微分積分 I 夏課題

平成 26 年 9 月 24 日

1

(1)

$$\begin{aligned} y &= (x^2+2x)(3x+1) \\ y' &= (2x+2)(3x+1)+(x^2+2x)(3) \\ &= 6x^2+8x+2+3x^2+6x \\ &= 9x^2+14x+2 \end{aligned}$$

(2)

$$\begin{aligned} y &= (2x^3+6x)(7x-5) \\ y' &= (6x^2+6)(7x-5)+(2x^3+6x)(7) \\ &= 42x^3-30x^2+42x-30+14x^3+42x \\ &= 56x^3-30x^2+84x-30 \end{aligned}$$

(3)

$$\begin{aligned} y &= (x+1)(x^2-4x-5) \\ y' &= 1(x^2-4x-5)+(x+1)(2x-4) \\ &= x^2-4x-5+2x^2-2x-4 \\ &= 3x^2-6x-9 \end{aligned}$$

(4)

$$\begin{aligned} y &= (x-1)(x-2)(x-3) \\ y' &= 1(x-2)(x-3)+1(x-1)(x-3) \\ &\quad +1(x-1)(x-2) \\ &= x^2-5x+6+x^2-4x+3+x^2-3x+2 \\ &= 3x^2-12x+11 \end{aligned}$$

2

(1)

$$\begin{aligned} y &= \frac{2x-3}{3x+1} \\ y' &= \frac{(2)(3x+1)-(2x-3)(3)}{(3x+1)^2} \\ &= \frac{6x+2-6x+9}{(3x+1)^2} \\ &= \frac{11}{(3x+1)^2} \end{aligned}$$

(2)

$$\begin{aligned} y &= \frac{x^2-1}{3x^2+1} \\ y' &= \frac{(2x)(3x^2+1)-(x^2-1)(6x)}{(3x^2+1)^2} \\ &= \frac{6x^3+2x-6x^3+6x}{(3x^2+1)^2} \\ &= \frac{8x}{(3x^2+1)^2} \end{aligned}$$

(3)

$$\begin{aligned} y &= \frac{1}{2x-3} \\ y' &= -\frac{2}{(2x-3)^2} \end{aligned}$$

(4)

$$\begin{aligned} y &= \frac{x+1}{x^2+2x+2} \\ y' &= \frac{(1)(x^2+2x+2)-(x+1)(2x+2)}{(x^2+2x+2)^2} \\ &= \frac{x^2+2x+2-2x^2-4x-2}{(x^2+2x+2)^2} \\ &= \frac{-x^2-2x}{(x^2+2x+2)^2} \\ &= -\frac{x(x+2)}{(x^2+2x+2)^2} \end{aligned}$$

3

(1)

$$\begin{aligned} y &= \frac{1}{x^2} \\ y' &= -\frac{2x}{x^4} \\ &= -\frac{2}{x^3} \end{aligned}$$

(2)

$$\begin{aligned} y &= \frac{3}{x}-\frac{2}{x^3} \\ y' &= -\frac{3}{x^2}+\frac{6x^2}{x^4} \end{aligned}$$

(3)

$$
\begin{aligned}
y &= (1-x^2)(1+\frac{1}{x^2}) \\
y' &= (-2x)(1+\frac{1}{x^2}) + (1-x^2)(-\frac{2}{x^3}) \\
&= -2x - \frac{2}{x} - \frac{2}{x^3} + \frac{2}{x} \\
&= -2x - \frac{2}{x^3}
\end{aligned}
$$

(4)

$$
\begin{aligned}
y &= (2x-3)^3 \\
y' &= 3(2x-3)^2(2) \\
&= 6(2x-3)^2
\end{aligned}
$$

(5)

$$
\begin{aligned}
y &= (x^2+1)^4 \\
y' &= 4(x^2+1)^3(2x) \\
&= 8x(x^2+1)^3
\end{aligned}
$$

(6)

$$
\begin{aligned}
y &= (3x^2-4x+1)^2 \\
y' &= 2(3x^2-4x+1)(6x-4) \\
&= 4(3x-2)(3x^2-4x+1)
\end{aligned}
$$

(7)

$$
\begin{aligned}
y &= \frac{1}{(3-4x)^2} \\
y' &= -\frac{2(3-4x)(-4)}{(3-4x)^4} \\
&= \frac{8(3-4x)}{(3-4x)^4} \\
&= \frac{8}{(3-4x)^3}
\end{aligned}
$$

(8)

$$
\begin{aligned}
y &= \frac{1}{(x^2-1)^4} \\
y' &= -\frac{4(x^2-1)^3(2x)}{(x^2-1)^8} \\
&= -\frac{8x(x^2-1)^3}{(x^2-1)^8} \\
&= -\frac{8x}{(x^2-1)^5}
\end{aligned}
$$

4

(1)

$$
\begin{aligned}
y &= \sqrt[3]{x^2} \\
y' &= \frac{2}{3}\sqrt[3]{x}
\end{aligned}
$$

(2)

$$
\begin{aligned}
y &= \sqrt{3x+1} \\
y' &= \frac{1}{2}(3x+1)^{-\frac{1}{2}}(3) \\
&= \frac{3}{2\sqrt{3x+1}}
\end{aligned}
$$

(3)

$$
\begin{aligned}
y &= \sqrt[3]{x+3} \\
y' &= \frac{1}{3\sqrt[3]{(x+3)^2}}
\end{aligned}
$$

(4)

$$
\begin{aligned}
y &= \sqrt{x^2+2} \\
y' &= \frac{2x}{2\sqrt{x^2+2}} \\
&= \frac{x}{\sqrt{x^2+2}}
\end{aligned}
$$

(5)

$$
\begin{aligned}
y &= \frac{\sqrt{x}}{x+1} \\
y' &= \frac{\frac{1}{2\sqrt{x}}(x+1) - \sqrt{x}(1)}{(x+1)^2} \\
&= \frac{x+1-2x}{2\sqrt{x}(x+1)^2} \\
&= \frac{1-x}{2\sqrt{x}(x+1)^2}
\end{aligned}
$$

(6)

$$
\begin{aligned}
y &= \frac{1}{\sqrt{2x+1}} \\
y' &= -\frac{\frac{2}{2\sqrt{2x+1}}}{2x+1} \\
&= -\frac{1}{\sqrt{2x+1}(2x+1)}
\end{aligned}
$$

5

(1)

$$
\begin{aligned}
y &= x^2(2x+3)^2 \\
y' &= 2x(2x+3)^2 + (x^2)2(2x+3)(2) \\
&= 2x(2x+3)(2x+3+2x) \\
&= 2x(2x+3)(4x+3)
\end{aligned}
$$

