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KELAS : IF1A

1. tentukan daerah definisi dan daerah fungsi-fungsi berikut:

1a. $g(x) = \frac{1}{2x}$

$$syarat \frac{f(x)}{g(x)}, g(x) \neq 0$$

$$2x \neq 0$$

$$x \neq \frac{0}{2}$$

$$x \neq 0$$

$$y = \frac{1}{2x}$$

$$y(2x) = 1$$

$$2xy = 1$$

$$x(2y) = 1$$

$$x = \frac{1}{2y}$$

$$2y \neq 0$$

$$y \neq 0$$

$$D = \{x|x \neq 0, x \ \in R\}$$

$$R = \{y|y \neq 0, y \in R\}$$

1b. $f(x) = \sqrt{1 - x^2}$

$$syarat \sqrt{f(x)}, f(x) \geq 0$$

$$1 - x^2 \geq 0$$

$$(1 + x)(1 - x) \geq 0$$

$$1 + x \geq 0$$

$x \geq -1$

$1 - x \geq 0$

$x \leq 1$

$y = \sqrt{1 - x^2}$

$1 - x^2 \geq y$

$(1 - x)(1 + x) \geq 0$

$1 - x \geq 0$

$x \leq 1$

$1 + x \geq 0$

$x \geq -1$

$D = \{x | -1 \leq x \leq 1, x \in R\}$

$R = \{y | -1 \leq y \leq 1, y \in R\}$

1c. $g(x) = 1 - x^2$

$y = 1 - x^2$

$y - 1 = -x^2$

$x^2 = -y + 1$

$x = \sqrt{-y + 1}$

$-y + 1 \geq 0$

$y \leq 1$

$D = \{x | x \in R\}$

$R = \{y | y \leq 1, y \in R\}$

1d. $f(x) = x^2 + 4$

$y = x^2 + 4$

$y - 4 = x^2$

$x^2 = y - 4$

$x = \sqrt{y - 4}$

$y - 4 \geq 0$

$y \geq 4$

$D = \{x | x \in R\}$

$R = \{y | y \geq 4, y \in R\}$

1e. $g(x) = \frac{1}{\sqrt{2+x}}$

$\frac{g(x)}{\sqrt{h(x)}}, h(x) \neq 0, \sqrt{h(x)} \geq 0$

$2 + x > 0$

$x > -2$

$$y = \frac{1}{\sqrt{2+x}}$$

$y^2(2 + x) = 1^2$

$$2 + x = \frac{1}{y^2}$$

$$x = \frac{1}{y^2} - 2$$

$$x = \frac{1}{y^2} - \frac{2y^2}{y^2}$$

$$x = \frac{1 - 2y^2}{y^2}$$

$\frac{f(x)}{g(x)}, g(x) \neq 0$

$y \neq 0$

$y \neq 0, artinya\ y\ harus\ lebih\ dari\ 0, (y > 0) atau\ y\ kurang\ dari\ , (y < 0)$

$jadi\ pasangan\ Domain\ x > -2\ yang\ memenuhi\ adalah\ range\ y > 0$

$D = \{x | x > -2, x \in R\}$

$R = \{y | y > 0, y \in R\}$

2. buktikan kesamaan berikut:

2a. $(1+sin\ x)(1-sin\ x)=\frac{1}{sec^2x}$

$$(1+sin\ x)(1-sin\ x)=\frac{1}{sec^2x}$$

$$1-sin\ x+sin\ x-sin^2x=\frac{1}{\left(\frac{1}{cos\ x}\right)^2}$$

$$1-si\ \ ^2x=\frac{1}{\frac{1}{co\ \ ^2x}}$$

$$cos^2x=1{:}\frac{1}{co\ \ ^2x}$$

$$cos^2x=1\times\frac{co\ \ ^2x}{1}$$

$$cos^2x=cos^2x$$

2b. $sec\ x-si\ \ \ x\ cos\ x=cos\ x$

$$sec\ x-sin\ x\ cos\ x-cos\ x=0$$

$$\frac{1}{cos\ x}-sin\ x\ cos\ x-cos\ x=0$$

$$\frac{1}{cos\ x}-\frac{cos\ x\ sin\ x\ cos}{cos\ x}-\frac{cos\ x\ cos\ x}{cos\ x}=0$$

$$\frac{1-cos^2x\ sin\ x-cos^2x}{cos\ x}=0$$

$$\frac{sin^2x-cos^2x\ sin\ x}{cos\ x}=0$$

$$sin^2x-cos^2x\ sin\ x=0$$

$$sin^2x-(1-sin^2x)sin\ x=0$$

$$sin^2x-(sin\ x-sin^3x)=0$$

$$sin^2x-sin\ x+sin^3x=0$$

$$sin\ x(sin^2x+sin\ x-1)=0$$

$sin\ x=0$

$sin^2x+sin\ x-1=0$

rumus persamaan kuadrat

$a = 1, b = 1, c - 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1 + 4}}{2}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

