Math 2552 Differential Equations

Space Colonization Population Dynamics

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Introduction

Our paper seeks to explore the limits of oxygen content and the population dynamics of humans and plants. The nature of population dynamics are well documented, however these are under a set of very specific circumstances. We seek to explore the limitations of oxygen content in hypothetical spatial settlements. Take for example the theorized O'Neill Cylinder. This structure would not receive outside oxygen and would need to be vigilant of the oxygen contents and coincidentally the plant population. As the population grows, more people would be born on average due to the growing population. This creates a positive feedback loop and would result in exponential growth. Eventually a limiting of resources would slow the growth and potentially cause an overshoot. After an arbitrarily long time the population would approach a constant deemed the carrying capacity, "K". This period of exponential growth followed by a gradual decrease in growth and arrival at carrying capacity is called logistic growth. Classical logistic growth is typically in the form $dN/dt=(a-bN)N$ (Vandermeer). This equation produces the S-curve indicative of logistic growth.

The part of the problem that is undocumented is the intricate dance between the carrying capacity, K, and the population, N. As plant population, P, increases, not only does K increase but so does dP/dt meaning that the carrying capacity is steadily changing with plant growth. Solving these equations concurrently would allow developers of future space settlements and be able to monitor how many people can be sustained based on the initial value of plants brought on board.

Analysis

In our analysis, we are making the assumption that the humans populating the O'Neil Cylinder or other types of space settlements will follow logistical growth. The logistical growth model (Svirin) for humans has the rate of change per a year of the human population proportional to the carrying capacity K and a relative growth factor a. The relative growth factor is the growth of the population without factoring the carrying capacity.

$$\frac{dN}{dt} = aN - \frac{aN^2}{K} \quad (1)$$

Since we decided to incorporate oxygen production as the main factor of the carrying capacity, K, we have created another equation for the change in carrying capacity over time as a function of the number of plants in the system, P. The variable o is the oxygen generated per plant in a year, and L is the amount of oxygen needed per person in a year.

$$K = \frac{oP}{L} \quad (2)$$

We can then incorporate this small equation into our original dN/dt formula to have an equation for the growth rate of the human population dependent on the number of plants in the system.

$$\frac{dN}{dt} = aN - \frac{aN^2}{\left(\frac{oP}{L}\right)} = aN - \frac{aN^2 L}{oP} \quad (3)$$

However, the growth rate of plants in relation with time, dP/dt, has not been factored in yet, so we have created a similar equation for the growth of the plant population using the logistical growth model. The variable b is the relative growth factor for plants, and M is the maximum capacity of plants, which we will independently set as a fixed constant.

$$\frac{dP}{dt} = bP - \frac{bP^2}{M} \quad (4)$$

By using both equations (3) and (4), we have created a non-linear system to represent the rate of growth of a theoretical space colony based on the growth rate of the plants that supply oxygen within said system. Through Euler's Method, we have approximate values for the population of humans, growth rate of humans, number of oxygen-producing plants, and the growth rate of plants at set time intervals of years.