(2)

$$
\begin{aligned}
y &= (x-1)^2(2x+1)^3 \\
y' &= 2(x-1)(2x+1)^3 + (x-1)^2 3(2x+1)^2(2) \\
&= (x-1)(2x+1)^2(2(2x+1)+6(x-1)) \\
&= (x-1)(2x+1)^2(10x-4) \\
&= 2(x-1)(2x+1)^2(5x-2)
\end{aligned}
$$

(3)

$$
\begin{aligned}
y &= \frac{(4+x^2)^2}{4-x^2} \\
y' &= \frac{2(4+x^2)(2x)(4-x^2)-(4+x^2)^2(-2x)}{(4-x^2)^2} \\
&= \frac{2x(4+x^2)(2(4-x^2)+(4+x^2))}{(4-x^2)^2} \\
&= \frac{2x(4+x^2)(8-2x^2+4+x^2)}{(4-x^2)^2} \\
&= \frac{2x(4+x^2)(-x^2+12)}{(4-x^2)^2}
\end{aligned}
$$

(4)

$$
\begin{aligned}
y &= \frac{(2x+1)^3}{(x^2-3)^2} \\
y' &= \frac{1}{(x^2-3)^2}\{3(2x+1)^2(2)(x^2-3)^2 \\
&\quad -(2x+1)^3 2(x^2-3)(2x)\} \\
&= \frac{6(2x+1)^2(x^2-3)^2-4x(2x+1)^3(x^2-3)}{(x^2-3)^4} \\
&= \frac{2(2x+1)^2(3(x^2-3)-2x(2x+1))}{(x^2-3)^3} \\
&= \frac{2(2x+1)^2(3x^2-9-4x^2-2x)}{(x^2-3)^3} \\
&= -\frac{2(2x+1)^2(x^2+2x+9)}{(x^2-3)^3}
\end{aligned}
$$

6

(1)

$$
\begin{aligned}
y &= \sqrt{2x+1} \\
y' &= \frac{2}{2\sqrt{2x+1}} \\
&= \frac{1}{\sqrt{2x+1}}
\end{aligned}
$$

(2)

$$
\begin{aligned}
y &= \frac{1+x^2}{\sqrt{x}} \\
y' &= \frac{2x\sqrt{x}-(1+x^2)\frac{1}{2\sqrt{x}}}{x} \\
&= \frac{4x^2-(1+x^2)}{2x\sqrt{x}} \\
&= \frac{3x^2-1}{2x\sqrt{x}}
\end{aligned}
$$

(3)

$$
\begin{aligned}
y &= \sqrt{x^2+3x+5} \\
y' &= \frac{2x+3}{2\sqrt{x^2+3x+5}}
\end{aligned}
$$

(4)

$$
\begin{aligned}
y &= \frac{1}{\sqrt{x^2+2x+2}} \\
y' &= -\frac{\frac{2x+2}{2\sqrt{x^2+2x+2}}}{x^2+2x+2} \\
&= \frac{x+1}{(x^2+2x+2)\sqrt{x^2+2x+2}}
\end{aligned}
$$

7

(1)

$$
\begin{aligned}
y &= \log(2x-3) \\
y' &= \frac{2}{2x-3}
\end{aligned}
$$

(2)

$$
\begin{aligned}
y &= log(x^2+2x+2) \\
y' &= \frac{2x+2}{x^2+2x+2}
\end{aligned}
$$

(3)

$$
\begin{aligned}
y &= \log\sqrt{1-x^2} \\
y' &= \frac{\frac{-2x}{2\sqrt{1-x^2}}}{\sqrt{1-x^2}} \\
&= \frac{x}{x^2-1}
\end{aligned}
$$

(4)

$$
\begin{aligned}
y &= x^2\log x \\
y' &= 2x\log x + x^2\frac{1}{x} \\
&= 2x\log x + x
\end{aligned}
$$

(5)

$$
\begin{aligned}
y &= \frac{1}{x+logx} \\
y' &= -\frac{1+\frac{1}{x}}{(x+\log x)^2} \\
&= -\frac{x+1}{x(x+\log x)^2}
\end{aligned}
$$

(6)

$$
\begin{aligned}
y &= x\log(x^2+1) \\
y' &= \log(x^2+1)+\frac{2x^2}{x^2+1}
\end{aligned}
$$

8

(1)

$$
\begin{aligned}
y &= e^{2x} \\
y' &= 2e^{2x}
\end{aligned}
$$

(2)

$$y = e^{2-3x}$$
$$y' = -3e^{2-3x}$$

(3)

$$y = xe^{-2x}$$
$$y' = e^{-2x} + x(-2e^{-2x})$$
$$= (1-2x)e^{-2x}$$

(4)

$$y = xe^{-x^2}$$
$$y' = e^{-x^2} + x(-2x)e^{-x^2}$$
$$= (1-2x^2)e^{-x^2}$$

(5)

$$y = (x+1)a^{2x}$$
$$y' = a^{2x} + (x+1)(\log a)a^{2x}(2)$$
$$= a^{2x}\{1 + 2(x+1)\log a\}$$

(6)

$$y = \frac{1}{1-e^{2x}}$$
$$y' = \frac{2e^{2x}}{(1-e^{2x})^2}$$

9

(1)

$$y = x^x$$
$$\frac{y'}{y} = (x\log x)'$$
$$= \log x + 1$$
$$y' = x^x(\log x + 1)$$

(2)

$$y = 3^{2x}$$
$$\frac{y'}{y} = (2x\log 3)'$$
$$= 2\log 3$$
$$y' = 2\log 3 \cdot 3^{2x}$$

10

(1)

$$y = \sin 3x$$
$$y' = 3\cos 3x$$

(2)

$$y = \cos 4x$$
$$y' = -4\sin 4x$$

(3)

$$y = \tan 4x$$
$$y' = 4\sec^2 4x$$

(4)

$$y = 2\sin 5x - 7$$
$$y' = 10\cos 5x - 7$$

(5)

$$y = \tan^2 x$$
$$y' = 2\tan x\sec^2 x$$

(6)

$$y = \frac{\cos x}{1+\sin x}$$
$$y' = \frac{-\sin x(1+\sin x) - \cos x(\cos x)}{(1+\sin x)^2}$$
$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2}$$
$$= -\frac{1}{1+\sin x}$$

(7)

$$y = \frac{1}{1-\cos x}$$
$$y' = -\frac{\sin x}{(1-\cos x)^2}$$

(8)

$$y = \sqrt{1+\cos 2x}$$
$$y' = \frac{-2\sin 2x}{2\sqrt{1+\cos 2x}}$$
$$= -\frac{\sin 2x}{\sqrt{1+\cos 2x}}$$