$jadi\ persamaan\ tersebut\ memiliki\ tiga\ penyelesaian$

$\sin x = 0$

$$x_1 = \frac{-1 + \sqrt{5}}{2}$$

$$x_2 = \frac{-1 - \sqrt{5}}{2}$$

2c.$cos\ 3y = 3 = 4cos^3 y - 3\ cos\ y$

$rumus\ sudut\ rangkap\ atau\ ganda, cos\ 3\alpha = 4cos^3\alpha - 3cos\ \alpha$

$cos\ 3y = 4co\ \ ^3y - 3cos\ y$

$cos\ 3y = cos\ 3y$

2d. $(1 + cos\ x)(1 - cos\ x) = sin^2 x$

$1 - cos\ x + cos\ x - co\ \ ^2x = si\ \ ^2x$

$1 - cos^2 x = sin^2 x$

$sin^2 x = sin^2 x$

2e. $(1 - cos^2 x)(1 + cot^2 x) = 1$

$$si\ \ ^2x \left(\frac{1}{sin\ x}\right)^2 = 1$$

$$sin^2 x\ \ \frac{1}{si\ \ ^2x} = 1$$

$$1 = 1$$

3. tentukan$(fog)(x)$ dan $(gof)(x)$ jika

3a. $f(x) = 1 - \sqrt{1+x}, \quad g(x) = \frac{1}{2x}$

$(fog)(x) = f\big(g(x)\big)$

$$= 1 - \sqrt{1 + \frac{1}{2x}}$$

$$= 1 - \sqrt{\frac{2x}{2x} + \frac{1}{2x}}$$

$$= 1 - \sqrt{\frac{2x+1}{2x}}$$

$$= 1 - \frac{\sqrt{2x+1}}{\sqrt{2x}}$$

$$= \frac{\sqrt{2x}}{\sqrt{2x}} - \frac{\sqrt{2x+1}}{\sqrt{2x}}$$

$$= \frac{\sqrt{2x} - \sqrt{2x+1}}{\sqrt{2x}}$$

$(gof)(x) = g\big(f(x)\big)$

$$= \frac{1}{2\big(1 - \sqrt{1+x}\big)}$$

$$= \frac{1}{2 - 2\sqrt{1+x}}$$

$$= \frac{1}{2 - 2\sqrt{1+x}} \times \frac{2 + 2\sqrt{1+x}}{2 + 2\sqrt{1+x}}$$

$$= \frac{1\big(2 + 2\sqrt{1+x}\big)}{4 - 4(1+x)}$$

$$= \frac{2 + 2\sqrt{1+x}}{4 - 4 - 4x}$$

$$= \frac{2 + 2\sqrt{1+x}}{-4x}$$

$$= \frac{1 + \sqrt{1+x}}{-2x}$$

3b. $f(x) = \sqrt{1-x^2}, \qquad g(x) = 1 - x^2$

$(fog)(x) = \sqrt{1 - (1-x^2)^2}$

$$= \sqrt{1 - \left((1 - x^2)(1 - x^2)\right)}$$

$$= \sqrt{1 - (1 - x^2 - x^2 + x^4)}$$

$$= \sqrt{1 - (1 - 2x^2 + x^4}$$

$$= \sqrt{1 - 1 + 2x^2 - x^4}$$

$$= \sqrt{2x^2 - x^4}$$

$$= \sqrt{2 - x^2}$$

$$(gof)(x) = 1 - \left(\sqrt{1 - x^2}\right)^2$$

$$= 1 - (1 - x^2)$$

$$= 1 - 1 + x^2$$

$$= x^2$$

3c. $f(x) = \frac{1}{2-x}, \quad g(x) = x^3 - 1$

$$(fog)(x) = \frac{1}{2 - (x^3 - 1)}$$

$$= \frac{1}{2 - x^3 + 1}$$

$$= \frac{1}{3 - x^3}$$

$$(gof)(x) = \left(\frac{1}{2 - x}\right)^3 - 1$$

$$= \frac{1^3}{(2 - x)^3} - 1$$

$$= \frac{1}{(2 - x)^3} - \frac{(2 - x)^3}{(2 - x)^3}$$

