Temperature Variability in Titan's Upper Atmosphere: The Role of Wave Dissipation

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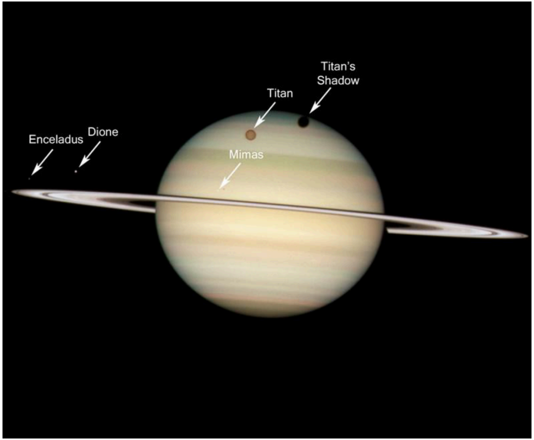
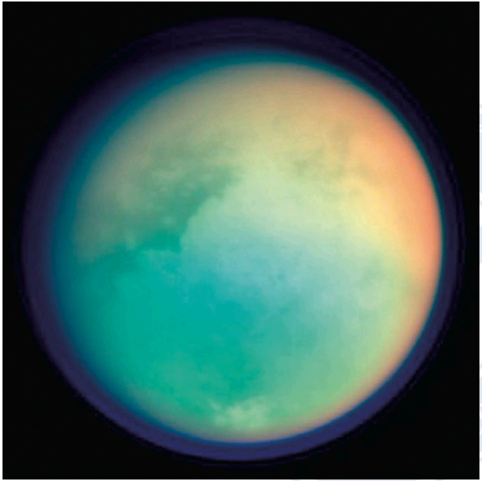
Outline

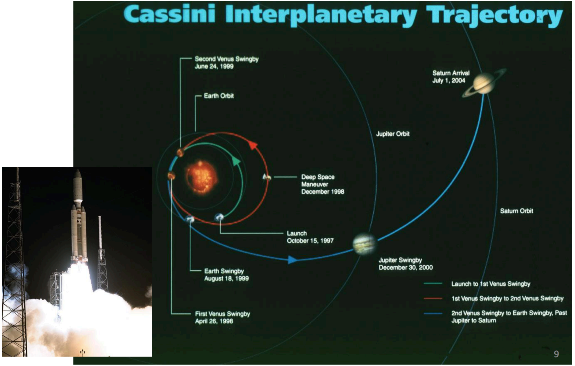
Saturnian System

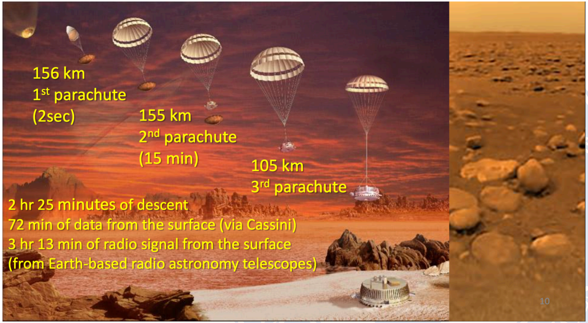




Titan has a primordial atmosphere, the main constituents of which are N_2, CH_4.

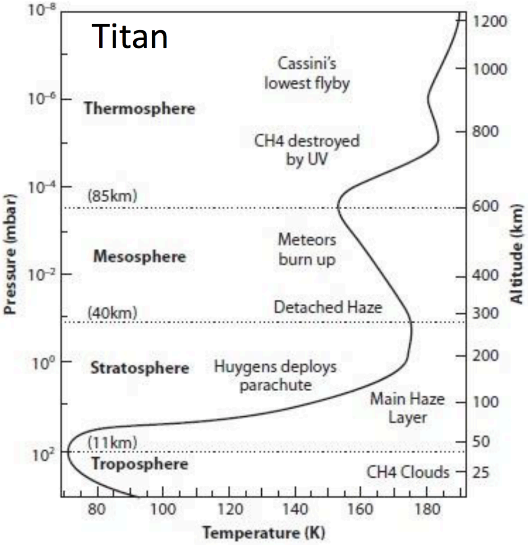
Cassini-Huygens

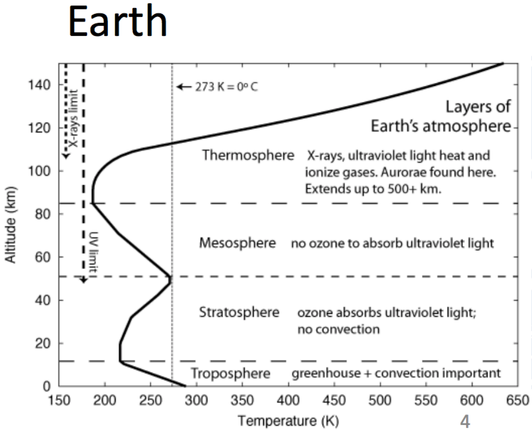




Entry, descent, and landing of Huygens (Jan. 14, 2005)

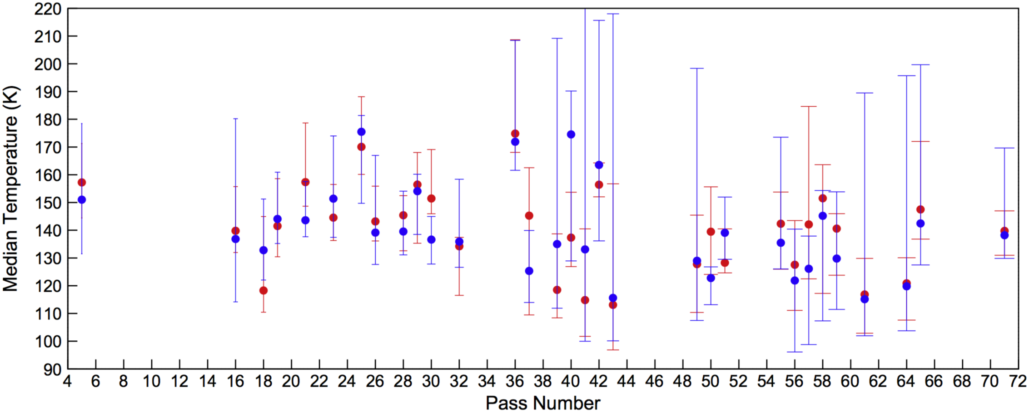
The Atmospheric Structures of Titan and Earth





Energy Crisis

- The analysis of INMS data showed that the median temperature on the upper atmosphere varies approximately 60 K.



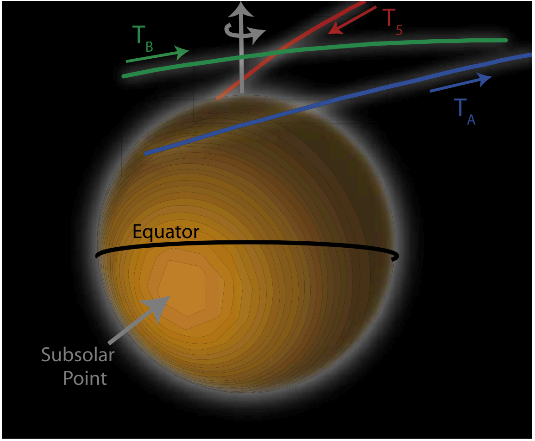
Snowden, D., Yelle, R.V., et al. 2013c

Energy Crisis

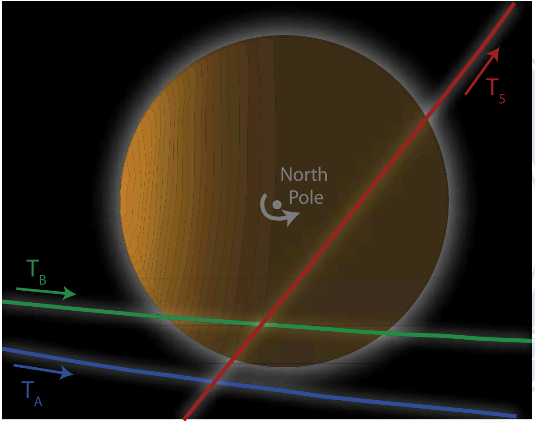
- For all the four giant planets in our solar system, the energy crisis is known to be present in their upper atmosphere: the temperatures observed to be 400-1000K but predicted to be 100-200K within the framework of the solar-driven scenario;
- For this energy crisis phenomenon, there are some scenarios to explain:
  1. solar EUV radiation;
  2. magnetospheric particle precipitation;
  3. Joule Heating;
  4. the variability in HCN abundance;
  5. wave dissipation;

