

微分積分Ⅱ冬課題

平成 27 年 1 月 5 日

1

(1)

$$\begin{aligned}\frac{x+5}{(x+2)(x-1)} &= \frac{a}{x+2} + \frac{b}{x-1} \\ x+5 &= (x-1)a + (x+2)b \\ x=-2 \text{ とすると } a &= -1 \\ x=1 \text{ とすると } b &= 2\end{aligned}$$

(2)

$$\begin{aligned}\frac{1}{(x-3)(x-1)} &= \frac{a}{x-3} + \frac{b}{x-1} \\ 1 &= (x-1)a + (x-3)b \\ x=3 \text{ とすると } a &= \frac{1}{2} \\ x=1 \text{ とすると } b &= -\frac{1}{2}\end{aligned}$$

(3)

$$\begin{aligned}\frac{2x}{(x-1)(x^2+1)} &= \frac{a}{x-1} + \frac{bx+c}{x^2+1} \\ 2x &= (x^2+1)a + (x-1)(bx+c) \\ x=1 \text{ とすると } a &= 1 \\ 2x &= ax^2 + a + bx^2 + cx - bx - c \\ &= (a+b)x^2 + (c-b)x + a - c \\ a=1 \text{ だから} \\ 2x &= (1+b)x^2 + (c-b)x + 1 - c \\ \text{係数を比較して} \\ a=1, b=-1, c &= 1\end{aligned}$$

2

(1)

$$\begin{aligned}(\text{証明})(\text{右辺}) &= \frac{1}{2}(1 - \cos 2x) \\ &= \frac{1}{2}(1 - \cos^2 x + \sin^2 x) \\ &= \frac{1}{2}(2 \sin^2 x) \\ &= \sin^2 x = (\text{左辺})\end{aligned}$$

(2)

$$\begin{aligned}(\text{証明})(\text{左辺}) &= \cos^2 \frac{x}{2} \\ &= \cos x + \sin^2 \frac{x}{2} \\ &= \cos x + \frac{1}{2} - \frac{\cos x}{2} \\ &= \frac{1}{2}(1 + \cos x) = (\text{右辺})\end{aligned}$$

3

(1)

$$\sin 2x \cos x = \frac{1}{2}(\sin 3x + \sin x)$$

(2)

$$\begin{aligned}\cos 2x \sin 3x &= \frac{1}{2}(\sin 5x - \sin -x) \\ &= \frac{1}{2}(\sin 5x + \sin x)\end{aligned}$$

(3)

$$\begin{aligned}\cos 3x \cos 5x &= \frac{1}{2}(\cos 8x + \cos -2x) \\ &= \frac{1}{2}(\cos 8x + \cos 2x)\end{aligned}$$

(4)

$$\sin 4x \sin 2x = -\frac{1}{2}(\cos 6x - \cos 2x)$$

4

(1)

$$\begin{aligned}\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 - 4x + 3} \\ &= \lim_{x \rightarrow 3} \frac{(x-3)(x+2)}{(x-3)(x-1)} \\ &= \frac{5}{2}\end{aligned}$$

(2)

$$\begin{aligned}\lim_{x \rightarrow 1} \frac{x^3 - 1}{3x^2 - x - 2} \\ &= \lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x + 1)}{(3x+2)(x-1)} \\ &= \frac{3}{5}\end{aligned}$$

(3)

$$\begin{aligned}
& \lim_{x \rightarrow \infty} \frac{2 + 5^x}{5^{x+1} - 3^x} \\
&= \lim_{x \rightarrow \infty} \frac{\frac{2}{5^x} + 1}{5 - \frac{3^x}{5^x}} \\
&= \frac{1}{5}
\end{aligned}$$

(4)

$$\begin{aligned}
& \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \\
&= \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \lim_{x \rightarrow 0} \frac{1}{1 + \cos x} \\
&= \frac{1}{2}
\end{aligned}$$

