Problem 1.3

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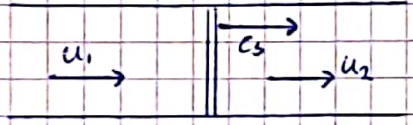
A few things should be noted that hold for problem 1.3.a – 1.3.cs.:

- We're considering a strong shock, so $\frac{\rho_2}{\rho_1} = 6$ and from conservation of mass we can therefor also say that the velocity in pre-shock state 1 and post-shock state 2 (V_i) has the following ratio in shock frame:
  $\frac{V_1}{V_2} = 6$
- The entropy condition must hold to have a valid shock

a.

Laboratory frame:

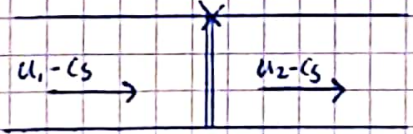


$c_s > 0, \quad u_1 > 0, \quad u_2 > 0$

$V_1 = u_1 - c_s$
$V_2 = u_2 - c_s$

$\frac{u_1 - c_s}{u_2 - c_s} = 6$ } $c_s = 1$, $u_1 = 7$, $u_2 = 2$

Shock frame:



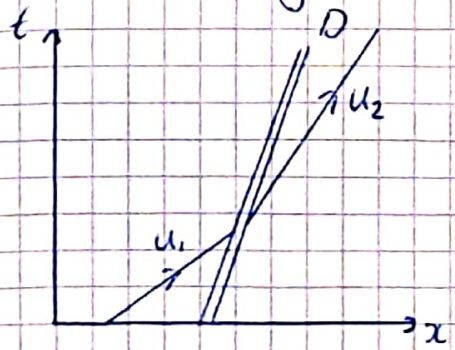
Entropy condition:

$(u_1 - a_1) > c_s > (u_2 - a_2)$

where, $-|u_1| > a_1$
$-|u_2| < a_2$

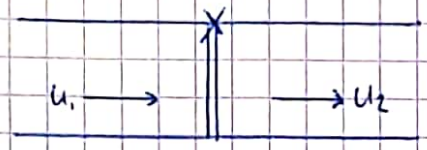
velocity ratios:

t, x - diagram:

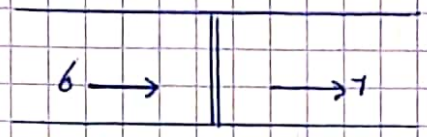


b. Laboratory frame:

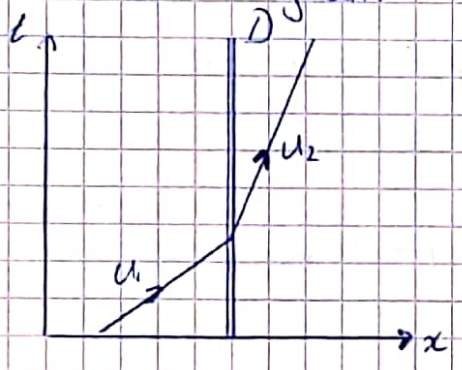
Shock frame:



Velocity ratios:



t, x - diagram:



$c_s = 0$, $u_1 > 0$, $u_2 > 0$

$v_1 = u_1$
$v_2 = u_2$

$\frac{u_1}{u_2} = 6$ } $u_1 = 6$, $u_2 = 1$

Entropy condition:

$|u_1| > a_1$
$|u_2| < a_1$

$(u_1 - a_1) > 0 > (u_2 - a_2)$

c. Laboratory frame:

Shock frame:

Velocity ratios

$c_s \neq 0$, $u_1 > 0$, $u_2 > 0$

$v_1 = u_1 + c_s$
$v_2 = u_2 + c_s$

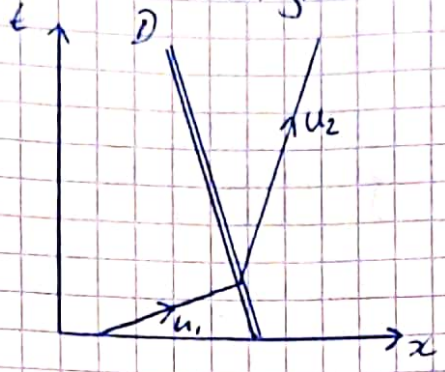
$\frac{u_1 + c_s}{u_2 + c_s} = 6$ } $|c_s| \neq 1$, $u_1 = 11$, $u_2 = 1$

Entropy condition:

$|u_1| > a_1$
$|u_2| < a_2$

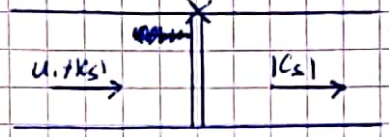
$(a_1 - u_1) < |c_s| < (a_2 - u_2)$

t, x - diagram:



C2. Laboratory frame:

Shock frame:



velocity ratios:

$c_s < 0$, $u_1 > 0$, $u_2 = 0$

$v_1 = u_1 + |c_s|$

$v_2 = |c_s|$

$\frac{u_1 + |c_s|}{|c_s|} = 6$ } $|c_s| = 1$, $u_1 = 5$, $u_2 = 0$

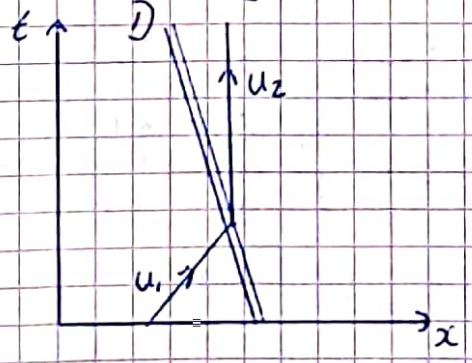
Entropy condition:

$|u_1| > a_1$

$|u_2| < a_2$

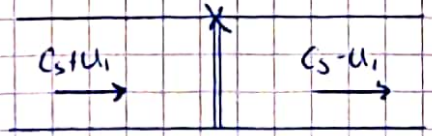
$(a_1 - u_1) < |c_s| < a_2$

t, x - diagram:



C3. Laboratory frame:

Shock frame:



$c_s < 0$, $u_1 > 0$, $u_2 < 0$

$v_1 = c_s + u_1$

$v_2 = c_s - u_1$

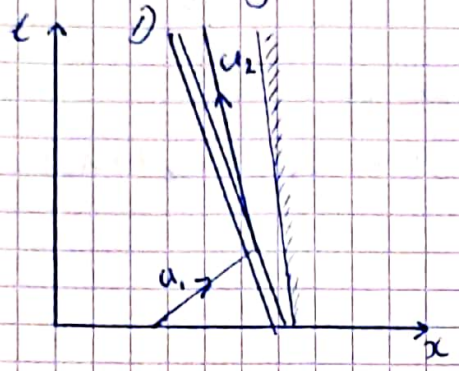
$\frac{c_s + u_1}{c_s - u_1} = 6$ } $c_s = 2$, $u_1 = 4$, $u_2 = 1$

velocity ratios

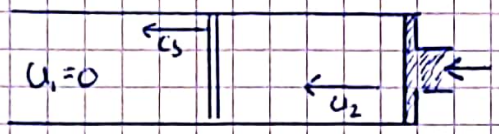
Entropy condition

$a_1 - u_1 < |c_s| < a_2 + |u_2|$

t,x - diagram:



Cs. Laboratory frame:



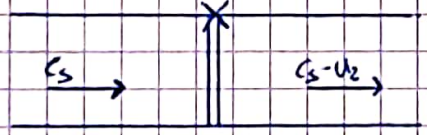
$c_s < 0, \; u_1 = 0, \; u_2 < 0$

$v_1 = c_s$

$v_2 = c_s - u_2$

$\frac{c_s}{c_s - u_2} = 6$ } $c_s = 6$, $u_1 = 0$, $u_2 = 5$

Shock frame:

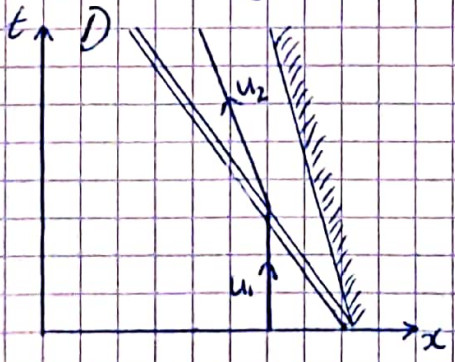


Entropy condition:

$a_1 < |c_s| < a_2 + |u_2|$

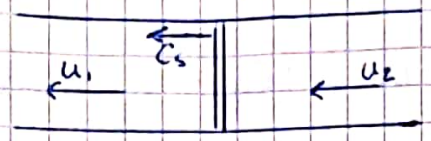
velocity ratios:

t,x - diagram:

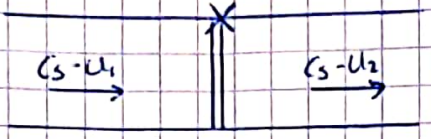


C5.

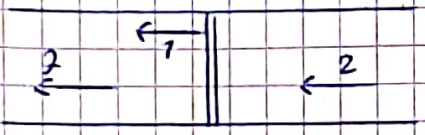
Laboratory frame



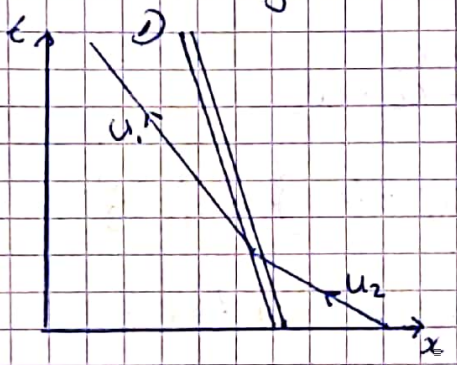
Shock frame:



velocity ratios



t,x - diagram



$c_s < 0$, $u_1 < 0$, $u_2 < 0$

$v_1 = c_s - u_1$

$v_2 = c_s - u_2$

$\frac{c_s - u_1}{c_s - u_2} = 6$ } $c_s = 1$, $u_1 = 2$, $u_2 = 2$

Entropy condition:

$a_1 + |u_1| < |c_s| < a_2 + |u_2|$