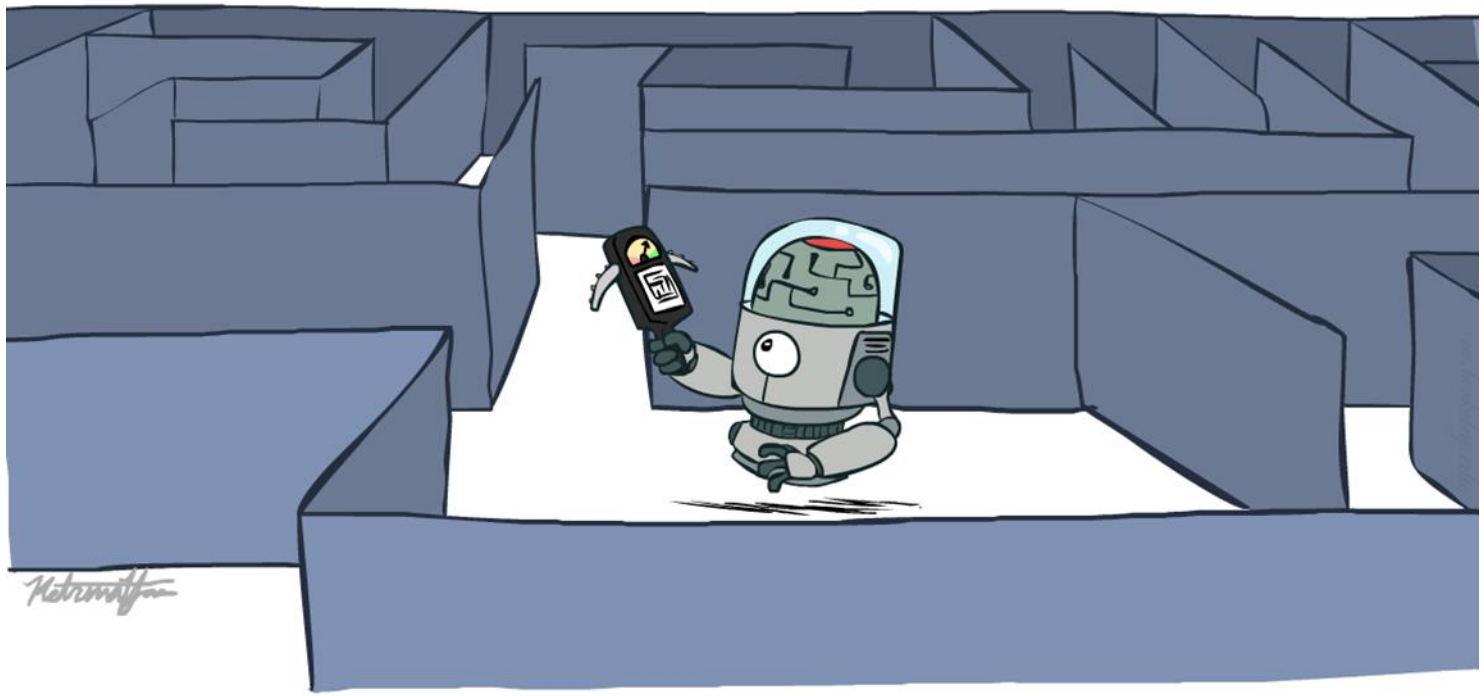
Informed Search

Heuristics Search



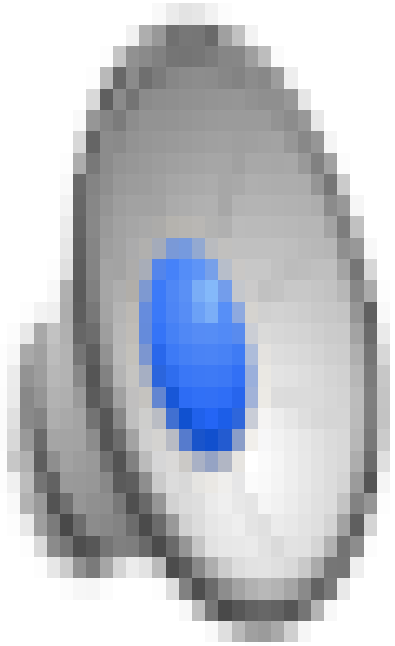
Recap: Search

- Search problem:
  - States (configurations of the world)
  - Actions and costs
  - Successor function (world dynamics)
  - Start state and goal test

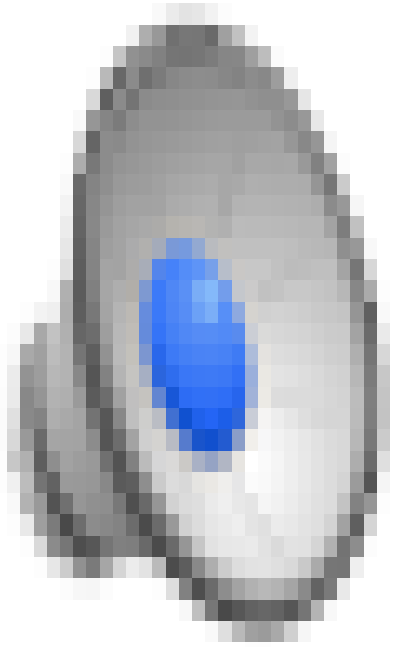
- Search tree:
  - Nodes: represent plans for reaching states
  - Plans have costs (sum of action costs)

- Search algorithm:
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)
  - Optimal: finds least-cost plans

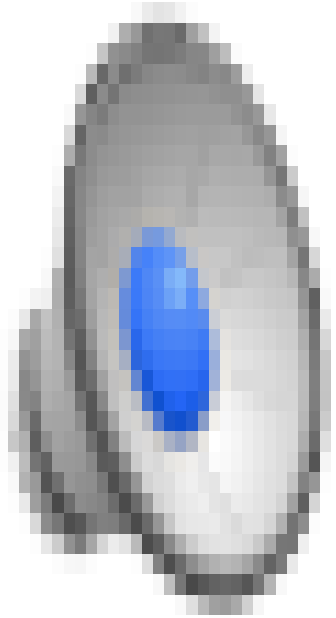
Video of Demo Maze Water DFS/BFS (part 1)



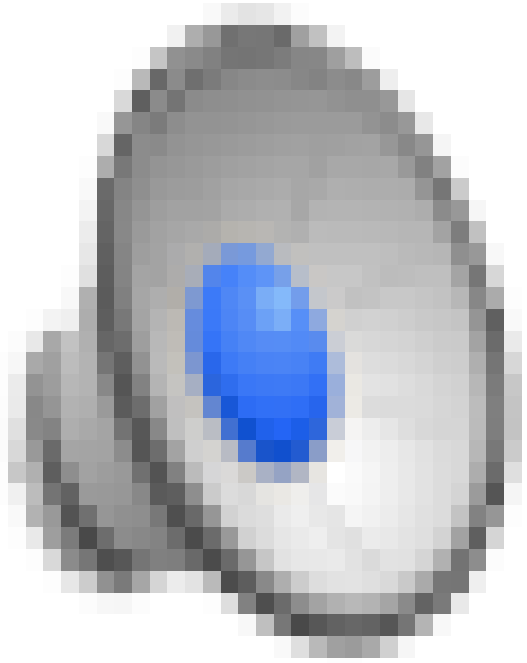
Video of Demo Maze Water DFS/BFS (part 2)



Video of Demo Contours UCS Empty



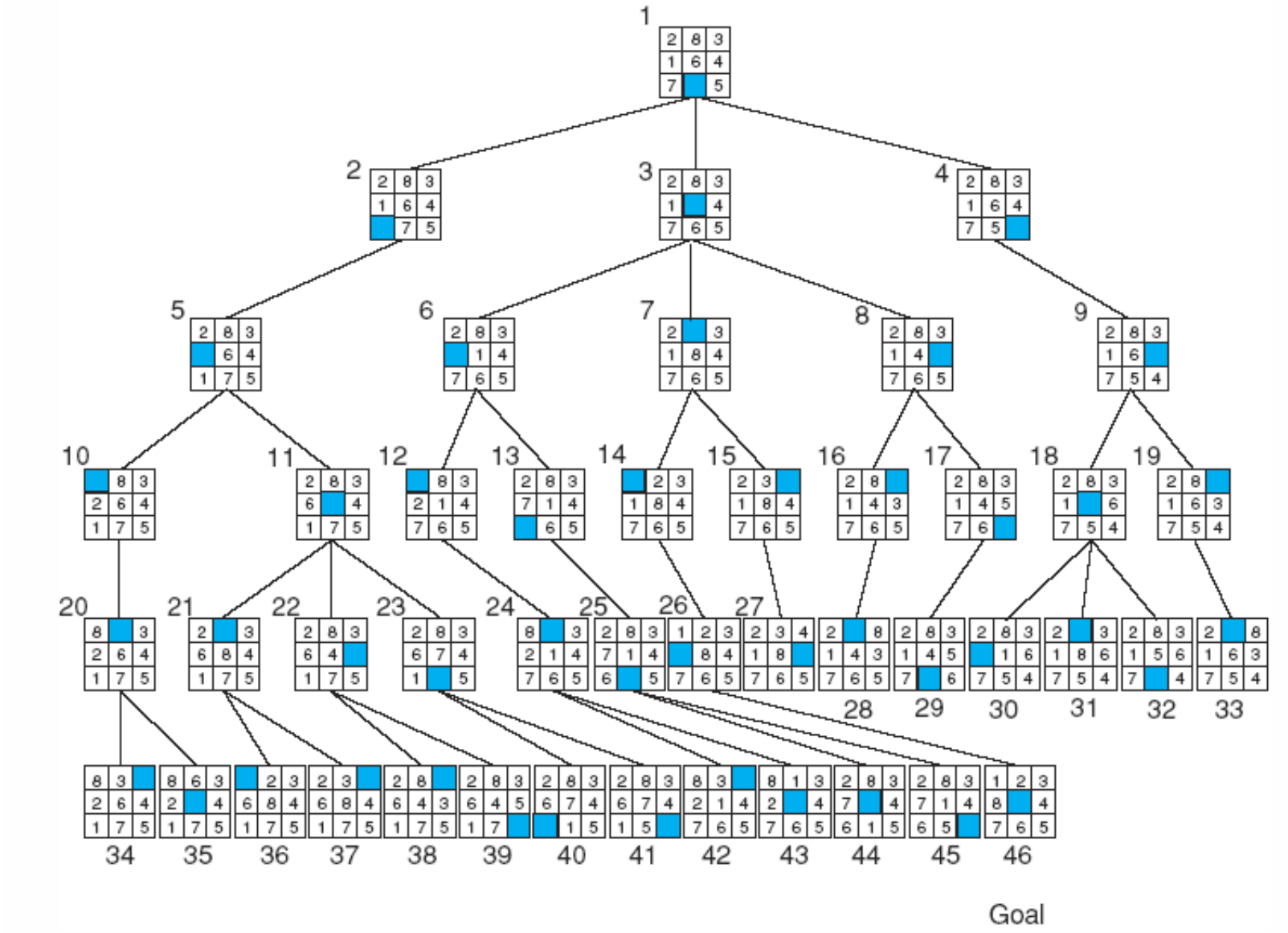
Video of Demo Contours UCS Pacman Small Maze



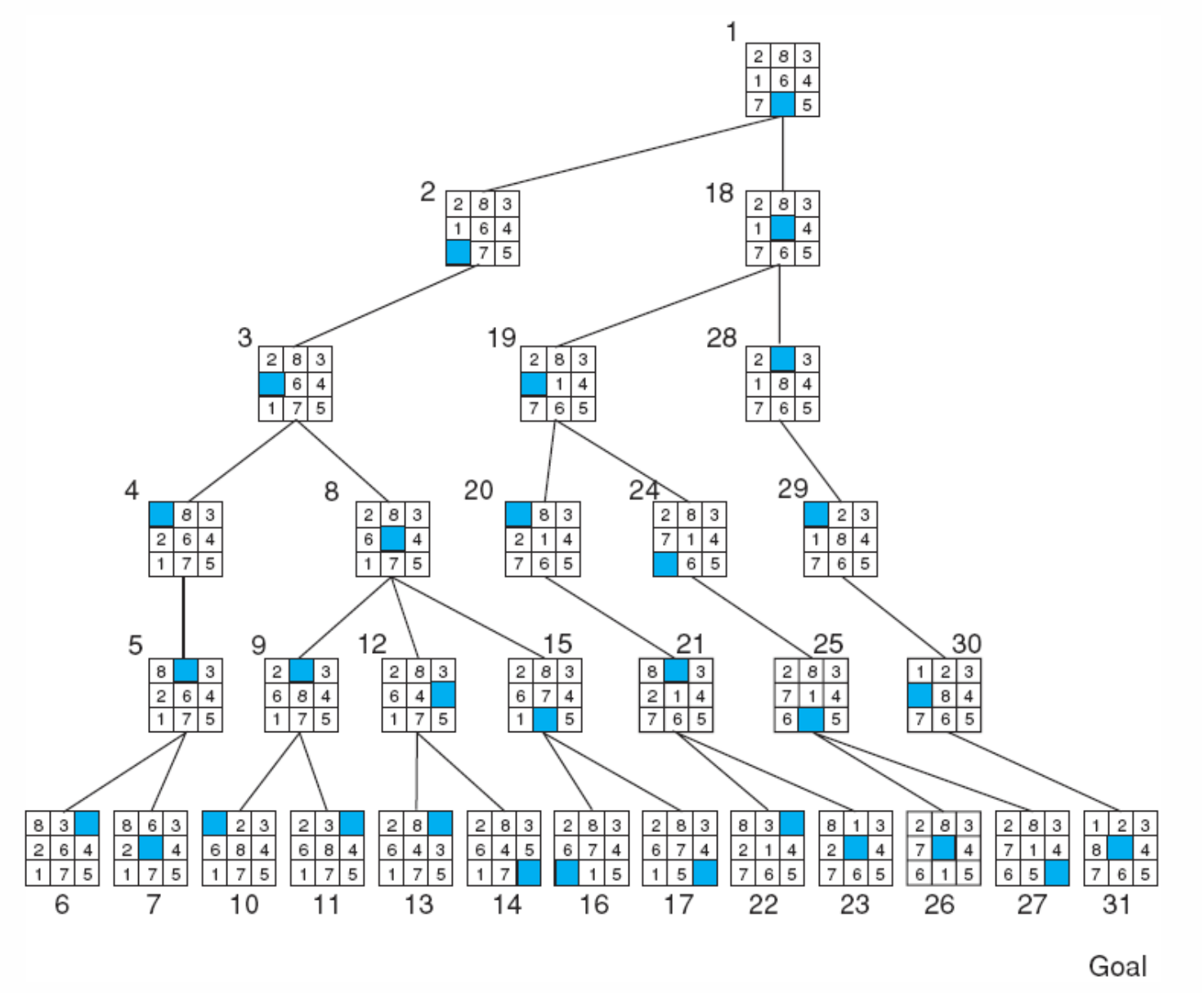
Today

- Informed Search
  - Heuristics
  - Greedy Search
  - A* Search

Breadth-first search of the 8-puzzle, showing order in which states were removed from open.



