

微分積分Ⅱ春課題

平成 27 年 3 月 26 日

1

(1)

$$\begin{aligned} & \lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x^3 + x^2 + 3x + 3} \\ &= \lim_{x \rightarrow -1} \frac{(x+2)(x+1)}{(x+1)(x^2+3)} \\ &= \lim_{x \rightarrow -1} \frac{x+2}{x^2+3} \\ &= \frac{1}{4} \end{aligned}$$

(2)

$$\begin{aligned} & \lim_{x \rightarrow \infty} \frac{2x^2 + 9x - 5}{3x^2 - x - 2} \\ &= \lim_{x \rightarrow \infty} \frac{2 + \frac{9}{x} - \frac{5}{x^2}}{3 - \frac{1}{x} - \frac{2}{x^2}} \\ &= \frac{2+0-0}{3-0-0} = \frac{2}{3} \end{aligned}$$

(3)

$$\begin{aligned} & \lim_{x \rightarrow \infty} \frac{1 - 3^x}{3^{x+1} + 2^x} \\ &= \lim_{x \rightarrow \infty} \frac{3^{-x} - 1}{3 + \frac{2^x}{3^x}} \\ &= \frac{0 - 1}{3 + 0} = -\frac{1}{3} \end{aligned}$$

(4)

$$\begin{aligned} & \lim_{x \rightarrow 3-0} \frac{|x-3|}{x^2 - x + 6} \\ &= \lim_{x \rightarrow 3-0} \frac{1}{-x - 2} \\ &= -\frac{1}{5} \end{aligned}$$

(5)

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sin 5x}{4x} \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin 5x}{5x} \cdot \frac{5x}{4x} \right) \\ &= 1 \cdot \frac{5}{4} = \frac{5}{4} \end{aligned}$$

(6)

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \\ &= \lim_{x \rightarrow 0} \frac{\sin x^2}{x^2(1 + \cos x)} \\ &= \lim_{x \rightarrow 0} \frac{\sin x}{x} \lim_{x \rightarrow 0} \frac{\sin x}{x} \lim_{x \rightarrow 0} \frac{1}{1 + \cos x} \\ &= 1 \cdot 1 \cdot \frac{1}{2} = \frac{1}{2} \end{aligned}$$

(7)

$$\begin{aligned} & \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x^2+1} - \sqrt{5}} \\ &= \lim_{x \rightarrow 2} \frac{(x-2)(\sqrt{x^2+1} + \sqrt{5})}{x^2 - 4} \\ &= \lim_{x \rightarrow 2} \frac{\sqrt{x^2+1} + \sqrt{5}}{x+2} \\ &= \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{4} \end{aligned}$$

(8)

$$\begin{aligned} & \lim_{h \rightarrow 0} (1 + 3h)^{\frac{1}{h}} \\ &= \lim_{h \rightarrow 0} (1 + 3h)^{\frac{3}{3h}} \\ &= e^3 \end{aligned}$$

2

(証明) $f(x) = x + \log_2(x^2 + 1) - 1 = 0$ とすると,
 $f(x)$ は $(-\infty, \infty)$ で連続であり
 $f(0) = \log_2 1 - 1 = -1 < 0$
 $f(1) = 1 + \log_2 2 - 1 = 1 > 0$
 よって中間値の定理より
 $f(x)$ は、0 と 1 の間に少なくとも 1 つの実数解を持つ
 Q.E.D.

3

$$\begin{aligned}
f(x) &= \sqrt{x} \\
f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \\
&= \lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} \\
&= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \\
&= \frac{1}{2\sqrt{x}}
\end{aligned}$$

4

(1)

$$\begin{aligned}
y &= (5x-3)^7 \\
y' &= 35(5x-3)^6
\end{aligned}$$

(2)

$$\begin{aligned}
y &= \sqrt{6x+5} \\
y' &= \frac{3}{\sqrt{6x+5}}
\end{aligned}$$

