

Removing the Astrophysical Gravitational Wave Foreground

Comparing the Efficacies of Signal Deletion and Signal Subtraction with Respect to Gravitational Wave Telescope Parameters

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Abstract

Gravitational waves from astrophysical sources have been detected and the next generation of gravitational wave telescopes might be able to detect gravitational waves from cosmic background sources. However we expect that these cosmic background signals will be obscured by a substantial astrophysical background which will make analyzing the cosmic background prohibitively difficult. In this paper we assess techniques for removing resolved astrophysical sources from gravitational wave data for the purpose of reducing the astrophysical background and easing analysis of the cosmic gravitational wave background.

Introduction

Advanced LIGO recently detected gravitational waves from a binary black hole merger [1]. While estimates for the number of coalescing binary black holes in the universe are imprecise, there are many such binaries [2]. This large number of binary black holes produces overlapping, incoherent signals which form a “background” of gravitational waves [3]. This background is statistically homogeneous and isotropic, despite the discrete nature of the sources which make it up [3]. With current advances in gravitational wave detection this background will be measured within the next few years [3].

!!! Insert description of background here. !!!

Binary black holes are not the only thing which contribute to the gravitational wave background. A number of other astrophysical sources contribute to the background including rotating neutron stars [4], neutron star phase transitions [5], instabilities in young neutron stars [6], and core collapse super-

novae [7]. There are also cosmological sources which contribute to the gravitational wave background including pre-Big Bang cosmological factors [8] [9] [10], the amplification of weak gravitational waves (including vacuum fluctuations) during inflation [11] [12], phase transitions in the early universe [13] [14] [15], and cosmic strings [16]. The analysis of gravitational waves produced by the aforementioned cosmological sources could provide insight into the early processes of the universe.

However the analysis of the gravitational wave background produced by cosmological sources is complicated by the inability to separate it from the gravitational wave background produced by astrophysical sources [3]. However the astrophysical background can be separated into two parts, an unresolvable astrophysical background in which every source generates signals with signal to noise ratios less than 1 and a resolvable astrophysical background in which every source generates signals with SNRs greater than or equal to 1 called the astrophysical foreground [17]. The signals generated by the astrophysical foreground can then

be identified and removed from the data, reducing the total astrophysical background and making it easier to identify any cosmic background which may exist.

This project is not concerned with how the astrophysical foreground signals are identified, which has been discussed in great detail elsewhere [18]. The goal of this project is to develop tools for determining the efficacy of signal deletion and signal subtraction as methods to remove resolved astrophysical signals and to use these tools to determine which method will be most effective for the next generation of gravitational wave telescopes. In Capstone I, as preparation for this, we analyze a simple model that assumes flawless signal removal.

Resolvable Energy Fraction

When removing astrophysical foreground signals, the point of the exercise is to reduce the energy of astrophysical signals recorded by a detector so that the astrophysical to cosmic signal energy ratio is as small as possible. After all, the less gravitational energy from astrophysical sources is recorded by the detector compared to cosmic sources, the easier it will be to accurately identify cosmic sources of gravitational energy.

This makes energy a sensible quantity with which to quantify total and resolved astrophysical signals. It also lends itself to a simple metric for quantifying the efficacy of signal removal techniques, we want the fraction of resolved energy over total energy to be as close to 1 as possible. That is we want the dimensionless quantity α to be as close to 1 as possible, as α is shown in Equation 1.

$$\alpha = \frac{E_{res}}{E_{tot}} \quad (1)$$

To do so we need to determine the total as-

trophysical gravitational wave energy incident on an arbitrary detector (E_{tot}) and the resolvable energy incident on an arbitrary detector (E_{res}).

For the purposes of this project we will be assuming that only astrophysical signals our detectors pick up come from black holes binaries, which is a reasonable approximation as black hole binaries are expected to constitute the vast majority of all astrophysical gravitational wave sources [4].

Astrophysical Signal Density

!!! Simplify following paragraph !!!

