# Analysis of the Hermeus Quarterhorse Hypersonic Aircraft

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The Mk III Quarterhorse, designed and planned to be fabricated by the American aerospace and defense corporation Hermeus, is designed to be a hypersonic unmanned aircraft capable of sustained high-speed flight to test hypersonic technologies and methods. The planned construction date is yet to be announced at the date of writing; however, the corporation has of now tested avionics and controls on their Mk.0 airframe.

#### **Nomenclature**

aerodynamic efficiency Ma Mach number Ttemperature P pressure Vvelocity k specific heat ratio heat capacity qmass flow rate  $\dot{m}$ dynamic pressure  $q_{pressure}$ local air density ρ oncoming air velocity и drag coefficient  $c_D$ lift coefficient  $c_L$ Aarea Reynolds number Re oncoming flow length of travel

x

dynamic viscosity μ

#### I. Introduction

ermeus is an aerospace defense and technology startup working to achieve controlled hypersonic flight. Controlled hypersonic flight is a major goal of aerospace research and design. Aircraft traveling at Mach 5 below 90 kilometers chemically dissociate the air around them. This dissociation introduces several additional design constraints to the aircraft. The airframe experiences extreme temperatures and pressures from the shockwaves generated by moving at such a speed. The biggest challenge for controlled hypersonic flight is efficient propulsion and material selection. After analysis of a proposed aircraft design from Hermeus, it is concluded that controlled hypersonic flight is possible.

## II. Background

Hermeus was founded in 2018 with the goal to break the manned flight speed record set by the Lockheed SR-71 Blackbird. Hermeus are attempting to break this record with a turbofan-ramjet engine they call the Chimera. The Chimera engine has completed ground testing, but it hasn't yet powered a real aircraft. Hermeus are currently testing the Quarterhorse Mk 0, a subsonic testbed. We are analyzing their final design, the Quarterhorse Mk III, based on promotional images.

## III. Computer Aided Design Modelling

Our first goal was to create a model of the Quarterhorse Mk III. We used the Computer Aided Design (CAD) software Catia V5 for this task. It was difficult to gather accurate information about the Quarterhorse MK3 since it hasn't been built yet. Many of the finer details are also classified and couldn't be used for our report.

Since information is limited, we used the following method. First, we vector traced promotional images of side, top, and front profiles of the aircraft<sup>4</sup>. Second, these images were then imported to Catia V5 to be modelled via cross sectioning and ribbing. Finally, the resulting generated solid was scaled to a total length of 40 feet based on public information. An isometric view of the model is presented below, with more in the appendix.



Figure 1: A sample image of the CAD model generated of the Hermeus' Mk III Quarterhorse hypersonic testbed.

We measured the following dimensions from the model using Catia. Table 1 shows these dimensions.

Table 1: Aircraft Dimensions

14000 11110 0 4/1 2 0 0000				
Geometry	Extent	Unit		
Overall Length	12.192	m		
Span	5.843	m		
Max Chord	4.828	m		
Mean Chord	2.414	m		
Air Intake Area	0.471	$m^2$		
Exhaust Area	0.386	$m^2$		
Total Wing Planform Area	13.604	$m^2$		
Fuselage Wetted Area	35.382	$m^2$		
Control Surface Wetted Area	5.948	$m^2$		
Total Vehicle Wetted Area	54.934	$m^2$		
Maximum Airframe Cross Section	1.493	$m^2$		
Total Vehicle Weight (estimated)	3428.676	kg		

## IV. Aerodynamic Efficiency

Analyzing the drag on an Outer Mold Line (OML) of an aircraft is very complicated. Typically, the best approach is using a wind tunnel and physical model or using Computational Fluid Dynamics (CFD) to simulate a wind tunnel. We can't use either method, so we are forced to estimate the drag by analyzing the main components of the OML.

Total drag can be approximated as the sum effect of the following contributions:

- I. Form drag drag induced by the maximum cross-sectional area of the vehicle normal to flow.
- II. Skin friction drag drag induced by skin roughness interacting with fluid flow boundary layer.
- III. Induced drag drag generated by the pressure differential created by wing lift generation.
- IV. Wave drag drag generated by shockwaves inducing flow separation.