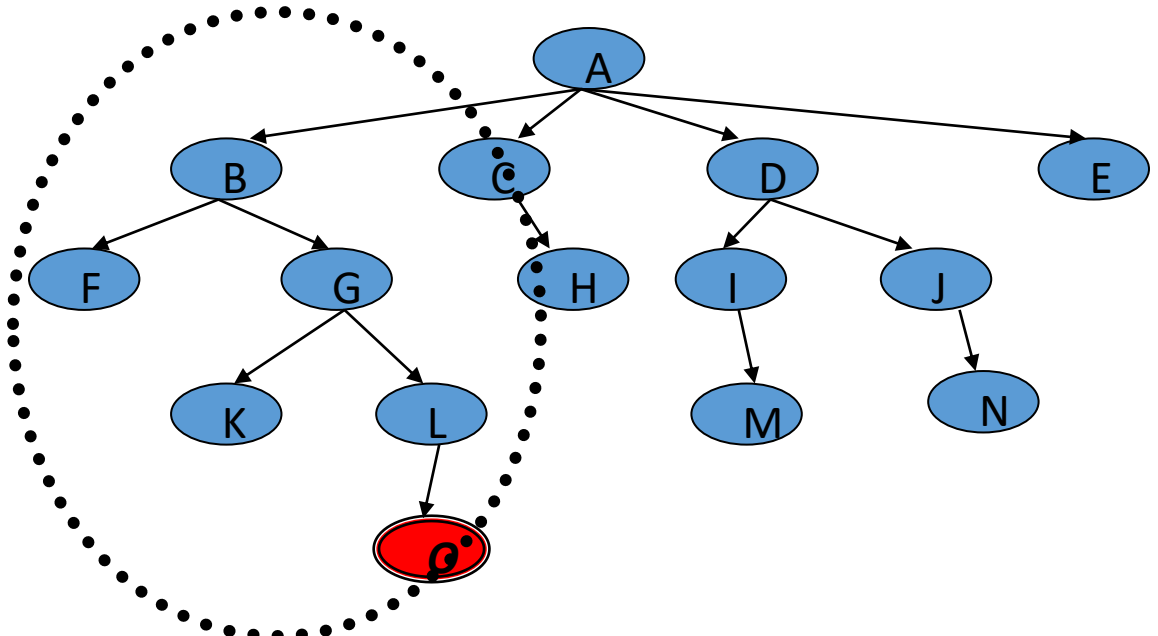
Depth-first search of the 8-puzzle with a depth bound of 5.



Using problem specific knowledge to aid searching

- With knowledge, one can search the state space as if he was given "hints" when exploring a maze.
  - Heuristic information in search = Hints
- Leads to dramatic speed up in efficiency.

Search only in this subtree!!



What are Heuristics?

- Heuristics are know-hows obtained through a lot of experiences.
- Heuristics often enable us to make decisions quickly without thinking deeply about the reasons.
- The more experiences we have, the better the heuristics will be.

When Heuristics are used?

AI problem solvers employ heuristics in two basic situations:

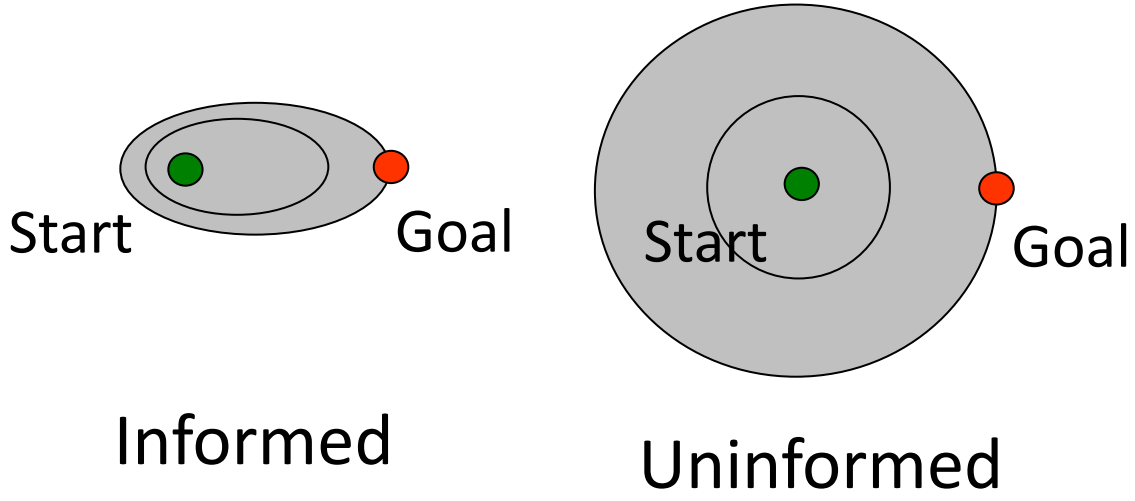
1. A **problem may not have an exact solution** because of inherent ambiguities in the problem statement or available data.
   - Medical diagnosis: Given symptoms may have several possible causes;
   - Doctors use heuristics to choose the most likely diagnosis and formulate a treatment plan.
2. A problem may have an exact solution, but the **computational cost of finding it may be prohibitive**.
   - Chess: state space growth is combinatorically explosive, with the number of possible states increasing with the depth of the search.
   - Heuristics attack this complexity by guiding the search along the most “promising” path through the space.

Why heuristic functions work?

- **Uninformed Search** : at most *b* choices at each node and a depth of *d* at the goal node
  - In the worst case, search around $O(b^d)$ nodes before finding a solution (Exponential Time Complexity).

- **Heuristics** improve the efficiency of search algorithms by reducing the effective branching factor from *b* to (ideally) a lower constant b^* such that $1 \leq b^* \ll b$

- Guide search ***towards the goal*** instead of ***all over the place***



Evaluation Function

- An evaluation function $f(n)$ gives an estimation on the "cost" of node n in a tree/graph
- So that the node with the least cost among all possible choices can be selected for expansion first
- Three approaches to define $f(n)$

Evaluation / Heuristic Function

- Evaluation function *f(n) = g(n) + h(n)*
  - *g(n)* = exact cost so far to reach n
  - *h(n)* = estimated cost to goal from n
  - *f(n)* = estimated total cost of cheapest path through n to goal

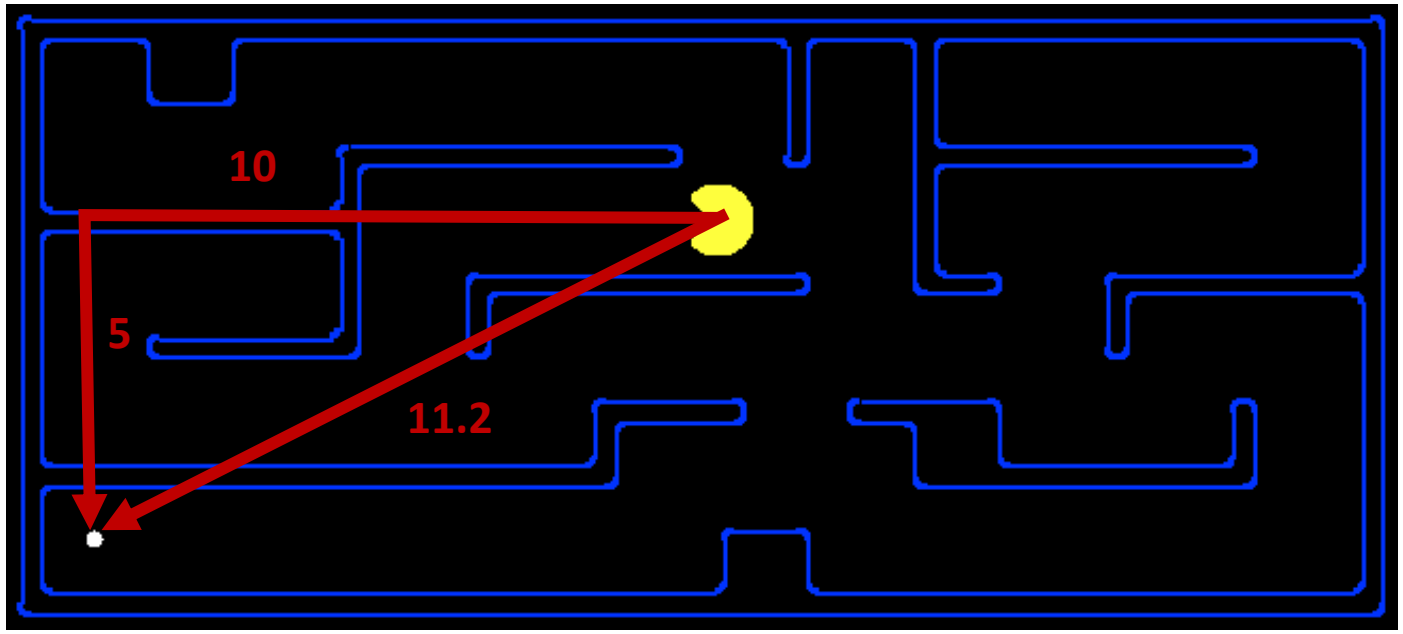
- Special cases:
  - Uniform Cost Search: $f(n) = g(n)$
  - Greedy (best-first) Search: $f(n) = h(n)$
  - A* Search: $f(n) = g(n) + h(n)$

How it works?

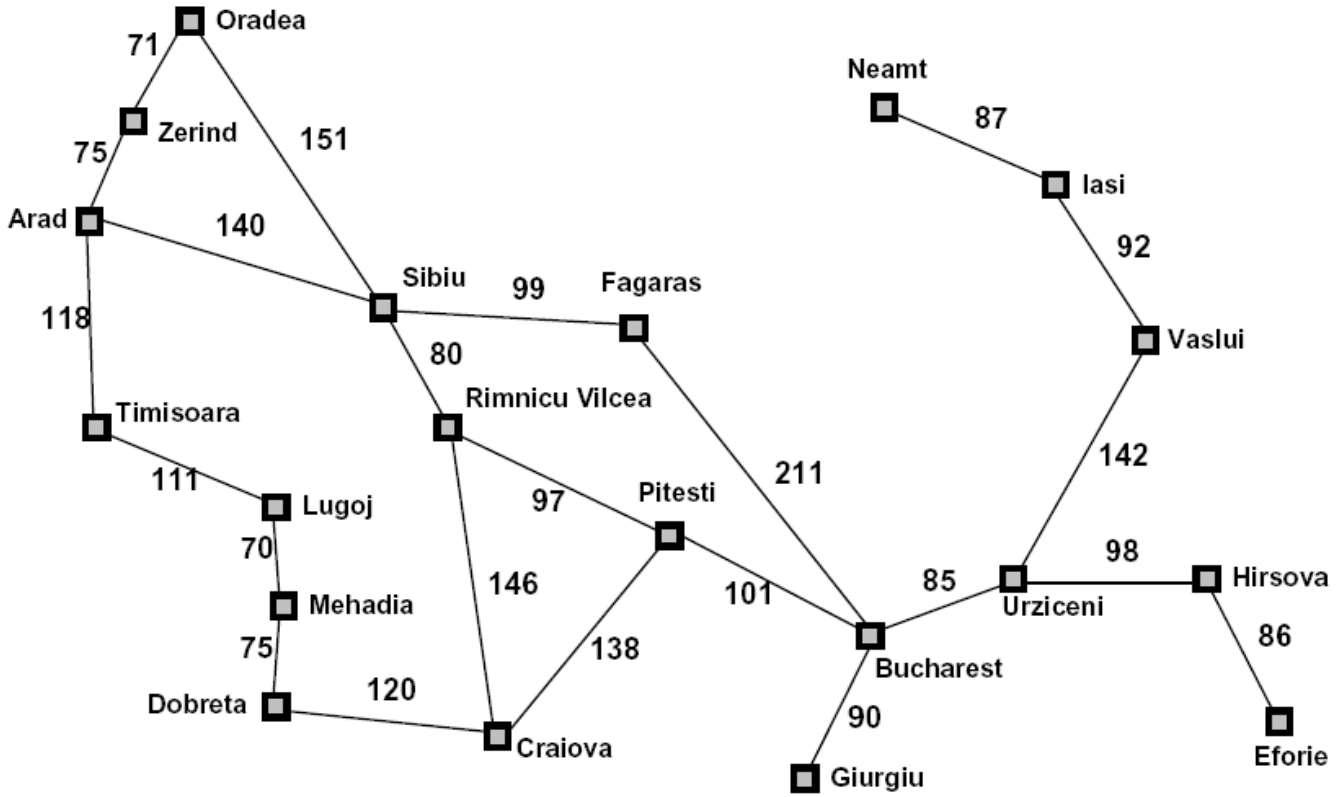
- Idea: use heuristic function for each node
  - that estimates desirability
- Expands most desirable unexpanded node
- **Implementation:**
  - fringe is a queue sorted in order of desirability
- **Special cases:**
  - Uniform Cost Search (uninformed)
  - Greedy (best-first) Search (informed)
  - A* Search (informed)