(3)

$$\begin{aligned}
y &= \frac{4x-3}{x+2} \\
y' &= \frac{4(x+2) - 4x+3}{(x+2)^2} \\
&= \frac{11}{(x+2)^2}
\end{aligned}$$

(4)

$$\begin{aligned}
y &= \frac{1}{3x^2-1} \\
y' &= -\frac{6x}{(3x^2-1)^2}
\end{aligned}$$

(5)

$$\begin{aligned}
y &= (x^2+1)^8 \\
y' &= 16x(x^2+1)^7
\end{aligned}$$

(6)

$$\begin{aligned}
y &= \frac{10}{3} \sqrt[5]{x^3} \\
y' &= \frac{10}{3} \cdot \frac{3}{5} \frac{1}{\sqrt[5]{x^2}} \\
&= \frac{2}{\sqrt[5]{x^2}}
\end{aligned}$$

(7)

$$\begin{aligned}
y &= \sqrt[4]{(2x^2+4x+3)^3} \\
y' &= \frac{3}{4} (2x^2+4x+3)^{-\frac{1}{4}} (4x+4) \\
&= \frac{3(x+1)}{\sqrt[4]{2x^2+4x+3}}
\end{aligned}$$

(8)

$$\begin{aligned}
y &= \frac{x}{(1-x)^2} \\
y' &= \frac{(1-x)^2 + 2x(1-x)}{(1-x)^4} \\
&= \frac{1+x}{(1-x)^3}
\end{aligned}$$

(9)

$$\begin{aligned}
y &= (x+1)\sqrt{2x-1} \\
y' &= \sqrt{2x-1} + \frac{x+1}{\sqrt{2x-1}} \\
&= \frac{3x}{\sqrt{2x-1}}
\end{aligned}$$

5

(1)

$$\begin{aligned}
y &= \cos \frac{x}{4} \\
y' &= -\frac{1}{4} \sin \frac{x}{4}
\end{aligned}$$

(2)

$$\begin{aligned}
y &= \sin^4 x \\
y' &= 4 \sin^3 x \cos x
\end{aligned}$$

(3)

$$\begin{aligned}
y &= \cos^3 5x \\
y' &= -15 \cos^2 5x \sin 5x
\end{aligned}$$

(4)

$$\begin{aligned}
y &= x^2 \cos \frac{1}{x} \\
y' &= 2x \cos \frac{1}{x} + x^2 \sin \frac{1}{x} \frac{1}{x^2} \\
&= 2x \cos \frac{1}{x} + \sin \frac{1}{x}
\end{aligned}$$

(5)

$$\begin{aligned}
y &= \frac{\cos x}{1 - \sin x} \\
y' &= \frac{-\sin x(1 - \sin x) + \cos^2 x}{(1 - \sin x)^2} \\
&= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1 - \sin x)^2} \\
&= \frac{1}{1 - \sin x}
\end{aligned}$$

(6)

$$\begin{aligned}
y &= \sin^2 \frac{1}{\sqrt{x}} \\
y' &= 2 \sin \frac{1}{\sqrt{x}} \cos \frac{1}{\sqrt{x}} \left(-\frac{1}{2x\sqrt{x}} \right) \\
&= -\frac{1}{x\sqrt{x}} \sin \frac{1}{\sqrt{x}} \cos \frac{1}{\sqrt{x}}
\end{aligned}$$

(7)

$$\begin{aligned}
y &= x^2 \tan x \\
y' &= 2x \tan x + x^2 \sec^2 x \\
&= x(2 \tan x + x \sec^2 x)
\end{aligned}$$

(8)

$$\begin{aligned}
y &= \frac{1 + \tan x}{1 - \tan x} \\
y' &= \frac{\sec^2 x (1 - \tan x) + \sec^2 x (1 + \tan x)}{(1 - \tan x)^2} \\
&= \frac{2 \sec^2 x}{(1 - \tan x)^2}
\end{aligned}$$

(9)

$$\begin{aligned}
y &= \arcsin \frac{x}{2} \\
y' &= \frac{\frac{1}{2}}{\sqrt{1 - \frac{x^2}{4}}} \\
&= \frac{1}{\sqrt{4 - x^2}}
\end{aligned}$$