Aerodynamic efficiency,  $\eta$ , is then presented as the ratio of the lift generated by the wings to the induced and wave drag by the wings alone.

#### A. Form Drag

Form drag can be calculated as the force resulting from dynamic pressure,  $q_{pressure}$ , applied across the maximum form cross section<sup>1</sup>. The following equation (equation 5 in appendix) is a restatement of Newton's third law F = ma, where  $\rho$  denotes air density, u denotes oncoming free stream air velocity,  $c_d$  denotes drag coefficient, and A denotes maximum vehicle form cross sectional area:

(1) 
$$F_{d,form} = \frac{1}{2} \rho u^2 c_d A$$

The coefficient of drag used here is calculated in section **C. Induced and Wave drag**. The value of form drag for our target cruise regime (discussed in sections V and VI) is included in the table at the end of this section, along with an example calculation in the appendix.

#### B. Skin Friction Drag

Skin friction drag comes from laminar and turbulent<sup>1</sup> flows interact to create a boundary layer of flowing air. This boundary layer drags against the surface of the airframe. The rougher the surface, the worse the drag. To determine the approximate skin friction drag on our airframe at the target cruise regime, we use the Prandtl one-seventh power solution for mostly turbulent flow. The Prandtl solution assumes the Prandtl boundary layer<sup>1</sup>, which itself assumes an isobaric boundary layer that is thin relative to dimension of the length of travel<sup>1</sup>. In this equation Re denotes the Reynolds number of flow, x represents the length of travel parallel with oncoming flow over a given surface, and  $\mu$  represents the dynamic viscosity of the free stream air:

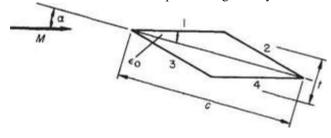
(2) 
$$Re_x = \frac{\rho u_0 x}{\mu}$$
  
(3)  $c_{d,skin} = \frac{0.031}{Re_x^{1/7}}$ 

(4) 
$$F_{d,skin} = \frac{1}{2}\rho u^2 c_{d,skin} A_{wetted}$$

The wetted area used can be found in section II *Table 1: Aircraft dimensions*. A sample calculation can be found in the appendix and values are detailed in the table at the end of this section.

#### C. Induced and Wave Drag

Induced drag and wave drag are caused by pressure differentials from shockwaves and Prantl-Meyer expansion on the wings<sup>1</sup>. For our analysis, we assume the wings use a double wedge airfoil. This assumption is supported by Hermeus' promotional material<sup>4</sup>. Pictured below is an example of this geometry at some angle of attack  $\alpha$ :



#### Figure 2: Double-wedge supersonic airfoil at some angle of attack

We use Busemann's theory of coefficients of pressure to determine lift. The following formulas are used to solve the pressure contributions of the shock waves and expansion fans generated at supersonic flight. Here  $M_{\infty}$  denotes the incoming Mach number,  $C_p$  denotes the coefficient of pressure on a surface,  $\theta$  denotes the angle of deflection of the flow,  $\delta$  denotes the half wedge angle, and k denotes the specific heat capacity of the local air.

(5) 
$$C_p = \frac{2\theta}{\sqrt{M_{\infty}^2 - 1}} + \left[ \frac{(k+1)M_{\infty}^4 - 4M_{\infty}^2 + 4}{2(M_{\infty}^2 - 1)^2} \right] \theta^2$$

From inspection of the CAD model, our angles used for the four surfaces as labeled in figure 2 are -1.812°, -8.188°, 8.188°, 1.812°, for surfaces 1, 2, 3, and 4 respectively at an angle of attack of 5 degrees.

These coefficients of pressure are then used to find the overall coefficient of lift and drag from the wings:

(6) 
$$C_L = \frac{\sum \pm c_{p,i} \cos \theta_i}{2 \cos \delta}$$
(7) 
$$C_D = \frac{\sum c_{p,i} \sin \theta_i}{2 \cos \delta}$$

The resulting coefficients of pressure, lift, and drag are detailed in the table at the end of this section with a calculation sample in the appendix.