Heuristic Search

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing



Example: Heuristic Function



Straight−line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
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Urziceni	80
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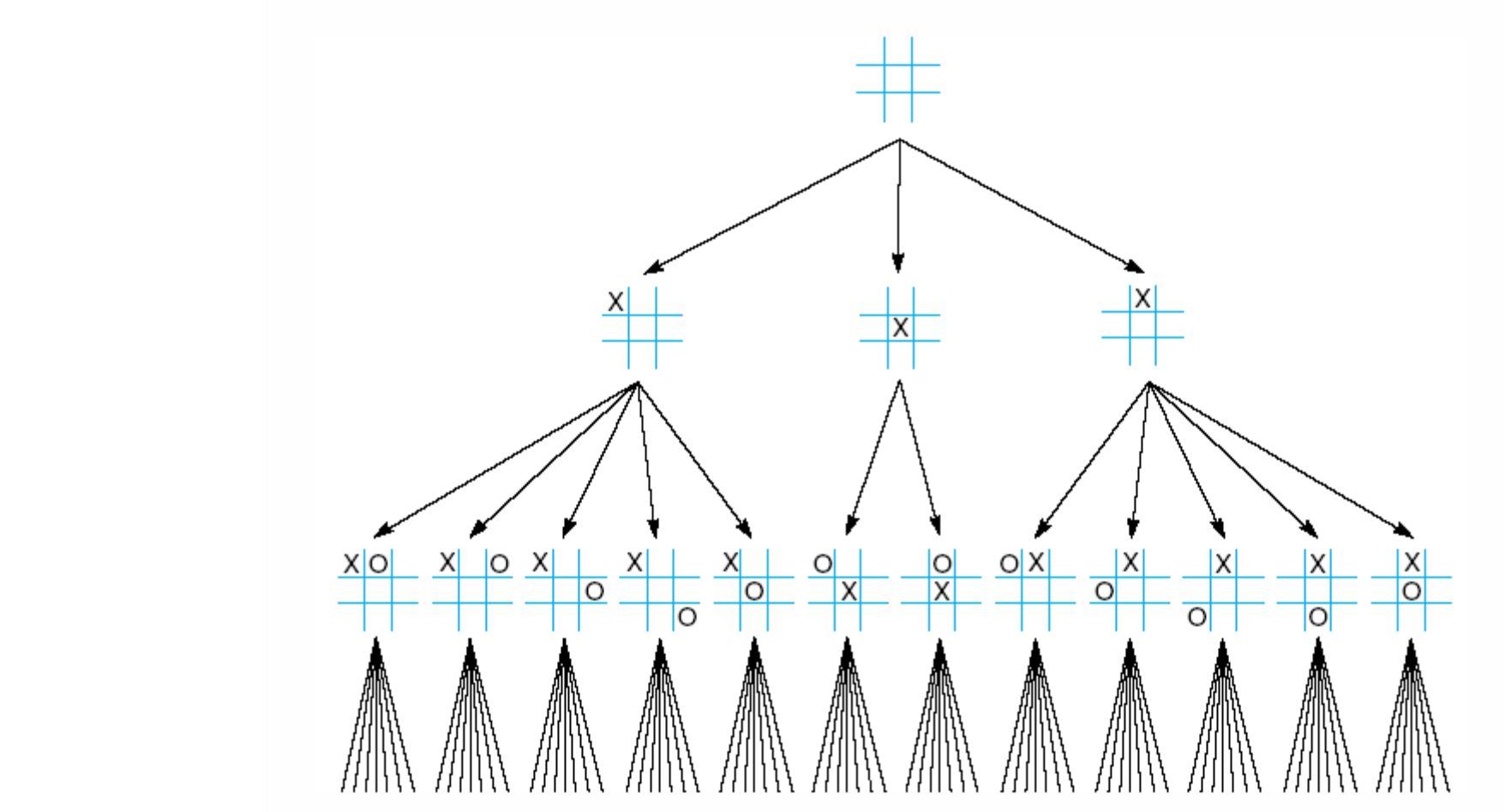
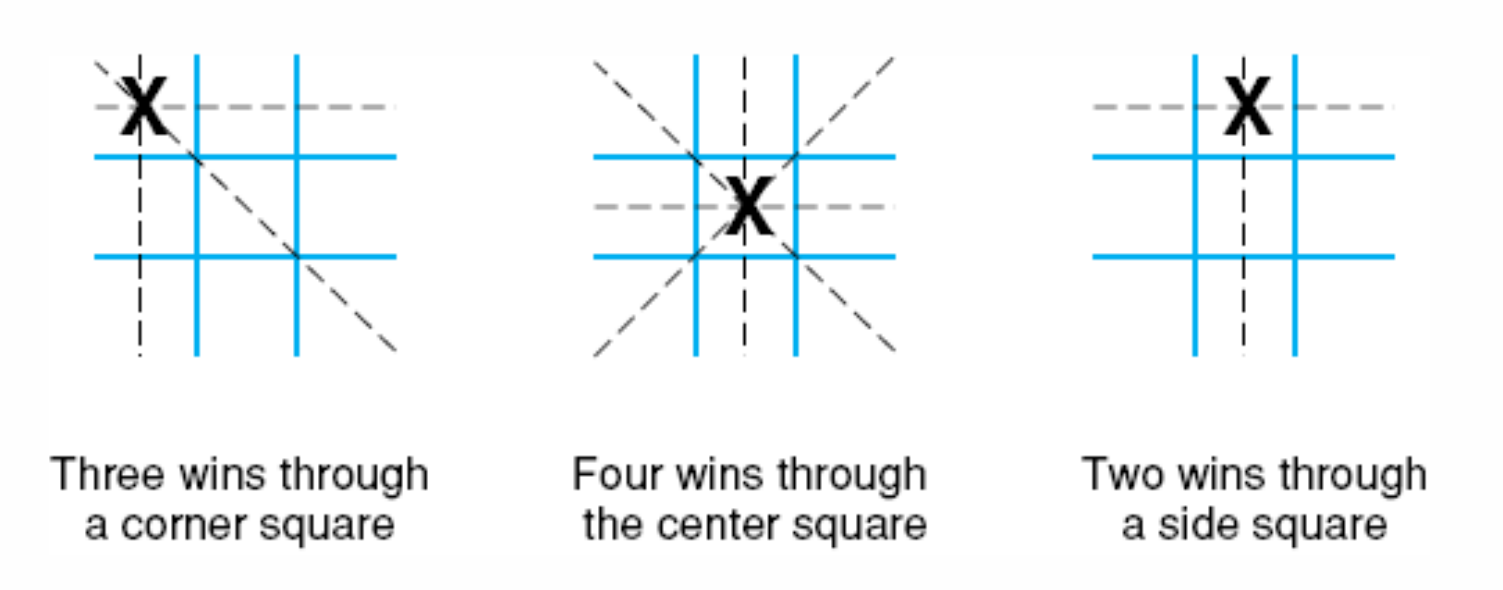
Zerind	374

h(x)

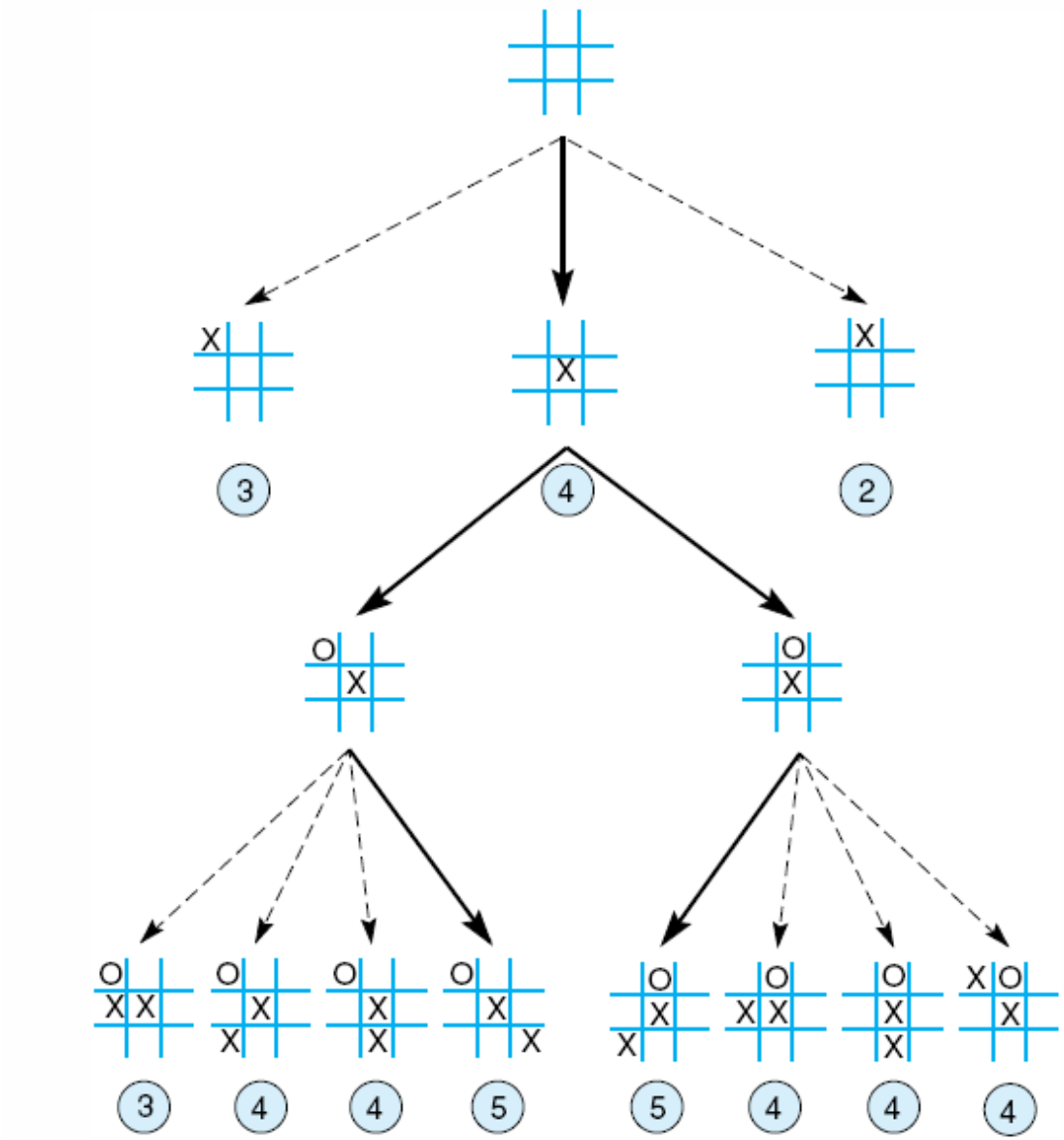
First three levels of the tic-tac-toe state space reduced by symmetry



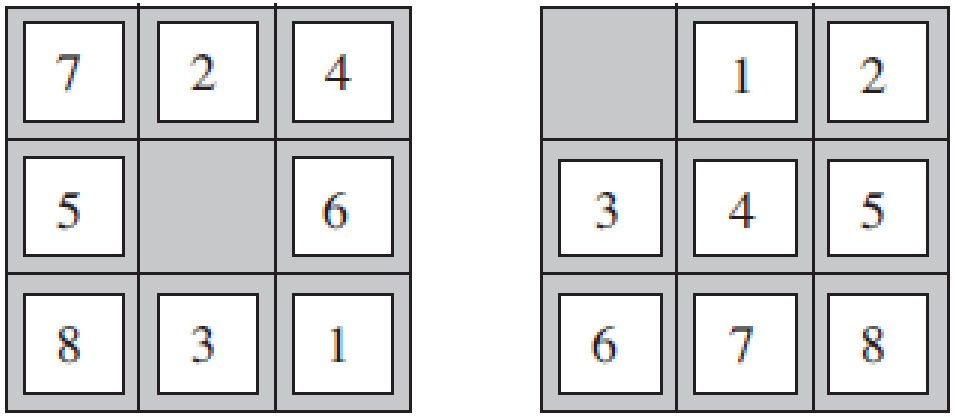
The “most wins” heuristic applied to the first children in tic-tac-toe.



Heuristically reduced state space for tic-tac-toe.



8–Puzzle: Heuristics?



- h1: The number of misplaced tiles (squares with number).
- h2: The sum of the distances of the tiles from their goal positions.