(5)

$$\begin{aligned}
& \lim_{x \rightarrow 2} \frac{x - 2}{\sqrt{x^2 + 1} - \sqrt{5}} \\
&= \lim_{x \rightarrow 2} \frac{(x - 2)(\sqrt{x^2 + 1} + \sqrt{5})}{x^2 + 1 - 5} \\
&= \lim_{x \rightarrow 2} \frac{(x - 2)(\sqrt{x^2 + 1} + \sqrt{5})}{(x - 2)(x + 2)} \\
&= \frac{\sqrt{5}}{2}
\end{aligned}$$

(6)

$$\begin{aligned}
& \lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - x) \\
&= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + 1} - x)(\sqrt{x^2 + 1} + x)}{(\sqrt{x^2 + 1} + x)} \\
&= \lim_{x \rightarrow \infty} \frac{1}{(\sqrt{x^2 + 1} + x)} \\
&= 0
\end{aligned}$$

5

(1)

$$\begin{aligned}
f(x) &= x^2 \\
f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\
&= \lim_{h \rightarrow 0} 2xh + h^2 \\
&= 2x
\end{aligned}$$

(2)

$$\begin{aligned}
f(x) &= \sqrt{x} \\
f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \\
&= \lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} \\
&= \frac{1}{2\sqrt{x}}
\end{aligned}$$

(3)

$$\begin{aligned}
f(x) &= \frac{1}{x} \\
f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\
&= \lim_{h \rightarrow 0} \frac{x - (x+h)}{hx(x+h)} \\
&= \lim_{h \rightarrow 0} \frac{-h}{hx(x+h)} \\
&= -\frac{1}{x^2}
\end{aligned}$$

6

(1)

$$\begin{aligned}
y &= (5x + 3)^7 \\
y' &= 35(5x + 3)^6
\end{aligned}$$

(2)

$$\begin{aligned}
y &= (x^2 + 3)^7 \\
y' &= 14x(x^2 + 3)^6
\end{aligned}$$

(3)

$$\begin{aligned}
y &= \sqrt{4x + 3} \\
y' &= \frac{2}{\sqrt{4x + 3}}
\end{aligned}$$

(4)

$$\begin{aligned}
y &= \sqrt[3]{6x + 1} \\
y' &= \frac{2}{\sqrt[3]{6x + 1}^2}
\end{aligned}$$

(5)

$$\begin{aligned}
y &= \frac{4x - 3}{x + 1} \\
y' &= \frac{4(x + 1) - (4x - 3)}{(x + 1)^2} \\
&= \frac{7}{(x + 1)^2}
\end{aligned}$$

(6)

$$\begin{aligned}
x &= \frac{1}{2t^2 - 1} \\
x' &= -\frac{4t}{(2t^2 - 1)^2}
\end{aligned}$$

(7)

$$\begin{aligned}
x &= \frac{t}{(1-t)^2} \\
x' &= \frac{(1-t)^2 - t(-2(1-t))}{(1-t)^4} \\
&= \frac{(1-t) + 2t}{(1-t)^3} \\
&= \frac{1+t}{(1-t)^3}
\end{aligned}$$

(8)

$$y = \cos \frac{t}{2}$$

$$y' = -\frac{1}{2} \sin \frac{t}{2}$$

(9)

$$y = \sin^4 x$$

$$y' = 4 \sin^3 x \cos x$$

(10)

$$y = \cos^3 5x$$

$$y' = -15 \cos^2 5x \sin 5x$$

7

(1)

$$y = e^{3x} \sin 2x$$

$$y' = 3e^{3x} \sin 2x + e^{3x} 2 \cos 2x$$

$$= (3 \sin 2x + 2 \cos 2x) e^{3x}$$

(2)

$$y = e^{-x} \cos 4x$$

$$y' = -e^{-x} \cos 4x - 4e^{-x} \sin 4x$$

$$= (-\cos 4x - 4 \sin 4x) e^{-x}$$

(3)

