Faculty of Aerospace Engineering

AE4140 - Gas Dynamics

Task 3

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Problem 11.1

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$$\frac{4fx\gamma}{D_H} = \frac{M^2 - M_0^2}{M^2 M_0^2} + \frac{\gamma+1}{2} \ln \frac{M_0^2 (1 + \frac{\gamma-1}{2} M^2)}{M^2 (1 + \frac{\gamma-1}{2} M_0^2)}$$

$$= 7,41$$

$$D_H = \sqrt{\frac{4A}{\pi}} \quad \text{where} \quad A = 0,025^2 \cdot \pi$$

$$D_H = 0,05$$

$$f = 0,005 \quad , \quad \gamma = \frac{7}{5}$$

$$\rightarrow \boxed{x = 13,25 \text{ m}}$$

$$p_1 = p_0 (1 + 0,2 M_1^2)^{-3,5}$$

$$p_1 = 234,86 \text{ kPa}$$

$$\frac{p_1}{p_0^*} = \frac{1}{M_1}\left[\frac{2 + 0,4 M_1^2}{2,4}\right]^{3} = 2,035$$

$$\frac{p_2}{p_0^*} = \frac{1}{M_2}\left[\frac{2 + 0,4 M_2^2}{2,4}\right]^{3} = 1,0021$$

$$p_2 = \frac{1,0021}{2,035} \cdot p_1$$

$$\boxed{p_2 = 115,66 \text{ kPa}}$$

$$L_{0,25} = 9,94$$

$$\frac{4f\gamma L_{0,25}}{D_H} = 5,5664 = \frac{M^2 - 0,3^2}{M^2 \cdot 0,3^2} + 1,2 \ln\left(\frac{0,3^2 (1 + 0,2 M^2)}{M^2 (1 + 0,2 \cdot 0,3^2)}\right)$$

Solving for M yields:

$$\boxed{M_{0,25} = 0.4721}$$

$$\frac{p_{2_{0,25}}}{p_0^*} = \frac{1}{M_{0,25}}\left[\frac{2 + 0,4 M_{0,25}^2}{2,4}\right]^3 = 1,397$$

$$\boxed{p_{2_{0,25}} = 161,25 \text{ kPa}}$$

Problem 11.2

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$$\frac{4fL^*}{D_H} = 0.336 = \frac{M_2^2 - 4}{4M_2^2} + \frac{1}{2}\ln\left[\frac{4(1+0.2M_2^2)}{M_2^2(1+0.2\cdot4)}\right]$$

$$\boxed{M_2 = 1,3}$$

$$p_1 = p_0(1+0.2M_1^2)^{-3,5} = 10.22 \text{ kPa}$$

$$\frac{p_1}{p^*} = \frac{1}{M_1}\left[\frac{2+0,4M_1^2}{2,4}\right]^3 = 1.6875$$

$$\frac{p_2}{p^*} = \frac{1}{M_2}\left[\frac{2+0,4M_2^2}{2,4}\right]^3 = 1,0663$$