11

(1)

$$y = \log(x^3 - 3x + 1)$$
$$y' = \frac{3x^2-3}{x^3-3x+1}$$
$$= \frac{3(x^2-1)}{x^3-3x+1}$$

(2)

$$y = \log(x + \sqrt{x^2+A})$$
$$y' = \frac{1 + \frac{2x}{2\sqrt{x^2+A}}}{(x+\sqrt{x^2+A})}$$
$$= \frac{\sqrt{x^2+A}+x}{\sqrt{x^2+A}(x+\sqrt{x^2+A})}$$
$$= \frac{1}{\sqrt{x^2+A}}$$

(3)

$$\begin{aligned} y &= x(\log x - 1) \\ &= x\log x - x \\ y' &= \log x + 1 - 1 \\ &= \log x \end{aligned}$$

(4)

$$\begin{aligned} y &= \frac{1}{2a}\log\frac{x-a}{x+a} \\ &= \frac{1}{2a}\{\log(x-a) - \log(x+a)\} \\ y' &= \frac{1}{2a}\{\frac{1}{x-a} - \frac{1}{x+a}\} \\ &= \frac{1}{x^2-a^2} \end{aligned}$$

(5)

$$\begin{aligned} y &= \frac{x}{e^x - 1} \\ y' &= \frac{(e^x-1) - x(e^x)}{(e^x-1)^2} \\ &= \frac{e^x(1-x) - 1}{(e^x-1)^2} \end{aligned}$$

(6)

$$\begin{aligned} y &= e^{sinx} \\ y' &= \cos x e^{\sin x} \end{aligned}$$

12

(1)

$$\begin{aligned} y &= a^{\sqrt{x}} \\ \frac{y'}{y} &= (\sqrt{x}\log a)' \\ &= \frac{1}{2\sqrt{x}}\log a \\ y' &= \frac{a^{\sqrt{x}}}{2\sqrt{x}}\log a \end{aligned}$$

(2)

$$\begin{aligned} y &= (\cos x)^{\sin x} \\ \frac{y'}{y} &= (\sin x \log\cos x)' \\ &= \cos x \log\cos x + \sin x \tan x \\ y' &= \{\cos x \log\cos x + \sin x\tan x\}(\cos x)^{\sin x} \end{aligned}$$

(3)

$$\begin{aligned} y &= x^{\log x} \\ \frac{y'}{y} &= (\log x \log x)' \\ &= \frac{2}{x}\log x \\ y' &= \frac{2}{x}\log x\, x^{\log x} \\ &= 2\log x\, x^{\log x - 1} \end{aligned}$$

(4)

$$\begin{aligned} y &= x^{\sqrt{x}} \\ \frac{y'}{y} &= (\sqrt{x}\log x)' \\ &= \frac{1}{2\sqrt{x}}\log x + \sqrt{x}\frac{1}{x} \\ &= \frac{1}{2\sqrt{x}}(\log x + 2) \\ y' &= \frac{1}{2\sqrt{x}}(\log x + 2)x^{\sqrt{x}} \end{aligned}$$

13

(1)

$$\begin{aligned} y &= x\cos x \\ y' &= \cos x - x\sin x \end{aligned}$$

(2)

$$\begin{aligned} y &= x^2 \sin\frac{1}{2} \\ y' &= 2x\sin\frac{1}{2} \end{aligned}$$

(3)

$$\begin{aligned} y &= \frac{1-\sin x}{1+\sin x} \\ y' &= \frac{(-\cos x)(1+\sin x) - (1-\sin x)(\cos x)}{(1+\sin x)^2} \\ &= \frac{-\cos x - \cos x\sin x - \cos x + \cos x\sin x}{(1+\sin x)^2} \\ &= \frac{-2\cos x}{(1+\sin x)^2} \end{aligned}$$

(4)

$$\begin{aligned} y &= \frac{\sin 2x}{1+\cos 2x} \\ y' &= \frac{(2\cos 2x)(1+\cos 2x) - (\sin 2x)(-2\sin 2x)}{(1+\cos 2x)^2} \\ &= \frac{2\cos 2x + 2\cos^2 2x + 2\sin^2 2x}{(1+\cos 2x)^2} \\ &= \frac{2}{1+\cos 2x} \end{aligned}$$

(5)

$$\begin{aligned} y &= \frac{\sin x + \cos x}{\sin x - \cos x} \\ y' &= \frac{1}{(\sin x - \cos x)^2}\{(\cos x - \sin x)(\sin x - \cos x) \\ &\qquad - (\sin x + \cos x)(\cos x + \sin x)\} \\ &= \frac{(2\sin x\cos x - 1) - (2\sin x\cos x + 1)}{(\sin x - \cos x)^2} \\ &= -\frac{2}{(\sin x - \cos x)^2} \end{aligned}$$

(6)

$$y = \log\left|\tan\frac{x}{2}\right|$$
$$y' = \frac{\frac{1}{2}\sec^2\frac{x}{2}}{\tan\frac{x}{2}}$$
$$= \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$
$$= \frac{1}{\sin x}$$

14

(1) $\begin{cases} x = t^2+1 \\ y = \log t \end{cases} \quad (t=1)$

$t=1$ における点を (x_1, y_1) とすると

$(x_1, y_1) = (2, 0)$.

接線の傾きは

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{1}{t}}{2t} = \frac{1}{2} \quad \because (t=1)$

よって接線の方程式は

$y - 0 = \frac{1}{2}(x-2)$

$y = \frac{1}{2}x - 1$

法線の傾きは

$-2 \quad \because \frac{dy}{dx} = \frac{1}{2}$

よって法線の方程式は

$y - 0 = -2(x-2)$

$y = -2x + 4$

(2)

$$\begin{cases} x = \frac{3t}{1+t^3} \\ y = \frac{3t^3}{1+t^3} \end{cases} \quad (t=2)$$

$t=2$ における点を (x_1, y_1) とすると

$$(x_1, y_1) = (\frac{2}{3}, \frac{8}{3})$$

接線の傾きは

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{(9t^2)(1+t^3)-(3t^3)(3t^2)}{(1+t^3)^2}}{\frac{(3)(1+t^3)-(3t)(3t^2)}{(1+t^3)^2}}$$
$$= \frac{(9t^2)(1+t^3)-(3t^3)(3t^2)}{(3)(1+t^3)-(3t)(3t^2)} = \frac{9t^2+9t^5-9t^5}{3+3t^3-9t^3}$$
$$= \frac{9t^2}{3-6t^3} = \frac{36}{-45} \quad \because (t=2) \quad = -\frac{4}{5}$$

よって接線の方程式は

$$y - \frac{8}{3} = -\frac{4}{5}(x - \frac{2}{3})$$
$$y = -\frac{4}{5}x + \frac{16}{5}$$