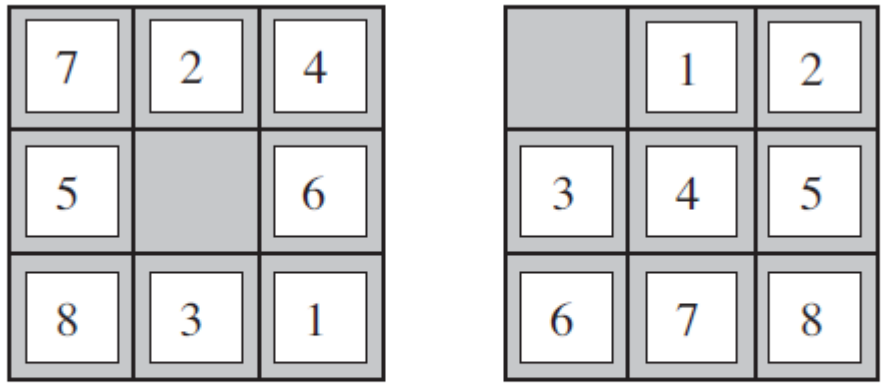
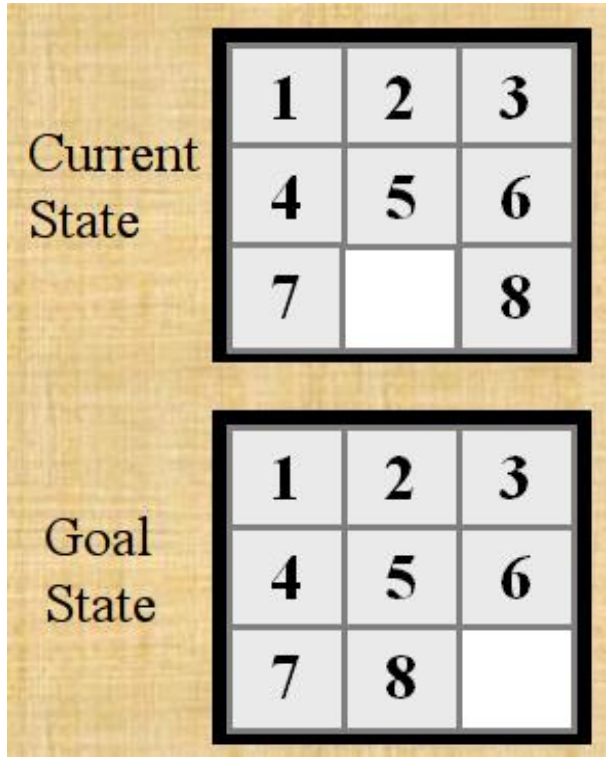


Figure 3.28 A typical instance of the 8-puzzle. The solution is 26 steps long.

- h_1 = the number of misplaced tiles. For Figure 3.28, all of the eight tiles are out of position, so the start state would have $h_1 = 8$. h_1 is an admissible heuristic because it is clear that any tile that is out of place must be moved at least once.
- h_2 = the sum of the distances of the tiles from their goal positions. Because tiles cannot move along diagonals, the distance we will count is the sum of the horizontal and vertical distances. This is sometimes called the **city block distance** or **Manhattan distance**. h_2 is also admissible because all any move can do is move one tile one step closer to the goal. Tiles 1 to 8 in the start state give a Manhattan distance of

$$h_2 = 3 + 1 + 2 + 2 + 2 + 3 + 3 + 2 = 18 \,.$$


Current State
1 2 3
4 5 6
7 8
Goal State
1 2 3
4 5 6
7 8

h1: The number of misplaced tiles (not including the blank)

Current State

1	2	3
4	5	6
7		8

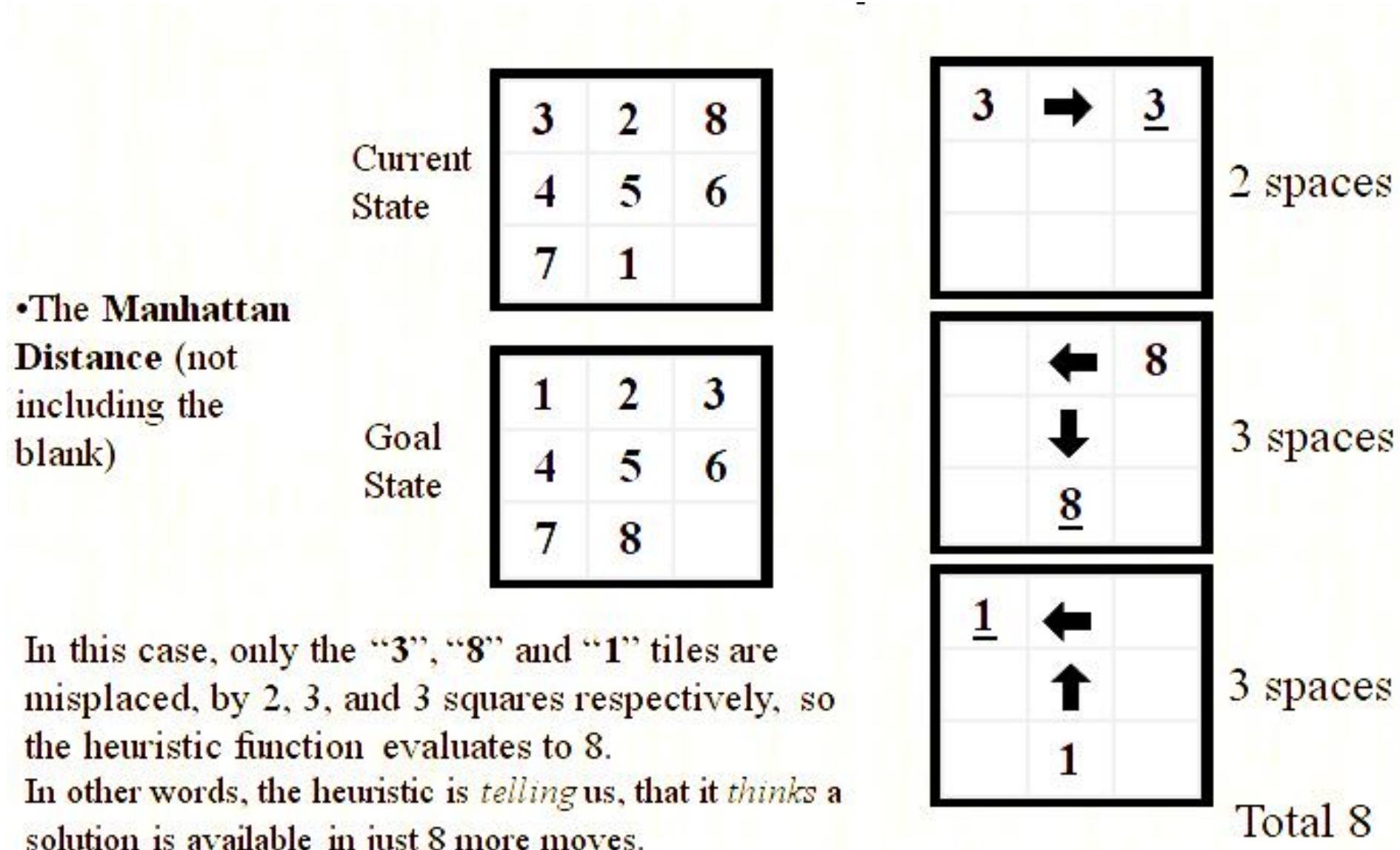
Goal State

1	2	3
4	5	6
7	8	

- *Notation:* $f(n) = h(n)$
- *h(current state) = 1*
- Because this state has one misplaced tile.

- Only "**8**" is misplaced
- So the heuristic function evaluates to 1.
- The heuristic is *telling* us, that it *thinks* a solution might be available in just 1 more move.

h2: The sum of the distances of tiles from goal positions



Notation: $f(n) = h(n)$, *h(current state) = 8*

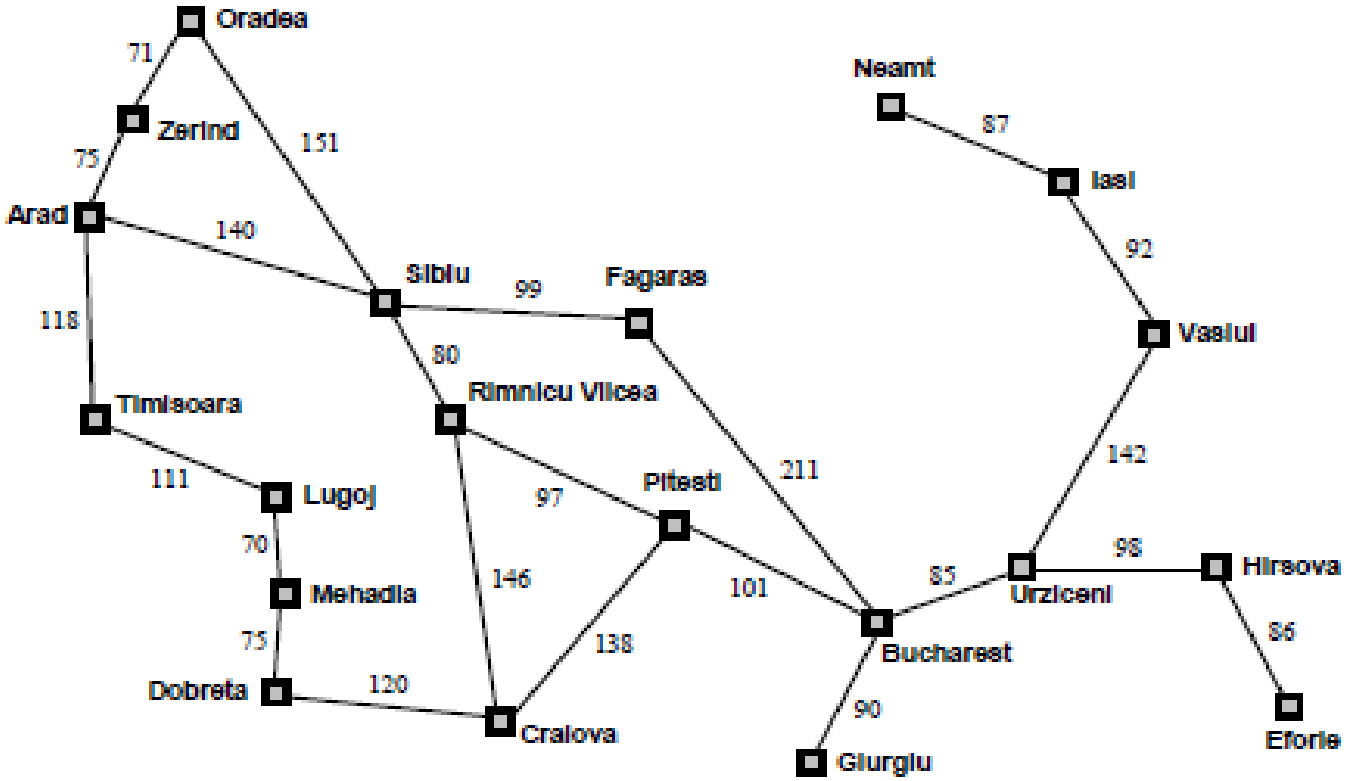
Greedy Best First Search

eval-fn: $f(n) = h(n)$

Greedy (Best-First) Search

- Idea: Expand node with the smallest estimated cost to reach the goal.
- Use heuristic function f(n) = h(n)
- **Best-first search** tries to expand the node that is closest to the goal, on the grounds that this is likely to lead to a solution quickly.