#### D. Aerodynamic Efficiency

The aerodynamic efficiency of the wings is found by taking the ratio of the lift force over the drag force generated by the wings, which is directly equal to the ratio of the coefficient of lift over the coefficient of drag as all terms besides these two cancel out in the ratioing.

(8) 
$$\eta = \frac{F_L}{F_D} = \frac{C_L}{C_D}$$

The aerodynamic efficiency of our vehicle at cruising flight regime and an angle of attack of 5 degrees is 8.10, which is substantial for a supersonic airfoil of this form.

## E. Aerodynamic Results

We made the following plot to show how the aerodynamic efficiency changes with angle of attack. This plot was calculated using our geometry, a cruise regime of Mach 5.0, and an altitude of 30km. This was done by iterating Busemann Theory calculations for the double wedge airfoil at all possible angles of attack on the given range. It is assumed that an angle of attack greater than 30 degrees is impossible.

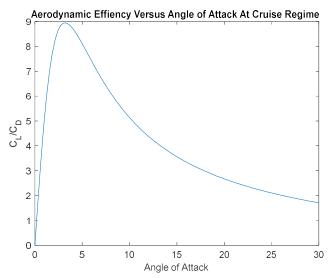


Figure 3: Aerodynamic efficiency with respect to angle of attack

The following table presents the numerical data gathered from the aerodynamic analysis of the Quarterhorse.

Table 2: Drag force.

	Extent	Unit
Form Drag	173	N
Fuselage Skin Friction Drag	3025	N
Wings Skin Friction Drag	1350	N
Control Surface Skin Friction Drag	623	N
Total Skin Drag	4999	N
Induced Wave Drag	203	N
Total Drag	5202	N
η for α of 5°	8.105	Unitless

#### V. Ramjet Propulsion Analysis

Ramjets are a simple form of hypersonic engine. A ramjet is made of a cone with an inlet that blocks the flow of air. This air is compressed as it enters the engine until the air flow strikes a surface. Once the air strikes a surface at a sufficient angle, a normal shockwave occurs. This shock wave suddenly drops the speed below Mach 1, and massively increases the pressure. A fuel injection and heat addition phase after the shock wave heats the air up to increase its speed, and the heated air exits the engine. The exit is quite small, and the flow of air accelerates as it is forced through the exit. This heated air exits at supersonic speeds and provides thrust to the plane.

Scramjets are more complex than ramjets. Scramjets use reflected shockwave to increase pressure instead of normal shockwaves. Reflected shock waves don't allow the flow of air to drop below supersonic speeds. This supersonic air is much harder to heat up with the fuel injection since the high speed flow tends to extinguish any flame that would burn the fuel. Scramjets have a major advantage over ramjets, despite the difficulties with burning the fuel. Scramjet engines can operate at much higher speeds than ramjets. Ramjets don't tend to produce net thrust above Mach 5<sup>5</sup>. Both ramjets and scramjets share a flaw. Neither jet will produce thrust when travelling much slower than Mach 3.

## A. Turbofan Implementation

The NASA X-43 was an experimental plane designed to test scramjets. The X-43 needed to boost to high Mach numbers using a rocket engine before the scramjet was deployed and tested<sup>3</sup>. Hermeus plans to remove the rocket boost by using a turbo jet at the earlier stages of flight. Hermeus acquired a Pratt and Whitney F100 engine to use

for the Quarterhorse Mk III. This engine was mainly used on the F-14 and F-16 jets, enabling a top speed of approximately Mach 2 for both planes. Turbofans like the F100 can't be used to achieve speeds above Mach 3 because the fan blades that power them can't spin fast enough to properly compress the air and achieve enough thrust<sup>5</sup>. Hermeus plans to use the F100 engine to accelerate to speeds where the ramjet can begin to function, and then use the geometry of the F100 as the inlet to their ramjet. Analysis of the F100 was outside the scope of this report, so we are willing to assume that they can reach speeds where the ramjet will begin working.