Solar EUV Radiation

- Two flybys in October 2004(TA) and April 2005(T5):



Side View of Titan



North Pole View of Titan

de La Haye, Waite, Johnson et al. 2007

Solar EUV Radiation

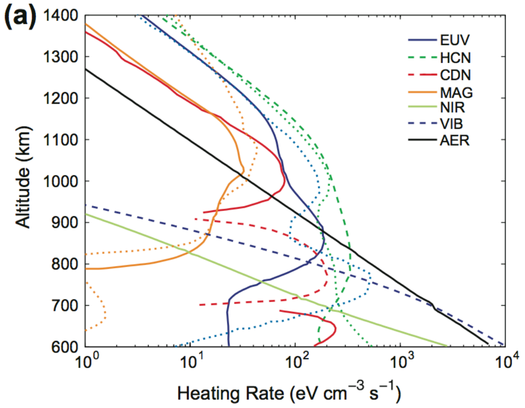
- Assuming solar EUV radiation alone, the dayside temperature in Titan's upper atmosphere should be higher than the nightside value, but such a feature has not been observed:

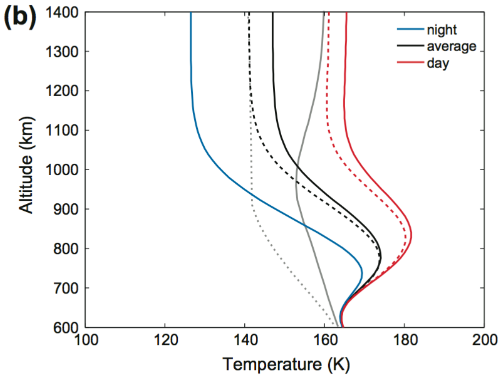
	T_A	T_B	T_5
	Temperature, K		
Combined data	152.8 ± 4.6	149.0 ± 9.2	157.4 ± 4.9
Ingress	150.0	Not enough data	162.3
Egress	157.4	149.0	154.1
	Eddy Diffusion Coefficient, $cm^2\ s^{-1}$		
Combined data	$(5.2^{+5.0}_{-2.9}) \times 10^9$	$(1.0^{+1.0}_{-0.58}) \times 10^{10}$	$(3.9^{+1.0}_{-0.9}) \times 10^9$
Ingress	2.3×10^9	Not enough data	3.0×10^9
Egress	1.2×10^{10}	1.0×10^{10}	4.9×10^9
Homopause altitude, km	1250 ± 60	1280 ± 120	1180 ± 30
Exobase altitude, km	1442 ± 7	1409 ± 14	1401 ± 2
Thickness of exobase layer, km	85 ± 2	81 ± 4	86 ± 3

de La Haye, Waite, Johnson et al. 2007

Charged Particle Precipitation

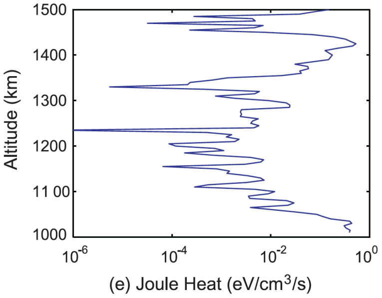
- Calculations shown that Titan’ s varying plasma environment contributes to a temperature variability of only 7 K; (Snowden, Yelle et al. 2014)
- Results confirm that magnetospheric particle precipitation can increase the temperature of Titan’ s thermosphere but cannot cause the very large temperature variations (60 K) .





Joule Heating

- Another potentially important source of energy in Titan's upper atmosphere is the Joule heating resulting from the electromagnetic interaction between Titan's ionosphere and Saturn's magnetosphere;
- Joule heating may be a significant energy source in the regions sampled by the outbound legs of T39, T41, and T43 because these passes sample low solar zenith angles, where the conductivity is large.

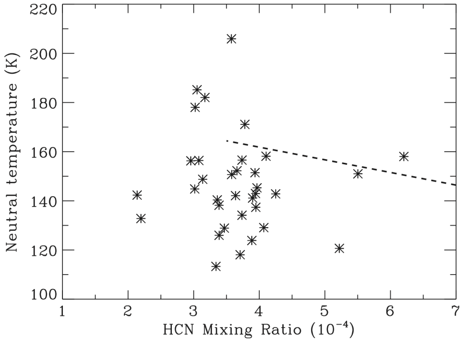


Snowden, Yelle et al. 2014

- This figure shows that the Joule heating rate due to electric field is less than 1 eV cm^{-3} s^{-1} at all altitudes;
- This heating rate is small compared to the energy deposition rates and is not strong enough to cause the large temperature increase observed during T43.

HCN Abundance

- The rotational line emission of HCN is thought to be the most important cooling mechanism in Titan's upper atmosphere;
- The model results of Snowden&Yelle (2014) indicate that a decrease in HCN volume mixing ratio about twice leads to an increase in temperature from 145 to 165 K above 1200 km.

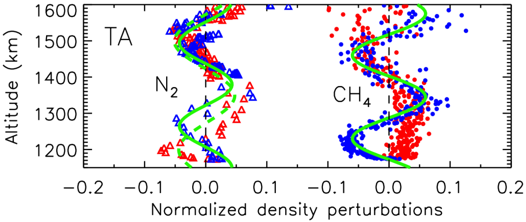


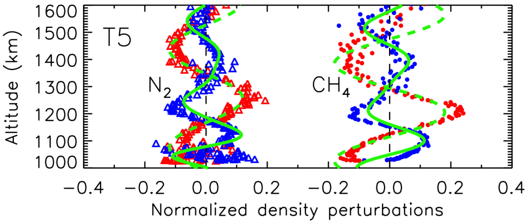
Cui, Cao, Lavvas et al. 2016

- The variability in HCN abundance can not to explain the temperature variability observed in Titan's upper atmosphere.

Wave Dissipation

- Wave dissipation results in the diffusive transport of energy and momentum;
- Waves not only carry momentum into the upper atmosphere but may also affect its energy balance;
- Therefore, waves may be another important energy source in Titan' s upper atmosphere.





Müller-Wodarg et al. 2006

Wave Dissipation

- Muller-Wodarg et.al. (2006) found that the energy and momentum were absorbed into Titan's upper atmosphere as wave dissipates, they analyzed the density wave perturbations of N_2 and CH_4 in Titan's upper atmosphere, and **they estimated the amount of energy carried by waves, and found that the maximum energy is about 1.25×10^9 eV cm^{-2} s^{-1}**;
- Further analysis by Snowden & Yelle (2014) showed that wave propagation and dissipation can heat or cool Titan's upper atmosphere. **However, they just estimated the viscous flux of kinetic energy on the order of 30 eV cm^{-3} s^{-1} at an altitude of 1300 km and the cooling rate due to the sensible heat flux is about -9 eV cm^{-3} s^{-1} for a weakly damped wave**;
- The analysis of INMS data in Cui et al.(2013) showed that the observed waves in Titan's upper atmosphere are upward propagating **gravity waves**.

Hydrostatic Gravity Wave Model

- The dissipation of vertically propagating gravity waves is entirely due to heat conduction and viscosity, and this model has been established by Matcheva & Strobel (1999), they used this model to analyze Jupiter's thermosphere;
- We analysis the gravity waves on Titan's upper atmosphere by using their model;
- According to Cui et al.(2013), the Coriolis frequency of Titan is smaller than the waves frequency, so we can assume that the Titan's atmosphere is stationary relative to gravity waves.The β-plane approximation is valid because we study the small scale gravity waves in Titan's upper atmosphere;
- Therefore, we can consider a small amplitude, hydrostatic gravity wave propagating in a dissipative, nonrotating, deep, compressible, hydrostatic atmosphere with no background zonal wind.The vertical and horizontal wave velocities $w^{'}$, $u^{'}$, the temperature perturbations $T^{'}$ and the pressure perturbations $p^{'}$ are the solutions of this theoretical hydrostatic gravity wave model.