法線の傾きは

$$\frac{5}{4} \quad \because \frac{dy}{dx} = -\frac{4}{5}$$

よって法線の方程式は

$$y - \frac{8}{3} = \frac{5}{4}(x - \frac{2}{3})$$
$$y = \frac{5}{4}x + \frac{11}{6}$$

15

(1)

$$x^2 + 2xy^2 - 3y^4 = 0$$
$$2x + 2y^2 + 4xyy' - 12y^3y' = 0$$
$$y'(4xy - 12y^3) = -2x - 2y^2$$
$$y' = -\frac{2x+2y^2}{4xy-12y^3}$$
$$y' = -\frac{x+y^2}{2y(x-3y^2)}$$

(2)

$$\frac{(x-1)^2}{4} + \frac{(y+3)^2}{9} = 1$$
$$\frac{2(x-1)}{4} + \frac{2y'(y+3)}{9} = 0$$
$$9(x-1) + 4y'(y+3) = 0$$
$$y' = -\frac{9(x-1)}{4(y+3)}$$

(3)

$$\sin(xy) = y$$
$$(y + xy')\cos(xy) = y'$$
$$y'\{x\cos(xy) - 1\} = -y\cos(xy)$$
$$y' = \frac{y\cos(xy)}{1 - x\cos(xy)}$$

(4)

$$\frac{1}{2}\log(x^2+y^2) = \tan^{-1}\frac{y}{x}$$
$$\frac{2x+2yy'}{2(x^2+y^2)} = \frac{\frac{y'x-y}{x^2}}{1+\frac{y^2}{x^2}}$$
$$\frac{x+yy'}{x^2+y^2} = \frac{y'x-y}{x^2+y^2}$$
$$yy' - y'x + x + y = 0$$
$$y'(y-x) = -x - y$$
$$y' = \frac{x+y}{x-y}$$

16

(1)

$$\begin{array}{r} x^2 \ -3x \quad +2 \\ \hline x^2+2x-1 \,)\, x^4 \ -x^3 \ -5x^2 \ +7x \ -2 \\ x^4 \ +2x^3 \ -x^2 \\ \hline -3x^3 \ -4x^2 \ +7x \\ -3x^3 \ -6x^2 \ +3x \\ \hline 2x^2 \ +4x \ -2 \\ 2x^2 \ +4x \ -2 \\ \hline 0 \end{array}$$

商 x^2-3x+2 余り 0

(2)

$$
\begin{array}{r}
x^4\ -x^3\ \ +x\ \ -1 \\
3x-2\ \overline{)\ 3x^5-5x^4+2x^3+3x^2-5x+5} \\
\underline{3x^5-2x^4} \\
-3x^4+2x^3 \\
\underline{-3x^4+2x^3} \\
3x^2 \\
\\
\underline{\qquad\qquad} \\
3x^2-5x \\
\underline{3x^2-2x} \\
-3x+5 \\
\underline{-3x+2} \\
3
\end{array}
$$

商 x^4-x^3+x-1 余り 3

(3)

$$
\begin{array}{r}
x^3-2x^2\ +3x\ \ -7 \\
x^2\ -3\ \overline{)\ x^5-2x^4\ \ \ \ -x^2\ +3x\ -1} \\
\underline{x^5\ \ \ -3x^3} \\
-2x^4+3x^3\ -x^2 \\
\underline{-2x^4\ \ \ +6x^2} \\
3x^3-7x^2\ +3x \\
\underline{3x^3\ \ \ -9x} \\
-7x^2+12x\ -1 \\
\underline{-7x^2\ \ \ +21} \\
12x-22
\end{array}
$$

商 x^3-2x^2+3x-7 余り $12x-22$

17

(1)

$$\frac{3x^2-4x+5}{x-2}=3x+2+\frac{9}{x-2}$$

(2)

$$\frac{3x^3+2x^2-5x+3}{x^2-2x+3}=3x+8+\frac{2x-21}{x^2-2x+3}$$

18

(1)

$$
\begin{aligned}
&\frac{2a^2+a-3}{a^2-2a-3}\div\frac{2a^2+5a+3}{a^2-4a+3}\times\frac{a^3+1}{a^3-1} \\
&=\frac{(2a+3)(a-1)(a-1)(a-3)(a+1)(a^2-a+1)}{(a-3)(a+1)(2a+3)(a+1)(a-1)(a^2+a+1)} \\
&=\frac{(a-1)(a^2-a+1)}{(a+1)(a^2+a+1)}
\end{aligned}
$$

(2)

$$
\begin{aligned}
&\frac{1}{x(x+1)}+\frac{1}{(x+1)(x+2)}+\frac{1}{(x+2)(x+3)} \\
&=\frac{(x+2)(x+3)+x(x+3)+x(x+1)}{x(x+1)(x+2)(x+3)} \\
&=\frac{x^2+5x+6+x^2+3x+x^2+x}{x(x+1)(x+2)(x+3)} \\
&=\frac{3x^2+9x+6}{x(x+1)(x+2)(x+3)} \\
&=\frac{3}{x(x+3)}
\end{aligned}
$$

(3)

$$
\begin{aligned}
&\frac{a}{(a-b)(a-c)}+\frac{b}{(b-c)(b-a)}+\frac{c}{(c-a)(c-b)} \\
&=\frac{-a(b-c)-b(c-a)-c(a-b)}{(a-b)(b-c)(c-a)} \\
&=\frac{-ab+ac-bc+ab-ac+bc}{(a-b)(b-c)(c-a)} \\
&=0
\end{aligned}
$$

19

(1)

$$
\begin{aligned}
&(3-2i)^3 \\
&=27-54i-36+8i \\
&=-9-46i
\end{aligned}
$$

(2)

$$
\begin{aligned}
&(1+i)(2+i)(3+i)(4+i) \\
&=(4+5i-1)(6+5i-1) \\
&=(3+5i)(5+5i) \\
&=-10+40i
\end{aligned}
$$

(3)

$$
\begin{aligned}
&1+\frac{1}{i}+\frac{1}{i^2}+\frac{1}{i^3} \\
&=1+\frac{1}{i}-\frac{1}{1}-\frac{1}{i} \\
&=0
\end{aligned}
$$

(4)

$$
\begin{aligned}
&\frac{2-i}{3+2i} \\
&=\frac{(2-i)(3-2i)}{9+4} \\
&=\frac{4}{13}-\frac{7}{13}i
\end{aligned}
$$

(5)

