Minimax Algorithm:

The **minimax algorithm** computes the minimax decision from the current state. It uses a simple recursive computation of the minimax values of each successor state, directly implementing the defining equations. The recursion proceeds all the way down to the leaves of the tree, and then the minimax values are **backed up** through the tree as the recursion unwinds. The minimax algorithm performs a complete depth-first exploration of the game tree.

```
function MINIMAX-DECISION(state) returns an action
  return arg max_{a ∈ ACTIONS(s)} MIN-VALUE(RESULT(state, a))

function MAX-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for each a in ACTIONS(state) do
     v ← MAX(v, MIN-VALUE(RESULT(s, a)))
  return v

function MIN-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← ∞
  for each a in ACTIONS(state) do
     v ← MIN(v, MAX-VALUE(RESULT(s, a)))
  return v
```

```
import math

def minimax (curDepth, nodeIndex,
             maxTurn, scores,
             targetDepth):

    # base case : targetDepth reached
    if (curDepth == targetDepth):
        return scores[nodeIndex]

    if (maxTurn):
        return max(minimax(curDepth + 1, nodeIndex * 2,
                    False, scores, targetDepth),
                   minimax(curDepth + 1, nodeIndex * 2 + 1,
                    False, scores, targetDepth))

    else:
        return min(minimax(curDepth + 1, nodeIndex * 2,
                     True, scores, targetDepth),
                   minimax(curDepth + 1, nodeIndex * 2 + 1,
                     True, scores, targetDepth))

# Driver code
scores = [3, 5, 2, 9, 12, 5, 23, 23]

treeDepth = math.log(len(scores), 2)

print("The optimal value is : ", end = "")
print(minimax(0, 0, True, scores, treeDepth))
```

Alpha-Beta Pruning

Alpha-Beta pruning is not actually a new algorithm, but rather an optimization technique for the minimax algorithm. It reduces the computation time by a huge factor. This allows us to search much faster and even go into deeper levels in the game tree. It cuts off branches in the game tree which need not be searched because there already exists a better move available. It is called Alpha-Beta pruning because it passes 2 extra parameters in the minimax function, namely alpha and beta.

Let’s define the parameters alpha and beta.

Alpha is the best value that the maximizer currently can guarantee at that level or above.

Beta is the best value that the minimizer currently can guarantee at that level or below.

```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, −∞, +∞)
  return the action in ACTIONS(state) with value v
---------------------------------------------------------------
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for each a in ACTIONS(state) do
    v ← MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    if v ≥ β then return v
    α ← MAX(α, v)
  return v
---------------------------------------------------------------
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← +∞
  for each a in ACTIONS(state) do
    v ← MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
    if v ≤ α then return v
    β ← MIN(β, v)
  return v
```

Code:

```
MAX, MIN = 1000, -1000
def minimax(depth, nodeIndex, maximizingPlayer, values, alpha, beta):
    if depth == 3:
        return values[nodeIndex]
    if maximizingPlayer:
        best = MIN

        for i in range(0, 2):
            val = minimax(depth + 1, nodeIndex * 2 + i, False, values, alpha, beta)
            best = max(best, val)
            alpha = max(alpha, best)
            if beta <= alpha:
                break
        return best
    else:
        best = MAX
        for i in range(0, 2):
            val = minimax(depth + 1, nodeIndex * 2 + i,True, values, alpha, beta)
            best = min(best, val)
            beta = min(beta, best)
            # Alpha Beta Pruning
            if beta <= alpha:
                break
        return best
    
# Driver Code
values = [3, 5, 6, 9, 1, 2, 0, -1]
print("The optimal value is :", minimax(0, 0, True, values, MIN, MAX))
```

TIC-TAC-TOE Game:

A Tic-Tac-Toe is a 2 player game, who take turns marking the spaces in a 3x3 grid. The player who succeeds in placing three of their marks in a horizontal, vertical, or diagonal row wins the game. The game is to be played between two people (in this program between HUMAN and COMPUTER). One of the player chooses ‘O’ and the other ‘X’ to mark their respective cells. The game starts with one of the players and the game ends when one of the players has one whole row/ column/ diagonal filled with his/her respective character (‘O’ or ‘X’). If no one wins, then the game is said to be draw. In our program the moves taken by the computer and the human are chosen randomly. We use rand () function for this.