Hydrostatic Gravity Wave Model

- The linearized set of governing equations in Cartesian coordinates for a small amplitude wave in a dissipative, nonrotating, deep, compressible, hydrostatic atmosphere with a constant zonal wind u_0 is given by:

$$\nabla \cdot \vec{V}' - \frac{w'}{H_\rho} = 0 \tag{1}$$

$$\left[\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x} - \frac{1}{\rho_0} \frac{\partial}{\partial z} \mu \frac{\partial}{\partial z}\right] u' = -\frac{1}{\rho_0} \frac{\partial p'}{\partial x} \tag{2}$$

$$\left[\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x} - \frac{1}{\rho_0} \frac{\partial}{\partial z} \mu \frac{\partial}{\partial z}\right] v' = -\frac{1}{\rho_0} \frac{\partial p'}{\partial y} \tag{3}$$

$$\left[\frac{\partial}{\partial t} + u_0 \frac{\partial}{\partial x} - \frac{1}{\rho_0 p_r} \frac{\partial}{\partial z} \mu \frac{\partial}{\partial z}\right] T' + \Gamma w' = 0 \tag{4}$$

$$\frac{\partial}{\partial z}\left(\frac{p'}{\rho_0}\right) = \frac{T' R}{H} \tag{5}$$

Hydrostatic Gravity Wave Model

- $w^{'}$ is solved by assuming that WKB approximation:

$$w^{'} = \Delta W(z)\, e^{i\varphi} \tag{6}$$

where the amplitude $\Delta W(z)$ and phase φ are defined as:

$$\Delta W(z) = \Delta W(z_0) \sqrt{\frac{k_{zr}(z_0)}{k_{zr}(z)}} \times exp\left[\int_{z_0}^{z} \left(\frac{1}{2H_\rho} - k_{zi}\right) dz\right] \tag{7}$$

$$\varphi = k_x x + k_y y + \int_{z_0}^{z} k_{zr} dz - \omega_0 t \tag{8}$$

Where $\omega_0^2 \ll N^2$, and $\frac{H-H_\rho}{H_\rho} \ll 1$, the parameters $H_\rho = \left(-\frac{1}{\rho_0}\frac{\partial \rho_0}{\partial z}\right)^{-1}$, $H = \left(-\frac{1}{p_0}\frac{\partial p_0}{\partial z}\right)^{-1}$ are the density and the pressure scale height, and ρ_0 is atmospheric background density of Titan. $\Gamma = \frac{\partial T_0}{\partial z} + \frac{g}{c_p}$ is the static stability coefficient and P_r is Prandtl number. z_0 is an arbitrary reference altitude, and $\Delta W(z_0)$ is reference amplitude. ω_0 is the wave frequency. k_x is the x-direction wavenumber. Similarly, k_y is the y-direction wavenumber.

Hydrostatic Gravity Wave Model

- Vertical wavenumber $k_z = k_{zr} + ik_{zi}$ is the root of the relation equation:

$$k_z^2 = \frac{k_h^2 N^2}{\hat{\omega}\left(\hat{\omega} + i\beta\right)} - \frac{1}{4H_\rho^2}\left(1 - 2\frac{dH_\rho}{dz}\right) \tag{9}$$

where $k_h = \sqrt{k_x^2 + k_y^2}$ is the horizontal wavenumber and $N = \sqrt{\frac{g\Gamma}{T_0}}$ is the buoyancy frequency and $\Gamma = \frac{\partial T_0}{\partial z} + \frac{g}{c_p}$ is the atmosphere static stability, T_0 is atmospheric background temperature of Titan. This relation equation is the six-order polynomial about k_z, the parameters $\hat{\omega}$, β are defined as:

$$\hat{\omega} = \omega_r + i\omega_i \tag{10}$$

$$\beta = \beta_r + i\beta_i \tag{11}$$

Hydrostatic Gravity Wave Model

- where:

$$\omega_r = -Im\left[\frac{1}{u'}\left(\frac{\partial}{\partial t} + u_0\frac{\partial}{\partial x} - \nu\frac{\partial^2}{\partial z^2}\right)u'\right] = \widetilde{\omega}_0 + 2k_{zr}\nu\left(\frac{1}{2H_\rho} - k_{zi}\right) \tag{12}$$

$$\omega_i = +Re\left[\frac{1}{u'}\left(\frac{\partial}{\partial t} + u_0\frac{\partial}{\partial x} - \nu\frac{\partial^2}{\partial z^2}\right)u'\right] = \nu\left[k_{zr}^2 - \left(\frac{1}{2H_\rho} - k_{zi}\right)^2\right] \tag{13}$$

$$\beta_r = Re\left[\frac{1}{T'}\left(1 - \frac{1}{Pr}\right)\nu\frac{d^2}{dz^2}T'\right] = \left(\frac{1}{Pr} - 1\right)\nu\left[k_{zr}^2 - \left(\frac{1}{2H_\rho} - k_{zi}\right)^2\right] \tag{14}$$

$$\beta_i = Im\left[\frac{1}{T'}\left(1 - \frac{1}{Pr}\right)\nu\frac{d^2}{dz^2}T'\right] = 2k_{zr}\nu\left(1 - \frac{1}{Pr}\right)\left(\frac{1}{2H_\rho} - k_{zi}\right) \tag{15}$$

Where $\widetilde{\omega}_0 = \omega_0 - u_0 k_x$ is the intrinsic frequency of the wave and $\nu = \mu/\rho_0$ is the kinetic viscosity. μ is the molecular dynamic viscosity and Pr is the Prandtl number. The parameter β_i was neglected in Matcheva & Strobel (1999). However, in our calculations we keep β_i.

Hydrostatic Gravity Wave Model

- u', T' and p' are related with w' by the polarization relations:

$$u' = -\frac{ik_x}{k_h^2}\left(-ik_z + \frac{1}{2H_\rho}\right)w' \tag{16}$$

$$T' = -\frac{i\Gamma}{\hat{\omega} + i\beta}w' \tag{17}$$

$$p' = \frac{i\rho_0\hat{\omega}}{k_h^2}\left(ik_z - \frac{1}{2H_\rho}\right)w' \tag{18}$$

- The initial parameters of this model are the wavelength λ_x, and the wave period τ, so we can get the horizontal wavenumber k_h by assuming that $\lambda_x = \lambda_y$, and the wave frequency $\omega_0 = \frac{2\pi}{\tau}$. Therefore, we can solve the eq (9) by giving the initial parameters.

Hydrostatic Gravity Wave Model

- The proof is as follows:

The following is the derivations of these equations:

First, we assume that:

$$\frac{\nu}{\delta u}\frac{d^2\delta u}{dz^2}=\frac{\nu}{\delta v}\frac{d^2\delta v}{dz^2}=\frac{\nu}{\delta T}\frac{d^2\delta T}{dz^2}=\lambda=\lambda_r+i\lambda_i \tag{14}$$

$$\left(\frac{1}{Pr}-1\right)\frac{\nu}{\delta T}\frac{d^2\delta T}{dz^2}=\left(\frac{1}{Pr}-1\right)\lambda=-\beta+i\gamma \tag{15}$$

Then, we have:

$$\frac{1}{u'}\left(\frac{\partial}{\partial t}+u_0\frac{\partial}{\partial x}-\nu\frac{\partial^2}{\partial z^2}\right)u'=\frac{1}{v'}\left(\frac{\partial}{\partial t}+u_0\frac{\partial}{\partial x}-\nu\frac{\partial^2}{\partial z^2}\right)v'$$

$$=\frac{1}{T'}\left(\frac{\partial}{\partial t}+u_0\frac{\partial}{\partial x}-\nu\frac{\partial^2}{\partial z^2}\right)T'=-i\widetilde{\omega}_0-\lambda \tag{16}$$

Where $\widetilde{\omega}_0=\omega_0-u_0k_x$.

