The Simulation of Kelvin-Helmholtz Instability by PYTHON

Reporter: Xing Wang

**State Key Laboratory of Lunar and Planetary Sciences,
Space Science Institute**

May 11, 2021

Contents

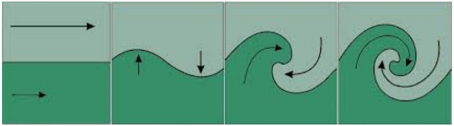
1 Introduction of Kelvin-Helmholtz Instability
1.1 Basic Concept of Kelvin-Helmholtz Instability
1.2 Natural Phenomena of Kelvin-Helmholtz Instability

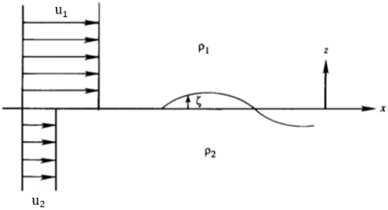
2 Finite Volume Method Simulation
2.1 Brief Introduction of Finite Volume Method
2.2 The Result of Simulation

Basic Concept of KHI

- Kelvin-Helmholtz instability (KHI) was first studied by Hermann von Helmholtz in 1868 and by William Thomson (Lord Kelvin) in 1871;

- the velocity **u** and density ρ profiles are uniform in each fluid layer;
- **u** and ρ are discontinuous at the interface: shear flow;



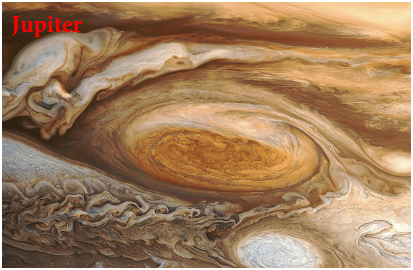


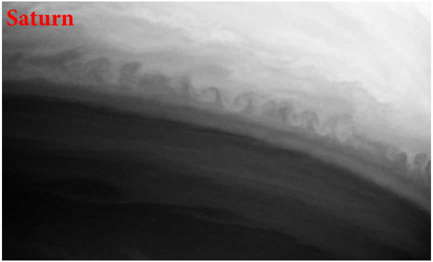
Natural Phenomena of KHI: Arts

Hiroshige Utagawa "Vortices in the Konaruto stream"

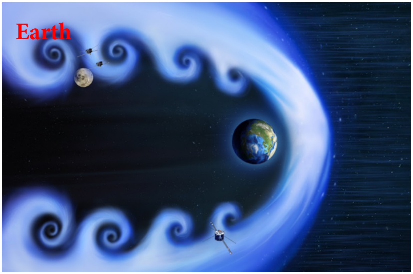
Vincent Van Gogh "La Nuit Etoilee"

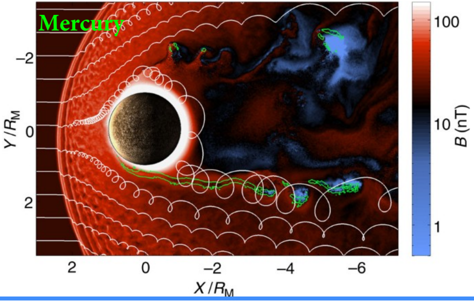
Ocean and Planetary Atmospheres

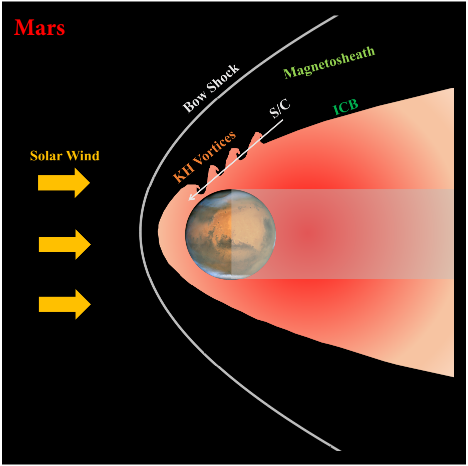




Planetary Space Environments







Hydrodynamics Equations

- Two-dimensional hydrodynamics equations:

$$\frac{\partial}{\partial \mathrm{t}}\begin{pmatrix}\rho \\ \rho \mathrm{v_x} \\ \rho \mathrm{v_y} \\ \rho \mathrm{e}\end{pmatrix} + \frac{\partial}{\partial \mathrm{x}}\begin{pmatrix}\rho \mathrm{v_x} \\ \rho \mathrm{v_x^2} + \mathrm{p} \\ \rho \mathrm{v_x v_y} \\ (\rho \mathrm{e} + \mathrm{p})\mathrm{v_x}\end{pmatrix} + \frac{\partial}{\partial \mathrm{y}}\begin{pmatrix}\rho \mathrm{v_y} \\ \rho \mathrm{v_y v_x} + \mathrm{p} \\ \rho \mathrm{v_y^2} + \mathrm{p} \\ (\rho \mathrm{e} + \mathrm{p})\mathrm{v_y}\end{pmatrix} = \mathbf{0} \tag{1}$$