0	X	0
0	X	X
X	0	X

Task: You are given with the basic functions of minimax and alpha-beta pruning. Just figure out the difference in implementation and implement the alpha-beta pruning for the given tic-tac-toe problem. Note: minimax function for tic-tac-toe is given.

```
from os import system
import time
from math import inf as infinity
from random import choice
import platform


HUMAN = -1
COMP = +1
board = [[0, 0, 0],
         [0, 0, 0],
         [0, 0, 0]]

def evaluate(state):
    if wins(state, COMP):
        score = +1
    elif wins(state, HUMAN):
        score = -1
    else:
        score = 0
    return score
def wins(state, player):
    win_state = [
        [state[0][0], state[0][1], state[0][2]],
        [state[1][0], state[1][1], state[1][2]],
        [state[2][0], state[2][1], state[2][2]],
        [state[0][0], state[1][0], state[2][0]],
        [state[0][1], state[1][1], state[2][1]],
        [state[0][2], state[1][2], state[2][2]],
        [state[0][0], state[1][1], state[2][2]],
        [state[2][0], state[1][1], state[0][2]]    ]

    if [player, player, player] in win_state:
        return True
    else:
        return False

def game_over(state):
    return wins(state, HUMAN) or wins(state, COMP)

def empty_cells(state):
    cells = []
    for x, row in enumerate(state):
        for y, cell in enumerate(row):
            if cell == 0:
                cells.append([x, y])
    return cells

def valid_move(x, y):
    if [x, y] in empty_cells(board):
        return True
    else:
        return False
```

```
def set_move(x, y, player):
    if valid_move(x, y):
        board[x][y] = player
        return True
    else:
        return False

def minimax(state, depth, player):
    if player == COMP:
        best = [-1, -1, -infinity]
    else:
        best = [-1, -1, +infinity]

    if depth == 0 or game_over(state):
        score = evaluate(state)
        return [-1, -1, score]

    for cell in empty_cells(state):
        x, y = cell[0], cell[1]
        state[x][y] = player
        score = minimax(state, depth - 1, -player)
        state[x][y] = 0
        score[0], score[1] = x, y

        if player == COMP:
            if score[2] > best[2]:
                best = score  # max value
        else:
            if score[2] < best[2]:
                best = score  # min value

    return best
```

```
def clean():
    os_name = platform.system().lower()
    if 'windows' in os_name:
        system('cls')
    else:
        system('clear')

def render(state, c_choice, h_choice):
    chars = {
        -1: h_choice,
        +1: c_choice,
        0: ' '
    }
    str_line = '---------------'

    print('\n' + str_line)
    for row in state:
        for cell in row:
            symbol = chars[cell]
            print(f'| {symbol} |', end='')
        print('\n' + str_line)
```

```python
def human_turn(c_choice, h_choice):
    depth = len(empty_cells(board))
    if depth == 0 or game_over(board):
        return

    # Dictionary of valid moves
    move = -1
    moves = {
        1: [0, 0], 2: [0, 1], 3: [0, 2],
        4: [1, 0], 5: [1, 1], 6: [1, 2],
        7: [2, 0], 8: [2, 1], 9: [2, 2],
    }

    clean()
    print(f'Human turn [{h_choice}]')
    render(board, c_choice, h_choice)

    while move < 1 or move > 9:
        try:
            move = int(input('Use numpad (1..9): '))
            coord = moves[move]
            can_move = set_move(coord[0], coord[1], HUMAN)

            if not can_move:
                print('Bad move')
                move = -1
        except (EOFError, KeyboardInterrupt):
            print('Bye')
            exit()
        except (KeyError, ValueError):
            print('Bad choice')
```

```python
def ai_turn(c_choice, h_choice):
    depth = len(empty_cells(board))
    if depth == 0 or game_over(board):
        return

    clean()
    print(f'Computer turn [{c_choice}]')
    render(board, c_choice, h_choice)

    if depth == 9:
        x = choice([0, 1, 2])
        y = choice([0, 1, 2])
    else:
        move = minimax(board, depth, COMP)
        x, y = move[0], move[1]

    set_move(x, y, COMP)
    time.sleep(1)
```

```
clean()
h_choice = ''  # X or O
c_choice = ''  # X or O
first = ''  # if human is the first
while h_choice != 'O' and h_choice != 'X':
    try:
        print('')
        h_choice = input('Choose X or O\nChosen: ').upper()
    except (EOFError, KeyboardInterrupt):
        print('Bye')
        exit()
    except (KeyError, ValueError):
        print('Bad choice')

if h_choice == 'X':
    c_choice = 'O'
else:
    c_choice = 'X'

clean() # Human may starts first
while first != 'Y' and first != 'N':
    try:
        first = input('First to start?[y/n]: ').upper()
    except (EOFError, KeyboardInterrupt):
        print('Bye')
        exit()
    except (KeyError, ValueError):
        print('Bad choice')

while len(empty_cells(board)) > 0 and not game_over(board):  # Main loop of this game
    if first == 'N':
        ai_turn(c_choice, h_choice)
        first = ''
    human_turn(c_choice, h_choice)
    ai_turn(c_choice, h_choice)
```

```
if wins(board, HUMAN): # Game over message
    clean()
    print(f'Human turn [{h_choice}]')
    render(board, c_choice, h_choice)
    print('YOU WIN!')
elif wins(board, COMP):
    clean()
    print(f'Computer turn [{c_choice}]')
    render(board, c_choice, h_choice)
    print('YOU LOSE!')
else:
    clean()
    render(board, c_choice, h_choice)
    print('DRAW!')
```