$$v = \frac{\log u}{u}$$

$$v' = \frac{\frac{1}{u} \cdot u - \log u}{u^2}$$

$$= \frac{1 - \log u}{u^2}$$

(4)

$$y = \log(x^2 + 3x + 1)$$

$$y' = \frac{2x + 3}{x^2 + 3x + 1}$$

(5)

$$y = \log \left| \frac{1+x}{1-x} \right|$$

$$y' = \frac{\frac{(1-x)+(1+x)}{(1-x)^2}}{\frac{1+x}{1-x}}$$

$$= \frac{2}{(1+x)(1-x)}$$

(6)

$$y = e^{\sin 2x}$$

$$y' = 2 \cos 2x e^{\sin 2x}$$

(7)

$$y = (e^{2t} + e^{-2t})^4$$

$$y' = 4(e^{2t} + e^{-2t})^3 (2e^{2t} - 2e^{-2t})$$

$$= 8(e^{2t} + e^{-2t})^3 (e^{2t} - e^{-2t})$$

(8)

$$y = \tan^{-1} 2x$$

$$y' = \frac{2}{1 + 4x^2}$$

(9)

$$y = \sin^{-1} \frac{3}{x}$$

$$y' = \frac{\frac{1}{3}}{\sqrt{1 - \frac{x^2}{9}}}$$

$$= \frac{1}{\sqrt{9 - x^2}}$$

(10)

$$x = \tan^{-1} \frac{2}{u}$$

$$x' = \frac{-\frac{2}{u^2}}{1 + \frac{4}{u^2}}$$

$$= \frac{-2}{u^2 + 4}$$

8

(1)

$$y = x^2 + 2x - 1$$

$$y' = 2x + 2$$

$$x = 1 \text{ の時 } y = 2 \text{ で傾き } y' = 4$$

$$\therefore y - 2 = 4(x - 1)$$

$$y = 4x - 2$$

(2)

$$y = \frac{1}{x}$$

$$y' = -\frac{1}{x^2}$$

$$x = 2 \text{ の時 } y = \frac{1}{2} \text{ で傾き } y' = -\frac{1}{4}$$

$$\therefore y - \frac{1}{2} = -\frac{1}{4}(x - 2)$$

$$y = -\frac{1}{4}x + 1$$

(3)

$$y = x^2$$

$$y' = 2x$$

$$x = -1 \text{ の時 } y = 1, y' = -2 \text{ なので傾き } m = \frac{1}{2}$$

$$\therefore y - 1 = \frac{1}{2}(x + 1)$$

$$y = \frac{1}{2}x + \frac{3}{2}$$

9

(1)

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{\cos t}{\sin t} = -\cot t$$

(2)

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sin t}{1 - \cos t}$$

(3)

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3 \sin^2 t \cos t}{-3 \cos^2 t \sin t} = -\tan t$$

10

(1)

$$y = x^3 - 5x^2 + 3x$$

$$y' = 3x^2 - 10x + 3$$

$$= (3x - 1)(x - 3)$$

$$y' = 0 \text{ を解くと } x = \frac{1}{3}, 3$$

x	\cdots	$\frac{1}{3}$	\cdots	3	\cdots
y'	+	0	-	0	+
y	\nearrow	$\frac{13}{27}$	\searrow	-9	\nearrow

$$\therefore x = \frac{1}{3} \text{ の時極大値 } y = \frac{13}{27}$$

$$x = 3 \text{ の時極小値 } y = -9$$

(2)

$$y = x \log x$$

$$y' = \log x + 1$$

$$y' = 0 \text{ を解くと } x = \frac{1}{e}$$

x	0	\cdots	$\frac{1}{e}$	\cdots
y'	\nearrow	-	0	+
y	\nearrow	\searrow	$-\frac{1}{e}$	\nearrow

$$\therefore x = -\frac{1}{e} \text{ の時極小値 } y = -\frac{1}{e}$$

11

(1)