(10)

$$\begin{aligned}
y &= \arctan \frac{2}{x} \\
y' &= \frac{-\frac{2}{x^2}}{1 + \frac{4}{x^2}} \\
&= -\frac{2}{x^2 + 4}
\end{aligned}$$

(11)

$$\begin{aligned}
y &= \log(x^2 + 3x + 1) \\
y' &= \frac{2x + 3}{x^2 + 3x + 1}
\end{aligned}$$

(12)

$$\begin{aligned}
y &= \log \left| \frac{1+x}{1-x} \right| \\
y' &= \frac{\frac{1-x+1+x}{(1-x)^2}}{\frac{1+x}{1-x}} \\
&= \frac{2}{1-x^2}
\end{aligned}$$

(13)

$$\begin{aligned}
x &= (e^{3t} + e^{-3t})^5 \\
x' &= 15(e^{3t} + e^{-5t})^4 (e^{3t} - e^{-3t})
\end{aligned}$$

6

(1)

$$\arcsin \frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

(2)

$$\arccos -1 = \pi$$

(3)

$$\arctan -\sqrt{3} = -\frac{\pi}{3}$$

7

(1)

$$\begin{aligned}
y &= \frac{(x+2)^5}{(x+1)^4} \\
\frac{y'}{y} &= \left(\log \frac{(x+2)^5}{(x+1)^4} \right)' \\
&= (5 \log(x+2) - 4 \log(x+1))' \\
&= \frac{5}{x+2} - \frac{4}{x+1} \\
y' &= \frac{(x+2)^4}{(x+1)^4} \left(5 - \frac{4(x+2)}{(x+1)} \right) \\
&= \frac{(x+2)^4(x+3)}{(x+1)^5}
\end{aligned}$$

(2)

$$\begin{aligned}
y &= x^{\sin x} \\
\frac{y'}{y} &= (\log x^{\sin x})' \\
&= (\sin x \log x)' \\
&= \cos x \log x + \frac{1}{x} \sin x \\
y' &= x^{\sin x} \left(\cos x \log x + \frac{1}{x} \sin x \right)
\end{aligned}$$

8

$$\begin{aligned}
x^2 + 2xy - 3y^2 + 6x - 1 &= 0 \\
2x + 2y + 2xy' - 6yy' + 6 &= 0 \\
y'(6y - 2x) &= 2x + 2y + 6 \\
y' &= \frac{x + y + 3}{3y - x}
\end{aligned}$$

9

$$\begin{aligned}
y &= (x+1)^2 \quad (x < -1) \\
x+1 &= \sqrt{y} \\
x &= \sqrt{y} - 1 \\
\therefore y &= \sqrt{x} - 1 \\
y' &= -\frac{1}{2\sqrt{x}}
\end{aligned}$$

10

(1)

$y = x^3 - 2x + 1$ 上に点 $(2, 1)$ は存在しない

$$\begin{aligned}
f(x) &= x^3 - 2x + 1 \\
f'(x) &= 3x^2 - 2 \\
f'(2) &= 10 \\
y - 1 &= 10(x - 2) \\
y &= 10x - 19
\end{aligned}$$

$$\begin{aligned}
f(x) &= x^2 - 2x + 1 \\
f'(x) &= 2x - 2 \\
f'(2) &= 2 \\
y - 1 &= 2(x - 2) \\
y &= 2x - 3
\end{aligned}$$

(2)

$$\begin{aligned}
f(x) &= \log(1 + x^2) \\
f'(x) &= \frac{2x}{1 + x^2} \\
f'(2) &= \frac{4}{5} \\
y - \log 5 &= \frac{4}{5}(x - 2) \\
y &= \frac{4}{5}x - \frac{8}{5} + \log 5
\end{aligned}$$