#### **B.** Assumptions

The exact ramjet geometry is unknown so we will need to make reasonable assumptions about jet. The ramjet equations begin with an oblique shock. All oblique shock equations, Equations (13-16) of the appendix of require an angle  $\delta$  which we can get from the plane's geometry. Any internal angles will need to be estimated, but we can assume that the designers will construct the ram so that a normal shockwave happens very quickly. We need to estimate how much the temperature will change during the fuel injection stage. Equation (17) of the appendix requires a fuel ratio F and a heat value q for our engine. Both F and q are standardized for jet engines. Our engine areas were calculated from the model geometry and are included in *Table 1: Plane Geometry* 

Table	3.	Ram	iot P	ropert	ios
Iuvie	J.	Num	iei I	roveru	<i>ues</i>

$\delta_1$	16°	°Degrees
$\delta_2$	36°	°Degrees
F	0.03	Unitless
q	44,000	kj/kg
F <sub>T</sub>	11627	N

#### C. Results

We found that the ram jet produces enough thrust to keep the Quarterhorse flying at Mach 5. At the service ceiling, we find that the engine is producing 11.63 kN of thrust. This is more than enough to overcome the 5kN of drag on the Quarterhorse travelling at Mach 5.

# VI. Speed Limitations

We can find a theoretical top speed by finding where the drag force is equivalent to the theoretical thrust of 11.627 kN. This ideal speed ignores factors like structural limitations. The Mach number where this force balance is achieved at Ma 8.455 which is 2585.1 m/s at 30km altitude. While this velocity has been achieved for short durations by in-atmosphere vehicles<sup>5</sup>, it is unlikely this airframe would ever survive this velocity before succumbing to aeroforces, thermal loads, and other structural impacts of this extreme flight regime.

## VII. Flight Trajectory Simulation

Our team needed several calculators to test Hermeus' claims about the Quarterhorse Mk3. We used these calculators to create flight simulations. The first calculator to be created was the ramjet analysis calculator. The second calculator was the drag calculator. Both MATLAB calculators are used to develop our simulators.

# A. Service Ceiling Calculator

The service ceiling simulator's purpose is to estimate the maximum service ceiling of the Hermeus Mk III. We defined the maximum service ceiling as the moment when the rate of climb slows to 100ft/min or lower, about 0.5 m/s. We set this condition to stop our simulator. This simulator is built using Euler's method of iteration and summation, instead of using integration for a more accurate plot. To set up the simulator we imported the previous drag and thrust calculators and iterated every 0.1 second for 1000 iterations. We also set up some initial values the table below.

Table 4: Initial Properties

$V_1$	1715	m/s
Ma∞	5	Unitless
Patm	101325	Pa
Tatm	288.15	K
θ	10	deg

Our sim has 4 set stopping conditions. The first is the rate of climb, mentioned earlier. When the rate of climb has decreased to below about 0.5 m/s the simulation will stop. The second stopping condition is a height check. If the rate of climb condition isn't met, the sim compares last marked height and the current height. If the current height is lower than the last, the simulation will stop. The third condition stops the simulation if the force of thrust becomes equal to the force of drag. The final condition is the completion of the iteration cycle. This final condition takes the longest to reach due to how long it takes for the calculator to iterate once. The result of the service ceiling simulator was relative to what we assumed, being off by only about 6km at roughly 24km, shown in figure 3. With this, we moved on to the flight trajectory.

