Problem 12.3

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a. We know that: • $T_3 = T_4$
• $S_3 = S_4$
• $M_2 > M_1$ (Rayleigh line property)

Because $M_2 > M_1$, from the Rayleigh total temperatures line we know:

$T_{02} > T_{01}$

Equal temperature at 4, 3 $\rightarrow$ $a_4 = a_3$

$\rightarrow$ $V_4 > V_3$

Now we use the isentropic relations:

$$\frac{\rho_{02}}{\rho_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{1}{\gamma-1}} \qquad \rho_{04} = \rho_{02}, \quad \rho_{03} = \rho_{01}$$

$$\rightarrow \frac{\rho_{04}}{\rho_{03}} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{\rho_{04}}{\rho_4} = \left(\frac{T_{04}}{T_4}\right)^{\frac{1}{\gamma-1}} \quad \text{and} \quad \frac{\rho_{03}}{\rho_3} = \left(\frac{T_{03}}{T_3}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{\frac{\rho_{04}}{\rho_4}}{\frac{\rho_{03}}{\rho_3}} = \frac{\left(\frac{T_{04}}{T_4}\right)^{\frac{1}{\gamma-1}}}{\left(\frac{T_{03}}{T_3}\right)^{\frac{1}{\gamma-1}}}$$

$$\frac{\rho_{04}}{\rho_{03}}\frac{\rho_3}{\rho_4} = \left(\frac{T_{04}}{T_{03}}\right)^{\frac{1}{\gamma-1}}\left(\frac{T_3}{T_4}\right)^{\frac{1}{\gamma-1}}, \quad \frac{T_3}{T_4} = 1$$

$$\rightarrow \frac{\rho_{04}}{\rho_{03}}\frac{\rho_3}{\rho_4} = \left(\frac{T_{04}}{T_{03}}\right)^{\frac{1}{\gamma-1}}, \quad \frac{\rho_{04}}{\rho_{03}} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{1}{\gamma-1}}$$

$$\rightarrow \frac{\rho_3}{\rho_4} = \left(\frac{T_{02}}{T_{02}}\right)^{\frac{1}{\gamma-1}}\left(\frac{T_{04}}{T_{03}}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{\rho_3}{\rho_4} = \frac{\rho_{01}}{\rho_{02}} \cdot \frac{\rho_{04}}{\rho_{03}} = 1$$

$$\rho_3 = \rho_4$$

$\dot{m}_3 = \dot{m}_4$

$\rho_3 A_3 V_3 = \rho_4 A_4 V_4$

$A_3 V_3 = A_4 V_4$

$\frac{A_3}{A_4} = \frac{V_4}{V_3} > 1$

$\boxed{A_3 > A_4}$

b. $M_3 = M_4 = 1$

$V = M\sqrt{\gamma R T}$

$\rightarrow V_3 = \sqrt{\gamma R T_3}$ and $V_4 = \sqrt{\gamma R T_4}$

$T_{02} = T_{04}$ and $T_{01} = T_{03}$

$M=1 \Rightarrow T = T_0 \left(1 + \frac{\gamma-1}{2}\right)^{-1}$

$T_{04} > T_{03}$ (question a)

$\rightarrow T_4 > T_3 \rightarrow \boxed{V_4 > V_3}$ (I)

$\dot{m}_3 = \dot{m}_4$

$A_3 \rho_3 V_3 = A_4 \rho_4 V_4$

$\frac{A_3}{A_4} = \frac{\rho_4 V_4}{\rho_3 V_3}$

$\rho_4 = \rho_{04}\left(1 + \frac{\gamma-1}{2} M_4^2\right)^{-\frac{1}{\gamma-1}}$

$\rho_3 = \rho_{03}\left(1 + \frac{\gamma-1}{2} M_4^2\right)^{-\frac{1}{\gamma-1}}$

From Rayleighs equations we know that:

$\rho_{04} > \rho_{03} \rightarrow \rho_4 > \rho_3$ (II)

From (I) and (II) we can conclude:

$\boxed{A_3 > A_4}$