- *Choose path with low heuristic value*



Straight-line distance to Bucharest	
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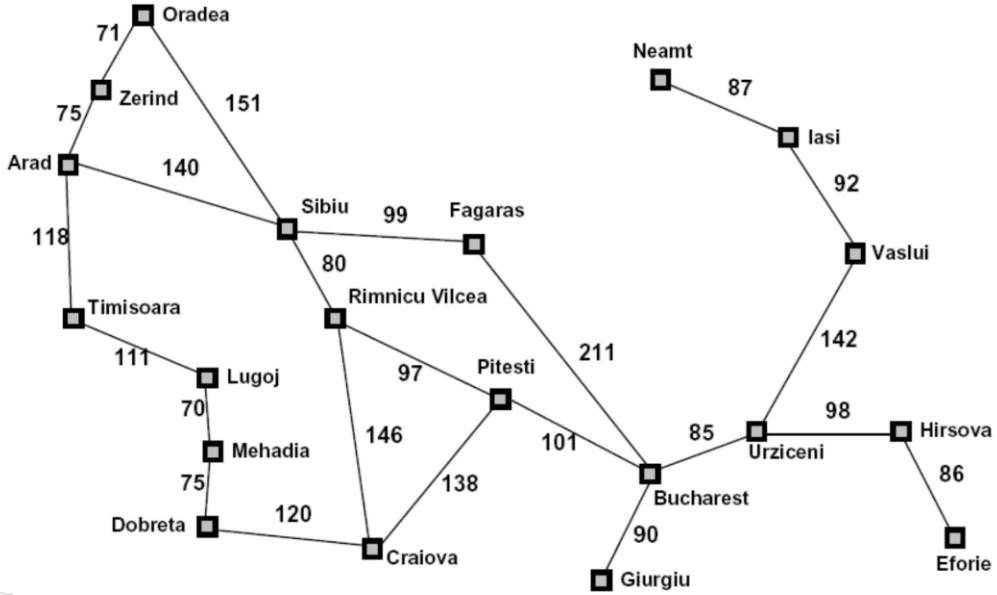
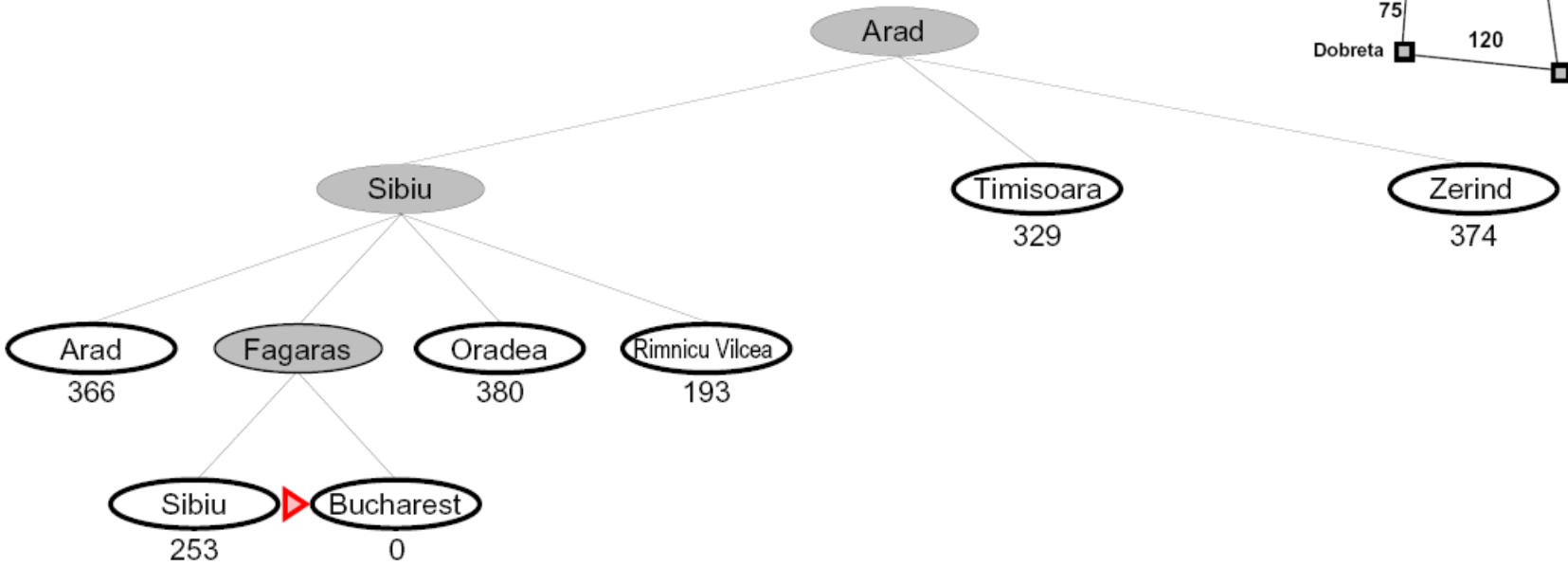
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy Search

- Expand the node that seems closest…



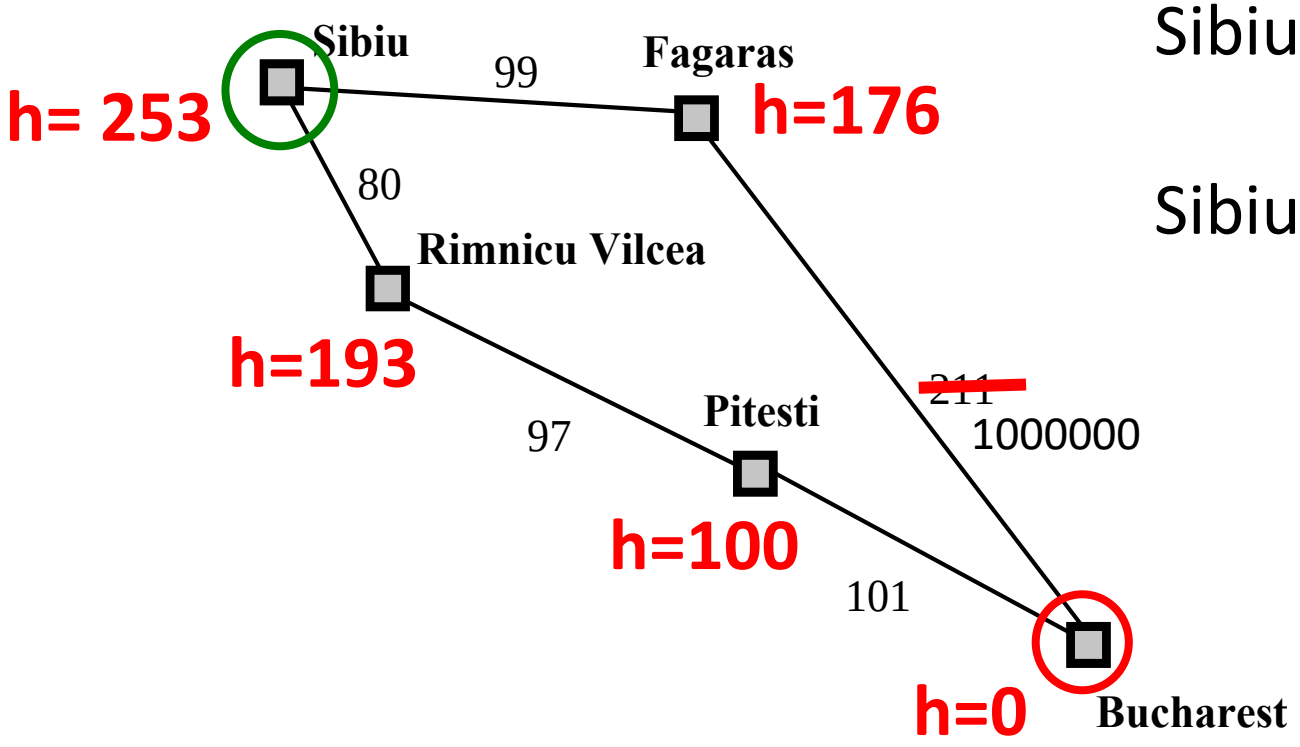


Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

- What can go wrong?

Greedy Search

- Expand the node that seems closest…(order frontier by h)
- What can possibly go wrong?

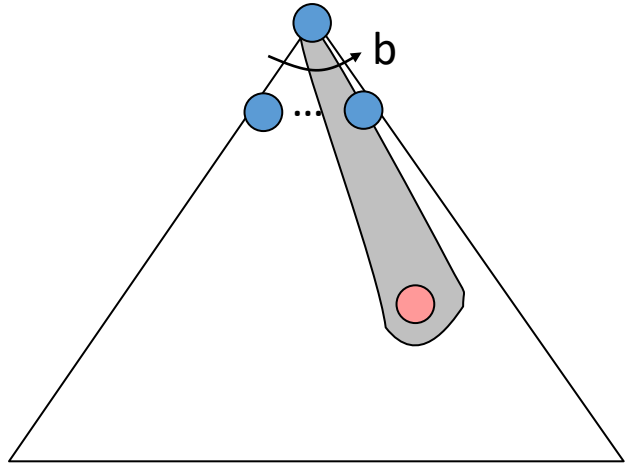


Sibiu-Fagaras-Bucharest =
99+211 = **310**

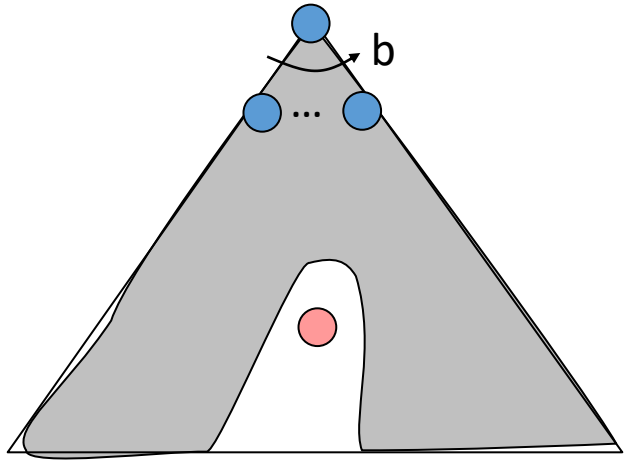
Sibiu-Rimnicu Vilcea-Pitesti-Bucharest =
80+97+101 = **278**

Greedy Search

- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state



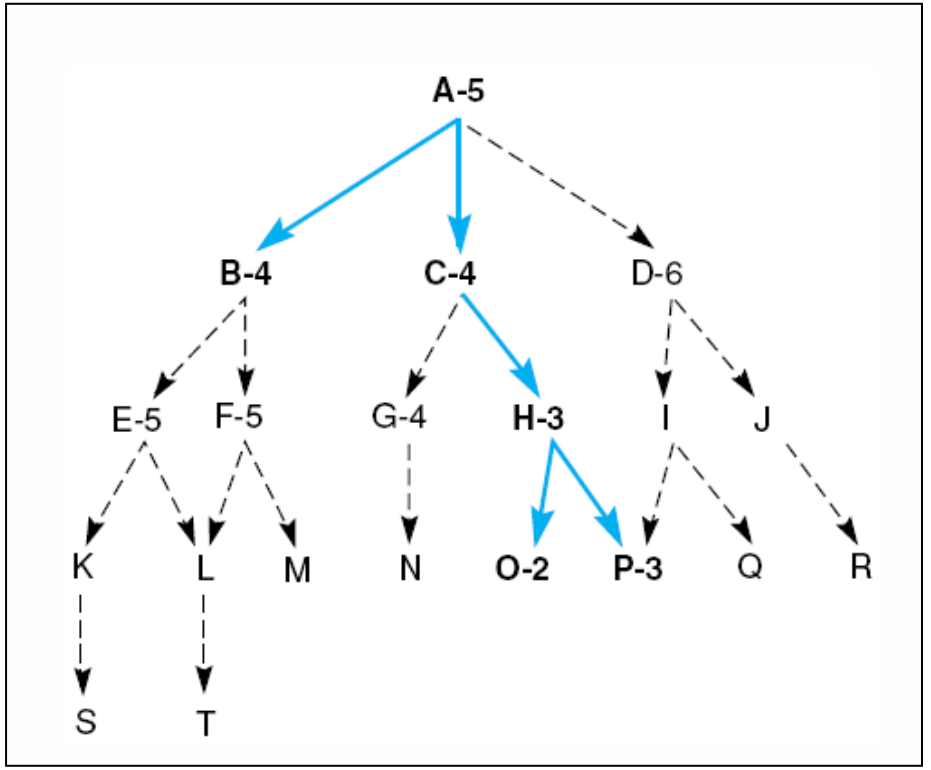
- A common case:
  - Best-first takes you straight to the (wrong) goal



- Worst-case: like a badly-guided DFS

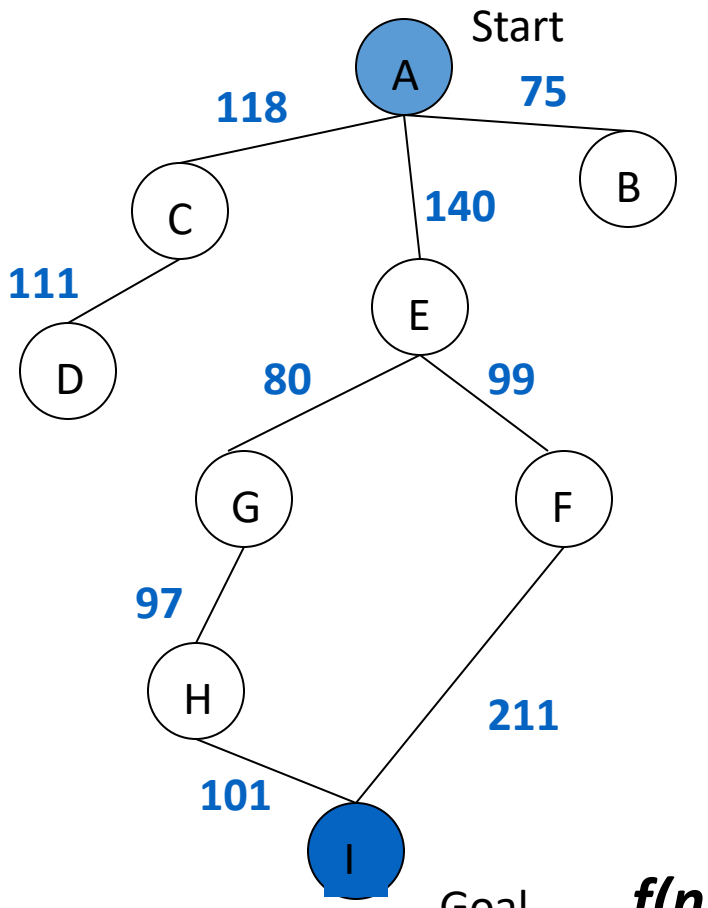
```
function best_first_search;

begin
   open := [Start];                                                    % initialize
   closed := [ ];
   while open ≠ [ ] do                                                 % states remain
      begin
         remove the leftmost state from open, call it X;
         if X = goal then return the path from Start to X
         else begin
                  generate children of X;
                  for each child of X do
                  case
                        the child is not on open or closed:
                           begin
                                 assign the child a heuristic value;
                                 add the child to open
                           end;
                        the child is already on open:
                           if the child was reached by a shorter path
                           then give the state on open the shorter path
                        the child is already on closed:
                           if the child was reached by a shorter path then
                              begin
                                 remove the state from closed;
                                 add the child to open
                              end;
                  end;                                                 % case
                  put X on closed;
                  re-order states on open by heuristic merit (best leftmost)
               end;
return FAIL                                                            % open is empty
end.
```