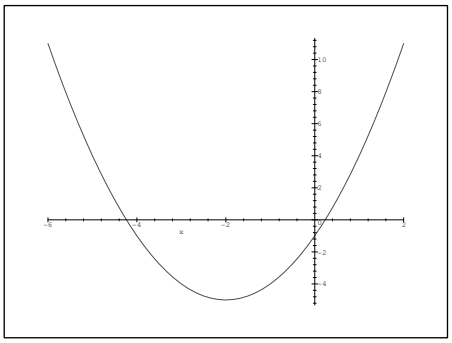
$$
\begin{aligned}
&\left(\frac{1-3i}{1+3i}-\frac{1+3i}{1-3i}\right)\times\frac{2+i}{2-i} \\
&=\frac{(-8-6i)-(-8+6i)}{10}\times\frac{2+i}{2-i} \\
&=\frac{-6i}{5}\frac{3+4i}{5} \\
&=\frac{24}{25}-\frac{18}{25}i
\end{aligned}
$$

20

(1)

$$\begin{aligned} y &= x^2 + 4x - 1 \\ &= (x+2)^2 - 5 \end{aligned}$$

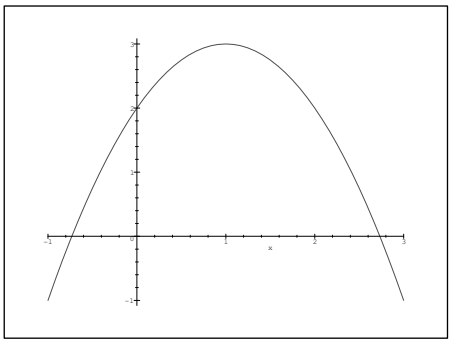
頂点 $(-2, -5)$



(2)

$$\begin{aligned} y &= -x^2 + 2x + 2 \\ &= -(x-1)^2 + 3 \end{aligned}$$

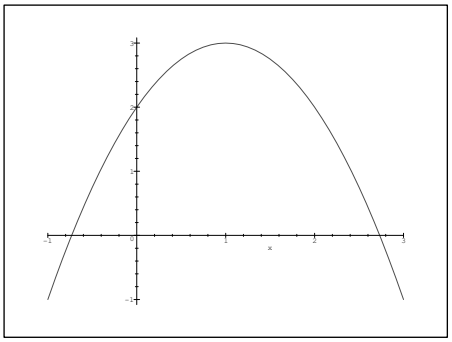
頂点 $(1, 3)$



(3)

$$\begin{aligned} y &= 2x^2 + 4x - 1 \\ &= 2(x+1)^2 - 3 \end{aligned}$$

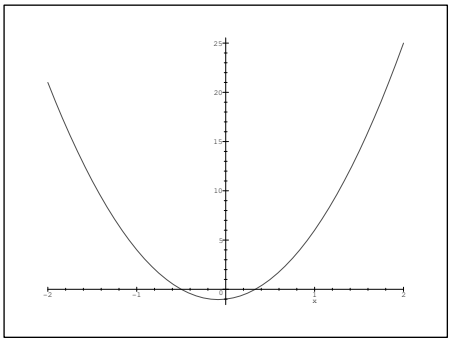
頂点 $(-1, -3)$



(4)

$$\begin{aligned} y &= 6x^2 + x - 1 \\ &= 6(x + \frac{1}{12})^2 - \frac{25}{24} \end{aligned}$$

頂点 $(-\frac{1}{12}, -\frac{25}{24})$



21

(1)

$$\begin{aligned} y &= x^2 - 4x + 1 \\ y' &= 2x - 4 = 0 \\ x &= 2 \end{aligned}$$

最大値なし，$x = 2$ のとき最小値 $y = -3$

(2)

$$\begin{aligned} y &= 4 - 2x - x^2 \\ y' &= -2 - 2x = 0 \\ x &= -1 \end{aligned}$$

$x = -1$ のとき最大値 $y = 5$，最小値なし

(3)

$$y = 6x^2 - x - 2$$
$$y' = 12x - 1 = 0$$
$$x = \frac{1}{12}$$

最大値なし，$x = \frac{1}{12}$ のとき最小値 $y = \frac{49}{24}$

(4)

$$y = 10 + 3x - x^2$$
$$y' = 3 - 2x = 0$$
$$x = \frac{3}{2}$$

$x = \frac{3}{2}$ のとき最大値 $\frac{49}{4}$，最小値なし

22

(1)

$$y = x^2 - 4x + 5, \quad (-1 \leqq x \leqq 3)$$
$$y' = 2x - 4 = 0$$
$$x = 2$$

$x = -1$ のとき最大値 $y = 10$，$x = 2$ のとき最小値 $y = 1$

(2)

$$y = -x^2 - 2x - 4, \quad (-2 \leqq x \leqq 1)$$
$$y' = -2x - 2 = 0$$
$$x = -1$$

$x = -1$ のとき最大値 $y = -3$，$x = 1$ のとき最小値 $y = -7$

(3)

$$y = x^2 - 4x + 2, \quad (-2 \leqq x \leqq 0)$$
$$y' = 2x - 4 = 0$$
$$x = 2$$

$x = -2$ の時最大値 $y = 14$，$x = 0$ の時最小値 $y = 2$

(4)

$$y = -2x^2 + 4x + 3, \quad (2 \leqq x \leqq 4)$$
$$y' = -4x + 4 = 0$$
$$x = 1$$

$x = -1$ のとき最大値 $y = -3$，$x = 4$ のとき最小値 $y = -13$

23

(1)

$$x^2 - 2x - 3 = 0$$
$$(x - 3)(x + 1) = 0$$
$$x = -1, 3$$

(2)

$$6x^2 + x - 2 = 0$$
$$(2x - 1)(3x + 2) = 0$$
$$x = -\frac{2}{3}, \frac{1}{2}$$

(3)

$$2x^2 - 2x - 1 = 0$$
$$x = \frac{2 \pm \sqrt{4 + 8}}{4}$$
$$= \frac{1 \pm \sqrt{3}}{2}$$

(4)

$$x^2 + 4x + 9 = 0$$
$$(x + 2)^2 + 5 = 0$$
$$(x + 2)^2 - 5i^2 = 0$$
$$(x + 2 + \sqrt{5}i)(x + 2 - \sqrt{5}i) = 0$$
$$x = -2 \pm \sqrt{5}i$$

(5)

$$50x^2 - 20x + 2 = 0$$
$$(5x + 1)(10x + 2) = 0$$
$$x = -\frac{1}{5}$$

(6)

$$x - 3 = 3x^2 + 2$$
$$3x^2 - x + 5 = 0$$
$$x = \frac{1 \pm \sqrt{1 - 60}}{6}$$
$$= \frac{1 \pm \sqrt{59}i}{6}$$

24

(1)

$$x^2 + 4x + 2a - 1 = 0$$
$$16 - 4(2a - 1) \geqq 0$$
$$20 \geqq 8a$$
$$a \leqq \frac{5}{2}$$