(3)

$$\begin{aligned}
f(x) &= x^2 - 2x + 1 \\
f(a) &= a^2 - 2a + 1 \\
f'(x) &= 2x - 2 \\
f'(a) &= 2a - 2 \\
y - f(a) &= f'(a)(x - a) \\
\text{この直線は点 } (1, -1) \text{ 上を通るから} \\
-1 - a^2 + 2a - 1 &= 2a - 2(1 - a) \\
a^2 + 2a &= 0 \\
\therefore a &= 0, 2 \\
a = 0 \text{ のとき } y - 1 &= -2(x - 0) \therefore y = -2x + 1 \\
a = 2 \text{ のとき } y - 1 &= 2(x - 2) \therefore y = 2x - 3
\end{aligned}$$

11

(1)

$$\frac{dy}{dx} = -\frac{3 \sin^2 t \cos t}{3 \cos^2 t \sin t} = -\tan t$$

(2)

$$\begin{aligned}
x &= \cos^3 t = \frac{1}{2\sqrt{2}} \\
y &= \sin^3 t = \frac{1}{2\sqrt{2}} \\
y - \frac{1}{2\sqrt{2}} &= -\left(x - \frac{1}{2\sqrt{2}}\right) \\
y &= -x + \frac{1}{\sqrt{2}}
\end{aligned}$$

12

(1)

$$\begin{aligned}
y &= \frac{1}{x} \\
y' &= -\frac{1}{x^2} \\
y - \frac{1}{a} &= -\frac{1}{a^2}(x - a) \\
y &= -\frac{x}{a^2} + \frac{2}{a}
\end{aligned}$$

(2)

この直線と x 軸の交点は

$$0 = -\frac{x}{a^2} + \frac{2}{a} \therefore x = 2a$$

この直線と y 軸の交点は

$$y = -\frac{0}{a^2} + \frac{2}{a} \therefore y = \frac{2}{a}$$

よって三角形の面積は

$$2a \cdot \frac{2}{a} \cdot \frac{1}{2} = 2 \text{ となり、常に一定である.}$$

13

$$\begin{aligned}
f(h) &\doteq f(0) + f'(0)h \\
f(h) &= \sqrt{1+h} \quad f(0) = 1 \\
f'(h) &= \frac{1}{2\sqrt{1+h}} \quad f'(0) = \frac{1}{2} \\
\therefore 1 + \frac{1}{2}h
\end{aligned}$$

14

$$y = x^4 - 2x^2 + 1 \quad (\text{証明}) y = e^x - 1 - x \quad (x > 0)$$

$$y' = 4x^3 - 4x \quad y' = 0 \text{ を解くと } x = 0, \pm 1 \quad y' = e^x - 1 \quad y' = 0 \text{ を解くと } x = 0$$

$$y'' = 12x^2 - 4 \quad y'' = 0 \text{ を解くと } x = \pm \frac{1}{\sqrt{3}} \quad x > 0 \text{ のとき } y > 0$$

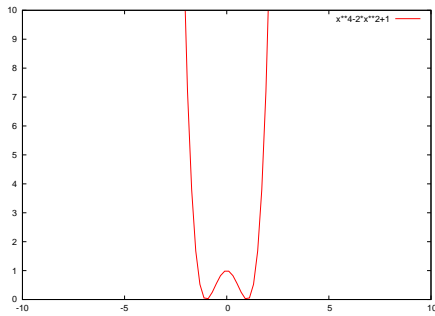
$$\therefore y > 0$$

$$\therefore e^x > 1 + x$$

Q.E.D.

x	\dots	-1	\dots	$-\frac{1}{\sqrt{3}}$	\dots	0	\dots	$\frac{1}{\sqrt{3}}$	\dots	1	\dots
y'	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$
y''	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
y	\searrow	0	\nearrow	$\frac{4}{9}$	\nearrow	1	\searrow	$\frac{4}{9}$	\searrow	0	\nearrow

$$\therefore \text{変曲点 } (\pm \frac{1}{\sqrt{3}}, \frac{4}{9})$$



15

$$y = x \log x$$

$$y' = \log x + 1 \quad y' = 0 \text{ を解くと } x = 0, \pm 1$$

x	0	\dots	$\frac{1}{e}$	\dots
y'	\nearrow	$-$	0	$+$
y	\nearrow	\searrow	0	\nearrow

