微分積分 II 春課題

平成 27 年 3 月 26 日

$\boxed{1}$

(1)

$$\lim_{x\to -1}\frac{x^2+3x+2}{x^3+x^2+3x+3}$$
$$=\lim_{x\to -1}\frac{(x+2)(x+1)}{(x+1)(x^2+3)}$$
$$=\lim_{x\to -1}\frac{x+2}{x^2+3}$$
$$=\frac{1}{4}$$

(2)

$$\lim_{x\to \infty}\frac{2x^2+9x-5}{3x^2-x-2}$$
$$=\lim_{x\to \infty}\frac{2+\frac{9}{x}-\frac{5}{x^2}}{3-\frac{1}{x}-\frac{2}{x^2}}$$
$$=\frac{2+0-0}{3-0-0}=\frac{2}{3}$$

(3)

$$\lim_{x\to \infty}\frac{1-3^x}{3^{x+1}+2^x}$$
$$=\lim_{x\to \infty}\frac{3^{-x}-1}{3+\frac{2^x}{3^x}}$$
$$=\frac{0-1}{3+0}=-\frac{1}{3}$$

(4)

$$\lim_{x\to 3-0}\frac{|x-3|}{x^2-x+6}$$
$$=\lim_{x\to 3-0}\frac{1}{-x-2}$$
$$=-\frac{1}{5}$$

(5)

$$\lim_{x\to 0}\frac{\sin 5x}{4x}$$
$$=\lim_{x\to 0}\left(\frac{\sin 5x}{5x}\frac{5x}{4x}\right)$$
$$=1\cdot\frac{5}{4}=\frac{5}{4}$$

(6)

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}$$
$$=\lim_{x\to 0}\frac{\sin x^2}{x^2(1+\cos x)}$$
$$=\lim_{x\to 0}\frac{\sin x}{x}\lim_{x\to 0}\frac{\sin x}{x}\lim_{x\to 0}\frac{1}{1+\cos x}$$
$$=1\cdot 1\cdot\frac{1}{2}=\frac{1}{2}$$

(7)

$$\lim_{x\to 2}\frac{x-2}{\sqrt{x^2+1}-\sqrt{5}}$$
$$=\lim_{x\to 2}\frac{(x-2)(\sqrt{x^2+1}+\sqrt{5})}{x^2-4}$$
$$=\lim_{x\to 2}\frac{\sqrt{x^2+1}+\sqrt{5}}{x+2}$$
$$=\frac{2\sqrt{5}}{4}=\frac{\sqrt{5}}{4}$$

(8)

$$\lim_{h\to 0}(1+3h)^{\frac{1}{h}}$$
$$=\lim_{h\to 0}(1+3h)^{\frac{3}{3h}}$$
$$=e^3$$

$\boxed{2}$

（証明）$f(x)=x+\log_2(x^2+1)-1=0$ とすると，

$f(x)$ は $(-\infty.\infty)$ で連続であり

$f(0)=\log_2 1-1=-1<0$

$f(1)=1+\log_2 2-1=1>0$

よって中間値の定理より

$f(x)$ は、0 と 1 の間に少なくとも 1 つの実数解を持つ

Q.E.D.

3

$$
\begin{aligned}
f(x) &= \sqrt{x} \\
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} \\
&= \lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x}} \\
&= \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}} \\
&= \frac{1}{2\sqrt{x}}
\end{aligned}
$$

4

(1)

$$
\begin{aligned}
y &= (5x-3)^7 \\
y' &= 35(5x-3)^6
\end{aligned}
$$

(2)

$$
\begin{aligned}
y &= \sqrt{6x+5} \\
y' &= \frac{3}{\sqrt{6x+5}}
\end{aligned}
$$

(3)

$$
\begin{aligned}
y &= \frac{4x-3}{x+2} \\
y' &= \frac{4(x+2)-4x+3}{(x+2)^2} \\
&= \frac{11}{(x+2)^2}
\end{aligned}
$$

(4)

$$
\begin{aligned}
y &= \frac{1}{3x^2-1} \\
y' &= -\frac{6x}{(3x^2-1)^2}
\end{aligned}
$$