$$= \frac{1}{(2 - x)^3} - \frac{((2 - x)(2 - x)(2 - x))}{(2 - x)^3}$$

$$= \frac{1}{(2 - x)^3} - \frac{(-x^3 + 6x^2 + 12x + 8)}{(2 - x)^3}$$

$$= \frac{1 + x^3 - 6x^2 - 12x - 8}{(2 - x)^3}$$

$$= \frac{x^3 - 6x^2 - 12x - 7}{(2 - x)^3}$$

4. tentukan nilai limit,

$$4a.\ \lim_{x \to -1} \frac{x^3 - 6x^2 + 11x - 6}{x^3 + 4x^2 - 19x + 14}$$

$$\lim_{x \to -1} \frac{x^3 - 6x^2 + 11x - 6}{x^3 + 4x^2 - 19x + 14} = \frac{-1^3 - 6(-1)^2 + 11(-1) - 6}{-1^3 + 4(-1)^2 - 19(-1) + 14}$$

$$= \frac{-1 - 6 - 11 - 6}{-1 + 4 + 19 + 14}$$

$$= \frac{-24}{36}$$

$$= -\frac{2}{3}$$

$$4b. \lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1}$$

$$\lim_{x \to 1} \frac{(x + 2)(x - 1)}{(x + 1)(x - 1)}$$

$$\lim_{x \to 1} \frac{x + 2}{x + 1} = \frac{1 + 2}{1 + 1}$$

$$= \frac{3}{2}$$

$$4c. \lim_{x \to 0} \sqrt{\frac{x^2 + 3x + 4}{x^3 + 10}}$$

$$\lim_{x \to 0} \sqrt{\frac{x^2 + 3x + 4}{x^3 + 10}} = \sqrt{\frac{0^2 + 3(0) + 4}{0^3 + 10}}$$

$$= \sqrt{\frac{4}{10}}$$

$$= \sqrt{\frac{2}{5}}$$

$$= \frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{10}}{5}$$

$$4d. \lim_{x \to 0} \frac{2 - \sqrt{4 - x}}{x}$$

$$\lim_{x\to 0}\frac{2-\sqrt{4-x}}{x}\times\frac{2+\sqrt{4-x}}{2+\sqrt{4-x}}$$

$$\lim_{x\to 0}\frac{4-(4-x)}{x(2+\sqrt{4-x})}$$

$$\lim_{x\to 0}\frac{x}{x\left(2+\sqrt{4-x}\right)}=\frac{1}{2+\sqrt{4-x}}$$

$$=\frac{1}{2+\sqrt{4-0}}$$

$$=\frac{1}{2+2}$$

$$=\frac{1}{4}$$

$$4e.\lim_{x\to 0}\frac{\sqrt{x+2}+\sqrt{2}}{x}$$

$$\lim_{x\to 0}\frac{\sqrt{x+2}+\sqrt{2}}{x}=\frac{\sqrt{0+2}+\sqrt{2}}{0}$$

$$=\frac{\sqrt{4}}{0}$$

$$=\frac{2}{0}, tidak\ terdefinisi$$

menggunakan perkalian akar sekawan

$$\lim_{x\to 0}\frac{\sqrt{x+2}+\sqrt{2}}{x}\times\frac{\sqrt{x+2}-\sqrt{2}}{\sqrt{x+2}-\sqrt{2}}$$

$$\lim_{x\to 0}\frac{x+2-2}{x(\sqrt{x+2}-\sqrt{2})}$$

$$\lim_{x\to 0}\frac{1}{\sqrt{x+2}-\sqrt{2}}=\frac{1}{\sqrt{0+2}-\sqrt{2}}$$

$$=\frac{1}{\sqrt{2}-\sqrt{2}}$$

$$=\frac{1}{0}, tidak\ terdefinis$$

$$4f.\lim_{x\to 0}\frac{\sqrt[3]{x+1}-\sqrt[3]{1}}{x}$$

$$\lim_{x\to 0}\frac{\sqrt[3]{x+1}-\sqrt[3]{1}}{x}=\frac{\sqrt[3]{0+1}-\sqrt[3]{1}}{0}$$