$$\therefore x = \frac{1}{e} \text{ のとき極小値 } -\frac{1}{e}, \text{ 極大値なし}$$

16

$$y = 2 \sin x - x$$

$$y' = 2 \cos x - 1 \quad y' = 0 \text{ を解くと } x = \frac{\pi}{3}, \frac{5\pi}{3}$$

x	0	\dots	$\frac{\pi}{3}$	\dots	$\frac{5\pi}{3}$	\dots	2π
y'	1	$+$	0	$-$	0	$+$	1
y	0	\nearrow	$\sqrt{3} - \frac{\pi}{3}$	\searrow	$-\sqrt{3} - \frac{5\pi}{3}$	\nearrow	-2π

$$\therefore x = \frac{\pi}{3} \text{ のとき最大値 } \sqrt{3} - \frac{\pi}{3}$$

$$x = \frac{5\pi}{3} \text{ のとき最小値 } -\sqrt{3} - \frac{5\pi}{3}$$

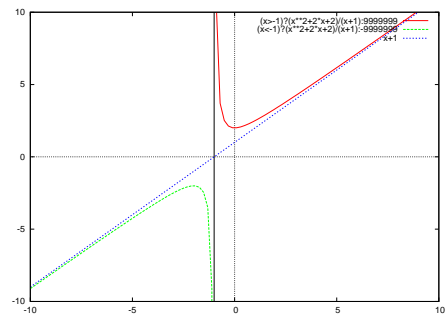
17

$$y = \frac{x^2 + 2x + 2}{x + 1} = x + 1 + \frac{1}{x + 1}$$

$$y' = 1 - \frac{1}{(x + 1)^2} = \frac{x^2 + 2}{(x + 1)^2}$$

$$y' = 0 \text{ を解くと } x = 0, 2, x \neq -1$$

x	\dots	-2	\dots	-1	\dots	0	\dots
y'	$+$	0	$-$	\nearrow	$-$	0	$+$
y	\nearrow	-2	\searrow	\nearrow	\searrow	2	\nearrow



19

(1)

$$\int (3x + s)^4 dx = \frac{1}{15} (3x + 2)^5$$

(2)

$$\int \frac{1}{\sqrt[3]{x}} dx = \frac{3}{2} \sqrt[3]{x^2}$$

(3)

$$\int \frac{1}{2x - 3} dx = \frac{1}{2} \log |2x - 3|$$

(4)

$$\begin{aligned} & \int \frac{1}{\cos^2 3x} dx \\ &= \frac{1}{3} \tan 3x \end{aligned}$$

(5)

$$\begin{aligned} & \int \frac{e^{4x} - e^x}{e^{2x}} dx \\ &= \int (e^{2x} - e^{-x}) dx \\ &= \frac{1}{2} e^{2x} + e^{-x} \end{aligned}$$

(6)

$$\begin{aligned} & \int \frac{1}{1+4x^2} dx \\ &= \frac{1}{2} \arctan 4x \end{aligned}$$

(7)

$$\begin{aligned} & \int \frac{1}{\sqrt{2-x^2}} dx \\ &= \arcsin \frac{x}{\sqrt{2}} \end{aligned}$$

(8)

$$\begin{aligned} & \int \frac{x^2-3}{x^2+3} dx \\ &= \int 1 - \frac{6}{x^2+3} \\ &= x - 2\sqrt{3} \arctan \frac{x}{\sqrt{3}} \end{aligned}$$

(9)

$$\begin{aligned} & \int \frac{x}{\sqrt{3+x} - \sqrt{3-x}} dx \\ &= \int \frac{x(\sqrt{3+x} + \sqrt{3-x})}{2x} dx \\ &= \frac{1}{2} \int (\sqrt{3+x} + \sqrt{3-x}) dx \\ &= \frac{1}{3} (\sqrt{3+x}^3 - \sqrt{3-x}^3) \end{aligned}$$

(10)

20**21****22****23****24****25**

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)