(5)

$$
\begin{aligned}
y &= (x^2+1)^8 \\
y' &= 16x(x^2+1)^7
\end{aligned}
$$

(6)

$$
\begin{aligned}
y &= \frac{10}{3}\sqrt[5]{x^3} \\
y' &= \frac{10}{3}\frac{3}{5}\frac{1}{\sqrt[5]{x^2}} \\
&= \frac{2}{\sqrt[5]{x^2}}
\end{aligned}
$$

(7)

$$
\begin{aligned}
y &= \sqrt[4]{(2x^2+4x+3)^3} \\
y' &= \frac{3}{4}(2x^2+4x+3)^{-\frac{1}{4}}(4x+4) \\
&= \frac{3(x+1)}{\sqrt[4]{2x^2+4x+3}}
\end{aligned}
$$

(8)

$$
\begin{aligned}
y &= \frac{x}{(1-x)^2} \\
y' &= \frac{(1-x)^2+2x(1-x)}{(1-x)^4} \\
&= \frac{1+x}{(1-x)^3}
\end{aligned}
$$

(9)

$$
\begin{aligned}
y &= (x+1)\sqrt{2x-1} \\
y' &= \sqrt{2x-1}+\frac{x+1}{\sqrt{2x-1}} \\
&= \frac{3x}{\sqrt{2x-1}}
\end{aligned}
$$

5

(1)

$$
\begin{aligned}
y &= \cos\frac{x}{4} \\
y' &= -\frac{1}{4}\sin\frac{x}{4}
\end{aligned}
$$

(2)

$$
\begin{aligned}
y &= \sin^4 x \\
y' &= 4\sin^3 x\cos x
\end{aligned}
$$

(3)

$$
\begin{aligned}
y &= \cos^3 5x \\
y' &= -15\cos^2 5x\sin 5x
\end{aligned}
$$

(4)

$$
\begin{aligned}
y &= x^2\cos\frac{1}{x} \\
y' &= 2x\cos\frac{1}{x}+x^2\sin\frac{1}{x}\frac{1}{x^2} \\
&= 2x\cos\frac{1}{x}+\sin\frac{1}{x}
\end{aligned}
$$

(5)

$$
\begin{aligned}
y &= \frac{\cos x}{1-\sin x} \\
y' &= \frac{-\sin x(1-\sin x)+cos^2x}{(1-\sin x)^2} \\
&= \frac{-\sin x+\sin^2 x+cos^2x}{(1-\sin x)^2} \\
&= \frac{1}{1-\sin x}
\end{aligned}
$$

(6)

$$\begin{aligned}
y &= \sin^2 \frac{1}{\sqrt{x}} \\
y' &= 2\sin\frac{1}{\sqrt{x}}\cos\frac{1}{\sqrt{x}}(-\frac{1}{2x\sqrt{x}}) \\
&= -\frac{1}{x\sqrt{x}}\sin\frac{1}{\sqrt{x}}\cos\frac{1}{\sqrt{x}}
\end{aligned}$$

(7)

$$\begin{aligned}
y &= x^2\tan x \\
y' &= 2x\tan x + x^2\sec^2 x \\
&= x(2\tan x + x\sec^2 x)
\end{aligned}$$

(8)

$$\begin{aligned}
y &= \frac{1+\tan x}{1-\tan x} \\
y' &= \frac{\sec^2 x(1-\tan x)+\sec^2 x(1+\tan x)}{(1-\tan x)^2} \\
&= \frac{2\sec^2 x}{(1-\tan x)^2}
\end{aligned}$$

(9)

$$\begin{aligned}
y &= \arcsin\frac{x}{2} \\
y' &= \frac{\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}} \\
&= \frac{1}{\sqrt{4-x^2}}
\end{aligned}$$

(10)

$$\begin{aligned}
y &= \arctan\frac{2}{x} \\
y' &= \frac{-\frac{2}{x^2}}{1+\frac{4}{x^2}} \\
&= -\frac{2}{x^2+4}
\end{aligned}$$

(11)

$$\begin{aligned}
y &= \log(x^2+3x+1) \\
y' &= \frac{2x+3}{x^2+3x+1}
\end{aligned}$$