Then, we substitute eq(6) in eq(1) to eq(5):

$$ik_x\delta u(z)+ik_y\delta v(z)+\frac{d\delta w(z)}{dz}=\frac{\delta w(z)}{H^*} \tag{17}$$

$$(-i\widetilde{\omega}_0-\lambda)\,\delta u=-\frac{ik_x}{\rho_0}\delta p \tag{18}$$

$$(-i\widetilde{\omega}_0-\lambda)\,\delta v=-\frac{ik_y}{\rho_0}\delta p \tag{19}$$

$$(-i\widetilde{\omega}_0-\lambda)\,\delta T+\Gamma\delta w=(-\beta+i\gamma)\,\delta T \tag{20}$$

$$\frac{1}{\rho_0}\frac{d\delta p(z)}{dz}=\frac{R}{H}\delta T(z) \tag{21}$$

And, according to eq(18) and eq(19), we have:

$$\frac{\delta u}{k_x}=\frac{\delta v}{k_y} \tag{22}$$

According to eq(20) and eq(21):

$$(-i\widetilde{\omega}_0-\lambda+\beta-i\gamma)\frac{1}{\rho_0}\frac{d\delta p}{dz}+N^2\delta w=0 \tag{23}$$

Substitute eq(23) in eq(18), we have:

$$\frac{d\delta u}{dz}=\frac{ik_xN^2\delta w}{(-i\widetilde{\omega}_0-\lambda)(-i\widetilde{\omega}_0-\lambda+\beta-i\gamma)} \tag{24}$$

Substitute eq(22) in eq(17):

$$\frac{ik_h^2}{k_x}\frac{d\delta u}{dz}+\frac{d^2\delta w}{dz^2}=-\frac{1}{H^{*2}}\frac{dH^*}{dz}\delta w+\frac{1}{H^*}\frac{d\delta w}{dz} \tag{25}$$

Where $N^2=\frac{R}{H}\Gamma=\frac{g}{T_0}\Gamma$, $k_h^2=k_x^2+k_y^2$.

And according to eq(24) and eq(25), we have:

$$\frac{d^2\delta w}{dz^2}-\frac{1}{H^*}\frac{d\delta w}{dz}+\frac{1}{H^{*2}}\frac{dH^*}{dz}\delta w-\frac{k_h^2N^2\delta w}{(-i\widetilde{\omega}_0-\lambda)(-i\widetilde{\omega}_0-\lambda+\beta-i\gamma)}=0 \tag{26}$$

However, according to eq(12), eq(13) and eq(16):

$$-i\widetilde{\omega}_0-\lambda=\omega_i-i\omega_r=-i\hat{\omega} \tag{27}$$

And substitute eq(27) in eq(26):

$$\frac{d^2\delta w}{dz^2}-\frac{1}{H^*}\frac{d\delta w}{dz}+\left(\frac{1}{H^{*2}}\frac{dH^*}{dz}+\frac{k_h^2N^2}{\hat{\omega}(\hat{\omega}+i\beta+\gamma)}\right)\delta w=0 \tag{28}$$

If we let $\gamma=0$(why?), and according to eq(9), we have:

$$\frac{d^2\delta w}{dz^2}-\frac{1}{H^*}\frac{d\delta w}{dz}+\left(\frac{1}{2H^{*2}}\frac{dH^*}{dz}+\frac{1}{4H^{*2}}+k_z^2\right)\delta w=0 \tag{29}$$

And substitute eq(8) in eq(29), we have get eq(7).

In the limits of the WKB approximation we obtain a wave-like solution:

$$\widetilde{w}(z)=\Delta W(z_0)\left(\frac{k_{zr}(z_0)}{k_{zr}(z)}\right)^{1/2}exp\left[-\int_{z_0}^{z}k_{zi}dz\right]exp\left[i\int_{z_0}^{z}k_{zr}dz\right] \tag{30}$$

Where $k_{zr}=Rek_z$, $k_{zi}=Imk_z$.

Then, according to eq(8) and eq(30), we have:

$$\delta w=\Delta W(z_0)\left(\frac{k_{zr}(z_0)}{k_{zr}(z)}\right)^{1/2}exp\left[\int_{z_0}^{z}\left(ik_z+\frac{1}{2H^*}\right)dz\right] \tag{31}$$

Hydrostatic Gravity Wave Model

- The proof is as follows:

If we assume that k_z is slowly change with z, then, we have:

$$\frac{d\delta w}{dz} = \left(ik_z + \frac{1}{2H^*}\right)\delta w \tag{32}$$

Now according to eq(17), eq(18) and eq(32), we have:

$$\frac{ik_h^2}{k_x}\delta u = \left(-ik_z + \frac{1}{2H^*}\right)\delta w \tag{33}$$

And:

$$\frac{ik_h^2}{k_x}\frac{d^2\delta u}{dz^2} = \left(-ik_z + \frac{1}{2H^*}\right)\frac{d^2\delta w}{dz^2} = \left(-ik_z + \frac{1}{2H^*}\right)\left(ik_z + \frac{1}{2H^*}\right)^2\delta w \tag{34}$$

Then we have:

$$\frac{1}{\delta u}\frac{d^2\delta u}{dz^2} = \left(ik_z + \frac{1}{2H^*}\right)^2 = \left(ik_{zr} + \frac{1}{2H^*} - k_{zi}\right)^2$$
$$= \left[\left(\frac{1}{2H^*} - k_{zi}\right)^2 - k_{zr}^2 + 2ik_{zr}\left(\frac{1}{2H^*} - k_{zi}\right)\right] \tag{35}$$

Now according to eq(14) and eq(27), we have:

$$\omega_r = \widetilde{\omega}_0 + \lambda_i = \widetilde{\omega}_0 + 2\nu k_{zr}\left(\frac{1}{2H^*} - k_{zi}\right) \tag{36}$$

$$\omega_i = -\lambda_r = \nu\left[{k_{zr}}^2 - \left(\frac{1}{2H^*} - k_{zi}\right)^2\right] \tag{37}$$

And according to eq(15), we have:

$$\beta = -\left(\frac{1}{Pr} - 1\right)\lambda_r = \left(\frac{1}{Pr} - 1\right)\omega_i$$
$$= \left(\frac{1}{Pr} - 1\right)\nu\left[{k_{zr}}^2 - \left(\frac{1}{2H^*} - k_{zi}\right)^2\right] \tag{38}$$

Using eq(6), eq(8) and eq(30) we obtain a final expression for the perturbed vertical velocity field:

$$w'(x,y,z,t) = \Delta W(z)\cos\varphi \tag{39}$$

Where:

$$\Delta W(z) = \Delta W(z_0)\left(\frac{k_{zr}(z_0)}{k_{zr}(z)}\right)^{1/2} exp\left[\int_{z_0}^{z}\left(\frac{1}{2H^*} - k_{zi}\right)dz\right] \tag{40}$$

$$\varphi = k_x x + k_y y + \int_{z_0}^{z} k_{zr}dz - \omega_0 t \tag{41}$$

The following is the derivations of eq(39) :

$$w'(x,y,z,t) = \delta w(z)\, exp\left[i\left(k_x x + k_y y - \omega_0 t\right)\right]$$
$$= \widetilde{w}(z)\, exp\left(\int\frac{dz}{2H^*}\right) exp\left[i\left(k_x x + k_y y - \omega_0 t\right)\right]$$
$$= \Delta W(z_0)\left(\frac{k_{zr}(z_0)}{k_{zr}(z)}\right)^{1/2} exp\left[\int\frac{dz}{2H^*} - \int_{z_0}^{z} k_{zi}dz\right]e^{i\varphi}$$
$$= \Delta W(z)\, e^{i\varphi} \tag{42}$$