(2)

$$2x^2 - 4ax + 3a - 1 = 0$$
$$16a^2 - 8(3a - 1) \geqq 0$$
$$16a^2 - 24a - 8 \geqq 0$$
$$2a^2 - 3a + 1 \geqq 0$$
$$(2a - 1)(a - 1) \geqq 0$$
$$\therefore \frac{1}{2} \geqq a, 1 \leqq a$$

25

(1)

$$x^2 + 2mx - m = 0$$
$$4m^2 + 4m = 0$$
$$m = -1, 0$$

$m = -1$ のとき

$$x^2 - 2x + 1 = 0$$
$$(x - 1)^2 = 0$$
$$x = 1$$

$m = 0$ のとき

$$x^2 = 0$$
$$x = 0$$

(2)

$$x^2(m + 3)x + m^2 = 0$$
$$(m + 3)^2 - 4m^2 = 0$$
$$3m^2 - 6m - 9 = 0$$
$$m^2 - 2m - 3 = 0$$
$$(m - 3)(m + 1) = 0$$
$$m = -1, 3$$

$m = -1$ のとき

$$x^2 + 2x + 1 = 0$$
$$(x + 1)^2 = 0$$
$$x = -1$$

$m = 3$ のとき

$$x^2 + 6x + 9 = 0$$
$$(x + 3)^2 = 0$$
$$x = -3$$

26

$$y = x^2 - 8x + k$$
$$64 - 4k = 0$$
$$k = 16$$

$k > 16$ のとき共有点なし
$k = 16$ のとき共有点 1 つ
$k < 16$ のとき共有点 2 つ

27

$$2x^2 - 3x - 4 = 0$$

二つの解をα, βとすると

$$\alpha + \beta = \frac{3}{2} \qquad \alpha\beta = -2$$

(1)

$$(x - 3\alpha + 1)(x - 3\beta + 1) = 0$$
$$x^2 + 2x - 3x(\alpha + \beta) - 3(\alpha + \beta) + 9\alpha\beta + 1 = 0$$
$$x^2 + 2x - \frac{3}{2}3x - \frac{3}{2}3 - 18 + 1 = 0$$
$$2x^2 + 4x - 9x - 9 - 36 + 2 = 0$$
$$2x^2 - 5x - 43 = 0$$

(2)

$$(x - \frac{1}{\alpha})(x - \frac{1}{\beta}) = 0$$
$$x^2 - x(\frac{1}{\alpha} + \frac{1}{\beta}) + \frac{1}{\alpha\beta} = 0$$
$$x^2 - x(\frac{\alpha + \beta}{\alpha\beta}) + \frac{1}{\alpha\beta} = 0$$
$$x^2 - \frac{\frac{3}{2}}{-2}x + \frac{1}{-2} = 0$$
$$4x^2 - 3x - 2 = 0$$

(3)

$$(x - \alpha^3)(x - \beta^3) = 0$$
$$x^2 - (\alpha^3 + \beta^3) + \alpha^3\beta^3 = 0$$
$$x^2 - (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)x + \alpha^3\beta^3 = 0$$
$$x^2 - (\alpha + \beta)\{(\alpha + \beta)^2 - 3\alpha\beta\}x + \alpha^3\beta^3 = 0$$
$$x^2 - \frac{3}{2}(\frac{9}{4} + 6)x - 8 = 0$$
$$8x^2 - 99x - 64 = 0$$

28

(1)

$$y = -2x^2 + 6x - 5, y = -3x + 2$$
$$2x^2 - 9x + 7 = 0$$
$$(2x - 7)(x - 1) - 0$$
$$x = 1, \frac{7}{2}$$

共有点 $(x, y) = (1, -1), (\frac{7}{2}, -\frac{17}{2})$

(2)

$$y = x^2 - 4x + 1, y = -x^2 + 2x + 1$$
$$2x^2 - 6x = 0$$
$$x(x - 3) = 0$$
$$x = 0, 3$$

共有点 $(x, y) = (0, 1), (3, -2)$

29

$$mx = x^2 + 1$$
$$x^2 - mx + 1 = 0$$
$$m^2 - 4 > 0$$
$$m > 2$$

30

(1)

$$4x - 1 < 7x + 2$$
$$-3x < 3$$
$$x > -1$$

(2)

$$x^2 + x - 12 \geq 0$$
$$(x + 4)(x - 3) \geq 0$$
$$-4 \geq x, \quad 3 \leq x$$

(3)

$$x^2 + 4 > 4x$$
$$x^2 - 4x + 4 < 0$$
$$(x - 2)^2 < 0$$
$$x \neq 2$$

(4)

$$x^2 - 3x + 4 \geq 0$$
$x =$ 全ての実数

(5) $\begin{cases} 2x + 1 \geq 3x - 2 \\ 6x^2 - 7x - 10 \geq 0 \end{cases}$

$\begin{cases} x \leq 3 \\ (6x + 5)(x - 2) \geq 0 \end{cases}$

解なし

(6) $\begin{cases} 2x^2 \leq 4x + 5 \\ 3^2 - 7x - 10 < 0 \end{cases}$

$\begin{cases} 2x^2 - 4x - 5 \leq 0 \\ (3x - 10)(x + 1) < 0 \end{cases}$

$\begin{cases} 1 \pm \frac{\sqrt{14}}{2} \leq 0 \\ (3x - 10)(x + 1) < 0 \end{cases}$

$1 - \frac{\sqrt{14}}{2} \leq x \leq \frac{\sqrt{14}}{2}$

31

(1)

$$mx^2 + 2mx + 3m - 4 > 0$$
$$4m^2 - 4m(3m - 4) > 0$$
$$m^2 - 3m^2 + 4m > 0$$
$$2m^2 - 4m < 0$$
$$2m(m - 2) < 0$$
$$0 < m < 2$$

(2)

$$3mx^2 + 12x + m + 1 < 0$$
$$144 - 4(3m)(m + 1) < 0$$
$$12 - m^2 - m < 0$$
$$m^2 + m - 12 > 0$$
$$(m + 4)(m - 3) > 0$$
$$-4 > m, \quad 3 < m$$
$3 < m$ は不適なので
$$-4 > m$$

32

(1)

$$\sin\theta = \frac{2}{5}$$
$$\cos^2\theta = 1 - \frac{4}{25} = \frac{21}{25}$$
θ は第 2 象限の角だから
$$\cos\theta = -\frac{\sqrt{21}}{5} \tan^2\theta = \frac{25}{21} - 1 = \frac{4}{21}$$
θ は第 2 象限の角だから
$$\tan\theta = -\frac{2}{\sqrt{21}}$$