To describe the energy (E) recorded by gravitational wave telescopes we use the integral of gravitational wave signal density ($\Omega_{gw}(f)$) with respect to frequency (f), shown in Equation 21. Signal density is a dimensionless quantity defined as the the energy density with respect to frequency ($f \frac{\partial \rho_{gw}(f)}{\partial f}$) of recorded gravitational waves as a proportion of the energy density needed to close the universe (ρ_c), as shown in Equation 3. [19]

$$E = \int_0^\infty \Omega_{gw}(f) df \quad (2)$$

$$\Omega_{gw}(f) = \frac{f \frac{\partial \rho_{gw}(f)}{\partial f}}{\rho_c} \quad (3)$$

Equation 3 is not a particularly workable form for signal density though. Before we continue further we want to find a more accessible way of describing $\Omega_{gw}(f)$.

Closure energy density (ρ_c) can be described with the Hubble constant (H_0), speed of light constant (c), and gravitational force constant (G), shown in Equation 4. [26]

$$\rho_c = \frac{3H_0c^2}{8\pi G} \quad (4)$$

The astrophysical energy density with respect to frequency ($f \frac{\partial \rho_{gw}(f)}{\partial f}$) can be divided by c to change area per volume into area per volume, which replaces the energy density with integrated flux ($\int \Phi(f, M_c) dz$) as shown in Equation 5. !!!!!!! Note that flux is dependent on chirp mass (M_c). Chirp mass (M_c) is a useful term for describing a binary's characteristics defined in terms of the masses of the binary's black holes (M_1 & M_2) as shown in Equation 6 [1].

$$f \frac{\partial \rho_{gw}(f)}{\partial f} = \frac{1}{c} \int \Phi(f, M_c) dz \quad (5)$$

$$M_c = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}} \quad (6)$$

Note that the transformation we make in Equation 5 introduces a chirp mass dependence into our description of signal density. This means any calculation of E that we make will have a chirp mass dependence, E will describe the gravitational wave energy of black hole binaries only of a given chirp mass rather than of all of them. This is dealt with later in the paper.

From here we can expand upon flux ($\int \Phi(f, M_c) dz$). Flux is a product of the energy density produced by a single source ($\frac{\partial E_{source}(f, M_c)}{\partial f}$), the apparent distribution of gravitational wave sources with respect to redshift ($R_z(z)$), and the inverse surface area of the sphere over which the gravitational waves are spread ($\frac{1}{4\pi((1+z)r(z))^2}$), all of which is shown in Equation 7. Note that the radius of the sphere over which gravitational waves are spread has radius $r(z)$, where $r(z)$ is the proper distance between the detector and the source.

$$\Phi(f, M_c) = \frac{\partial E_{source}(f, M_c)}{\partial f} R_z(z) \frac{1}{4\pi(1+z)r(z)} \quad (7)$$

The energy density produced by a single source ($\frac{\partial E_{source}(f, M_c)}{\partial f}$) is described in [20], as shown in Equation 8. Unfortunately this equation for energy density assumes a Newtonian orbit, which means any signal density equation we define using this energy density will be restricted to low signal frequencies where the black holes are orbiting one another at large distances and it is reasonable to make a Newtonian approximation of their orbit.

$$\frac{\partial E_{source}(f, M_c)}{\partial f} = \frac{(G\pi)^{2/3}}{3} ((1+z)M_c)^{5/3} f^{-1/3} \quad (8)$$

The apparent distribution of gravitational wave sources with respect to redshift ($R_z(z)$) is a product of the actual distribution of gravitational wave sources ($R_V(z)$), the expansion of space-time with respect to redshift ($\frac{\partial V(z)}{\partial z}$), and a constant fudge factor λ which can accounts for the observed incidence of black hole binaries, all of which is shown in Equation 9.

$$R_z(z) = \lambda R_V(z) \frac{\partial V(z)}{\partial z} \quad (9)$$

The actual distribution of gravitational wave source $R_V(z)$ is not well determined, for the purposes of this paper we will use the function for $R_V(z)$ described in [21], shown in Equation 10.