$$\boxed{p_2 = \frac{1,6875}{1,0663}\cdot p_1 = 16,17 \text{ kPa}}$$

$$T_1 = T_0(1+0.2M_1^2)^{-1} = 162,8 \text{ K}$$

$$\frac{T_1}{T^*} = \frac{2,4}{2+0,4M_1^2} = 0,667$$

$$\frac{T_2}{T^*} = \frac{2,4}{2+0,4M_2^2} = 0,896$$

$$\boxed{T_2 = \frac{0,896}{0,667}T_1 = 218,9 \text{ K}}$$

$$\frac{A}{A^*} = 3 = 0,5787\frac{(1+0,2M^2)^3}{M}$$

$$\boxed{M = 0,197}$$

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Problem 11.3

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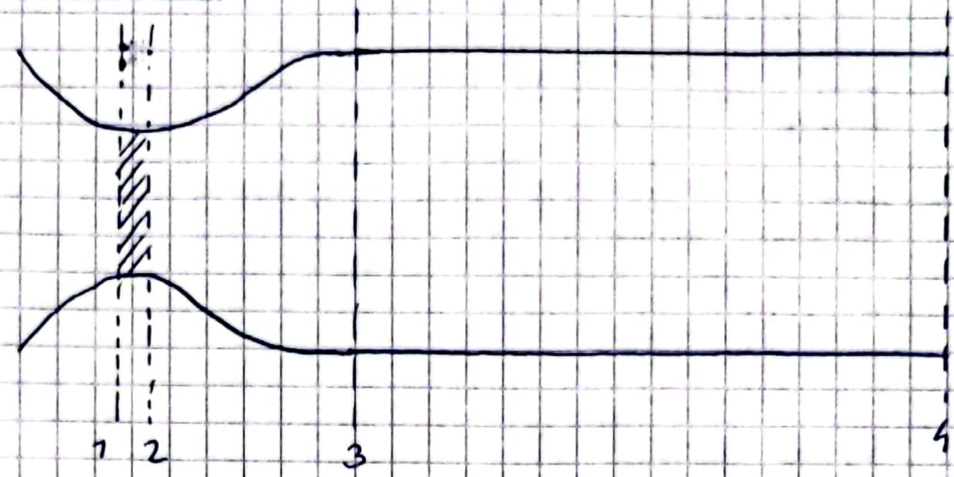
a. I.)



Sonic conditions in nozzle : $M = 1$

$\frac{A}{A^*} = 3 \rightarrow M_3 = 0.194$

$\frac{4 f x \gamma}{D_H} = \frac{4 f \, 12 D \gamma}{\sqrt{2} D} = \frac{48}{\sqrt{2}} f \gamma = \cancel{0.2488} \; 0.168$

$\rightarrow M_4 = 0.1945$

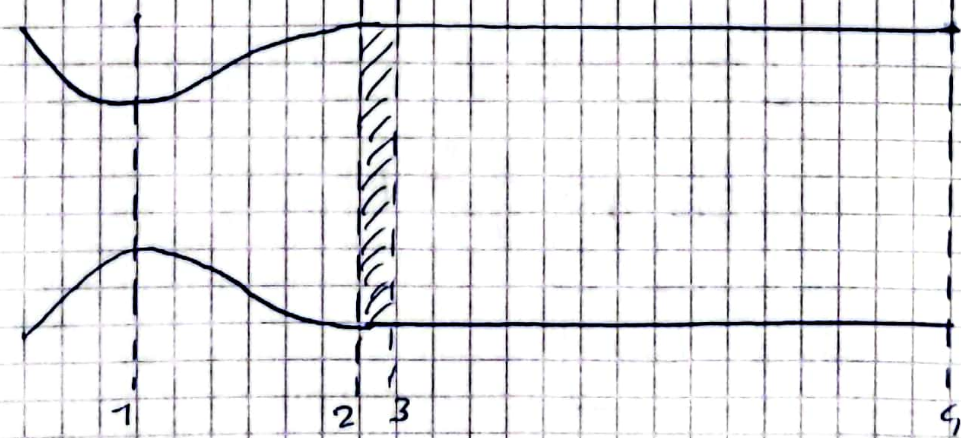
$p_3 = p_t (1 + 0.2 M_3^2)^{-3.5} = 9.738$ bar

$\frac{p_4}{p^*} = \frac{1}{M_4} \left[\frac{2 + 0.4 M_4^2}{2.4} \right]^3 = 3.0434$

$\frac{p_3}{p^*} = \frac{1}{M_3} \left[\frac{2 + 0.4 M_3^2}{2.4} \right]^3 = 3.0508$

$\rightarrow \boxed{p_{res} = 9.7141 \text{ bar}}$

II.)