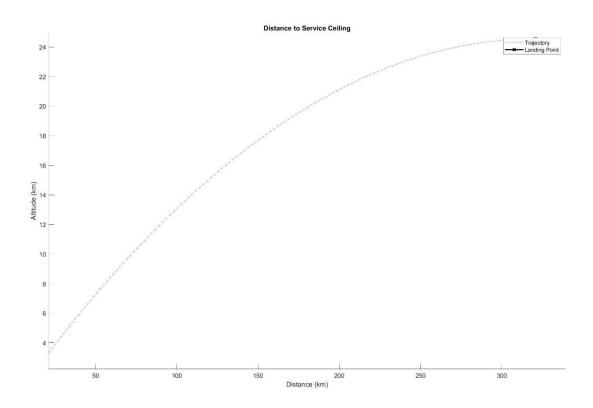


Figure 3: A plot of the Quarterhorse's trajectory from the ground to its service ceiling. This plot assumes that the plane began its flight at Mach 5, so the distance travelled isn't accurate. The max height was about 24 km in this simulation.

To estimate the flight trajectory of the Hermeus Mk III, we turned to the X-43 to aid us in creating some specifications for our simulator. The X-43 was reported to fly hypersonic at its max service ceiling for at least 10 seconds before it slowed to a cruise for 14 minutes and landed in the ocean. During the X-43's 10 seconds of hypersonic flight it travelled an incredible distance of 24km. Our trajectory, while not being an exact replication of that flight path should roughly follow that part of the flight path. Since both the service ceiling and flight trajectory

calculators were developed in tandem, we assumed a service ceiling of 30km and a top speed of Mach 5. We then merged our two calculators. We had to assume a value for lift so that our simulator could produce a reasonable result. The simulator did indeed produce a reasonable trajectory, flying quickly to 20 km, slowing down, and starting to climb down, since it was no longer maintain speeds to stay at the maximum service ceiling, and then it eventually crashed after about 160km down the stretch. The trajectory is shown in Figure 4. For at least the sustained portion of the graph, the trajectory matches what we've found for the sustained hypersonic flight of the X-43. The flight trajectory we simulate is reasonable, further proving the Hermeus MK III likely capable of its goals.

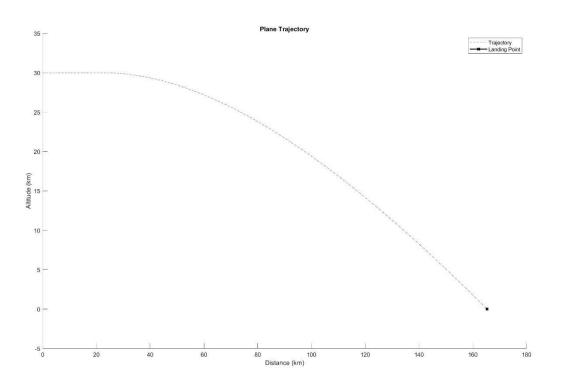


Figure 4: A plot of the Quarterhorse trajectory after achieving Mach 5 at a ceiling of 30km.

#### VIII. Conclusion

Hermeus' claim of achieving Mach 5.5 flight seems reasonable given the geometry we have analyzed. The drag forces aren't excessive, and the ramjet is operating within historical standards. Their claims of achieving this flight after using a turbojet engine and swapping it to a ramjet are outside the scope of this project, though ground tests show that the system works<sup>4</sup>. Manned flight won't significantly change the way this plane flies, though it's currently unknown how a pilot would respond to hypersonic flight. If Hermeus can get their ramjet to fire, it will break the SR-71's manned flight record of Mach 3.3. The maximum service ceiling we were able to find was around 24km, traveling at Mach 5. Based on the data we have from the x-43 and similar planes; the service ceiling is a reasonable estimate and a feasible achievement for the Mk III.

# Acknowledgments

A special thank you to Dr. Post, Haakon Evers, and August Schneider for their effort, input, critiques, and helpfulness in this project.

## References

- <sup>1</sup>Anderson, J. D., Fundamentals of aerodynamics, New York, NY: McGraw-Hill Education, 2017.
- <sup>2</sup>Gerhold, C., Brown, M., and Jasinski, C., "Evaluation of skin friction drag for liner applications in ...," Evaluation of Skin Friction Drag for Liner Applications in Aircraft Available: https://ntrs.nasa.gov/api/citations/20160007691/downloads/20160007691.pdf.
- <sup>3</sup>Dinius, D., "X-43A hyper-X," *X-43A Hyper-X* Available: https://www.nasa.gov/reference/x-43a/.
- <sup>4</sup>Hermeus, "Quarterhorse Mk 0 begins testing," *Hermeus* Available: https://www.hermeus.com/blog-quarterhorse-mk-ready-for-test.
- <sup>5</sup>Anderson, J., "Hypersonic flight," *Homepage* Available: https://airandspace.si.edu/stories/editorial/hypersonic-flight.