The corresponding expressions for the temperature and pressure perturbation fields are obtained as well:

$$T'(x,y,z,t) = \frac{\Gamma}{\omega_r}\Delta W(z)\cos\theta\cos\left(\varphi - \frac{\pi}{2} - \theta\right) \tag{43}$$

$$p'(x,y,z,t) = -\left(\frac{\omega_r k_{zr}}{k_h^2}\right)\Delta W\left[1 - \frac{\omega_i}{\omega_r}\left(\frac{k_{zi}}{k_{zr}} + \frac{1}{2H^* k_{zr}}\right)\right]\frac{\cos\left(\varphi - \theta'\right)}{\cos\theta'} \tag{44}$$

Where θ and θ' are define as:

$$tan\theta = \frac{1}{P_r}\frac{\omega_i}{\omega_r} \tag{45}$$

$$tan\theta' = \left(\frac{\omega_i}{\omega_r} + \frac{k_{zi}}{k_{zr}} + \frac{1}{2H^* k_{zr}}\right)\left[1 - \frac{\omega_i}{\omega_r}\left(\frac{k_{zi}}{k_{zr}} + \frac{1}{2H^* k_{zr}}\right)\right]^{-1} \tag{46}$$

The following is the derivations of eq(43) and eq(44) :

Hydrostatic Gravity Wave Model

- The proof is as follows:

According to eq(20), we have:

$$\delta T = \frac{-\Gamma\delta w}{-i\tilde{\omega}_0 - \lambda + \beta} = \frac{-i\Gamma\delta w}{\hat{\omega} + i\beta} \tag{47}$$

Then:

$$T^{'}(x,y,z,t) = \frac{-i\Gamma}{\hat{\omega} + i\beta}\Delta W(z)\, e^{i\varphi} \tag{48}$$

And according to eq(15) and eq(27):

$$\frac{-i}{\hat{\omega}+i\beta}e^{i\varphi} = \frac{exp\left[i\left(\varphi-\frac{\pi}{2}\right)\right]}{\omega_r + i\frac{\omega_i}{Pr}} = \frac{1}{\omega_r\left(1+i\frac{1}{Pr}\frac{\omega_i}{\omega_r}\right)}exp\left[i\left(\varphi-\frac{\pi}{2}\right)\right]$$

$$= \frac{1}{\omega_r(1+itan\theta)}exp\left[i\left(\varphi-\frac{\pi}{2}\right)\right] = \frac{cos\theta}{\omega_r(cos\theta+isin\theta)}exp\left[i\left(\varphi-\frac{\pi}{2}\right)\right]$$

$$= \frac{cos\theta}{\omega_r}exp\left[i\left(\varphi-\frac{\pi}{2}-\theta\right)\right]$$

Then, eq(48) become:

$$T^{'}(x,y,z,t) = \frac{\Gamma}{\omega_r}\Delta W(z)\, cos\theta exp\left[i\left(\varphi-\frac{\pi}{2}-\theta\right)\right] \tag{49}$$

For eq(44):

Firstly, according to eq(33):

$$u^{'}(x,y,z,t) = -\frac{ik_x}{k_h^2}\left(-ik_z + \frac{1}{2H^*}\right)\Delta W(z)\, e^{i\varphi} \tag{50}$$

Similarly, we have:

$$v^{'}(x,y,z,t) = -\frac{ik_y}{k_h^2}\left(-ik_z + \frac{1}{2H^*}\right)\Delta W(z)\, e^{i\varphi} \tag{51}$$

And according to eq(18) and eq(50), we have:

$$p^{'}(x,y,z,t) = -\frac{\rho_0}{ik_x}\left(-i\tilde{\omega}_0 - \lambda\right)u^{'}(x,y,z,t)$$

$$= \frac{i\rho_0\hat{\omega}}{k_h^2}\left(ik_z - \frac{1}{2H^*}\right)\Delta W(z)\, e^{i\varphi}$$

Now we consider:

$$\hat{\omega}\left(ik_z - \frac{1}{2H^*}\right) = (\omega_r + i\omega_i)\left(ik_{zr} - \frac{1}{2H^*} - k_{zi}\right)$$

$$= \omega_r k_{zr}\left(1 + i\frac{\omega_i}{\omega_r}\right)\left[i - \left(\frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}\right)\right]$$

$$= \omega_r k_{zr}\left[i - \left(\frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}\right) - \frac{\omega_i}{\omega_r} - i\frac{\omega_i}{\omega_r}\left(\frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}\right)\right]$$

$$= \omega_r k_{zr}\left\{-\left(\frac{\omega_i}{\omega_r} + \frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}\right) + i\left[1 - \frac{\omega_i}{\omega_r}\left(\frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}\right)\right]\right\}$$

$$= \omega_r k_{zr}\left[1 - \frac{\omega_i}{\omega_r}\left(\frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}\right)\right]\left\{i - \frac{\frac{\omega_i}{\omega_r} + \frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}}{1 - \frac{\omega_i}{\omega_r}\left(\frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}\right)}\right\}$$

$$= \omega_r k_{zr}\left[1 - \frac{\omega_i}{\omega_r}\left(\frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}\right)\right]\left(i - tan\theta^{'}\right)$$

$$= \omega_r k_{zr}\left[1 - \frac{\omega_i}{\omega_r}\left(\frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}\right)\right]\frac{icos\theta^{'} - sin\theta^{'}}{cos\theta^{'}}$$

$$= \omega_r k_{zr}\left[1 - \frac{\omega_i}{\omega_r}\left(\frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}\right)\right]\frac{ie^{i\theta^{'}}}{cos\theta^{'}}$$

Where $tan\theta^{'} = \left(\frac{\omega_i}{\omega_r} + \frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}\right)\left[1 - \frac{\omega_i}{\omega_r}\left(\frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}\right)\right]^{-1}$.

Then, we have:

$$p^{'}(x,y,z,t) = -\frac{\rho_0\omega_r k_{zr}}{k_h^2}\Delta W(z)\left[1 - \frac{\omega_i}{\omega_r}\left(\frac{1}{2H^*k_{zr}} + \frac{k_{zi}}{k_{zr}}\right)\right]\frac{e^{i\left(\varphi+\theta^{'}\right)}}{cos\theta^{'}} \tag{52}$$

Reference

Matcheva;Strobel (1999). Heating of jupiter's thermosphere by dissipation of gravity waves due to molecular viscosity and heat conduction. Icarus, 140.

Propagation and Amplitude of Gravity Wave

- The analysis by Cui et al.(2013) show that the range of gravity wave period is $\approx$ 80 min to $\approx$ 10 h;
- In Muller-Wodarg et al.(2006) , the specific gas constant R is 290 J (kg K)$^{-1}$, the specific heat at constant pressure c_p is 1450 J (kg K)$^{-1}$, and the molecular viscosity μ is 1$\times$ 10^{-5} kg (m s)$^{-1}$. The acceleration of gravity $g = GM/(z + R_T)^2$ is varies with altitude z, where M is the mass of Titan and R_T is the radius of Titan;
- The value of Prandtl number Pr is set to 0.69 in Matcheva &Strobel (1999);
- In our calculation, we take the same values for these parameters.

Propagation and Amplitude of Gravity Wave

- We select 9 wave models and the properties of these waves are listed in table 1, the wave periods are 3,6,9 h and horizontal wavelengths are 600, 900, 1200 km;
- For each wave model, the wave frequency $\omega_0 = 2\pi/\tau$ is positive, so the real vertical wavenumber k_{zr} must be negative when gravity waves are propagated upwardly. The horizontal phase speed $c_h = \lambda_h/\tau$ also listed in this table.
- The temperature perturbation can be sorted into $T' = \Delta T(z)\, e^{i\theta_T}$ by rewriting eq (17).The amplitude $\Delta T(z)$ and phase θ_T are defined as:

$$\Delta T(z) = \frac{\Gamma \Delta W(z)\, cos\theta}{\frac{\omega_r}{Pr} + \left(1 - \frac{1}{Pr}\right)\tilde{\omega}_0} \tag{19}$$