- $\frac{A}{A^*} = 3 \rightarrow M_2 = 2.64$
  Shock table $\rightarrow M_3 = 0.5005$ $\Big\}$ $\frac{p_{03}}{p_{02}} = 0.4452$

→ $p_{02} = 10$ bar , $p_{03} = 4,452$ bar

$M_4 = 0.5114$ (fanno line)

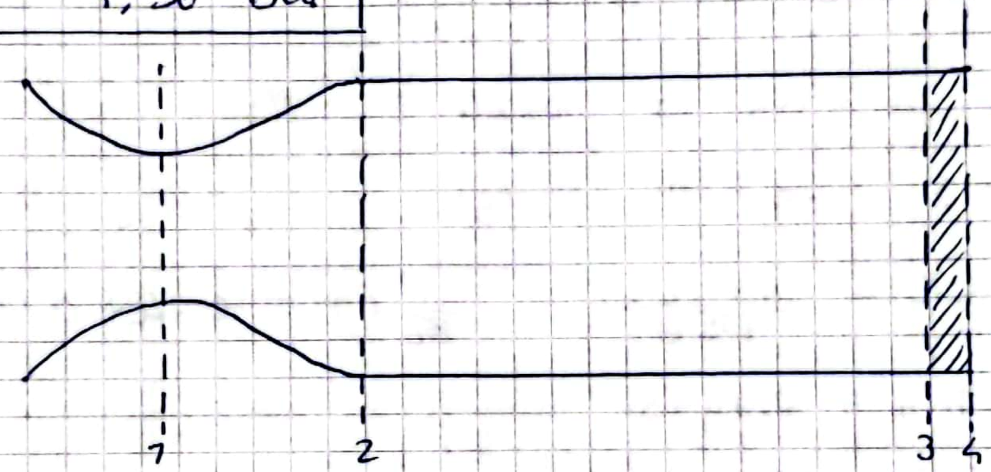
$$\frac{p_{04}}{p_0^*} = \frac{1}{M_4}\left[\frac{2+0,4M_4^2}{2,4}\right]^3 = 1,3186$$

$$\frac{p_{03}}{p_0^*} = \frac{1}{M_3}\left[\frac{2+0,4M_3^2}{2,4}\right]^3 = 1,3389$$

$$p_{04} = \frac{p_{04}}{p_0^*}\cdot\frac{p_0^*}{p_{03}}\cdot p_{03}$$

$$\boxed{p_{04} = 4,36 \text{ bar}}$$

III.)



$M_2 = 2,64$, $M_3 = 2,122$ (fanno line)

$M_4 = 0,5583$ (shock table)

→ $\frac{p_{04}}{p_{03}} = 0,6649$

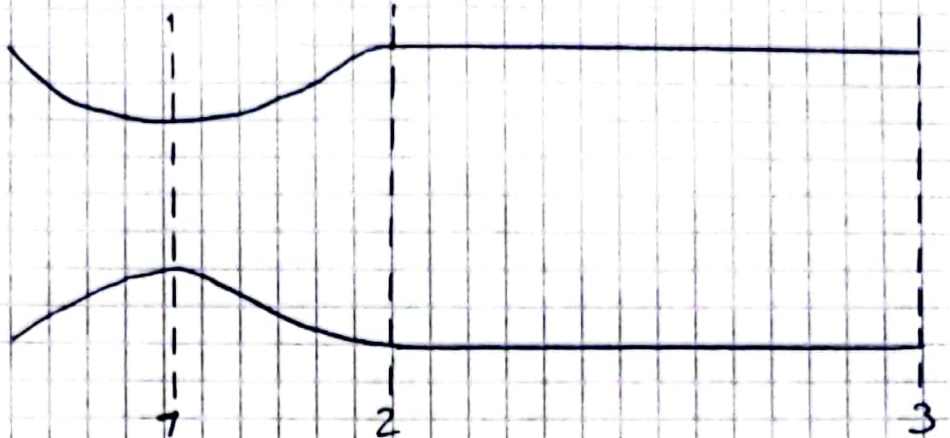
$$\frac{p_{02}}{p_0^*} = \frac{1}{M_2}\left[\frac{2+0,4M_2^2}{2,4}\right]^3 = 3,0073$$

$$\frac{p_{03}}{p_0^*} = \frac{1}{M_3}\left[\frac{2+0,4M_3^2}{2,4}\right]^3 = 1,8723$$

$$p_{res} = p_{04} = \frac{p_{04}}{p_{03}}\,\frac{p_{03}}{p_0^*}\,\frac{p_0^*}{p_{02}}\,p_{02}$$

$$\boxed{p_{res} = 4,14 \text{ bar}}$$

b.



