OpenMC Module: Monte Carlo Theory

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Solving the Neutron Transport Equation

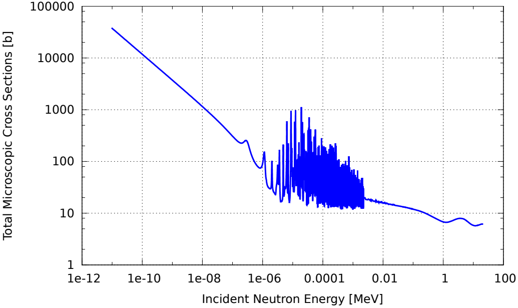
- Assume $\partial\psi/\partial t = 0$ and scale fission term

$$\begin{aligned}\mathbf{\Omega}\cdot\nabla\psi_k + \Sigma_t(\mathbf{r},E)\psi_k(\mathbf{r},\mathbf{\Omega},E) = \\ \iint dE'\,d\Omega'\,\Sigma_s(\mathbf{r},\mathbf{\Omega}\cdot\mathbf{\Omega}',E'\to E)\psi_k(\mathbf{r},\mathbf{\Omega}',E') \\ + \frac{1}{k}\frac{\chi(E)}{4\pi}\int dE'\,\nu\Sigma_f(\mathbf{r},E')\phi_k(\mathbf{r},E')\end{aligned}$$

- Eigenvalue problem for k and ψ_k
  - where $\phi_k = \int d\Omega\,\psi_k(\mathbf{\Omega})$
- Monte Carlo method invovles simulating individual neutrons
- Random numbers are sampled from probability distributions that represent physics

Cross Section Data Needed

- Microscopic cross sections and other physics laws are input into Monte Carlo codes
- Data contained in ACE-formatted files
- These files are commonly generated with NJOY proecessing code
- Currently can be obtained via MCNP/Serpent Software or directly from NEA (JEFF)



Basic Monte Carlo Algorithm – Eigenvalue

Guess initial source distribution and k

for $i = 1 \rightarrow n_{batches}$ **do**

    for $j = 1 \rightarrow n_{particles}$ **do**

        Sample neutron from source distribution

        while Neutron is alive **do**

            Sample distance to collision

            Determine isotope in collision

            Sample physics

            Bank fission sites

        end while

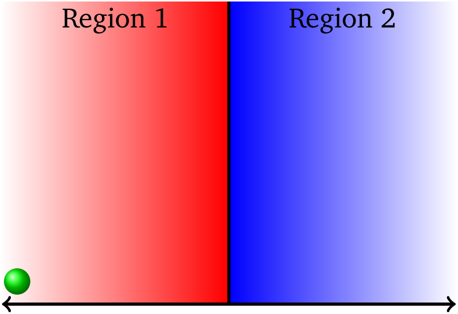
    end for

    Sample neutrons from fission sites collected

    Calculate k

end for

Distance to Neutron Collision



- Neutron is born or exits collision
- Neutrons are sampled from:

$$p(x)dx = \Sigma_t e^{-\Sigma_t x} dx$$

- If a surface is crossed, neutron is resampled from this surface
- If a surface is not crossed, a neutron has collided

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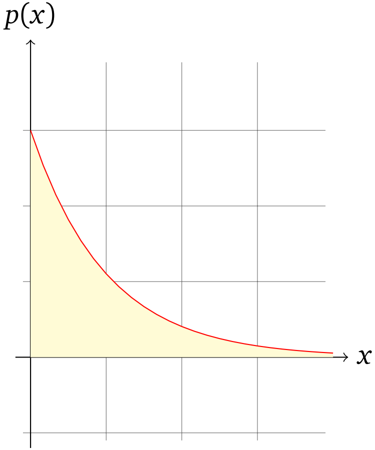

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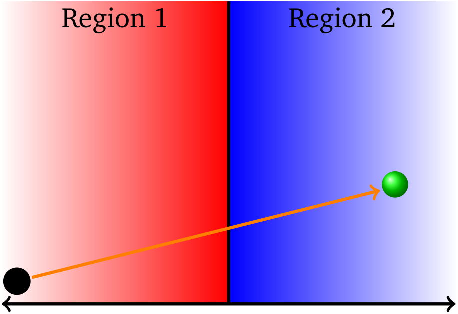