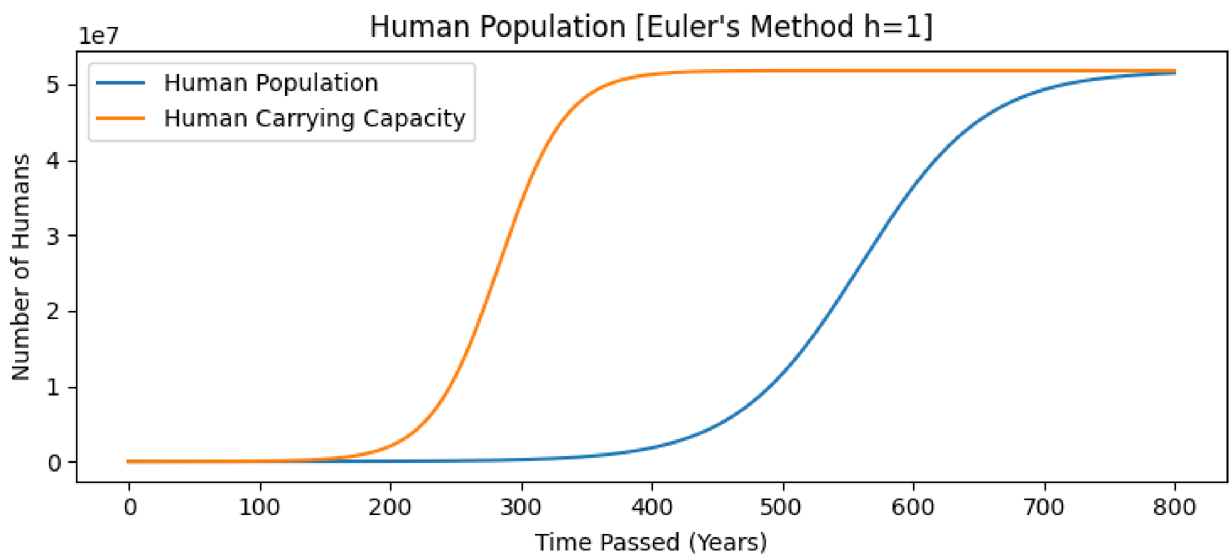
In our simulation, we are planning to set a = 0.021, based on the highest population growth rate humans have hit globally on Earth, which occurred in 1962 (Roser). Additionally, we are going to model our data around usage of the Douglas-fir tree, which have a growth rate of b = 0.0392 (qualityforest.com) and will have a maximum capacity of M = 383317040 trees. We got this value by multiplying the number of trees per acre, as found by the USDA (fs.usda.gov) is 2960 trees/ha, and the available area of the O 'Neill Cylinder, 500 miles squared (space.nss.org). We set the annual amount of oxygen needed per person, L = 740 kg oxygen / year (The Conscious Club), and the the oxygen production per tree, o = 100 kg oxygen / year (Villazon).

$$\frac{dN}{dt} = 0.021N - \frac{0.021N^2L}{oP} \tag{5}$$

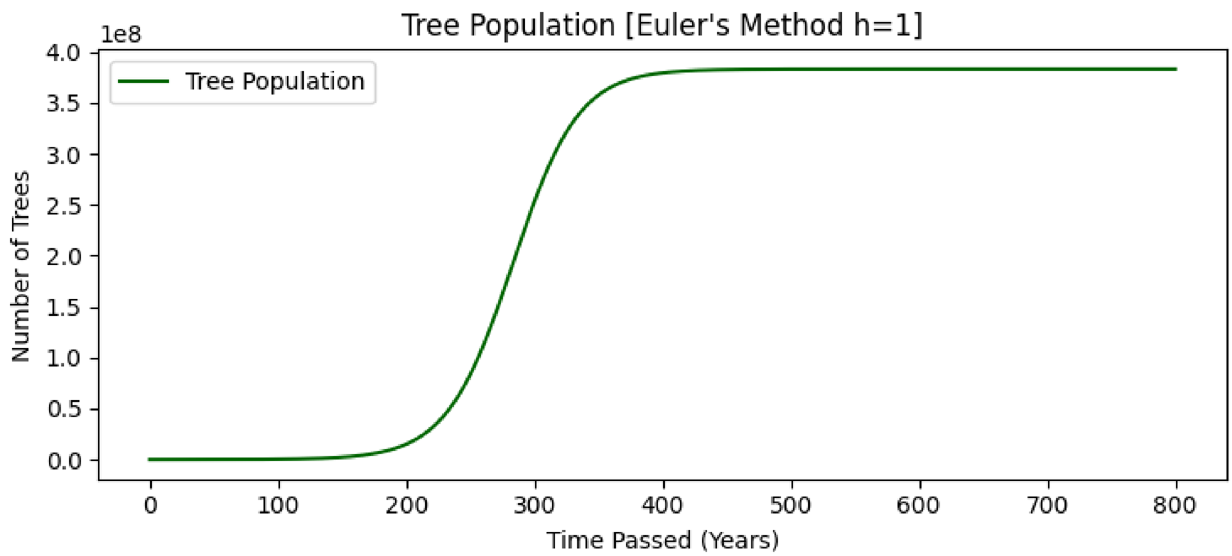
$$\frac{dP}{dt} = 0.0392P - \frac{0.0392P^2}{383317040} \tag{6}$$