(2)

$$\cos\theta = \frac{1}{3}$$
$$\sin^2\theta = 1 - \frac{1}{9} = \frac{8}{9}$$

θ は第 4 象限の角だから

$$\sin\theta = \frac{-2\sqrt{2}}{3}$$
$$\tan^2\theta = 9 - 1 = 8$$

θ は第 4 象限の角だから

$$\tan\theta = -2\sqrt{2}$$

(3)

$$\tan\theta = \frac{3}{4}$$
$$\frac{9}{16} + 1 = \frac{1}{\cos\theta}$$
$$\cos^2\theta = \frac{16}{25}$$

θ は第 3 象限の角だから

$$\cos\theta = -\frac{4}{5}$$
$$\sin^2\theta = 1 - \frac{16}{25} = \frac{9}{25}$$

θ は第 3 象限の角だから

$$\sin\theta = -\frac{3}{5}$$

33

(1)

$$\begin{aligned}(\text{左辺}) &= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} \\ &= \frac{1 - \frac{\sin\theta}{\cos\theta}}{1 + \sin\theta\cos\theta} \\ &= \frac{1 - \tan\theta}{1 + \tan\theta} = (\text{右辺})\end{aligned}$$

(2)

$$\begin{aligned}(\text{左辺}) &= \frac{\sin\theta + 1}{\cos\theta} \\ &= \frac{-\cos^2\theta}{\cos\theta(\sin\theta - 1)} \\ &= \frac{\cos\theta}{1 - \sin\theta} = (\text{右辺})\end{aligned}$$

(3)

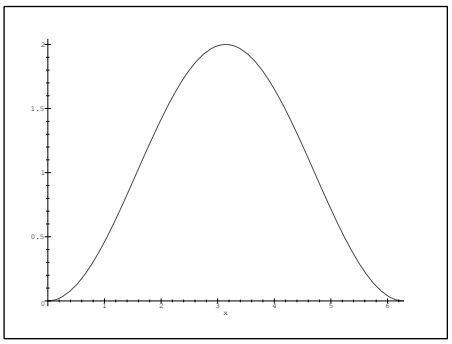
$$\begin{aligned}(\text{左辺}) &= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta) \\ &= \cos^2\theta - \sin^2\theta \\ &= \cos 2\theta = (\text{右辺})\end{aligned}$$

(4)

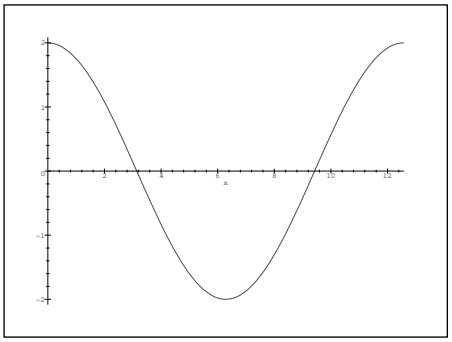
$$\begin{aligned}(\text{左辺}) &= \frac{1 - (\cos^2\theta - \sin^2\theta)}{2\sin\theta\cos\theta} \\ &= \frac{2\sin^2\theta}{2\sin\theta\cos\theta} \\ &= \frac{\sin\theta}{\cos\theta} \\ &= \tan\theta = (\text{右辺})\end{aligned}$$

34

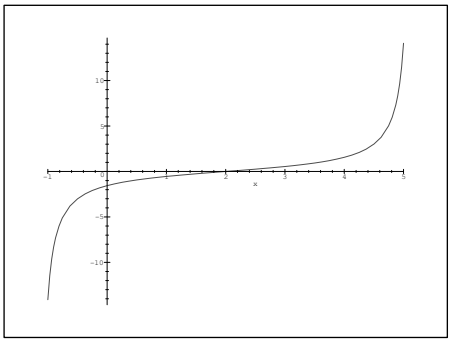
(1)



(2)



(3)



35

(1)

$$\sqrt{2}\cos x = -1$$
$$\cos x = -\frac{1}{\sqrt{2}}$$
$$x = \frac{3}{4}\pi, \frac{5}{4}\pi$$

(2)

$$2\sin\left(x+\frac{\pi}{3}\right) = 1$$
$$\sin\left(x+\frac{\pi}{3}\right) = \frac{1}{2}$$
$$x+\frac{\pi}{3} = \frac{\pi}{6}, \frac{5}{6}\pi$$
$$x = \frac{11}{6}\pi, \frac{1}{2}\pi$$

(3)

$$\sqrt{2}\cos \leq -1$$
$$\cos x \leq -\frac{1}{\sqrt{2}}$$
$$\frac{3}{4}\pi \leq x \leq \frac{5}{4}\pi$$

(4)

$$2\sin\left(x+\frac{\pi}{3}\right) > 1$$
$$\sin\left(x+\frac{\pi}{3}\right) > \frac{1}{2}$$
$$\frac{\pi}{6} < x+\frac{\pi}{3} < \frac{5}{6}\pi$$
$$\frac{11}{6}\pi < x < 2\pi, \quad 0 \leq x < \frac{1}{2}\pi$$

(5)

$$\cos 3x = -\frac{sqrt3}{2}$$
$$3x = \frac{5}{6}\pi, \frac{7}{6}\pi, \frac{17}{6}\pi, \frac{19}{6}\pi, \frac{29}{6}\pi, \frac{31}{6}\pi$$
$$x = \frac{5}{18}\pi, \frac{7}{18}\pi, \frac{17}{18}\pi, \frac{19}{18}\pi, \frac{29}{18}\pi, \frac{31}{18}\pi$$

(6)

$$2\sin^2 x - 7\sin x + 3 = 0$$
$$(2\sin x - 1)(\sin x - 3) = 0$$
$$\sin x \frac{1}{2}, 3$$
$$x = \frac{\pi}{6}, \frac{5}{6}\pi$$

(7)

$$2\sin 2x > \sqrt{3}$$
$$\sin 2x > \frac{\sqrt{3}}{2} \quad \frac{\pi}{3} < 2x < \frac{2}{3}\pi, \quad \frac{7}{3}\pi < 2x < \frac{8}{3}\pi$$
$$\frac{\pi}{6} < x < \frac{1}{3}\pi, \quad \frac{7}{6}\pi < x < \frac{4}{3}\pi$$

(8)

$$2\cos^2 x - 3\cos x - 2 \leq 0$$
$$(2\cos x + 1)(\cos x - 2) \leq 0$$
$$-\frac{1}{2} \leq \cos x \leq 2$$
$$0 \leq x \leq \frac{2}{3}\pi, \quad \frac{4}{3}\pi \leq x < 2\pi$$

36

$$\cos^2\alpha = 1 - \frac{9}{25} = \frac{16}{25} \qquad 0 < \alpha < \frac{\pi}{2} \text{より}$$
$$\cos\alpha = \frac{4}{5}$$
$$\sin^2\beta = 1^{\frac{16}{25}} = \frac{9}{25} \qquad 0 < \beta < \frac{\pi}{2} \text{より}$$
$$\sin\beta = \frac{3}{5}$$