$$y = x^3 - 3x$$

$$y' = 3x^2 - 3$$

$$y' = 0 \text{ を解くと } x = \pm 1$$

x	-2	\cdots	-1	\cdots	0
y'	9	+	0	-	-3
y	-2	\nearrow	2	\searrow	0

$$\therefore x = -2 \text{ の時最小値 } y = -2$$

$$x = -1 \text{ の時最大値 } y = 2$$

(2)

$$y = xe^{-x}$$

$$y' = e^{-x} - xe^{-x}$$

$$y' = 0 \text{ を解くと } x = 1$$

x	0	\cdots	1	\cdots	2
y'	1	+	0	-	$-\frac{1}{e^2}$
y	0	\nearrow	$\frac{1}{e}$	\searrow	$\frac{2}{e^2}$

$$\therefore x = 0 \text{ の時最小値 } y = 0$$

$$x = 1 \text{ の時最大値 } y = \frac{1}{e}$$

(3)

$$y = x - 2 \sin x$$

$$y' = 1 - 2 \cos x$$

$$y' = 0 \text{ を解くと } x = \frac{\pi}{3}, \frac{5\pi}{3}$$

x	0	\cdots	$\frac{\pi}{3}$	\cdots	$\frac{5\pi}{3}$	\cdots	2π
y'	-1	-	0	+	0	-	-1
y	0	\searrow	$\frac{\pi}{3} - \sqrt{3}$	\nearrow	$\frac{5\pi}{3} - \sqrt{3}$	\searrow	2π

$$\therefore x = \frac{\pi}{3} \text{ の時最小値 } y = \frac{\pi}{3} - \sqrt{3}$$

$$x = \frac{5\pi}{3} \text{ の時最大値 } y = \frac{5\pi}{3} - \sqrt{3}$$

12

(1)

$$y = x^4 - 2x^3$$

$$y' = 4x^3 - 6x^2$$

$$= 2x^2(2x - 3)$$

$$y' = 0 \text{ を解くと } x = \frac{3}{2}, 0$$

$$y'' = 12x^2 - 12x$$

$$= 12x(x - 1)$$

$$y'' = 0 \text{ を解くと } x = 1, 0$$

x	\cdots	0	\cdots	1	\cdots	$\frac{3}{2}$	\cdots
y'	-	0	-	-	-	0	+
y''	+	0	-	0	+	+	+
y	\searrow	0	\searrow	-1	\searrow	$\frac{27}{16}$	\nearrow

$$\therefore \text{変曲点 } (0, 0), (1, -1)$$

(2)

$$y = \frac{1}{x^2 + 1}$$

$$y' = -\frac{2x}{(x^2 + 1)^2}$$

$$y' = 0 \text{ を解くと } x = 0$$

$$y'' = \frac{-2(x^2 + 1)^2 + 8x^2(x^2 + 1)}{(x^2 + 1)^4}$$

$$= \frac{6x^2 - 2}{(x^2 + 1)^3}$$

$$y'' = 0 \text{ を解くと } x = \pm \frac{1}{\sqrt{3}}$$

x	\cdots	$-\frac{1}{\sqrt{3}}$	\cdots	0	\cdots	$\frac{1}{\sqrt{3}}$	\cdots
y'	$+$	$+$	$+$	0	$-$	$-$	$-$
y''	$+$	0	$-$	$-$	$-$	0	$+$
y	\nearrow	$\frac{3}{4}$	\searrow	1	\searrow	$\frac{3}{4}$	\searrow

\therefore 変曲点 $(-\frac{1}{\sqrt{3}}, \frac{3}{4}), (\frac{1}{\sqrt{3}}, \frac{3}{4})$

(3)

$$y = x^4 - 4x^3 + 6x^2$$

$$y' = 4x^3 - 12x^2 + 12x$$

$$y' = 0 \text{ を解くと } x = 0$$

$$y'' = 12x^2 - 24x + 12$$

$$= 12(x - 1)^2$$

$$y'' = 0 \text{ を解くと } x = 1$$

x	\cdots	0	\cdots	1	\cdots
y'	$-$	0	$+$	$+$	$+$
y''	$+$	$+$	$+$	0	$+$
y	\searrow	0	\nearrow	3	\nearrow