Using the given constants and equations we found, we inserted them into our custom Euler's method code, Appendix 2, in order to find the population of humans, N, and number of plants, P, at certain time intervals. We started with initial values of the human population and the tree population at N = 1000 humans (Salotti) and P = 10000 trees. The size of the steps, h = 1 year, reflects the relatively small growth rate of humans that we have selected, leading to only changes, albeit small, year by year. This effect of the growth rate leads to a higher step size, as major fluctuations can be only found over larger time intervals, and we felt it not necessary to have a smaller h due to this.

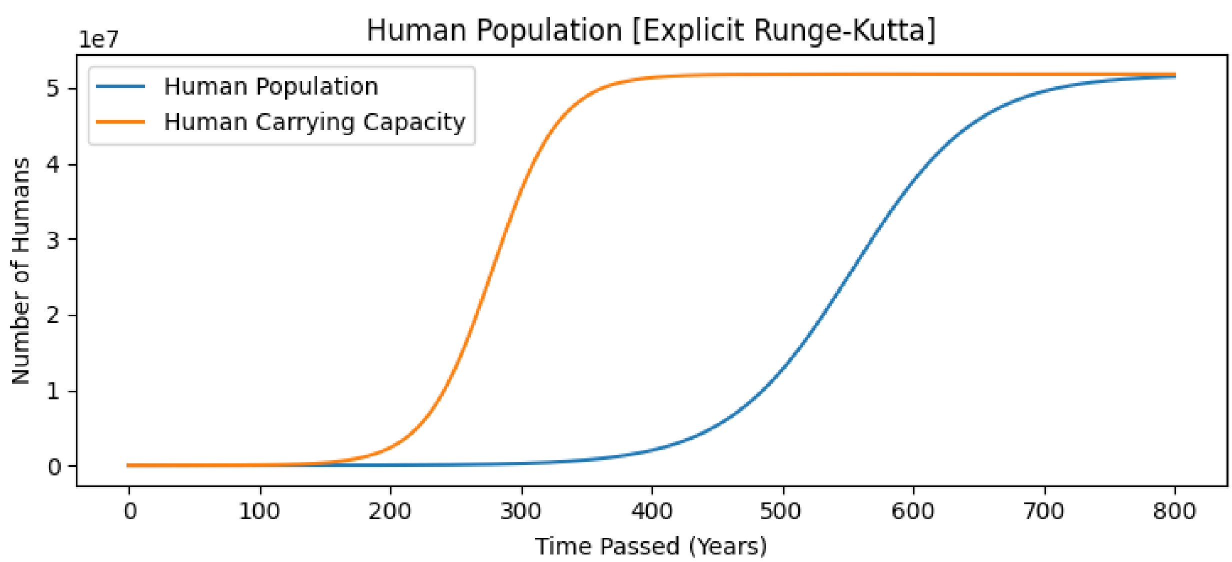
	Time	N	dN/dt	P	dP/dt
0	100	4.016003e+03	75.361466	3.391755e+05	1.272089e+04
1	200	2.953694e+04	598.693364	1.498580e+07	5.420794e+05
2	300	2.319435e+05	4739.150158	2.529885e+08	3.395854e+06
3	400	1.794424e+06	35667.079021	3.795955e+08	1.494439e+05
4	500	1.158039e+07	186629.733845	3.832467e+08	2.855241e+03
5	600	3.632075e+07	229840.833665	3.833157e+08	5.347462e+01
6	700	4.926877e+07	51525.944522	3.833170e+08	1.001128e+00
7	800	5.148262e+07	6755.955175	3.833170e+08	1.874253e-02