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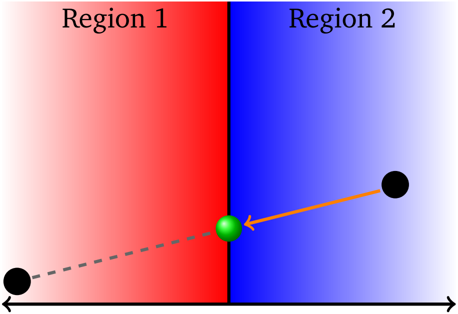

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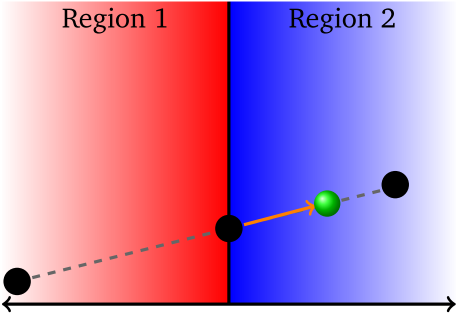

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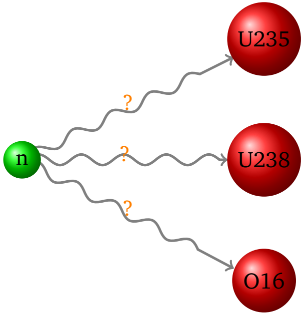
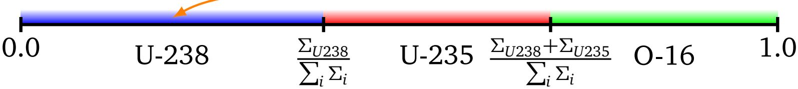
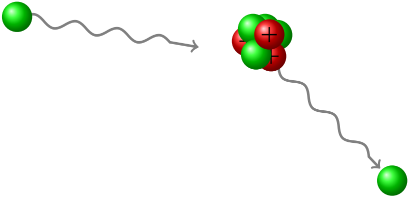
Choosing the isotope involved in collision



- Select random number (0,1), ξ



Sampling the Interaction Physics



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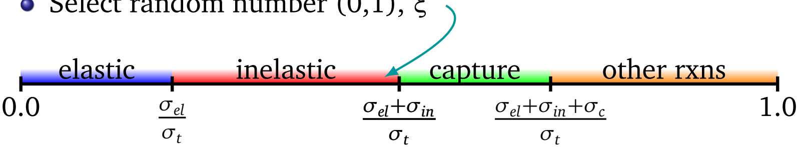

- Physics is performed by sampling from ACE data

Implicit Fission Model – Eigenvalue Calculations

- Fission is not explicity sampled, like other reactions
- At every collision, fission neutrons are implicitly sampled

$$n = \left\lfloor w \frac{\nu \Sigma_f}{\Sigma_t} + \xi \right\rfloor \quad \text{fission sites}$$

- Sites are banked for next batch of neutrons

```
type Bank
  real(8) :: wgt ! weight of bank site
  real(8) :: xyz(3) ! location of bank particle
  real(8) :: uvw(3) ! diretional cosines
  real(8) :: E ! energy
end type Bank
```

- Note: w is the statistical neutron weight

Estimating Effective Multiplication Factor

- Analog

$$k_{ana} = \sum_{i \in \text{bank}} \frac{w_i}{W}$$

- Collision

$$k_{col} = \sum_{i} \frac{w_i \nu \Sigma_f}{W \Sigma_t}$$

- Absorption

$$k_{abs} = \sum_{i} \frac{w_i \nu \Sigma_f}{W \Sigma_a}$$

- Track-length

$$k_{track} = \sum_{i} \frac{w_i \nu \Sigma_f d}{W}$$

Tallying Physics

- Method to extract quantities out of Monte Carlo run (e.g., flux, reaction rates, current, etc.)
- Tallies are made using different estimators (e.g., analog, collision, track-length)
- When an event occurs, the following formula is used for tallies:

$$\text{tally} = \sum_{i \in \text{events}} \frac{R_i w_i \phi_i}{W}$$

- w_i: neutron statistical weight, where W is total starting weight
- R_i: response function (flux=1, reaction rates=Σ)
- ϕ: estimator (analog=1, collision=Σ_t^{-1}, track=d)

Eigenvalue Calculations – Converging Fission Source

- Monte Carlo eigenvalue problem solved with Power Iteration
- Source must be guessed and iterated until converged
- May take many iterations (inactive batches) to converge depending on dominance ratio