- where $\mathrm{e} = \epsilon + \frac{\mathrm{v}^2}{2}, \mathrm{p} = (\gamma - 1)\rho\epsilon$.
- The equations can be rewritten by:

$$\frac{\partial}{\partial \mathrm{t}}\mathbf{U} + \nabla \cdot \mathbf{F}(\mathbf{U}) = \mathbf{0} \tag{2}$$

Finite Volume Method in 2-D

- Divide computational domain into disjoint cells: $\Omega = \cup_i C_i$;
- For Eq(2),using divergence theorem, we can Integral it for cell C_i:

$$\frac{\partial}{\partial t}\int_{C_i} \mathbf{U}\mathrm{d}x\mathrm{d}y = -\oint_{\partial C_i} \mathbf{F}\cdot \mathrm{d}\mathbf{S} \tag{3}$$

- Cell average value:

$$\mathbf{U}_i(t) = \frac{1}{|C_i|}\int_{C_i} \mathbf{U}(x,y,t)\mathrm{d}x\mathrm{d}y \tag{4}$$

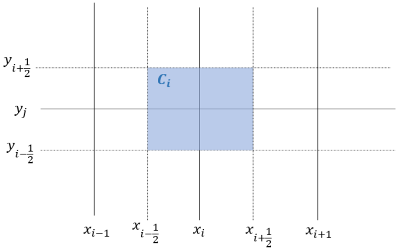
where $|C_i|$ is the area of cell C_i;

- $$\oint_{\partial C_i} \mathbf{F}\cdot \mathrm{d}\mathbf{S} = \sum_{k\in N(i)} F_{ik}\Delta S_{ik} \tag{5}$$

where $N(i)$={$k : C_k$ and C_i share a common face}.

- According to Eq(3)-(5):

$$|C_i|\frac{U_i^{(n+1)} - U_i^{(n)}}{\Delta t} = -\sum_{k\in N(i)} F_{ik}\Delta S_{ik}$$



- Timestep(CFL condition):

$$\Delta t = C_{CFL}\min_i\left(\frac{\Delta x}{c_i + |v_i|}\right)$$

- Gradient:

$$\frac{\partial f_{i,j}}{\partial x} = \frac{f_{i+1,j} - f_{i-1,j}}{2\Delta x}, \qquad \frac{\partial f_{i,j}}{\partial y} = \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta y}$$

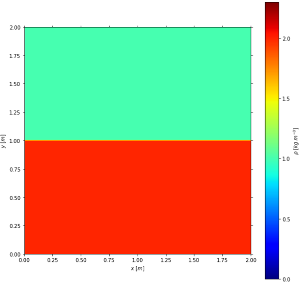
- Second-Order Extrapolation in Space:

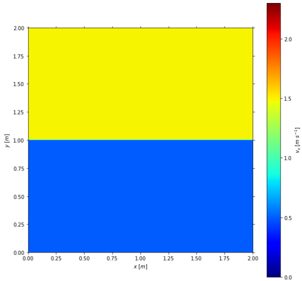
$$f_{i+\frac{1}{2},j} = f_{i,j} + \frac{\partial f_{i,j}}{\partial x}\frac{\Delta x}{2}, f_{i-\frac{1}{2},j} = f_{i,j} - \frac{\partial f_{i,j}}{\partial x}\frac{\Delta x}{2}$$

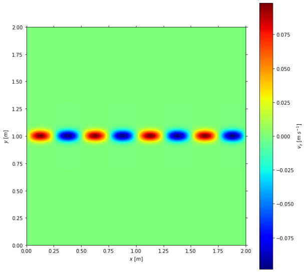
Initial Conditions

$$\rho(y) = \begin{cases} 2 & 0 < y < 1 \\ 1 & 1 < y < 2 \end{cases} \qquad v_x(y) = \begin{cases} 0.5 & 0 < y < 1 \\ 1.5 & 1 < y < 2 \end{cases}$$

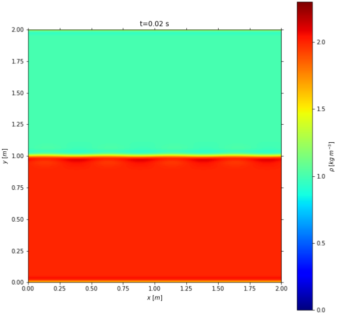
$$vy(y) = 0.1\sin(4\pi x)e^{-400(y-1)^2}, \quad L_x \times L_y = [0,2]\times[0,2], \quad \gamma = \frac{5}{3}, \quad p = 2.5$$







The result of simulation:



Thanks!