## **Appendix**

#### **Aerodynamics Section Sample Calculations**

Form Drag sample calculation:

$$F_{d,form} = \frac{1}{2}\rho u^2 c_d A = 0.5 * \frac{0.0104 kg}{m^3} * \left(\frac{1682.6m}{s}\right)^2 * 0.0079 * 1.493 m^2 = 173.06 N$$

Skin Drag sample calculation:

$$Re_{x} = \frac{\rho u_{0}x}{\mu} = \frac{\frac{0.0104kg}{m^{3}} * \frac{1682.6m}{s} * 13.38072m}{1.52306E - 5 Pa * s} = 1.5305905E7$$

$$c_{d,skin} = \frac{0.031}{Re_{x}^{1/7}} = 0.002917$$

$$F_{d,skin} = \frac{1}{2} \rho u^2 c_{d,skin} A_{wetted} = 0.5 * \frac{0.0104 kg}{m^3} * \left(\frac{1682.6m}{s}\right)^2 * 0.002917 * 54.934 m^2 = 3025.55 \, N$$

Induced & Wave Drag sample calculation:

For angle of attack of 5 degrees, theta values are given in Aerodynamics section, Mach number of 5, k of 1.4, and delta of 3.19 degrees:

$$C_{p} = \frac{2\theta}{\sqrt{M_{\infty}^{2} - 1}} + \left[ \frac{(k+1)M_{\infty}^{4} - 4M_{\infty}^{2} + 4}{2(M_{\infty}^{2} - 1)^{2}} \right] \theta^{2} = [-0.0105, -0.0280, 0.0777, 0.0129]$$

$$C_{L} = \frac{\sum \pm C_{p,i} \cos \theta_{i}}{2 \cos \delta}$$

$$= \frac{0.0105 \cos(-1.812^{\circ}) + 0.0280 \cos(-8.188^{\circ}) + 0.0777 \cos(8.188^{\circ}) + 0.0129 \cos(1.812^{\circ})}{2 \cos(3.19^{\circ})}$$

$$= 0.06409$$

$$C_{D} = \frac{\sum C_{p,i} \sin \theta_{i}}{2 \cos \delta}$$

$$= \frac{-0.0105 \sin(-1.812^{\circ}) - 0.0280 \sin(-8.188^{\circ}) + 0.0777 \sin(8.188^{\circ}) + 0.0129 \sin(1.812^{\circ})}{2 \cos(3.19^{\circ})}$$

$$= 0.00791$$

$$\eta = \frac{F_{L}}{F_{D}} = \frac{C_{L}}{C_{D}} = \frac{0.06409}{0.00791} = 8.105$$

# **Equations Used**

$$(1) Re_x = \frac{\rho u_0 x}{u}$$

(1) 
$$Re_x = \frac{\rho u_0 x}{\mu}$$
  
(2)  $c_{d,skin} = \frac{0.031}{Re_x^{1/7}}$ 

(3) 
$$F_{d,skin} = \frac{1}{2}\rho u^2 c_{d,skin} A_{wetted}$$

$$(4) \quad F_{d,form} = \frac{1}{2} \rho u^2 c_d A$$

(4) 
$$F_{d,form} = \frac{1}{2}\rho u^2 c_d A$$
  
(5)  $C_p = \frac{2\theta}{\sqrt{M_{\infty}^2 - 1}} + \left[ \frac{(k+1)M_{\infty}^4 - 4M_{\infty}^2 + 4}{2(M_{\infty}^2 - 1)^2} \right] \theta^2$ 

