微分積分 II 春課題

平成 27 年 3 月 22 日

$\boxed{\mathbf{1}}$

(1)

$$\lim_{x\to-1}\frac{x^2+3x+2}{x^3+x^2+3x+3}$$
$$=\lim_{x\to-1}\frac{(x+2)(x+1)}{(x+1)(x^2+3)}$$
$$=\lim_{x\to-1}\frac{x+2}{x^2+3}$$
$$=\frac{1}{4}$$

(2)

$$\lim_{x\to\infty}\frac{2x^2+9x-5}{3x^2-x-2}$$
$$=\lim_{x\to\infty}\frac{2+\frac{9}{x}-\frac{5}{x^2}}{3-\frac{1}{x}-\frac{2}{x^2}}$$
$$=\frac{2+0-0}{3-0-0}=\frac{2}{3}$$

(3)

$$\lim_{x\to\infty}\frac{1-3^x}{3^{x+1}+2^x}$$
$$=\lim_{x\to\infty}\frac{3^{-x}-1}{3+\frac{2^x}{3^x}}$$
$$=\frac{0-1}{3+0}=-\frac{1}{3}$$

(4)

$$\lim_{x\to3-0}\frac{|x-3|}{x^2-x+6}$$
$$=\lim_{x\to3-0}\frac{1}{-x-2}$$
$$=-\frac{1}{5}$$

(5)

$$\lim_{x\to0}\frac{\sin 5x}{4x}$$
$$=\lim_{x\to0}(\frac{\sin 5x}{5x}\frac{5x}{4x})$$
$$=1\cdot\frac{5}{4}=\frac{5}{4}$$

(6)

$$\lim_{x\to0}\frac{1-\cos x}{x^2}$$
$$=\lim_{x\to0}\frac{\sin x^2}{x^2(1+\cos x)}$$
$$=\lim_{x\to0}\frac{\sin x}{x}\lim_{x\to0}\frac{\sin x}{x}\lim_{x\to0}\frac{1}{1+\cos x}$$
$$=1\cdot1\cdot\frac{1}{2}=\frac{1}{2}$$

(7)

$$\lim_{x\to2}\frac{x-2}{\sqrt{x^2+1}-\sqrt{5}}$$
$$=\lim_{x\to2}\frac{(x-2)(\sqrt{x^2+1}+\sqrt{5})}{x^2-4}$$
$$=\lim_{x\to2}\frac{\sqrt{x^2+1}+\sqrt{5}}{x+2}$$
$$=\frac{2\sqrt{5}}{4}=\frac{\sqrt{5}}{4}$$

(8)

$$\lim_{h\to0}(1+3h)^{\frac{1}{h}}$$
$$=\lim_{h\to0}(1+3h)^{\frac{3}{3h}}$$
$$=e^3$$

$\boxed{\mathbf{2}}$

（証明）$f(x)=x+\log_2(x^2+1)-1=0$ とすると，

$f(x)$ は $(-\infty.\infty)$ で連続であり

$f(0)=\log_2 1-1=-1<0$

$f(1)=1+\log_2 2-1=1>0$

よって中間値の定理より

$f(x)$ は、0 と 1 の間に少なくとも 1 つの実数解を持つ

Q.E.D.

$\boxed{\mathbf{3}}$

$$
\begin{aligned}
f(x) &= \sqrt{x} \\
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \\
&= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} \\
&= \frac{1}{2\sqrt{x}}
\end{aligned}
$$

$\boxed{\mathbf{4}}$

(1)

$$
\begin{aligned}
y &= (5x-3)^7 \\
y' &= 35(5x-3)^6
\end{aligned}
$$

(2)

$$
\begin{aligned}
y &= \sqrt{6x+5} \\
y' &= \frac{3}{\sqrt{6x+5}}
\end{aligned}
$$

(3)

$$
\begin{aligned}
y &= \frac{4x-3}{x+2} \\
y' &= \frac{4(x+2)-4x+3}{(x+2)^2} \\
&= \frac{11}{(x+2)^2}
\end{aligned}
$$

(4)

$$
\begin{aligned}
y &= \frac{1}{3x^2-1} \\
y' &= -\frac{6x}{(3x^2-1)^2}
\end{aligned}
$$

(5)

$$
\begin{aligned}
y &= (x^2+1)^8 \\
y' &= 16x(x^2+1)^7
\end{aligned}
$$

(6)

$$
\begin{aligned}
y &= \frac{10}{3}\sqrt[5]{x^3} \\
y' &= \frac{10}{3}\frac{3}{5}\frac{1}{\sqrt[5]{x^2}} \\
&= \frac{2}{\sqrt[5]{x^2}}
\end{aligned}
$$