$$\theta_T = \varphi - \theta - \frac{\pi}{2} \tag{20}$$

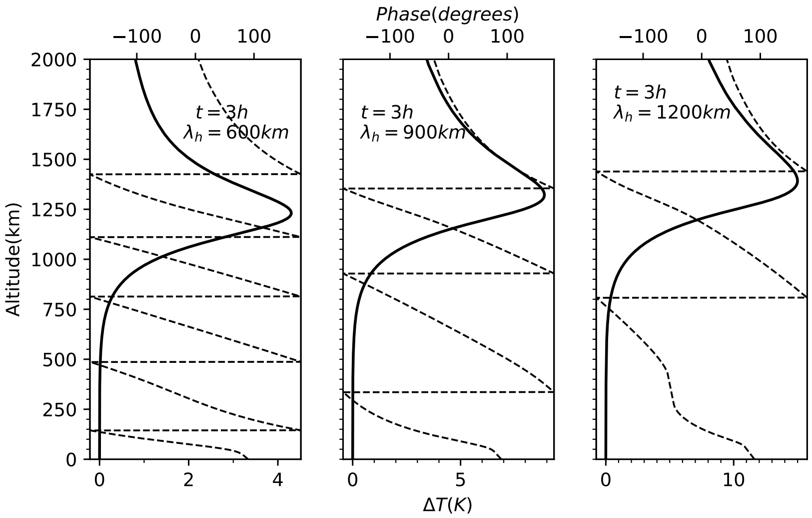
And $tan\theta = \frac{\omega_i}{\omega_r + (Pr-1)\tilde{\omega}_0}$.

Propagation and Amplitude of Gravity Wave

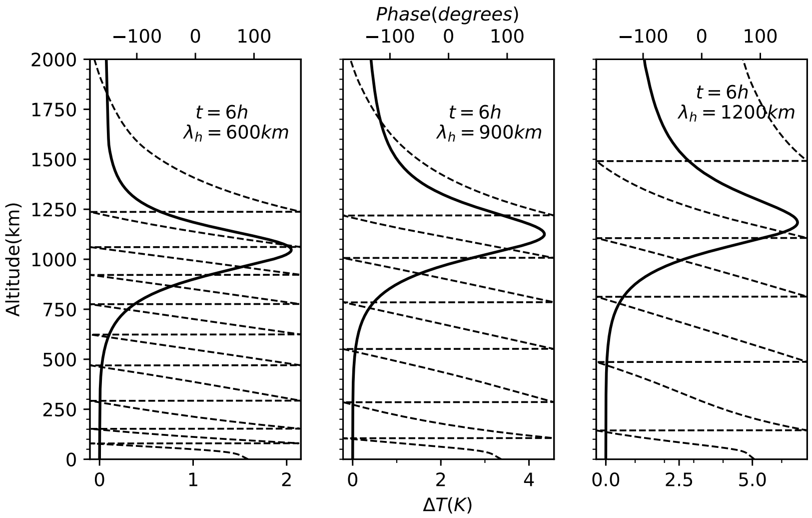
表: Table 1: Summary of gravity wave parameters and the maximum temperature amplitudes and the altitudes at which the waves have their maximum amplitudes

Period τ (h)	λ_h (km)	c_h (km s^{-1})	$(\Delta T)_{max}$(K)	$z_{(\Delta T)_{max}}$ (km)
3	600	0.06	4.3	1230
3	900	0.08	8.9	1321
3	1200	0.11	15.0	1396
6	600	0.03	2.0	1046
6	900	0.04	4.4	1125
6	1200	0.06	6.5	1184
9	600	0.02	1.6	947
9	900	0.03	2.5	1021
9	1200	0.04	4.6	1076

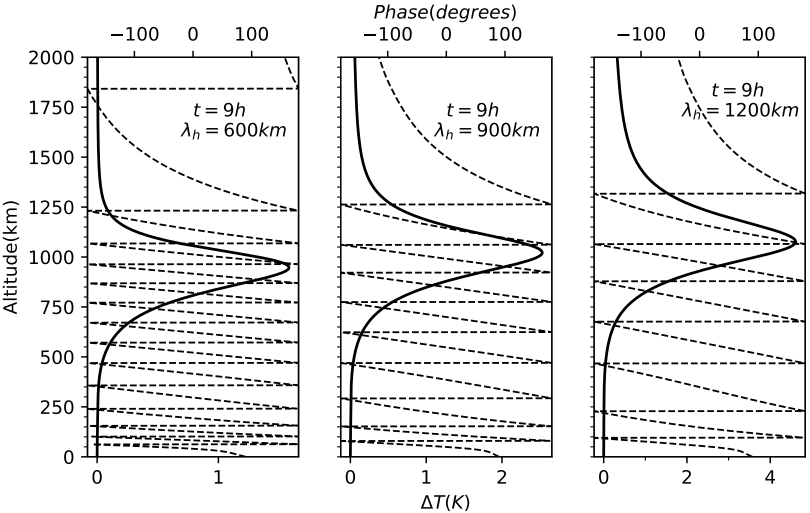
Propagation and Amplitude of Gravity Wave



Propagation and Amplitude of Gravity Wave



Propagation and Amplitude of Gravity Wave



Heating or Cooling Rate of Gravity Waves

- The vertically propagating gravity waves can heating or cooling the Titan's atmosphere because they can transport energy and momentum stresses upwardly;
- In a conservative atmosphere, the imaginary parts of the vertical wave number k_{zi} and ω_i vanish and θ equal 0 in eq (20). Therefore there is no vertical transport of internal energy on average due to the difference between θ_T and φ becomes $\pi/2$;
- However, in a dissipative atmosphere, the phase difference between $T^{'}$ and $w^{'}$ is $\theta + \frac{\pi}{2}$, then the time average of $w^{'}T^{'}$ is **nonzero and the gravity waves can transport internal energy**.
- The total wave energy flux $F_z^t = F_z^w + F_z^h$, where $F_z^w = \overline{p^{'}w^{'}}$ is the classical vertical wave energy flux, it is equal to **the work done by the pressure forces** and $F_z^h = \rho_0 c_p \overline{w^{'}T^{'}}$ is the sensible heat flux. The overline means $\overline{\alpha\beta} = \frac{1}{2}Re(\alpha\beta^*)$, where α and β are arbitrary complex functions and '*' denotes the complex conjugate.

Heating or Cooling Rate of Gravity Waves

- According to eq (18), eq (19) and eq (20), F_z^w and F_z^h can be expressed as follows:

$$F_z^w = -\frac{1}{2}\rho_0 \frac{\omega_r k_{zr}}{k_h^2}(\Delta W)^2 \left[1 - \frac{\omega_i}{\omega_r}\left(\frac{k_{zi}}{k_{zr}} + \frac{1}{2k_{zr}H_\rho}\right)\right] \tag{21}$$

$$F_z^h = -\frac{1}{2}\frac{c_p\rho_0}{\Gamma}\frac{\omega_i}{Pr}(\Delta T)^2 \tag{22}$$

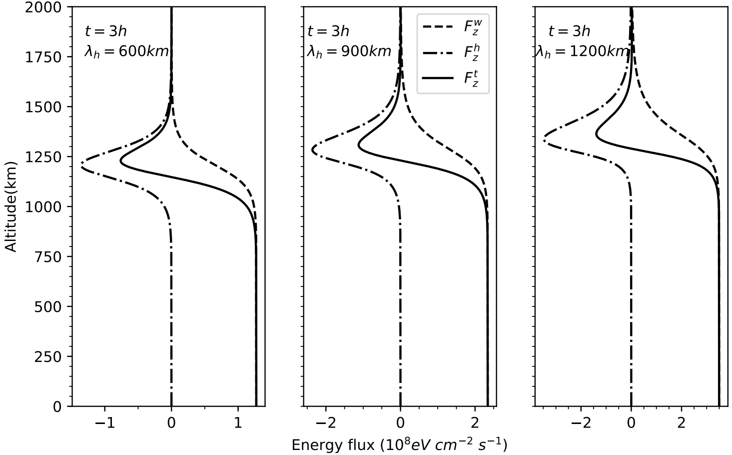
- From above equations we can know that the sign of F_z^w is determined by the sign of k_{zr}, which is the direction of wave propagation.
- The waves we analysis are propagating upwardly, so $k_{zr} < 0$ and the sign of F_z^w always positive.
- The sign of sensible heat flux F_z^h is independent of the direction of wave propagation and always negative, but the absolute of F_z^h is follows the profile of the temperature amplitude ΔT.