(1)

$$\sin(\alpha+\beta) = \frac{3}{5}\cdot\frac{4}{5} + \frac{3}{5}\cdot\frac{3}{5} = \frac{24}{25}$$

(2)

$$\cos(\beta-\alpha) = \frac{4}{5}\cdot\frac{4}{5} + \frac{3}{5}\cdot\frac{3}{5} = 1$$

(3)

$$\sin(\alpha-\frac{\pi}{4}) = \sin\alpha\cos\frac{\pi}{4} - \cos\alpha\sin\frac{\pi}{4}$$
$$= \frac{3}{5}\cdot\frac{1}{\sqrt{2}} - \frac{4}{5}\cdot\frac{1}{\sqrt{2}}$$
$$= -\frac{1}{5\sqrt{2}}$$

(4)

$$\tan\beta + \frac{2}{3}\pi = \frac{\tan\beta + \tan\frac{2}{3}\pi}{1-\tan\beta\tan\frac{2}{3}\pi}$$
$$= \frac{\frac{3}{4}-\sqrt{3}}{1-\frac{3}{4}\frac{\sqrt{3}}{-\frac{1}{2}}}$$
$$= \frac{\frac{3}{4}-\sqrt{3}}{1+\frac{3\sqrt{3}}{4}}$$
$$= \frac{(3-4\sqrt{3})(4-3\sqrt{3})}{(4+3\sqrt{3})(4-3\sqrt{3})}$$
$$= \frac{12-16\sqrt{3}-9\sqrt{3}+36}{11}$$
$$= \frac{48-25\sqrt{3}}{11}$$

(5)

$$\sin 2\beta = 2\sin\beta\cos\beta$$
$$= 2\cdot\frac{4}{5}\cdot\frac{3}{5}$$
$$= \frac{24}{25}$$

(6)

$$\begin{aligned}\cos^2\frac{\alpha}{2} &= \frac{1}{2}(1+\cos\alpha)\\ &= \frac{1}{2}(1+\frac{4}{5})\\ &= \frac{1}{2}\cdot\frac{9}{5}\\ &= \frac{9}{10}\end{aligned}$$

$0<\alpha<\dfrac{\pi}{2}$より

$$\cos\frac{\alpha}{2}=\frac{3}{\sqrt{10}}$$

37

(1)

$$\sin\frac{x}{8}+\sin\frac{3}{8}x = 2\sin 4x\sin 2x$$

(2)

$$\cos 6x-\cos 2x = -2\sin 4x\sin 2x$$

(3)

$$\begin{aligned}\sin\frac{3}{4}x\sin\frac{x}{4} &= -\frac{1}{2}\{\cos x\cos\frac{1}{2}x\}\\ &= \frac{1}{2}\cos\frac{x}{2}-\frac{1}{2}\cos x\end{aligned}$$

(4)

$$\begin{aligned}\sin 5x\cos 7x &= \frac{1}{2}\{\sin 12x\sin -2x\}\\ &= \frac{1}{2}\sin 12x-\frac{1}{2}\sin 2x\end{aligned}$$

38

(1)

$$\begin{aligned}\cos x-\sin x &= 1\\ -\sin x+\cos x &= 1\\ \sqrt{2}\sin(x+\frac{3}{4}\pi) &= 1\\ \sin(x+\frac{3}{4}\pi) &= \frac{1}{\sqrt{2}}\\ x+\frac{3}{4}\pi &= \frac{\pi}{4},\frac{3}{4}\pi\\ x &= 0,-\frac{3}{2}\pi\end{aligned}$$

(2)

$$\begin{aligned}2\sin(x+\frac{\pi}{6}) &= \sqrt{3}\\ \sin(x+\frac{\pi}{6}) &= \frac{\sqrt{3}}{2}\\ x+\frac{\pi}{6} &= \frac{\pi}{3},\frac{2}{3}\pi\\ x &= \frac{\pi}{6},\frac{\pi}{2}\end{aligned}$$

(3)

$$\begin{aligned}\sin x-\cos x &\geq 1\\ \sqrt{2}\sin(x-\frac{\pi}{4}) &\geq 1\\ \sin(x-\frac{\pi}{4}) &\geq \frac{1}{\sqrt{2}}\\ \frac{\pi}{4}\leq x-\frac{\pi}{4} &\leq \frac{2}{4}\pi\\ \frac{\pi}{2}\leq x &\leq \pi\end{aligned}$$

(4)

$$\begin{aligned}\sqrt{3}\cos x-\sin x &< 1\\ -\sin x+\sqrt{3}\cos x &< 1\\ 2\sin(x+\frac{2}{3}\pi) &< 1\\ \sin(x+\frac{2}{3}\pi) &< \frac{1}{2}\\ \frac{5}{6}\pi < x+\frac{2}{3}\pi &< \frac{13}{6}\pi\\ \frac{\pi}{6} &< x < \frac{3}{2}\pi\end{aligned}$$

39

(1)

$$\begin{aligned}y &= \sin x+\sqrt{3}\cos x\\ &= 2\sin(x+\frac{\pi}{3})\end{aligned}$$

$x=\dfrac{\pi}{6}$のとき，最大値 2

$x=\dfrac{7}{6}\pi$のとき，最小値 -2

(2)

$$\begin{aligned}y &= \sin x-\cos(x-\frac{\pi}{6})\\ &= \sin x-(\cos x\cos\frac{\pi}{6}+\sin x\sin\frac{\pi}{6})\\ &= \sin x-\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x\\ &= \frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x\\ &= \sin(x-\frac{\pi}{3})\end{aligned}$$

$x=\dfrac{5}{6}\pi$のとき，最大値 1

$x=\dfrac{11}{6}\pi$のとき，最小値 -1

(3)

$$
\begin{aligned}
y &= \cos^2 x + \cos 2x \\
&= \frac{1}{2}(1 + \cos 2x) + \cos 2x \\
&= \frac{1}{2}(1 + 3\cos 2x)
\end{aligned}
$$

$x = \dfrac{\pi}{2}, \dfrac{2}{3}\pi$のとき，最大値 2

$x = 0$ のとき，最小値$\dfrac{1}{2}$

(4)

$$
\begin{aligned}
y &= 2\cos -2x + 2\sin x \cos x \\
&= 1 + \cos 2x + \sin 2x
\end{aligned}
$$

$x = \dfrac{\pi}{8}, \dfrac{9}{8}\pi$のとき，最大値 $1 + \sqrt{2}$

$x = \dfrac{5}{8}\pi, \dfrac{13}{8}\pi$のとき，最小値 $1 - \sqrt{2}$