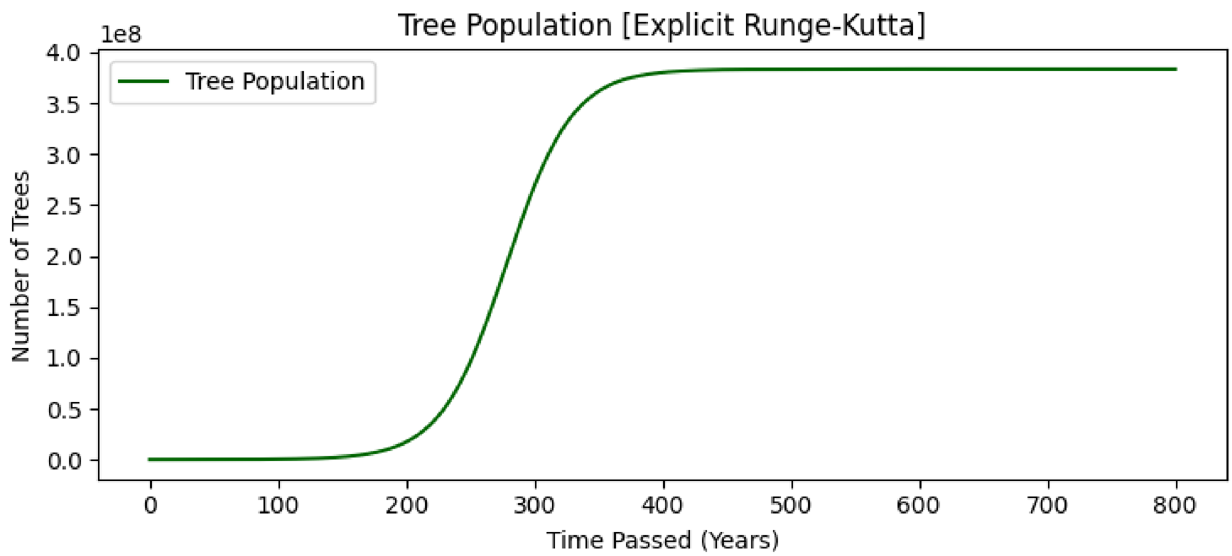


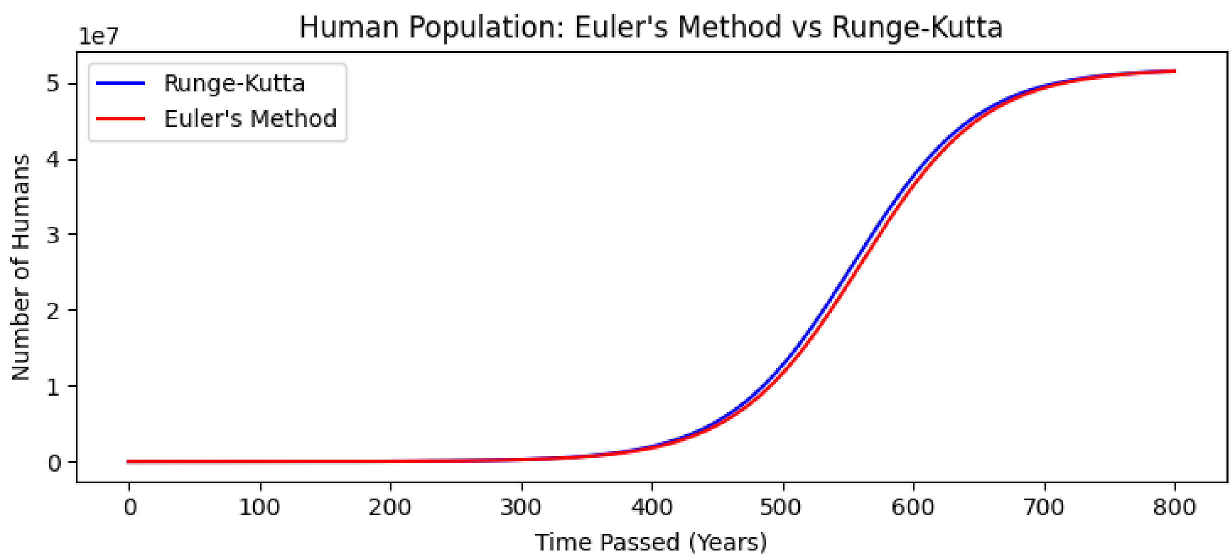
Based on this simulation, it shows the growth rate in the first few hundred years is relatively slow but, once it passes the 400 year mark, it starts accelerating eventually reaching a carrying capacity of around 50 million, which far exceeds the theoretical maximum of objects like the O'Neil cylinder based on population density. This indicates that the area in an O'Neill cylinder, based off oxygenation data, can support up to 50 million people sustainably. We theorize that the growth rate of the human population is hindered by the relatively low carrying capacity due to the starting number of trees. The people have to wait until the number of trees can support a significantly larger population, which after a certain period (around 150-200 years) will kickstart exponential growth. It can also be observed that, as the human population reaches the carrying capacity, its growth rate seems to decrease, as expected.