$M_2 = 2.64$, $M_3 = 2.122$

$$\frac{p_{02}}{p_0^*} = \frac{1}{M_2}\left[\frac{2 + 0.4 M_2^2}{2.4}\right]^3 = 3.0073$$

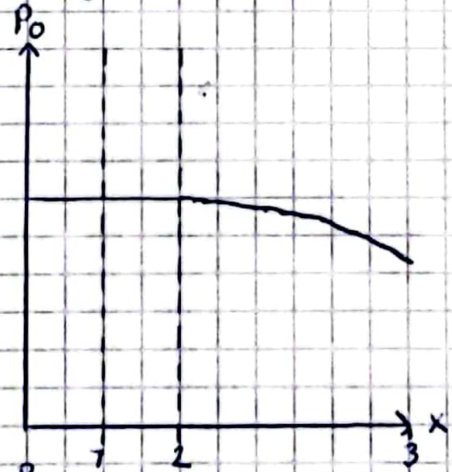
$$\frac{p_{03}}{p_0^*} = \frac{1}{M_3}\left[\frac{2 + 0.4 M_3^2}{2.4}\right]^3 = 1.8723$$

$$p_{res} = \frac{p_{03}}{p_0^*}\frac{p_0^*}{p_{02}}\, p_{02}$$

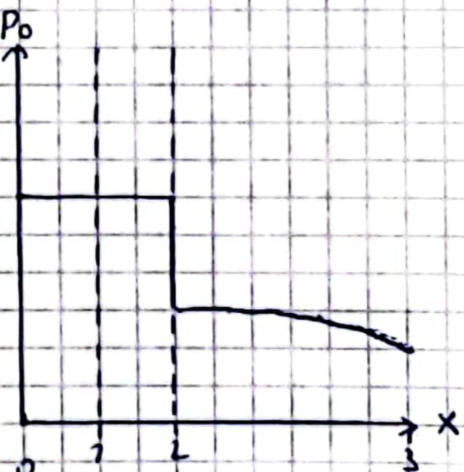
$$\boxed{p_{res} = 6.23 \text{ bar}}$$

c. Use figure above for points 1, 2, 3

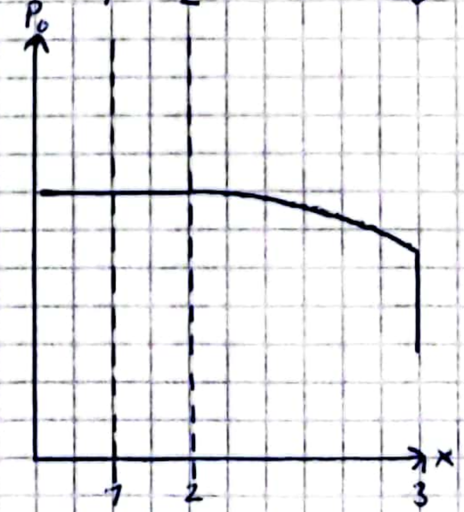
aI.)



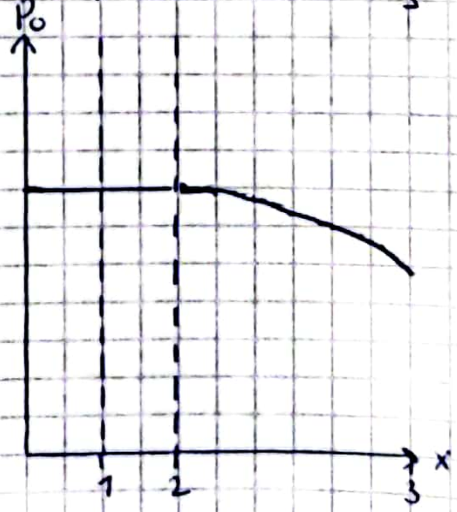
aII.)



aIII.)



b.)



