Problem 2.4

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a. J^+: $M_2 + \tilde{S}_2 = M_1 + \tilde{S}_1$

J^-: $M_1 - \tilde{S}_1 = M_0 - \tilde{S}_0 = 0 \rightarrow M_1 = \tilde{S}_1$

$\rightarrow M_2 + \tilde{S}_2 = 2M_1$

J^-: $M_2 - \tilde{S}_2 = M_0 - \tilde{S}_0 = 0 \rightarrow M_2 = \tilde{S}_2$

$\rightarrow M_1 = \tilde{S}_1 = M_2 = \tilde{S}_2 = \mathcal{E}$

$M_2 + \tilde{S}_2 = 2\mathcal{E}$

J^-: $M_2 - \tilde{S}_2 = M_R - \tilde{S}_{R,2}$

$\tilde{S}_2 = \tilde{S}_{R,2} \rightarrow M_2 = M_R = \frac{u_R(t)}{a_0}$

$\tilde{S}_2 = 2\mathcal{E} - M_2 = 2\mathcal{E} - M_R$

$$\boxed{\tilde{S}_2 = 2\mathcal{E} - \frac{u_R(t)}{a_0}}$$

J^-: $M_3 - \tilde{S}_3 = M_0 - \tilde{S}_0 = 0 \rightarrow M_3 = \tilde{S}_3$

J^-: $M_3 - \tilde{S}_3 = M_R - \tilde{S}_{R,3}$

$\tilde{S}_3 = \tilde{S}_{R,3} \rightarrow M_3 = M_R$

$\tilde{S}_3 = M_3 = M_R = \frac{u_R(t)}{a_0}$

$$\boxed{\tilde{S}_3 = \frac{u_R(t)}{a_0}}$$

b. $\Sigma F = m \cdot a$

$\Sigma F = F_R^- - F_R^+ = (p_R^- - p_R^+)A$

acoustic wave: $a^2 = \Delta p / \Delta\rho \rightarrow \Delta p = a_0^2 \Delta\rho$

$\Sigma F = (p_R^- - p_R^+)A = (\Delta p_R^- - \Delta p_R^+)A = (\Delta\rho_R^- - \Delta\rho_R^+) A a_0^2$

And we know: $\tilde{S} = \Delta\rho / \rho_0 \rightarrow \Delta\rho = \tilde{S}\rho_0$

$\Sigma F = (\tilde{S}_{R,2} - \tilde{S}_{R,3}) a_0^2 A \rho_0$

$\Sigma F = (\tilde{S}_2 - \tilde{S}_3) a_0^2 A \rho_0$

Now, we have:

$(\tilde{S}_2 - \tilde{S}_3) a_0^2 A \rho_0 = m \cdot a = m \frac{d u_R(t)}{dt}$

From c_1, we know $\tilde{S}_2$ and $\tilde{S}_3$:

$$\left(2\varepsilon - \frac{u_R(t)}{a_0} - \frac{u_R(t)}{a_0}\right) a_0^2 A \rho_0 = m \frac{du_R(t)}{dt}$$

$$2\varepsilon a_0^2 A \rho_0 - 2\frac{u_R(t)}{a_0} a_0^2 A \rho_0 = m \frac{du_R(t)}{dt}$$

$$\frac{du_R(t)}{dt} + \frac{2 A a_0 \rho_0}{m} u_R(t) = \frac{2 A a_0 \rho_0}{m} \varepsilon a_0$$

$$\boxed{\frac{du_R(t)}{dt} + \lambda u_R(t) = \lambda \varepsilon a_0 \quad \text{with} \quad \lambda = \frac{2 A a_0 \rho_0}{m}}$$

c. $u_R(0) = 0$ for $0 \le t < \frac{L}{a_0}$

$u_R(0) = Ce^0 + a_0 \varepsilon = 0$

$\rightarrow C = -a_0 \varepsilon$ for $0 \le t < \frac{L}{a_0}$

$u_R(\frac{L}{a_0}) = 0$

$u_R(\frac{L}{a_0}) = Ce^{-\lambda \frac{L}{a_0}} + a_0 \varepsilon = 0$

$\rightarrow C = -\frac{a_0 \varepsilon}{e^{-\lambda(L/a_0)}}$ for $t \ge \frac{L}{a_0}$

d. $\lim_{t \to \infty} u_R(t) = a_0 \varepsilon$

This means that $u_R(t)$ approaches $a_0 \varepsilon$ as time goes to infinity