Heating or Cooling Rate of Gravity Waves

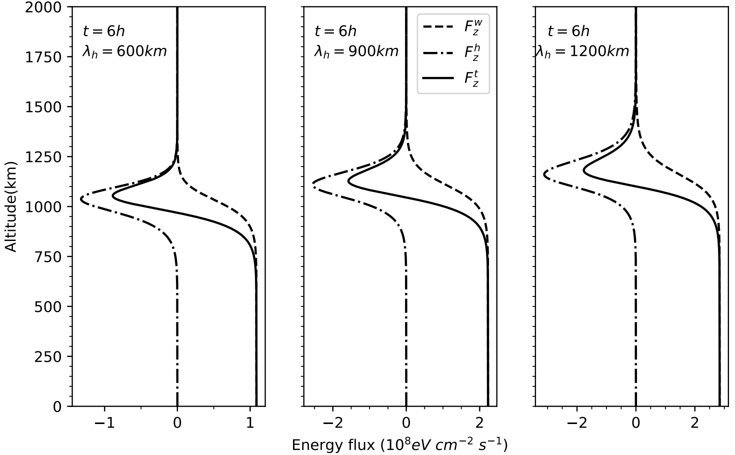
表: Table 2: Summary of the maximum heating rates(MHR)/ maximum cooling rates(MCR) of the total vertical energy fluxes and the altitudes at which the waves have their maximum heating/cooling rates

τ (h)	λ_h(km)	MHR (eV cm^{-3} s^{-1})	z at MHR (km)	MCR (eV cm^{-3} s^{-1})	z at MCR (km)
3	600	14.70	1155	-5.62	1285
3	900	16.62	1223	-7.30	1375
3	1200	41.91	1276	-8.82	1427
6	600	14.44	987	-8.76	1101
6	900	28.05	1059	-13.19	1184
6	1200	34.44	1105	-13.24	1238
9	600	19.70	893	-14.76	1002
9	900	21.95	961	-13.46	1076
9	1200	40.65	1013	-20.79	1133

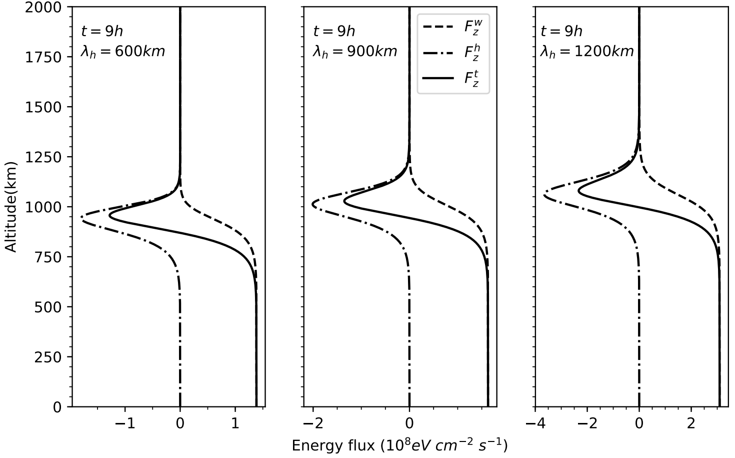
Heating or Cooling Rate of Gravity Waves



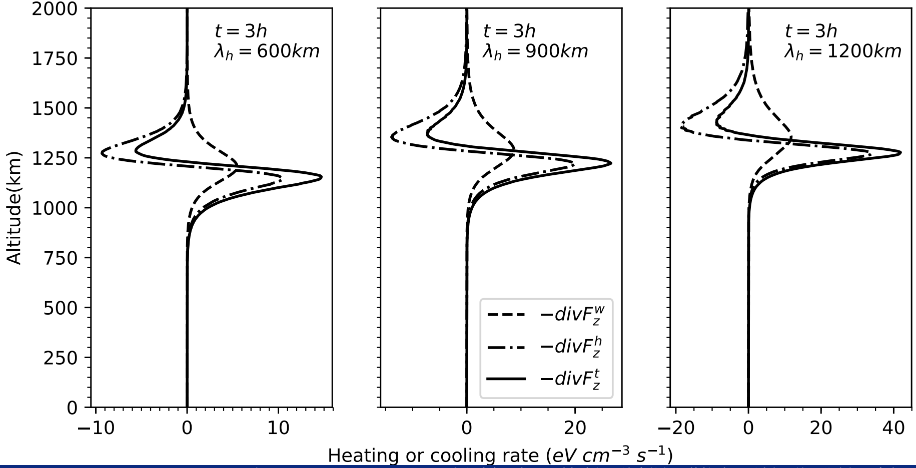
Heating or Cooling Rate of Gravity Waves



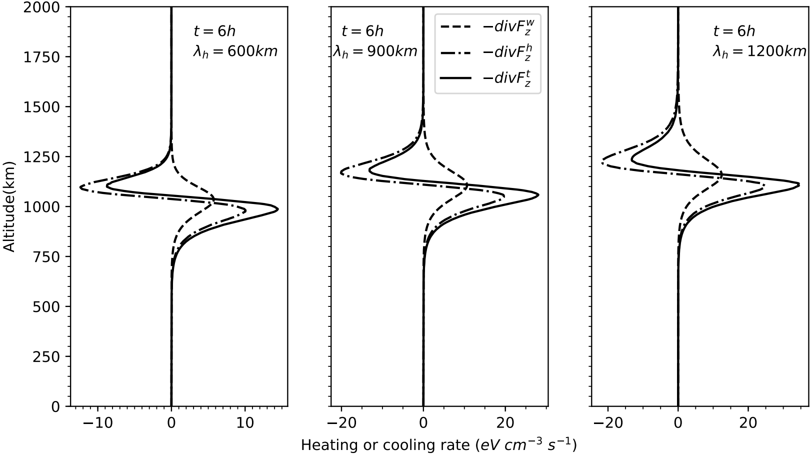
Heating or Cooling Rate of Gravity Waves



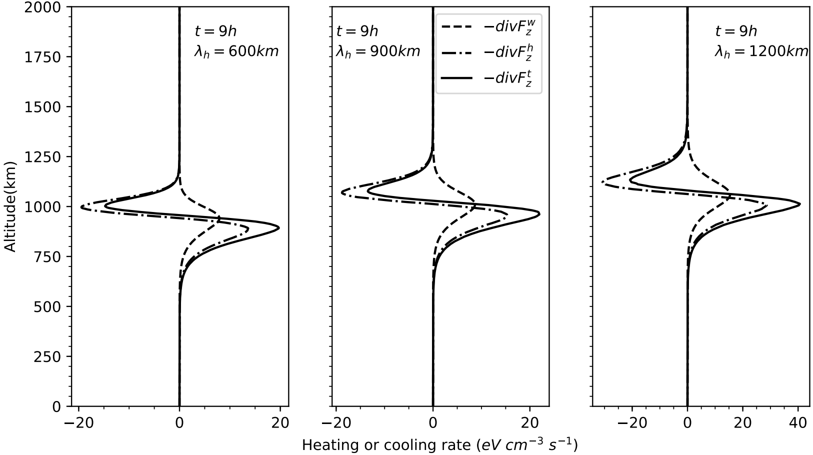
Heating or Cooling Rate of Gravity Waves



Heating or Cooling Rate of Gravity Waves



Heating or Cooling Rate of Gravity Waves



Discussion

- By analysis the wave energy fluxes and their divergences of above the 9 waves models, we can find that **the classical energy fluxes are always positive and keep constant in lower region while are zero in upper region**, they have heating effect at all altitude for all 9 waves models because they decrease with altitude monotonically, **their negative divergences generates only heating at all height**;
- **The sensible heat fluxes are negative and follow the profile of the temperature amplitude, they reaches local minimum value at upper region**.Their divergence can heat and cool the Titan's atmosphere in different altitude intervals, generally, **they generate heating rate in lower atmosphere and generate cooling rate in upper atmosphere**;
- The total vertical energy fluxes are the sum of the classical energy fluxes and the sensible heat fluxes, and their altitude profiles are very close to the profiles of the sensible heat fluxes.Also, the profiles of the heating/cooling rates by the total vertical energy fluxes are closely matches the heating/cooling rates by the sensible heat fluxes.