$$R_V(z) = 0.015 \frac{(1+z)^{2.7}}{1 + [\frac{1+z}{2.9}]^{5.6}} \quad (10)$$

The expansion of space-time with respect to redshift ($\frac{\partial V(z)}{\partial z}$) can be described as the proper surface area of the sphere over which the gravitational waves are spread ($4\pi r(z)^2$) divided by the Hubble Constant in units of distance ($\frac{H_0}{c}$) and a factor which describes the the dependence of the volume of spacetime surrounding an observed source with respect to redshift ($E(\Omega_M, \Omega_\Lambda, z)$), as shown in Equation 11.

$$\frac{\partial V(z)}{\partial z} = \frac{4\pi r(z)^2 c}{H_0 E(\Omega_M, \Omega_\Lambda, z)} \quad (11)$$

For $E(\Omega_M, \Omega_\Lambda, z)$ we use the function that is described in [21], which is well agreed upon. This is shown in Equation 12.

$$E(\Omega_M, \Omega_\Lambda, z) = \sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda} \quad (12)$$

Now we have finally split $\Omega_{gw}(f, M_c)$ into a slew of parts which we can numerically evaluate (for low frequencies at least). The last thing we need to do to get a numerical description of $\Omega_{gw}(f, M_c)$ is to describe the values of the constants embedded in those parts.

The speed of light (c) is $3 * 10^8 \frac{m}{s}$. The gravitational force constant (G) is $6.674 * 10^{-11} \frac{m^3}{kg \cdot s^2}$. Using the standard Λ CDM cosmology [29] we can define the Hubble constant (H_0) as $70 \frac{km}{s \cdot Mpc}$, Ω_M as 0.3, and Ω_Λ as 0.7. Lastly we need to define λ . Unfortunately not enough black hole binaries have been detected yet to accurately determine λ , so we are forced to simply make a best guess. !!!

Pulling together Equations 3, 4, 5, 7, 8, 9, 10, 11, and 12 we can construct Equation 13 as an equation with which to describe signal density for low frequencies.

$$\Omega_{gw}^{low-f}(f, M_c) = \frac{8\lambda(\pi G)^{5/3}}{9H_0^3 c^2} M_c^{5/3} f^{2/3} \int_0^{z_{max}} \frac{R_V(z) dz}{(1+z)^{1/3} E[\Omega_M, \Omega_\Lambda, z]} \quad (13)$$

This is a reasonable low-frequency approximation for gravitational wave signal density, but to turn signal density into an energy quantity we need to integrate signal density from 0 Hz to ∞ Hz so a low-frequency approximation just isn't enough. (Especially since $\Omega_{gw}^{low-f}(f, M_c)$ would integrate to infinity under such bounds).

To define signal density in a way that is valid for all frequencies, let us first ask what happens to black hole binaries at high frequency.

As black hole binary systems radiate energy in the form of gravitational waves the black holes spiral inward towards one another and the frequency with which they orbit one another increases. However at some point the black holes will be so close that their Schwarzschild radii intersect and the two black holes merge into one. Once the black hole binary has merged into a single black hole it ceases to emit gravitational wave energy of a magnitude that gravitational wave telescopes can detect. So black hole binaries will not radiate energy beyond a particular cutoff frequency, the highest orbital frequency the black holes reach before merging.

Furthermore at higher frequencies where the black holes are very close to one another, relativistic effects begin to dominate and our Newtonian approximation of flux ($\Phi(f, M_c)$) does not adequately match reality.

As it so happens detailed models of binary strain ($h(f, M_c)$) exist [27] which show an appropriate cutoff at a given binary's merger frequency and which take general relativity into account. Furthermore strain can be related to flux ($\Phi(f)$) with respect to frequency (f), which gives us a way to use these models of binary strain to describe the behavior of signal density for all frequencies.