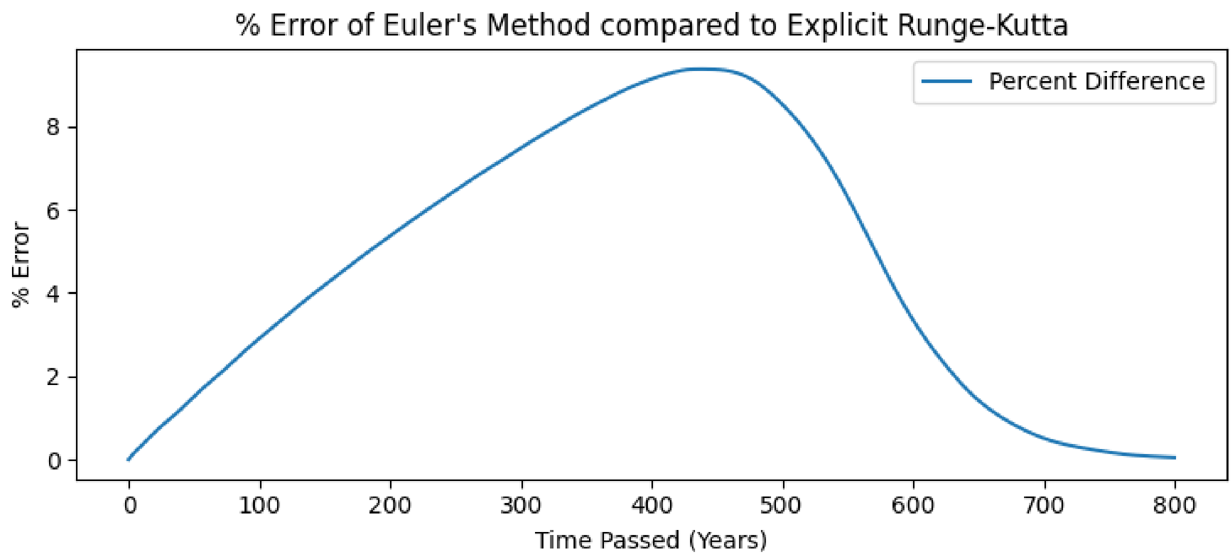
The tree population reaches its carrying capacity based on surface area much quicker than the human population.







Comparing the two graphs, the Runge-Kutta deviates the most during the period of highest growth rate, but both return to similar values as the human population reaches the carrying capacity.



Comparing these two methods, the error climbs linearly during the growth phase of the human population, but as the growth rate starts to slow and the population eventually reaches carrying capacity, the error starts returning back to zero.