(6) 
$$C_{L} = \frac{\sum \pm c_{p,i} \cos \theta_{i}}{2 \cos \delta}$$
(7) 
$$C_{D} = \frac{\sum c_{p,i} \sin i}{2 \cos \delta}$$
(8) 
$$\eta = \frac{F_{L}}{F_{D}} = \frac{C_{L}}{C_{D}}$$

$$(7) \quad C_D = \frac{\sum C_{p,i} \operatorname{si}^{i}}{2 \operatorname{cos}^{\delta}}$$

(8) 
$$\eta = \frac{F_L}{F_D} = \frac{C_L}{C_D}$$

(9) 
$$Ma_2 = \sqrt{\frac{Ma_1^2 + \frac{2}{k-1}}{\frac{2k}{k-1}Ma_1^2 - 1}}$$

$$(10)T_2 = T_1 \frac{1 + \frac{k-1}{2} M a_1^2}{1 + \frac{k-1}{2} M a_2^2}$$

$$(11)P_2 = P_1 \frac{\frac{1+kMa_1^2}{1+kMa_2^2}}{\frac{1+kMa_2^2}{1+kMa_2^2}}$$

$$(12)F_t = \dot{m}(V_6 - \bar{V}_1) + A_{inlet}(P_6 - P_1)$$

$$(11)P_2 = P_1 \frac{1 + kMa_1^2}{1 + kMa_2^2}$$

$$(12)F_t = \dot{m}(V_6 - V_1) + A_{inlet}(P_6 - P_1)$$

$$(13)\tan(\delta) = \frac{\left(\frac{2}{tan\theta}\right)(Ma_1^2sin^2\theta - 1)}{Ma_1^2(k + cos2\theta) + 2}$$

$$(14) Ma_2^2 sin^2(\theta - \delta) = \frac{\frac{Ma_1^2 sin^2\theta + \frac{2}{k-1}}{\frac{2k}{k-1} Ma^2 sin^2\theta - 1}}{\frac{2k}{k-1} Ma^2 sin^2\theta - 1}$$

$$(15)P_2 = P_1 \left( \frac{2k}{k+1} M a_1^2 \sin^2 \theta - \frac{k-1}{k+1} \right)$$

$$(16)T_2 = T_1 \left( \frac{\left[ \frac{7Ma_1^2 \sin^2 \theta - 1}{36(Ma_1^2 \sin^2 \theta)} \right] for \ k = 1.4 \right]}{36(Ma_1^2 \sin^2 \theta)}$$

$$(15)P_{2} = P_{1} \left( \frac{2k}{k+1} M a_{1}^{2} sin^{2} \theta - \frac{k-1}{k+1} \right)$$

$$(16)T_{2} = T_{1} \left( \frac{[7Ma_{1}^{2} sin^{2} \theta - 1][Ma_{1}^{2} sin^{2} \theta + 5]}{36(Ma_{1}^{2} sin^{2} \theta)} \right) for k = 1.4$$

$$(17)T_{2} + \frac{T_{2}^{2} \left( \frac{Ma_{1} \sqrt{kRT_{1}}}{T_{1}} \right)^{2}}{2 \cdot 1005} = T_{1} + \frac{(Ma_{1} \cdot \sqrt{kRT_{1}})^{2}}{2 \cdot 1005} + \frac{q \cdot f}{1.005}$$

$$(18) \frac{A_{2}}{A_{1}} = \frac{Ma_{1}}{Ma_{2}} \cdot \left( \frac{1 + 0.2Ma_{2}^{2}}{1 + 0.2Ma_{1}^{2}} \right)^{3}$$

$$(18) \frac{A_2}{A_1} = \frac{Ma_1}{Ma_2} \cdot \left(\frac{1 + 0.2Ma2^2}{1 + 0.2Ma1^2}\right)^3$$