$$= \frac{1-1}{0}$$

$$= \frac{0}{0}, bentuk\ tak\ tentu$$

Menggunakan aturan L,Hopital

$$\lim_{x \to 0} \frac{\sqrt[3]{x+1} - \sqrt[3]{1}}{x} \times \frac{\sqrt[3]{(x+1)^2} + \sqrt[3]{x+1} + 1}{\sqrt[3]{(x+1)^2} + \sqrt[3]{x+1} + 1}$$

$$\lim_{x \to 0} \frac{\left(\sqrt[3]{x+1} - 1\right)\left(\sqrt[3]{(x+1)^2} + \sqrt[3]{x+1} + 1\right)}{x\left(\sqrt[3]{(x+1)^2} + \sqrt[3]{x+1} + 1\right)}$$

$$\lim_{x \to 0} \frac{\sqrt[3]{(x+1)^3} + \sqrt[3]{(x+1)^2} + \sqrt[3]{x+1} - \sqrt[3]{(x+1)^2} - \sqrt[3]{x+1} - 1}{x\left(\sqrt[3]{(x+1)^2} + \sqrt[3]{x+1} + 1\right)}$$

$$\lim_{x \to 0} \frac{x+1-1}{x\left(\sqrt[3]{(x+1)^2} + \sqrt[3]{x+1} + 1\right)}$$

$$\lim_{x \to 0} \frac{x}{x\left(\sqrt[3]{(x+1)^2} + \sqrt[3]{x+1} + 1\right)}$$

$$\lim_{x \to 0} \frac{1}{\sqrt[3]{(x+1)^2} + \sqrt[3]{x+1} + 1} = \frac{1}{\sqrt[3]{(0+1)^2} + \sqrt[3]{0+1} + 1}$$

$$= \frac{1}{\sqrt[3]{1} + \sqrt[3]{1} + 1}$$

$$= \frac{1}{1+1+1}$$

$$= \frac{1}{3}$$

$$4g. \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}, jika\ f(x) = x^3 + 2\sqrt{x}$$

$$misal\ \Delta x = h$$

$$f(x) = x^3 + 2\sqrt{x}$$

$$f(x+h) = (x+h)^3 + 2\sqrt{x+h}$$

$$= (x+h)(x+h)(x+h) + 2\sqrt{x+h}$$

$$= (x^2 + 2hx + h^2)(x+h) + 2\sqrt{x+h}$$

$$= x^3 + 3hx^2 + 3h^2x + h^3 + 2\sqrt{x+h}$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0}\frac{x^3+3hx^2+3h^2x+h^3+2\sqrt{x+2}-\left(x^3+2\sqrt{x}\right)}{h}$$

$$\lim_{h\to 0}\frac{x^3+3h^2x+h^3+2\sqrt{x+h}-x^3-2\sqrt{x}}{h}$$

$$\lim_{h\to 0}\frac{3hx^2}{h}+\frac{3h^2x}{h}+\frac{h^3}{h}+\frac{2\sqrt{x+h}}{h}-\frac{2\sqrt{x}}{h}$$

$$\lim_{h\to 0}3x^2+3hx+h^2+\frac{2\sqrt{x+h}}{h}-\frac{2\sqrt{x}}{h}=3x^2+3(0)x+(0)^2+\frac{2\sqrt{x+0}}{h}-\frac{2\sqrt{x}}{h}$$

$$=3x^2+\frac{2\sqrt{x}}{h}-\frac{2\sqrt{x}}{h}$$

$$=3x^2$$

$4h.\ \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}, jika\ f(x)=\frac{1}{\sqrt{1-2x}}$

$misal\ \Delta x = h$

$$f(x)=\frac{1}{\sqrt{1-2x}}$$

$$f(x+h)=\frac{1}{\sqrt{1-2(x+h)}}$$

$$f(x+h)=\frac{1}{\sqrt{1-2x-h}}$$

$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$\lim_{h\to 0}\frac{\frac{1}{\sqrt{1-2x-h}}-\frac{1}{\sqrt{1-2x}}}{h}=\frac{\frac{1}{\sqrt{1-2x-0}}-\frac{1}{\sqrt{1-2x}}}{(0)}, tidak\ terdefinisi$$

$4i.\ \lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}=\frac{\sqrt[3]{1}-1}{\sqrt{1}-1}$$

$$=\frac{1-1}{1-1}$$

$$=\frac{0}{0}, tidak\ terdefinisi$$