Problem 11.4

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a. $\frac{4\bar{\gamma} f}{D} x = 4\bar{\gamma} \cdot 5{,}3 = 29{,}68$

$\rightarrow M_2 = 0{,}17$ (fanno line)

$\frac{p_2}{p^*} = \frac{1}{M_2}\left[\frac{2 + 0{,}4 M_2^2}{2{,}4}\right]^3 = 3{,}464$

$p_2 = p_t (1 + 0{,}2 M_2^2)^{-3{,}5} = 6{,}86$ bar

$p^* = \frac{p^*}{p_2} p_2 = 1{,}98$ bar

$p^* = p_{exit} = p_{rec}$

$\boxed{p_{exit} = 6{,}86 \text{ bar}}$

b. $\frac{4\bar{\gamma} f}{D} \frac{1}{5} x = 5{,}936$

$\rightarrow M_2 = 0{,}326$ (fanno line)

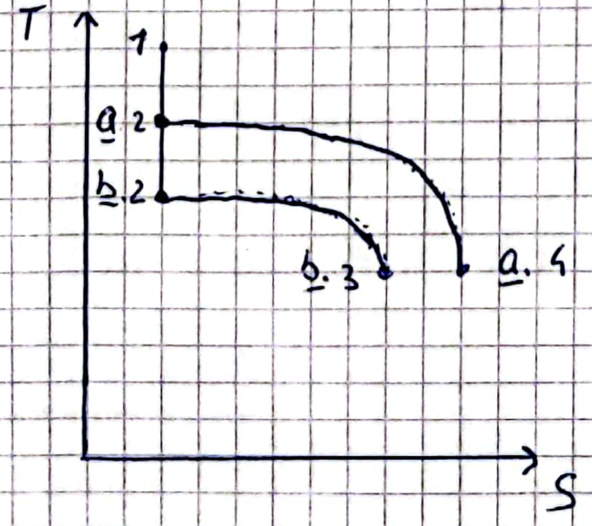
$\frac{p_2}{p^*} = \frac{1}{M_2}\left[\frac{2 + 0{,}4 M_2^2}{2{,}4}\right]^3 = 2{,}6118$

$p_2 = p_t (1 + 0{,}2 M_2^2)^{-3{,}5} = 6{,}5032$ bar

$p^* = \frac{p^*}{p_2} p_2 = 2{,}49$ bar, $p^* = p_{exit}$

$\boxed{p_{exit} = 2{,}49 \text{ bar}}$

c.



Problem 12.1

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$T_1 = 303\ K$, $p_1 = 50\ kPa$, $v_1 = 80\ m/s$

$T_{t,1} = T_1(1 + \frac{\gamma-1}{2} M_1^2)$

We know $\gamma = 7/5$. what is M_1?

$M_1 = \frac{V_1}{a_1}$, where $a_1 = \sqrt{\gamma R T_1} = 348.95\ m/s$

$M_1 = 0.229$

$T_{t,1} = 303(1 + 0.2 \cdot 0.229^2) = 316.18\ K$

$T_{t,2} = T_{t,1} + \frac{q}{c_p}$

$\rightarrow\ q = 40\ MJ/40 = 1 \cdot 10^6\ J$

$\rightarrow\ c_p = \frac{\gamma R}{\gamma - 1} = 1004.67\ J/kgK$

$\boxed{T_{t,2} = 1311.53\ K}$

$\frac{T_{t,1}}{T_t^*} = \frac{2(1+\gamma)^2 M_1^2}{(1+\gamma M_1^2)^2}(1 + \frac{\gamma-1}{2} M_1^2) = 0.5258$

$T_t^* = 600.558\ K$

$\frac{T_{t2}}{T_t^*} = \frac{2(1+\gamma)^2 M_2^2}{(1+\gamma M_2^2)^2}(1 + \frac{\gamma-1}{2} M_2^2) = 2.1838$