Best First Search



1. **open = [A5]; closed = [ ]**
2. **evaluate A5; open = [B4,C4,D6]; closed = [A5]**
3. **evaluate B4; open = [C4,E5,F5,D6]; closed = [B4,A5]**
4. **evaluate C4; open = [H3,G4,E5,F5,D6]; closed = [C4,B4,A5]**
5. **evaluate H3; open = [O2,P3,G4,E5,F5,D6]; closed = [H3,C4,B4,A5]**
6. **evaluate O2; open = [P3,G4,E5,F5,D6]; closed = [O2,H3,C4,B4,A5]**
7. **evaluate P3; the solution is found!**

Greedy Search

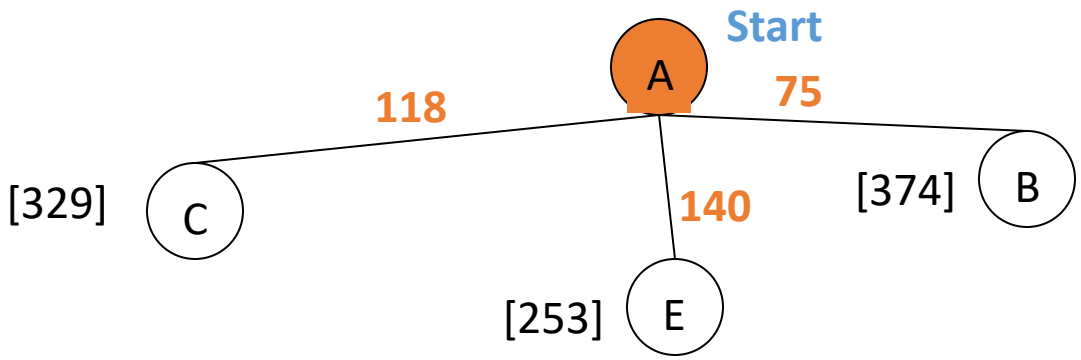


State	Heuristic: h(n)
A	366
B	374
C	329
D	244
E	253
F	178
G	193
H	98
I	**0**

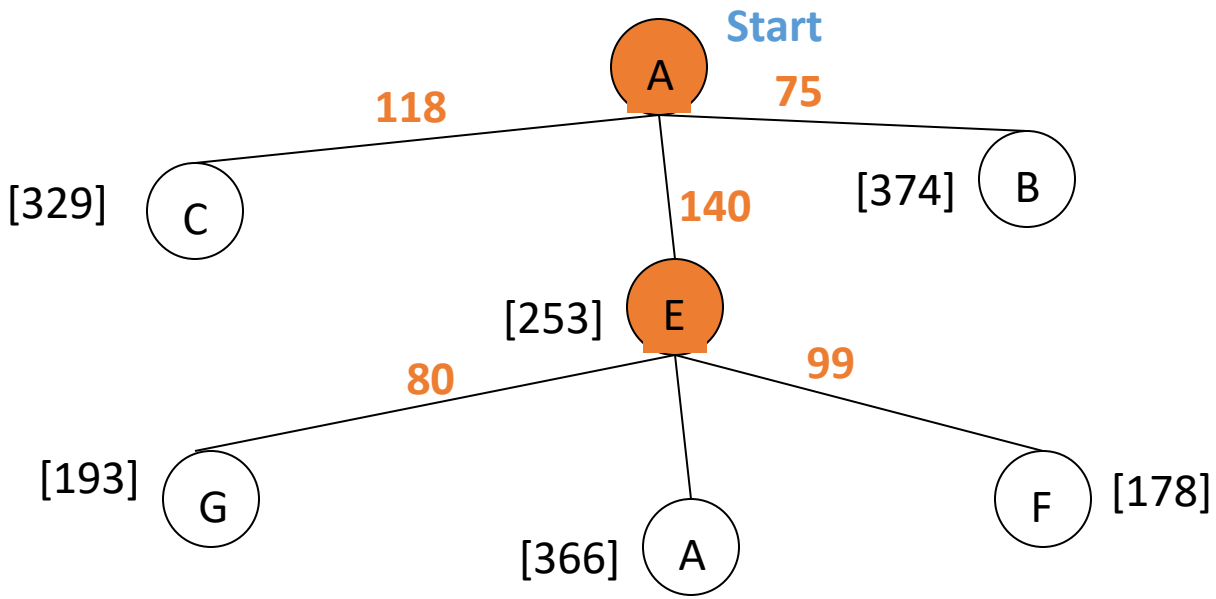
***f(n) = h (n)* = straight-line distance heuristic**

Greedy Search: Tree Search

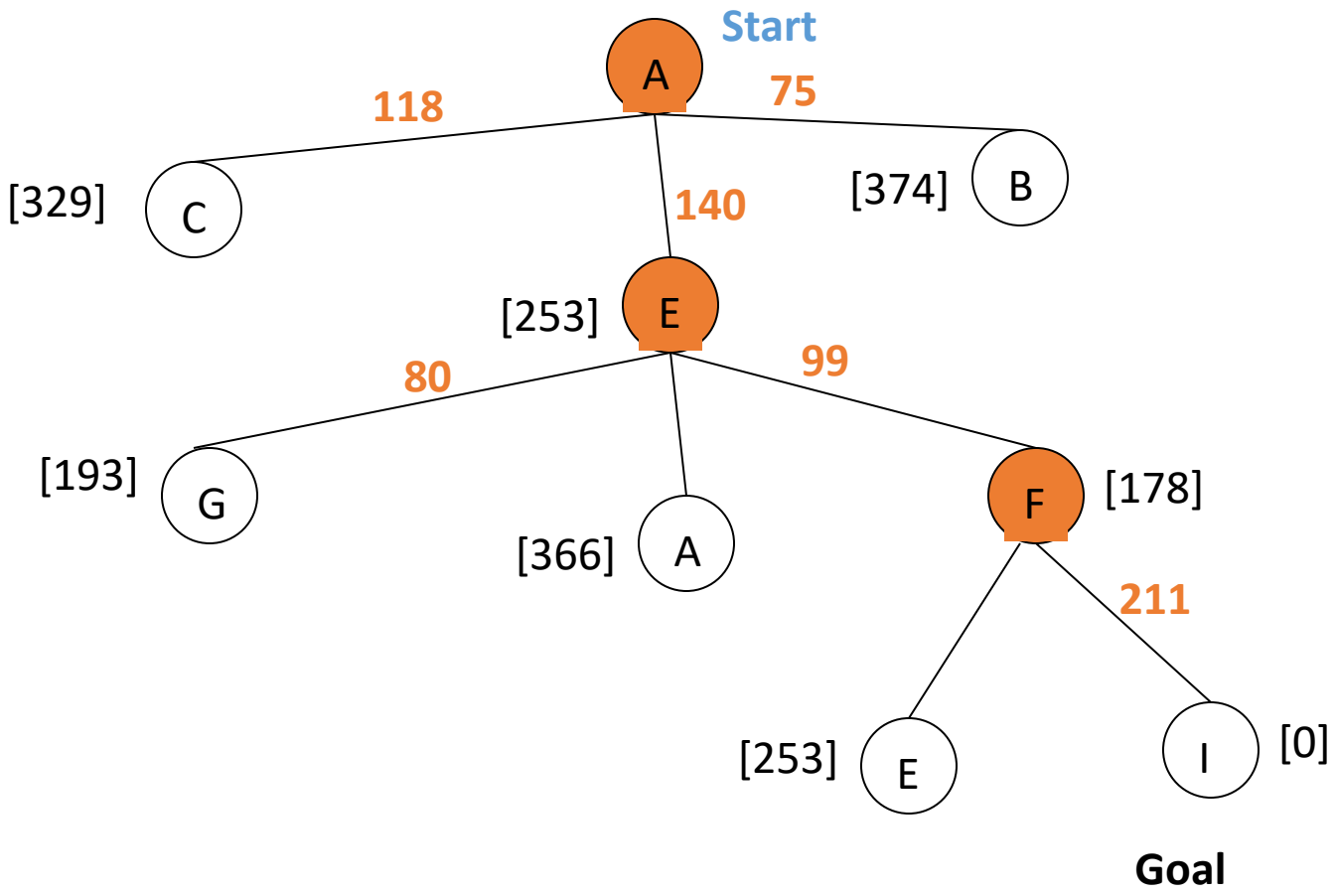
Greedy Search: Tree Search



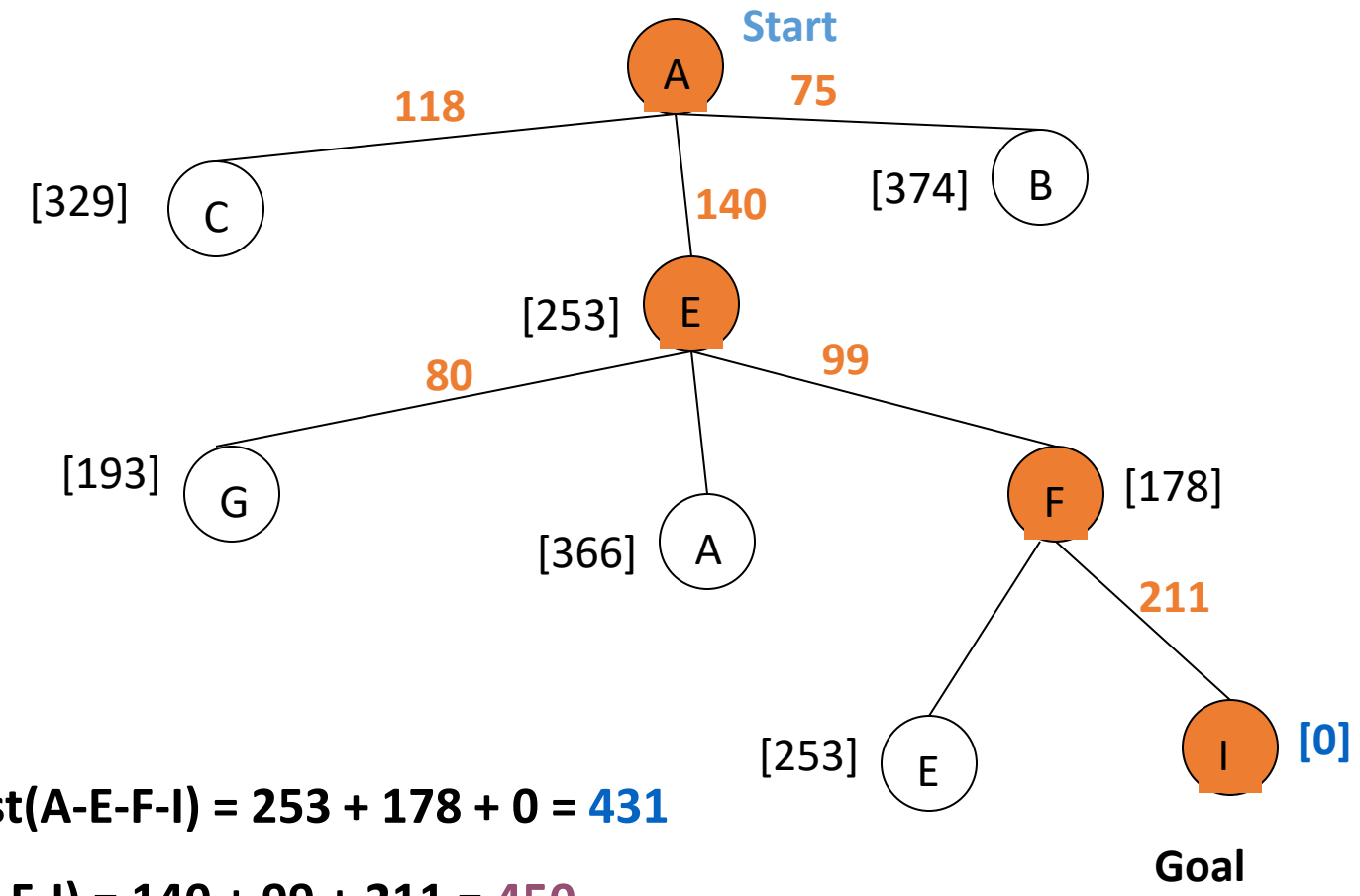
Greedy Search: Tree Search



Greedy Search: Tree Search



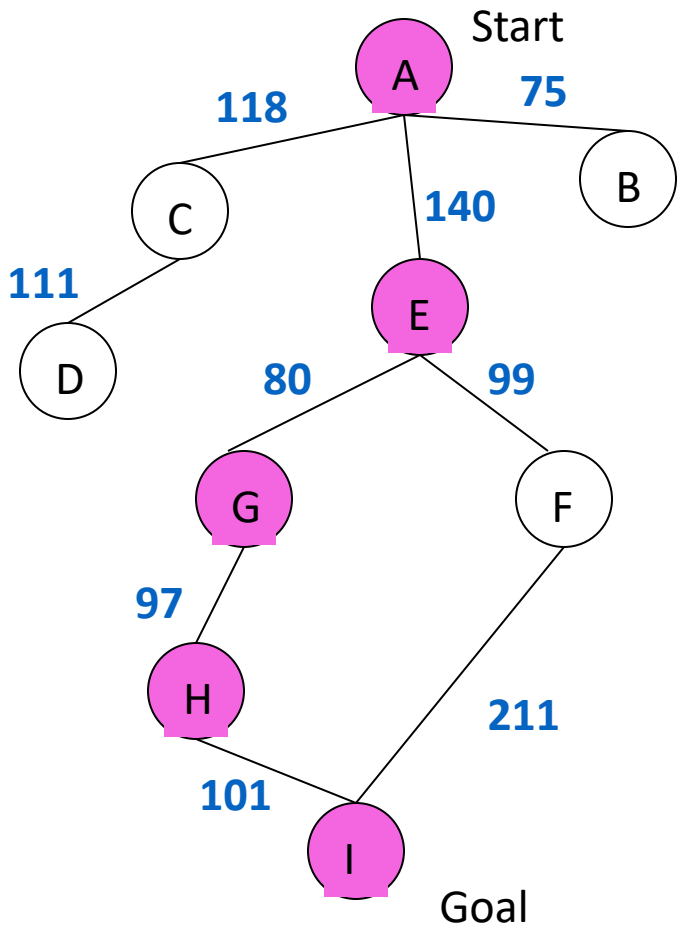
Greedy Search: Tree Search



Path cost(A-E-F-I) = 253 + 178 + 0 = 431

dist(A-E-F-I) = 140 + 99 + 211 = 450

Greedy Search: Optimal?