From [26] we can retrieve the proportionalities in Equation 14, which taken together show the proportionality of flux to strain.

$$\Phi(f, M_c) \propto \frac{\partial E}{\partial f} \propto f^2 |h(f, M_c)|^2 \quad (14)$$

This does not fully describe the frequency dependence of the proportionality between flux and strain though. As shown above in !!!!!!! flux has a frequency dependence of $f^{-1/3}$ (for a more intuitive explanation of why this is see Appendix 0.1).

Therefore the proportionality between strain and the non-frequency components of flux

is as shown in Equation 15.

$$\Phi(M_c) \propto f^{7/3} |h(f, M_c)|^2 \quad (15)$$

Because $\Phi(M_c)$ is part of our low-frequency approximation of signal density and not dependent on frequency, we can say that $\Omega_{gw}^{low-f}(f, M_c)$ is proportional without respect to frequency to the non-frequency components of flux. This allows us to describe a proportionality between signal density and strain, shown in Equation 16.

$$\Omega_{gw}(f, M_c) \propto \Phi(M_c) \propto f^{7/3} |h(f, M_c)|^2 \quad (16)$$

From this we can immediately create another proportionality, shown in Equation 17.

$$\frac{\Omega_{gw}(f, M_c)}{\Omega_{gw}^{low-f}(10Hz, M_c)} \propto \frac{f^{7/3} |h(f, M_c)|^2}{(10Hz)^{7/3} |h(10Hz, M_c)|^2} \quad (17)$$

At 10 Hz Newtonian effects dominate the inspiral of binary black holes and $\Omega_{gw}^{low-f}(10Hz, M_c)$ returns accurate values. So by solving the proportionality in Equation 17 using our equation for $\Omega_{gw}^{low-f}(10Hz, M_c)$ and using the values for $h(f)$ given in [27] we can calculate high frequency values for signal density, as shown in Equation 18.

$$\Omega_{gw}(f, M_c) = \left(\frac{f}{10Hz}\right)^{7/3} \frac{|h(f, M_c)|^2}{|h(10Hz, M_c)|^2} \Omega_{gw}^{low-f}(f, M_c) \quad (18)$$

This equation accurately describes the gravitational wave signal density for black hole binaries for all frequency values, though as mentioned earlier it still has an unwanted chirp mass dependence that needs to be dealt with before it can be integrated with respect to frequency to find an energy quantity.

Total Astrophysical Signal Energy

Resolvable Astrophysical Signal Energy

Appendix

0.1 Flux Frequency Dependence

As described in Equation 14, $\Phi(f, M_c) \propto \frac{\partial E}{\partial f}$. Because our $\Phi(f, M_c)$ operates off the assumption of a Newtonian orbit, we can choose to find $\frac{\partial E}{\partial f}$ using Kepler's laws of orbital mechanics, shown in Equation 19 and Equation 20.

$$E = -\frac{M^2}{2a} \quad (19)$$

$$2\pi f = \left(\frac{M}{a^3}\right)^{1/2} \quad (20)$$

In Equations 19 and 20 M describes the masses of each orbiting body, assuming the orbiting bodies have identical mass. Which is a reasonable enough approximation to make for this qualitative description of flux's frequency dependence. Also, a is the centripetal acceleration of the orbiting bodies, E is the gravitational potential energy contained within their orbit, and f is their orbital frequency (which is the same thing as their signal frequency).

From Equations 19 and 20 we can derive Equation 21 and take its derivative with respect to frequency, f , to find Equation 22.

$$E = -\frac{M}{2}(M\pi f)^{2/3} \quad (21)$$

$$\frac{\partial E}{\partial f} = \frac{M^2\pi}{3}(M\pi f)^{-1/3} \quad (22)$$

Therefore we can say that $\frac{\partial E}{\partial f} \propto f^{-1/3}$ and from there say that $\Phi(f, M_c) \propto f^{-1/3}$. This is hopefully a more satisfactory explanation of why flux has a frequency dependence of $f^{-1/3}$ than simply citing some equation pulled from the aether.

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