# **Additional CAD Model Images**



Staged View



Side Profile



Front Profile



Isometric Profile



Aft profile

## MATLAB Code used for Max Speed Calculation and Aerodynamics Section Results

```
format long g
 M_{inf} = 8.45;
 gamma = 1.4;
t = 0.1027176;
 area = 3.401;
 chord = sqrt(area);
 AOA = deg2rad(5); %linspace(0,pi/6,100); %Use for polar generation
 delta = atan((t/2)/(chord/2))
  delta =
         0.0556407591743525
 theta = [delta-AOA; -(delta+AOA); delta+AOA; -(delta-AOA)]
  theta = 4 \times 1
        -0.031625703425364
        -0.142907221774069
         0.142907221774069
         0.031625703425364
 C_p = ((2.*theta)./(sqrt(M_inf^2-1)))+(((gamma+1)*M_inf^4-
4*M inf^2+4)/(2*(M inf^2-1)^2)).*theta.^2
  C p = 4 \times 1
      -0.00633220046203531
      -0.00943562170295895
       0.0586915135113428
       0.00874449438067509
 eta = zeros(1,100);
for i=1:1:1 %1:1:100 for polar generation
     C_L = (-C_p(1,i)*cos(theta(1,i)) - C_p(2,i)*cos(theta(2,i)) +
C_p(3,i)*cos(theta(3,i)) + C_p(4,i)*cos(theta(4,i))) * (1/(2*cos(delta)));
     C_D_{ift} = (C_p(1,i)*sin(theta(1,i)) + C_p(2,i)*sin(theta(2,i)) +
C_p(3,i)*sin(theta(3,i)) + C_p(4,i)*sin(theta(4,i))) * (1/(2*cos(delta)));
     eta(i) = C L./C D lift;
 end
 %%plot(rad2deg(AOA),eta)
 %%title("Aerodynamic Effiency Versus Angle of Attack At Cruise Regime")
 %%xlabel("Angle of Attack")
 %%ylabel("C_L/C_D")
 mu = 0.0000152304;
```

```
rho = 0.0103541;
U inf = M inf * sqrt(1.4*287*232.928)
 U_inf =
            2585.0712877474
 L_{wing} = 4.722266;
Re_wing = (rho*U_inf*L_wing)/mu
 Re_wing =
           8298966.59318721
C_D_skinFrictionWing = 0.031/(Re_wing^(1/7))
 C_D_skinFrictionWing =
        0.00318368206952626
 F_D_skinFrictionWing = 2*C_D_skinFrictionWing*0.5*rho*(4*area)*U_inf^2
 F_D_skinFrictionWing =
           2996.77210754187
 L_fuselage = 13.38072;
area_fuselage = 18.382+17;
Re_fuselage = (rho*U_inf*L_fuselage)/mu
 Re_fuselage =
           23515436.926423
C_D_skinFrictionFuselage = 0.031/(Re_fuselage^(1/7))
 C D skinFrictionFuselage =
        0.00274353975549818
F_D_skinFrictionFuselage =
2*C_D_skinFrictionFuselage*0.5*rho*(area_fuselage)*U_inf^2
 F D skinFrictionFuselage =
           6716.6240319151
 L_cs = 3.219602;
area_cs = 5.948;
Re_cs = (rho*U_inf*L_cs)/mu
```

```
Re_cs =
```

5658166.95657524

# $C_D_skinFrictionCS = 0.031/(Re_cs^(1/7))$

C\_D\_skinFrictionCS =
 0.00336274339541505

# F\_D\_skinFrictionCS = C\_D\_skinFrictionCS\*0.5\*rho\*(area\_cs)\*U\_inf^2

F\_D\_skinFrictionCS = 1383.9553897057

# formdrag = 0.5\*rho\*U\_inf^2\*C\_D\_lift\*1.493

formdrag =

263.30294024541

# inducedDrag = 0.5\*rho\*(U\_inf^2)\*C\_D\_lift\*5.843\*0.3

# F\_D\_skinFrictionFuselage + F\_D\_skinFrictionWing + F\_D\_skinFrictionCS

ans =

11097.3515291627

# F\_D\_total = F\_D\_skinFrictionFuselage + F\_D\_skinFrictionWing + F\_D\_skinFrictionCS + formdrag + inducedDrag

F\_D\_total =

11669.7929315355