State	Heuristic: h(n)
A	366
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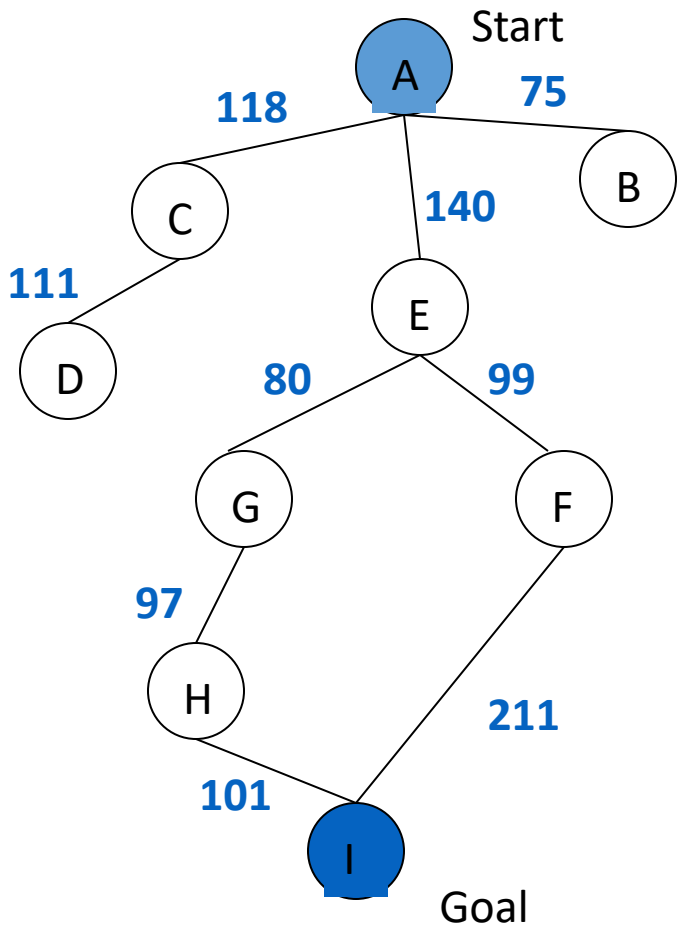
H	98
I	0

Not optimal!

***f(n) = h (n)* = straight-line distance heuristic**

dist(A-E-G-H-I) =140+80+97+101=418

Greedy Search: Complete ?

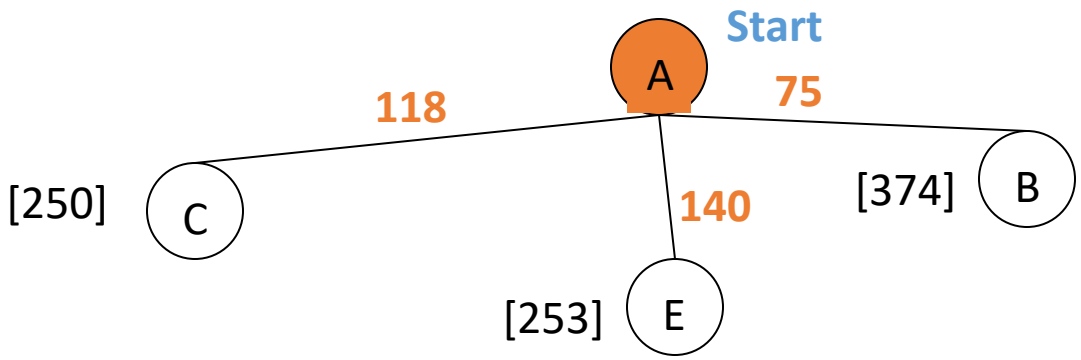


State	Heuristic: h(n)
A	366
B	374
** C	**250**
D	244
E	253
F	178
G	193
H	98
I	0

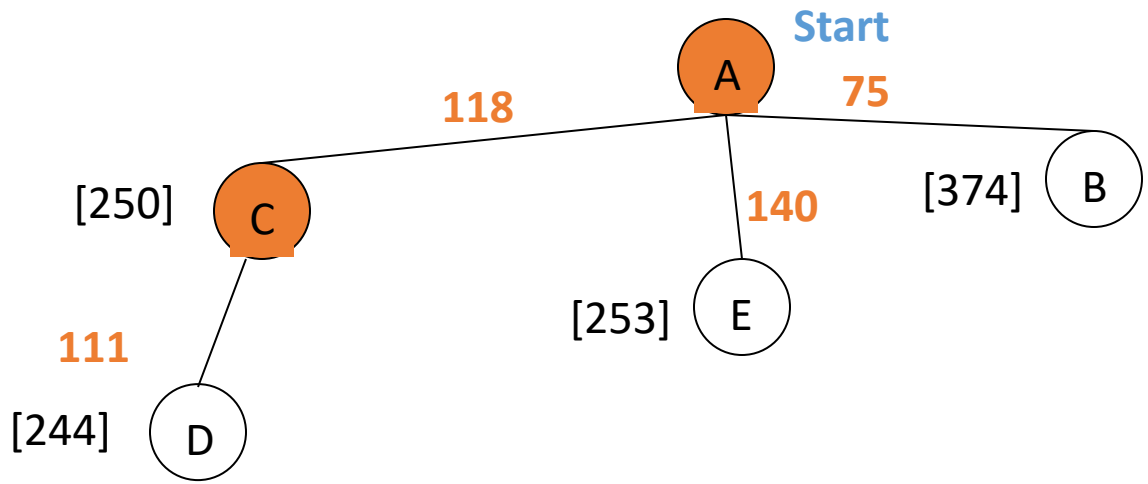
***f(n) = h (n)* = straight-line distance heuristic**