Conclusion

This process required finding numerical solutions to a nonlinear system of differential equations. MATH-2552 has let us learn how to set up systems to analyze such problems like the various methods of modeling population dynamics.Through the class we learned a variety of methods to solve these systems. We learned that such systems are quite tedious and often infeasible to complete manually. Methods like Euler, improved Euler, or Runge-Kutta are required to attain practical answers.

For a moderate space population of about 1000, our results show that when using the most optimal trees the population will increase much slower than the carrying capacity thus oxygen will never be of much concern. However, this conclusion is based on many assumptions. Provided we have the technology to transfer an initial tree population this large would be a major concern. We also do not have a way to take into account latent growth rates in space since we don't have enough experimental data on space colonies. Interestingly, our data has found that in the first few hundred years, the human population growth is limited to around a 100 people a year, assuming they utilize sustainable oxygen sources, indicating the very early stages of space colonization must be supplemented with oxygen to counter very slow and inefficient growth rates. Another factor we did not take into account was the changing rate at which each tree will produce oxygen, in which we set as a constant rate. The oxygen production rate would depend on the age of the tree, and would thus be complicated to incorporate, as all trees will end up having different ages that have to be taken into account. However, it can be estimated that if considered, the changing oxygen production rates will lead to a slower population growth than observed.

The growth rate of humans, as observed, seems to be small, especially with the initial rates and conditions. To supplement the oxygen production from the trees in order to aid in the initial growth of the population, we have considered methods like water electrolysis, which is the system currently used by the ISS to produce oxygen. However, the process itself is considerably inefficient, so it may actually inhibit resources which can go into other systems to help grow the colony.

Appendices

Appendix 1: Code for Custom Euler Method

```
import numpy as np

def euler_method(f, S_0, t0, tf, h):

    n = int((tf - t0) / h) + 1
    t = np.linspace(t0, tf, n)
    y = np.zeros((n, len(S_0)))
    y[0] = S_0

    for i in range(1, n):
        y[i] = np.add(y[i-1], np.multiply( f(t[i-1], y[i-1]), h)
)

    return t, y
```

Appendix 2: Code for Generating and Plotting Figures

```
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp
import numpy as np

# oxygen(kg) a person consumes per year
L = 740
# oxygen(kg) per douglas fir tree produces per year
o = 100
# human population relative growth factor
a = 0.0021
# plant population relative growth factor
b = 0.0039
# surface area of space colony in km^2
```

```
surface_area = 1294.99
# number of douglas-fir trees per hecta acre
tree_density = 2960
# hecta acre to km^2
ha_to_km2 = 0.01

# max number of plants
max_plants = tree_density * (1/ha_to_km2) * surface_area

print(max_plants)

def population_dynamics(t, S):
    n, p = S
    return [a*n - ((a * (n**2)) / ( (o/L)*p) ),
            b*p - (b * (p**2)) / max_plants]

n_0 = 1000
p_0 = 1000*L/o
S_0 = [n_0, p_0]
tf = 3000

t = np.linspace(0, tf, tf+1)

eulerSol = euler_method(population_dynamics, S_0, 0, tf, 1)
n_list = []
p_list = []
for data in eulerSol[1]:
    n_list.append(data[0])
    p_list.append(data[1])

print(n_list)
rungeKuttaSol = solve_ivp(population_dynamics, t_span=(0,
max(t)), y0=S_0, t_eval=t).y
print(len(rungeKuttaSol[0]))
```

```
#Plotting method changes based off which figure we are graphing
#plt.rcParams["figure.figsize"] = [7.50, 3.50]
#plt.rcParams["figure.autolayout"] = True
#plt.plot(t, rungeKuttaSol[0], label = "Number of Humans")
#plt.plot(t, rungeKuttaSol[1], label = "Tree Population")
#plt.legend()
#plt.ylabel('Number of Trees')
#plt.xlabel('Time Passed (Years)')
#plt.title("Tree Population [Explicit Runge-Kutta]")
plt.show()
```

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