$\boxed{M_2 = 0.702}$

$\frac{p_1}{p^*} = \frac{2.4}{1 + 1.4 M_1^2} = 2.236$

$\frac{p_2}{p^*} = \frac{2.4}{1 + 1.4 M_2^2} = 1.4201$

$p_2 = \frac{p_2}{p^*} \cdot \frac{p^*}{p_1} \cdot p_1$

$\boxed{p_2 = 31.76\ kPa}$

Problem 12.2

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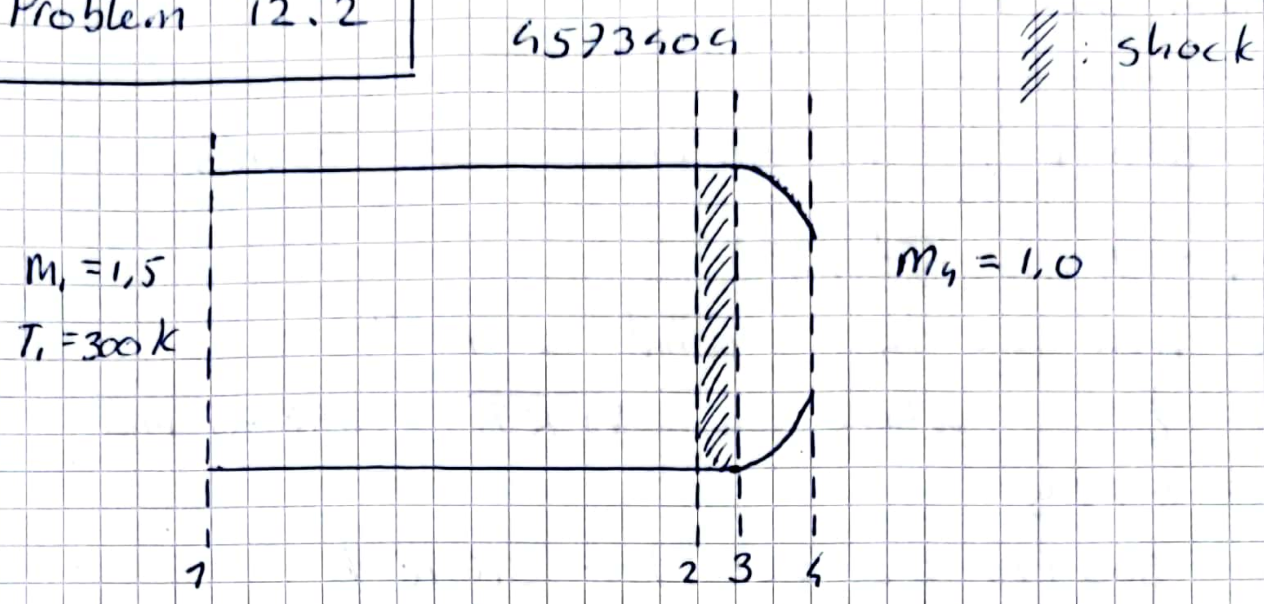

$$\frac{A^*}{A} = 0{,}5787^{-1} \frac{(1+0{,}2M_3^2)^{-3}}{M_3^{-1}} = 0{,}98$$

$\rightarrow M_3 = 0{,}98$ (isentropic flow)

$\rightarrow M_4 = 1{,}19$ (shock tables)

$T_{t,1} = T_1(1+0{,}2M_1^2) = 435$ K

$$T_t^* = T_{t,1}\left[\frac{2(2{,}4)^2 M_1^2}{(1+1{,}4M_1^2)^2}(1+0{,}2M_1^2)\right]^{-1}$$

$T_t^* = 199.33$ K

$$T_{t,2} = T_t^*\left[\frac{2(2{,}4)^2 M_2^2}{(1+1{,}4M_2^2)^2}(1+0{,}2M_2^2)\right]$$

$T_{t,2} = 469$ K

$q = (T_{t,2} - T_{t,1})\,c_p$, where $c_p = \frac{\gamma R}{\gamma - 1} = 1004{,}67$ kJ/kgK

$$\boxed{q = 34{,}16 \text{ kJ}}$$

Problem 12.3

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a. We know that:
- $T_3 = T_4$
- $S_3 = S_4$
- $M_2 > M_1$ (Rayleigh line property)

