2}$; $J(\Theta) = M\left[\frac{(x-\Theta)^2}{\Theta^4}\right] =$

$= M\left[\frac{x^2 - 2x\Theta + \Theta^2}{\Theta^4}\right] = M\left[\frac{x^2}{\Theta^4}\right] - M\left[\frac{2x}{\Theta^3}\right] + M\left[\frac{1}{\Theta^2}\right] =$

$= \frac{1}{\Theta^4} M[x^2] - \frac{2}{\Theta^3} M[x] + \frac{1}{\Theta^2} = \frac{1}{\Theta^2}$

$D[\tilde{\Theta}_1] \geq \frac{1}{n\frac{1}{\Theta^2}} = \frac{\Theta^2}{n} = D[\tilde{\Theta}_1]$ //

r – эффективна

№4

$\xi \sim R(\Theta; 2\Theta)$; $p(x,\Theta) = \frac{1}{\Theta}$ $(\Theta, 2\Theta)$

$F(x) = \begin{cases} 0, & x < \Theta \\ \frac{x-\Theta}{\Theta}, & \Theta \leq x \leq 2\Theta \end{cases}$