Greedy Search: Tree Search

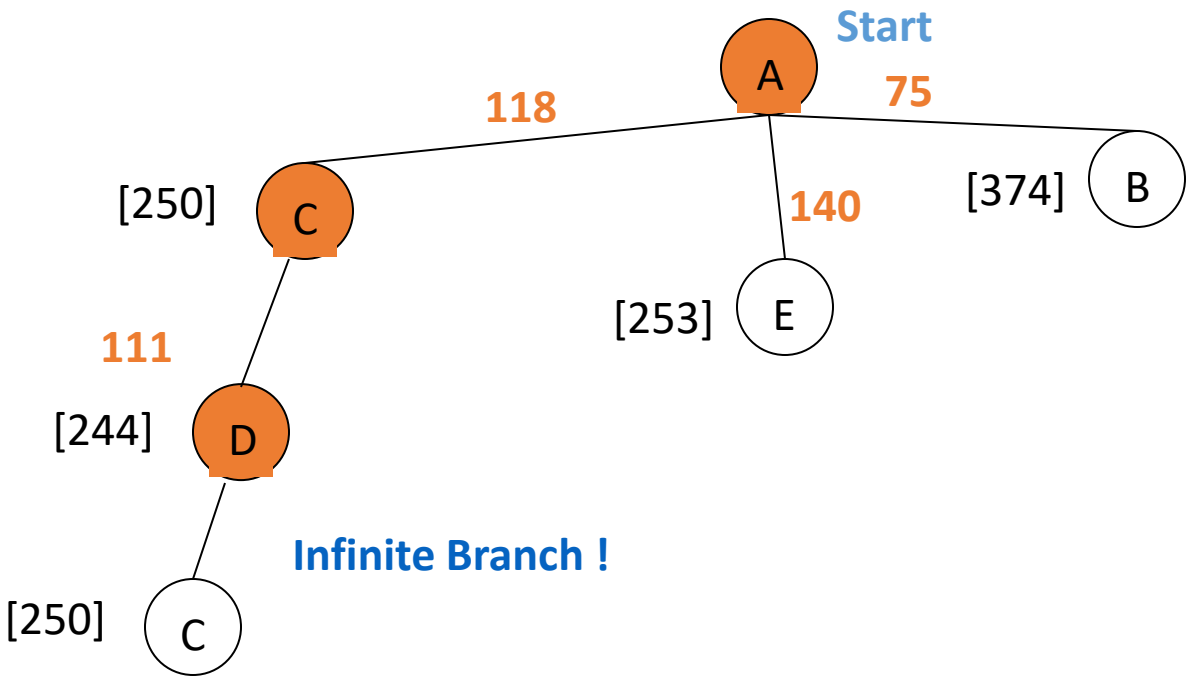
Greedy Search: Tree Search



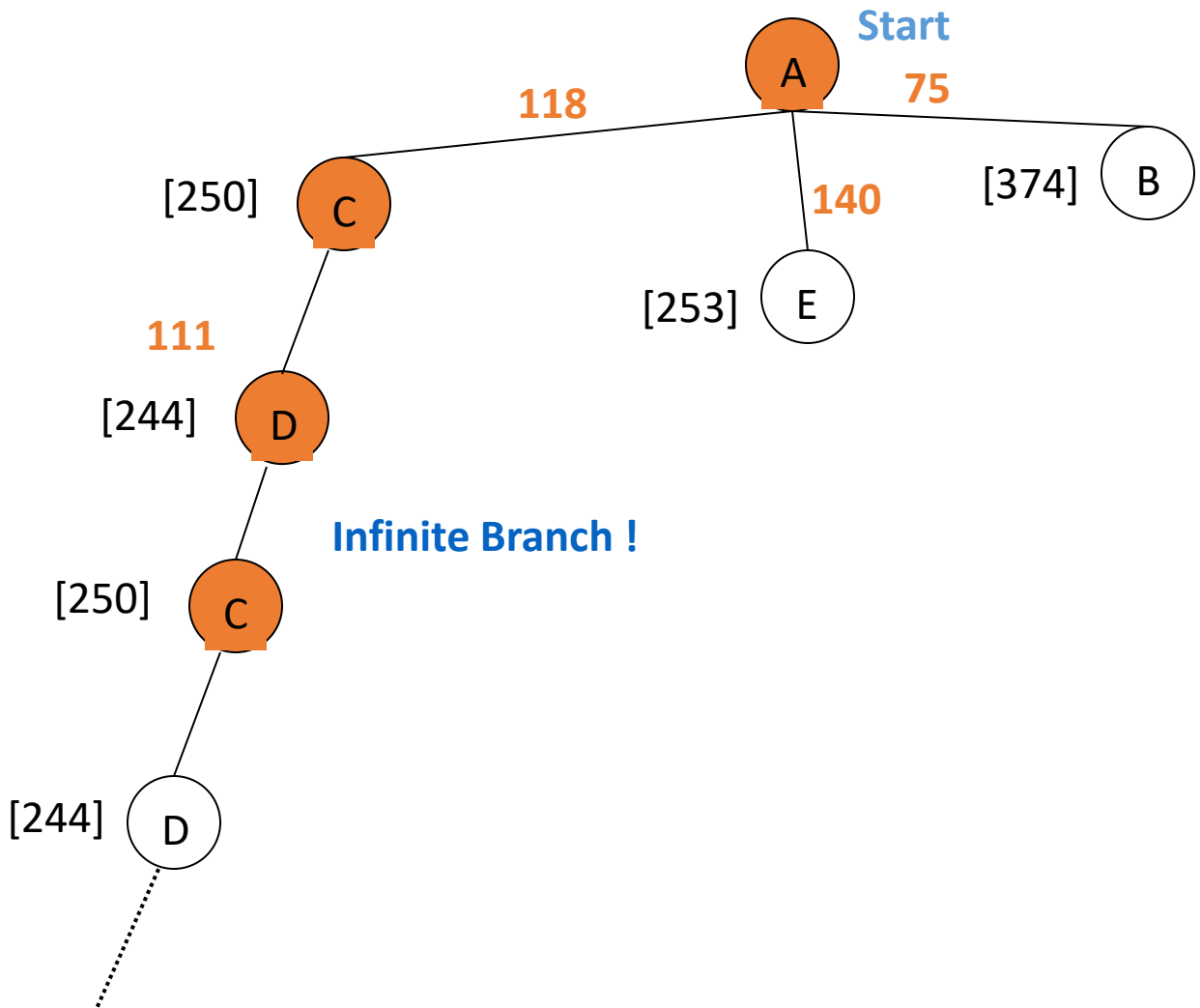
Greedy Search: Tree Search



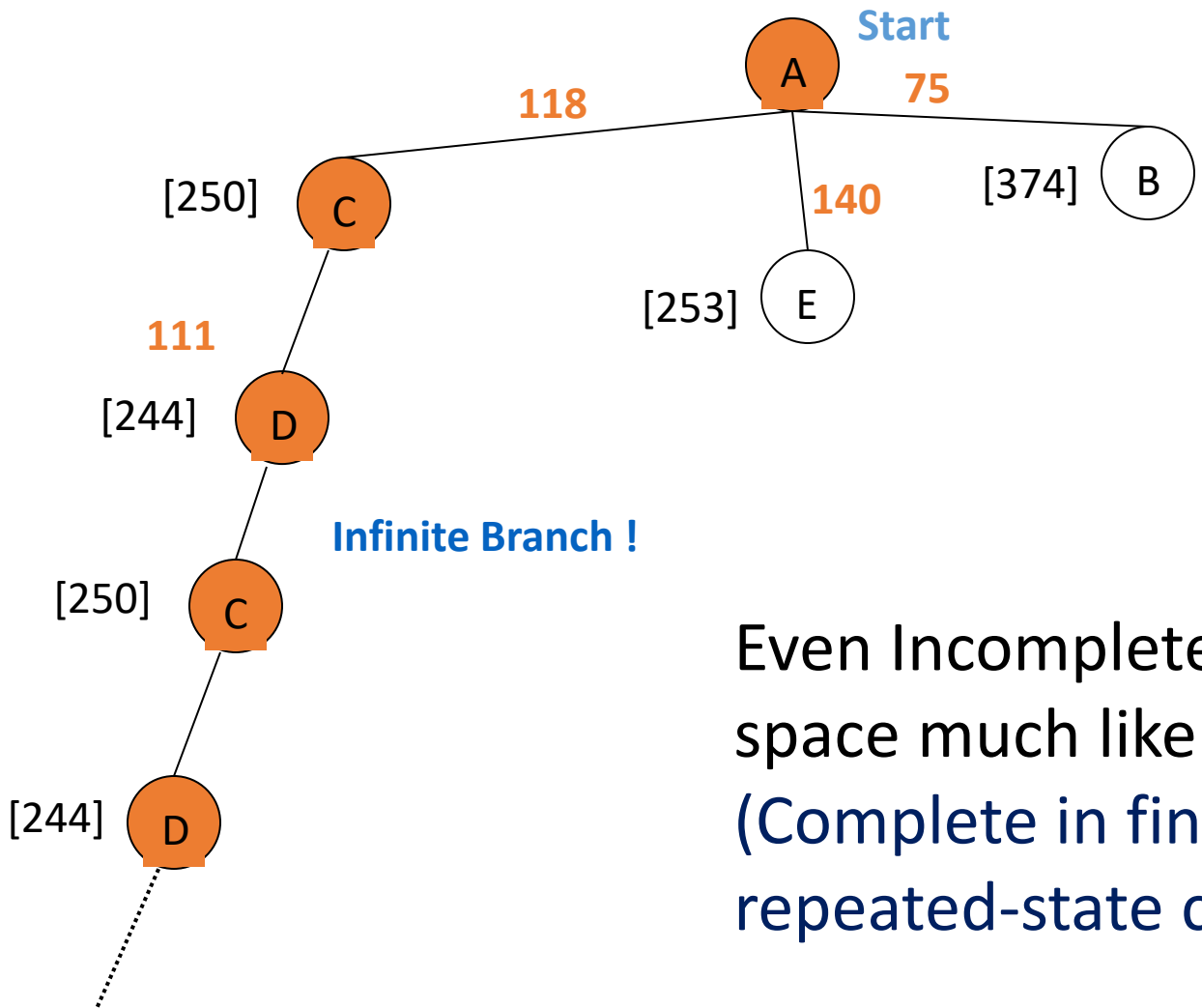
Greedy Search: Tree Search



Greedy Search: Tree Search

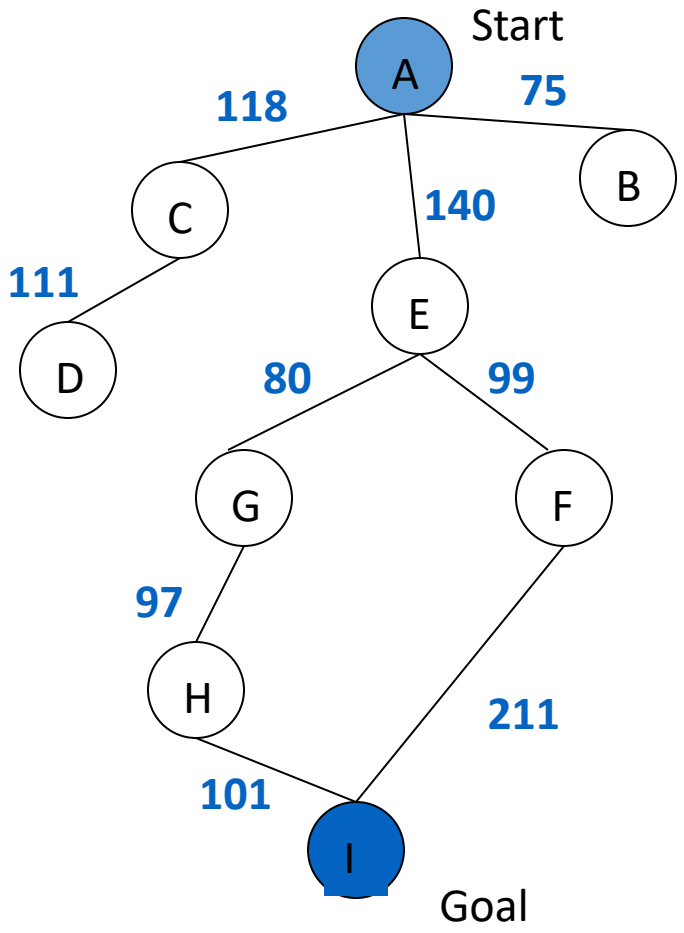


Greedy Search: Tree Search

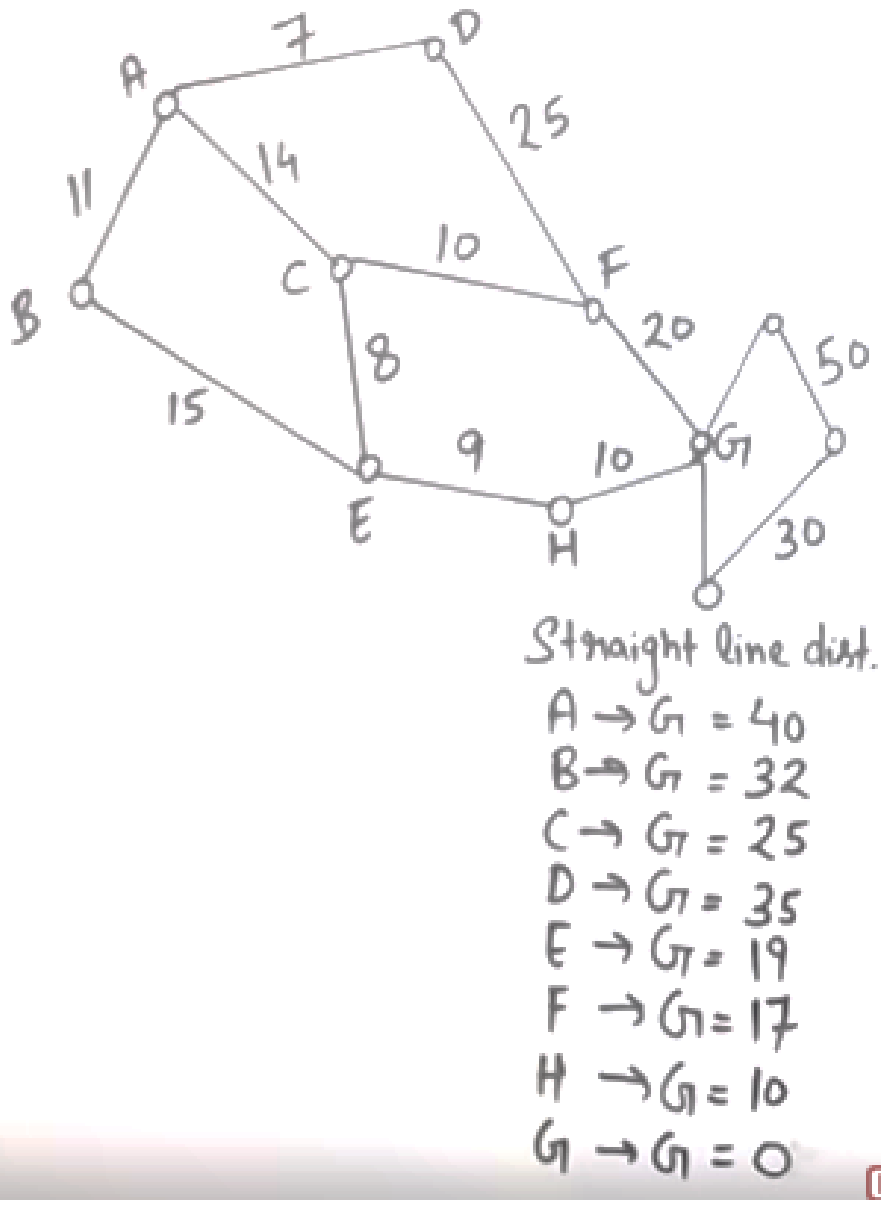


Even Incomplete in finite state space much like depth first search (Complete in finite space with repeated-state checking)

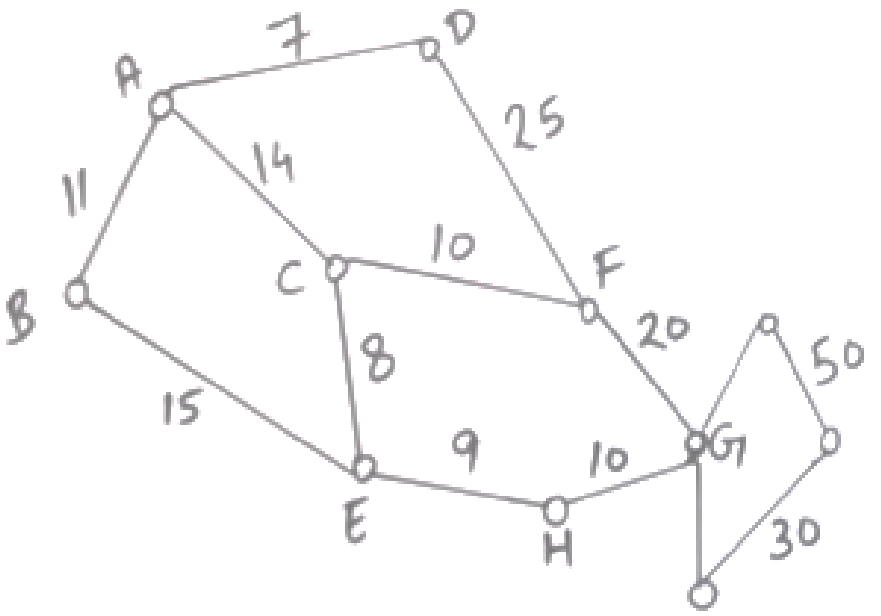
Greedy Search: Time and Space Complexity ?



- Greedy search is not optimal.
- Greedy search is incomplete
- In the worst case, the Time and Space Complexity of Greedy Search are both $O(b^m)$
- Where b is the branching factor and m the maximum path length



	Frontier (Greedy)	Expand	Explored
1	(A,40)	A	Empty
2	(A-B,32)(A-C,25)(A-D,35)	C	A
3	(A-B,32)(A-D,35) (A-C-E,19)(A-C-F,17)	F	A,C
4	(A-B,32)(A-D,35) (A-C-E,19)(A-C-F-G,0)	G	A,C,F
5	(A-C-F-G,0) Goal Found		



Straight line dist.

A → G = 40
B → G = 32
C → G = 25
D → G = 35
E → G = 19
F → G = 17
H → G = 10
G → G = 0

	Frontier (Greedy)	Expand	Explored
1	(A,40)	A	Empty
2	(A-C,25) (A-B,32)(A-D,35)	C	A
3	(A-B,32)(A-D,35)(A-C-F,17)(A-C-E,19)	F	A,C
4	(A-B,32)(A-D,35)(A-C-F-G,0)(A-C-E,19)	G	A,C,F
5	(A-C-F-G) Goal Found		A,C,F,G

	Frontier (UCS)	Expand	Explored
1	(A,0)	A	
2	(A-B,11)(A-C,14)(A-D,7)	D	A
3	(A-B,11)(A-C,14)(A-D-F,32)	B	A,D
4	(A-B-E,26)(A-C,14)(A-D-F,32)	C	A,D,B
5	(A-B-E,26)(A-C-F,24)(A-C-E,22)(A-D-F,32)	E	A,D,B,C
6	(A-B-E,26)(A-C-F,24)(A-C-E-H,31)(A-D-F,32)	F	A,D,B,C,E
7	(A-B-E,26)(A-C-F-G,44)(A-C-E-H,31)(A-D-F,32)	E	A,D,B,C,E,F
8	~~(A-B-E,26)~~(A-C-F-G,44)(A-C-E-H,31)(A-D-F,32)	F	
9	~~(A-B-E,26)~~(A-C-F-G,44)(A-C-E-H,31)~~(A-D-F,32)~~	H	
10	A-C-E-H-G,41 Goal Found		

Heuristics

- Problem:
  - A heuristic is only an informed guess of the next step to be taken in solving a problem.
  - It is often based on experience or intuition.
  - Heuristics use limited information, such as knowledge of the present situation or descriptions of states currently on the open list.
  - They are not always able to predict the exact behavior of the state space farther along in the search.
  - A heuristic can lead a search algorithm to a suboptimal solution or fail to find any solution at all. This is an **inherent limitation** of heuristic search.