\therefore 変曲点なし

13

(1)

$$\int (3x + 2)^4 dx$$

$$= \frac{(3x + 2)^5}{15}$$

(2)

$$\int \frac{1}{\sqrt[3]{x}} dx$$

$$= \frac{3}{2} \sqrt[3]{x^2}$$

(3)

$$\int \frac{1}{1 - 2x} dx$$

$$= -\frac{1}{2} \log |1 - 2x|$$

(4)

$$\int (\cos 2x + \sin 3x) dx$$

$$= \frac{1}{2} \sin 2x - \frac{1}{3} \cos 3x$$

(5)

$$\int \frac{e^{4x} + e^x}{e^{2x}} dx$$

$$= \int (e^{2x} + e^{-x}) dx$$

$$= \frac{1}{2} e^{2x} - e^{-x}$$

(6)

$$\int \frac{1}{4 + x^2}$$

$$= \frac{1}{2} \tan^{-1} \frac{x}{2}$$

(7)

$$\int \frac{dt}{\sqrt{9 - t^2}}$$

$$= \sin^{-1} \frac{t}{3}$$

(8)

$$t = \sqrt{x + 1}$$

$$dt = \frac{dx}{2\sqrt{x + 1}} = \frac{dx}{2t}$$

$$\int x\sqrt{x + 1} dx$$

$$= \int (t^2 - 1)t 2t dt$$

$$= \int (2t^4 - 2t^2) dt$$

$$= \frac{2}{5} t^5 - \frac{2}{3} t^3$$

$$= \frac{2}{15} t^3 (3t^2 - 5)$$

$$= \frac{2}{15} \sqrt{x + 1}^3 (3x - 2)$$

(9)

$$t = x^2 + 3$$

$$dt = 2x dx$$

$$\int x\sqrt{x^2 + 3} dx$$

$$= \int \sqrt{tx} \frac{dt}{2x}$$

$$= \frac{1}{2} \int \sqrt{t} dt$$

$$= \frac{1}{3} \sqrt{(x^2 + 3)}^3$$

(10)

$$\int \frac{x}{x^2 + 1} dx$$

$$= \frac{1}{2} \int \frac{(x^2 + 1)'}{x^2 + 1} dx$$

$$= \frac{1}{2} \log |x^2 + 1|$$

(11)

$$\begin{aligned}
t &= \log x \\
dt &= \frac{1}{x} dx \\
\int \frac{1}{x \log x} dx \\
&= \int \frac{1}{xt} x dt \\
&= \log t \\
&= \log |\log x|
\end{aligned}$$

(12)

$$\begin{aligned}
&\int x \cos x dx \\
&= x \sin x - \int \sin x dx \\
&= x \sin x + \cos x
\end{aligned}$$

(13)

$$\begin{aligned}
&\int x \sin x dx \\
&= -x \cos x - \int -\cos x \\
&= -x \cos x + \sin x
\end{aligned}$$

(14)

$$\begin{aligned}
\int x e^x dx &= x e^x - \int e^x dx \\
&= x e^x - e^x \\
&= (x - 1) e^x
\end{aligned}$$

(15)

$$\begin{aligned}
&\int \log x dx \\
&= x \log x - \int x \frac{1}{x} dx \\
&= x \log x - x
\end{aligned}$$

(16)

$$\begin{aligned}
I &= \int e^x \cos x dx \\
I &= e^x \sin x - \int e^x \sin x dx \\
&= e^x \sin x - (-e^x \cos x - \int e^x \cos x dx) \\
&= e^x \sin x + e^x \cos x - \int e^x \cos x dx \\
2I &= e^x \sin x + e^x \cos x \\
I &= \frac{1}{2} e^x (\sin x + \cos x)
\end{aligned}$$