Because $M_2 > M_1$, from the Rayleigh total temperatures line we know:

$T_{02} > T_{01}$

Equal temperature at 4, 3 $\rightarrow$ $a_4 = a_3$

$\rightarrow$ $V_4 > V_3$

~~Now we use the entropy equation~~

Now we use the isentropic relations:

$$\frac{\rho_{02}}{\rho_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{1}{\gamma-1}} \qquad \rho_{04} = \rho_{02}, \quad \rho_{03} = \rho_{01}$$

$$\rightarrow \frac{\rho_{04}}{\rho_{03}} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{\rho_{04}}{\rho_4} = \left(\frac{T_{04}}{T_4}\right)^{\frac{1}{\gamma-1}} \quad \text{and} \quad \frac{\rho_{03}}{\rho_3} = \left(\frac{T_{03}}{T_3}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{\frac{\rho_{04}}{\rho_4}}{\frac{\rho_{03}}{\rho_3}} = \frac{\left(\frac{T_{04}}{T_4}\right)^{\frac{1}{\gamma-1}}}{\left(\frac{T_{03}}{T_3}\right)^{\frac{1}{\gamma-1}}}$$

$$\frac{\rho_{04}}{\rho_{03}}\frac{\rho_3}{\rho_4} = \left(\frac{T_{04}}{T_{03}}\right)^{\frac{1}{\gamma-1}}\left(\frac{T_3}{T_4}\right)^{\frac{1}{\gamma-1}}, \qquad \frac{T_3}{T_4} = 1$$

$$\rightarrow \frac{\rho_{04}}{\rho_{03}}\frac{\rho_3}{\rho_4} = \left(\frac{T_{04}}{T_{03}}\right)^{\frac{1}{\gamma-1}}, \qquad \frac{\rho_{04}}{\rho_{03}} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{1}{\gamma-1}}$$

$$\rightarrow \frac{\rho_3}{\rho_4} = \left(\frac{T_{01}}{T_{02}}\right)^{\frac{1}{\gamma-1}}\left(\frac{T_{04}}{T_{03}}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{\rho_3}{\rho_4} = \frac{\rho_{01}}{\rho_{02}} \cdot \frac{\rho_{04}}{\rho_{03}} = 1$$

$$\rho_3 = \rho_4$$

$$\dot{m}_3 = \dot{m}_4$$

$$\rho_3 A_3 V_3 = \rho_4 A_4 V_4$$

$$A_3 V_3 = A_4 V_4$$

$$\frac{A_3}{A_4} = \frac{V_4}{V_3} > 1$$

$$\boxed{A_3 > A_4}$$

b. $M_3 = M_4 = 1$

$$V = M\sqrt{\gamma R T}$$

$\rightarrow V_3 = \sqrt{\gamma R T_3}$ and $V_4 = \sqrt{\gamma R T_4}$

$T_{02} = T_{04}$ and $T_{01} = T_{03}$

$M = 1$, $T = T_0 \left(1 + \frac{\gamma - 1}{2}\right)^{-1}$

$T_{04} > T_{03}$ (question a)

$\rightarrow T_4 > T_3 \rightarrow \boxed{V_4 > V_3}$ I

$$\dot{m}_3 = \dot{m}_4$$

$$A_3 \rho_3 V_3 = A_4 \rho_4 V_4$$

$$\frac{A_3}{A_4} = \frac{\rho_4 V_4}{\rho_3 V_3}$$

$$\rho_4 = \rho_{04}\left(1 + \frac{\gamma-1}{2} M_4^2\right)^{-\frac{1}{\gamma-1}}$$

$$\rho_3 = \rho_{03}\left(1 + \frac{\gamma-1}{2} M_4^2\right)^{-\frac{1}{\gamma-1}}$$

From Rayleighs equations we know that:

$\rho_{04} > \rho_{03} \rightarrow \rho_4 > \rho_3$ II

From I and II we can conclude:

$$\boxed{A_3 > A_4}$$

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Problem 12.4

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