Discussion

- Snowden estimated the viscous flux of kinetic energy on order of 30 eV cm^{-3} s^{-1} approximately near 1300 km altitude and the wave cooling rate due to the sensible heat flux is -9.0 eV cm^{-3} s^{-1} in Snowden & Yelle (2014), there are very close to the values by our calculation for the 3-h wave with horizontal wavelength $\lambda_h = 1200$ km;
- Besides, the heating energy flux of EUV is about 4$\times$ 10^{8} eV cm^{-2} s^{-1} above 1200 km altitude by Snowden calculation, and the analysing by Müller-Wodarg showed that the globally averaged EUV energy into the Titan's thermosphere is 3.5 to 5$\times$ 10^{9} eV cm^{-2} s^{-1} in Müller-Wodarg et al.(2006);
- However, in our calculations, the maximum of the total vertical energy fluxes for the 9 waves models is about 3.5$\times$ 10^{8} eV cm^{-2} s^{-1}, which is lower than the energy flux of EUV by Snowden and Müller-Wodarg calculated;
- To compare the heating energy flux between gravity waves and EUV in detail, we simply calculated the altitude profiles of the EUV heating rate due to solar radiation.

EUV Heating Rate

- The major constituents of Titan's atmosphere are N_2 and CH_4, and minor constituents are organic molecules, such as C_2H_6 and C_2H_2.In the next analysis, we calculate the photoionization heating rates $H(z,\chi)$ of these four neutral species via

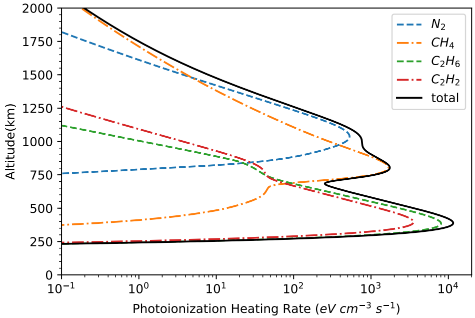
$$H(z,\chi) = \sum_i \int_0^{\lambda_{max}} \sigma_i(\lambda)\, n_i(z)\, F_\infty(\lambda)\, e^{-\tau(\lambda,z,\chi)} d\lambda \tag{23}$$

where z is the altitude, χ is the solar zenith angle (SZA), λ is the photo wavelength, σ_i, n_i are photoionization cross section and number density for neutral species i. $F_\infty(\lambda)$ is the solar radiation flux at the top of the Titan's atmosphere. $\tau(\lambda,z,\chi)$ is the optical depth given by

$$\tau(\lambda,z,\chi) = \sum_i \int_z^\infty \frac{\sigma_i(\lambda)\, n_i(z)}{cos\chi} dz \tag{24}$$

EUV Heating Rate

- The results of the photoionization heating rates for these neutral species are showed in figure, In our calculation, we set the SZA χ=45°. The heating rate due to N_2 and CH_4 reaches maximum at upper region, while the heating rate of organic molecules reaches maximum at lower region. For the neutral N_2, the maximum of photoionization heating rate is 5.29×10^2 eV cm^{-3} s^{-1} at 1037 km altitude, while the heating rate due to neutral CH_4 maximize near 804 km altitude with the value 1.68×10^3 eV cm^{-3} s^{-1};



- For organic molecules C_2H_6 and C_2H_2, the altitudes of the maximum heating rate by these two neutral species are 385, 396 km, which are lower than 500 km altitude, so we neglect photoionization heating rates by these two neutral specie when we compare the heating rate with the gravity waves models.

EUV Heating Rate

- The maximum heating rate of above 9 waves models is 41.91 eV cm^{-3} s^{-1} at 1276 km altitude, while the total photoionization heating rate is 82.95 eV cm^{-3} s^{-1} at the same altitude, and the integrated the total photoionization heating rate is about 1.78×10^{9} eV cm^{-2} s^{-1} above 1200 km altitude, **these values are all higher than the values by dissipations of above 9 waves models**;
- These results indicates that **the dissipations of gravity waves can't explain the large temperature variability in Titan's upper atmosphere**.

Monte Carlo Simulation

- Our above calculations show that **the heating rate by the dissipations of the gravity wave in Titan's atmosphere is close to the value by the EUV energy flux**, and **their heating rates are insufficient to reproduce the large temperature variability of $\approx$ 60 K**.We use the monte carlo method to simulate the variability of temperature in Titan's upper atmosphere;
- The average temperature of Titan's upper atmosphere varies from 112 to 175 K by analysing the INMS data from 32 Cassini passes in Snowden et al. (2013), and they founded that these passes exhibit wave-like temperature perturbations in Titan's atmosphere, which have wavelengths between 150 and 420 km and amplitudes between 3% and 22%;
- Further analysis by Cui et al.(2013) showed that it is exhibit density waves in various constituents of Titan's upper atmosphere, the perturbation amplitude of major constituent N_2 is between 4% and 16%, with a mean of 8.3%, for CH_4, the amplitude range from 1.5% to 11%, with a mean of 5.7%, and the observed vertical wavelength is $\approx$ 240-400 km.

Monte Carlo Simulation

- For constituent i, the perturbation of density is expressed as:

$$\delta_i = \frac{n_i - \overline{n}_i}{\overline{n}_i} = A_i cos\left(k_i\left(z - z_0\right) + \phi_i\right) \tag{25}$$

where A_i, k_i and ϕ_i are amplitudes, wave numbers, and phases. z_0 is the bottom altitude, $\overline{n}_i$ is the mean-state number density.The range of altitude is 1000-1500 km in Snowden et al.(2013), so z_0=1000 km in our calculation. Now we set the random variable ϕ_i is uniform distribution:$\phi_i \sim U(0, 2\pi)$, and set the amplitudes of major constituent N_2 is normal distribution:$A_{N_2} \sim U(4\%, 16\%)$.To do the simulation, we select N=10^4 sample points randomly, and take three different values for vertical wavelength, there are 240, 300, 400 km.

Monte Carlo Simulation

表: Table 3: Summary of Results of Mento Carlo Simulations for N_2

Vertical Wavelength (km)	T_{min} (K)	T_{max} (K)	ΔT (K)
240	150.09	156.07	5.98
300	150.32	155.81	5.49
400	150.23	155.87	5.64

- The results of simulation for N_2 are listed in table 3;
- The mean-state number density can expressed as $\overline{n}_i \propto exp\left[\frac{GMm}{k_B T}\left(\frac{1}{r} - \frac{1}{r_0}\right)\right]$,so the mean-state temperature $\overline{T}$ can be obtained by using the least squares method for $\overline{n}_i$.The value of $\overline{T}$ is 153.01 K;
- **The results from above monte carlo simulations are also insufficient to explain the large temperature variability in Titan's upper atmosphere.**

Conclusions

- Existing studies on the Titan's energy crisis have investigated a number of driving forces including solar EUV heating, magnetospheric electron impact heating, Joule heating, HCN abundance, but none of them contributes to the observed temperature variability;
- We investigate in this study the role of wave dissipation. For this purpose, we construct a simple linearized model of wave propagation in Titan' s upper atmosphere based on the WKB approximation, from which the energy flux and heating rate are calculated as a function of altitude for several selected wave modes likely present on Titan;
- Our calculations reveal a maximum heating rate of 40 eV cm^3 s^{-1} at around 1280 km, which is only half of the solar EUV heating rate at the same altitude. **This implies that wave dissipation is not very likely to be a viable mechanism causing substantial temperature variability in Titan' s upper atmosphere**;
- We further speculate that such a temperature variability is an observational bias provided that the characteristic wavelength is comparable with the vertical extent over which the INMS sampled the ambient atmosphere along a typical Cassini encounter with Titan.A Monte Carlo simulation is implemented to verify the above speculation, but we find that, for a reasonable choice of wave parameters, **the modeled temperature variability is far insufficient to account for the observations**.

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