(7)

$$
\begin{aligned}
y &= \sqrt[4]{(2x^2+4x+3)^3} \\
y' &= \frac{3}{4}(2x^2+4x+3)^{-\frac{1}{4}}(4x+4) \\
&= \frac{3(x+1)}{\sqrt[4]{2x^2+4x+3}}
\end{aligned}
$$

(8)

$$
\begin{aligned}
y &= \frac{x}{(1-x)^2} \\
y' &= \frac{(1-x)^2+2x(1-x)}{(1-x)^4} \\
&= \frac{1+x}{(1-x)^3}
\end{aligned}
$$

(9)

$$
\begin{aligned}
y &= (x+1)\sqrt{2x-1} \\
y' &= \sqrt{2x-1}+\frac{x+1}{\sqrt{2x-1}} \\
&= \frac{3x}{\sqrt{2x-1}}
\end{aligned}
$$

$\boxed{\mathbf{5}}$

(1)

$$
\begin{aligned}
y &= \cos\frac{x}{4} \\
y' &= -\frac{1}{4}\sin\frac{x}{4}
\end{aligned}
$$

(2)

$$
\begin{aligned}
y &= \sin^4 x \\
y' &= 4\sin^3 x\cos x
\end{aligned}
$$

(3)

$$
\begin{aligned}
y &= \cos^3 5x \\
y' &= -15\cos^2 5x\sin 5x
\end{aligned}
$$

(4)

$$
\begin{aligned}
y &= x^2\cos\frac{1}{x} \\
y' &= 2x\cos\frac{1}{x}+x^2\sin\frac{1}{x}\frac{1}{x^2} \\
&= 2x\cos\frac{1}{x}+\sin\frac{1}{x}
\end{aligned}
$$

(5)

$$
\begin{aligned}
y &= \frac{\cos x}{1-\sin x} \\
y' &= \frac{-\sin x(1-\sin x)+cos^2x}{(1-\sin x)^2} \\
&= \frac{-\sin x+\sin^2 x+cos^2x}{(1-\sin x)^2} \\
&= \frac{1}{1-\sin x}
\end{aligned}
$$

(6)

$$\begin{aligned} y &= \sin^2 \frac{1}{\sqrt{x}} \\ y' &= 2\sin\frac{1}{\sqrt{x}}\cos\frac{1}{\sqrt{x}}(-\frac{1}{2x\sqrt{x}}) \\ &= -\frac{1}{x\sqrt{x}}\sin\frac{1}{\sqrt{x}}\cos\frac{1}{\sqrt{x}} \end{aligned}$$

(7)

$$\begin{aligned} y &= x^2\tan x \\ y' &= 2x\tan x + x^2\sec^2 x \\ &= x(2\tan x + x\sec^2 x) \end{aligned}$$

(8)

$$\begin{aligned} y &= \frac{1+\tan x}{1-\tan x} \\ y' &= \frac{\sec^2 x(1-\tan x)+\sec^2 x(1+\tan x)}{(1-\tan x)^2} \\ &= \frac{2\sec^2 x}{(1-\tan x)^2} \end{aligned}$$

(9)

$$\begin{aligned} y &= \arcsin\frac{x}{2} \\ y' &= \frac{\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}} \\ &= \frac{1}{\sqrt{4-x^2}} \end{aligned}$$

(10)

$$\begin{aligned} y &= \arctan\frac{2}{x} \\ y' &= \frac{-\frac{2}{x^2}}{1+\frac{4}{x^2}} \\ &= -\frac{2}{x^2+4} \end{aligned}$$

(11)

$$\begin{aligned} y &= \log(x^2+3x+1) \\ y' &= \frac{2x+3}{x^2+3x+1} \end{aligned}$$

(12)

$$\begin{aligned} y &= \log\left|\frac{1+x}{1-x}\right| \\ y' &= \frac{\frac{1-x+1+x}{(1-x)^2}}{\frac{1+x}{1-x}} \\ &= \frac{2}{1-x^2} \end{aligned}$$

(13)

$$\begin{aligned} x &= (e^{3t}+e^{-3t})^5 \\ x' &= 15(e^{3t}+e^{-5t})^4(e^{3t}-e^{-3t}) \end{aligned}$$

$\boxed{\textbf{6}}$

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

$\boxed{\textbf{7}}$

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

$\boxed{\mathbf{8}}$

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

$\boxed{\mathbf{9}}$

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

$\boxed{\mathbf{10}}$

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

$\boxed{\mathbf{11}}$

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

12

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

13

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)