(12)

$$\begin{aligned}
y &= \log\left|\frac{1+x}{1-x}\right| \\
y' &= \frac{\frac{1-x+1+x}{(1-x)^2}}{\frac{1+x}{1-x}} \\
&= \frac{2}{1-x^2}
\end{aligned}$$

(13)

$$\begin{aligned}
x &= (e^{3t}+e^{-3t})^5 \\
x' &= 15(e^{3t}+e^{-5t})^4(e^{3t}-e^{-3t})
\end{aligned}$$

6

(1)

$$\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

(2)

$$\arccos -1 = \pi$$

(3)

$$\arctan -\sqrt{3} = -\frac{\pi}{3}$$

7

(1)

$$\begin{aligned}
y &= \frac{(x+2)^5}{(x+1)^4} \\
\frac{y'}{y} &= \left(\log\frac{(x+2)^5}{(x+1)^4}\right)' \\
&= (5\log(x+2) - 4\log(x+1))' \\
&= \frac{5}{x+2} - \frac{4}{x+1} \\
y' &= \frac{(x+2)^4}{(x+1)^4}\left(5 - \frac{4(x+2)}{(x+1)}\right) \\
&= \frac{(x+2)^4(x+3)}{(x+1)^5}
\end{aligned}$$

(2)

$$\begin{aligned}
y &= x^{\sin x} \\
\frac{y'}{y} &= (\log x^{\sin x})' \\
&= (\sin x\log x)' \\
&= \cos x\log x + \frac{1}{x}\sin x \\
y' &= x^{\sin x}\left(\cos x\log x + \frac{1}{x}\sin x\right)
\end{aligned}$$

8

$$\begin{aligned}
x^2+2xy-3y^2+6x-1 &= 0 \\
2x+2y+2xy'-6yy'+6 &= 0 \\
y'(6y-2x) &= 2x+2y+6 \\
y' &= \frac{x+y+3}{3y-x}
\end{aligned}$$

9

$$y=(x+1)^2\ (x<-1)$$
$$x+1=\sqrt{y}$$
$$x=\sqrt{y}-1$$
$$\therefore y=\sqrt{x}-1$$
$$y'=-\frac{1}{2\sqrt{x}}$$

10

(1)

$y=x^3-2x+1$ 上に点 $(2,1)$ は存在しない

$$f(x)=x^3-2x+1$$
$$f'(x)3x^2-2$$
$$f'(2)=10$$
$$y-1=10(x-2)$$
$$y=10x-19$$

$$f(x)=x^2-2x+1$$
$$f'(x)=2x-2$$
$$f'(2)=2$$
$$y-1=2(x-2)$$
$$y=2x-3$$

(2)

$$f(x)=\log\left(1+x^2\right)$$
$$f'(x)=\frac{2x}{1+x^2}$$
$$f'(2)=\frac{4}{5}$$
$$y-\log 5=\frac{4}{5}(x-2)$$
$$y=\frac{4}{5}x-\frac{8}{5}+\log 5$$

(3)

$$f(x)=x^2-2x+1$$
$$f(a)=a^2-2a+1$$
$$f'(x)=2x-2$$
$$f'(a)=2a-2$$
$$y-f(a)=f'(a)(x-a)$$

この直線は点 $(1,-1)$ 上を通るから

$$-1-a^2+2a-1=2a-2(1-a)$$
$$a^2+2a=0$$
$$\therefore a=0,2$$

$a=0$ のとき $y-1=-2(x-0)\therefore y=-2x+1$

$a=2$ のとき $y-1=2(x-2)\therefore y=2x-3$

11

(1)

$$\frac{dy}{dx}=-\frac{3\sin^2 t\cos t}{3\cos^2 t\sin t}=-\tan t$$

(2)

$$x=\cos^3 t=\frac{1}{2\sqrt{2}}$$
$$y=\sin^3 t=\frac{1}{2\sqrt{2}}$$
$$y-\frac{1}{2\sqrt{2}}=-\left(x-\frac{1}{2\sqrt{2}}\right)$$
$$y=-x+\frac{1}{\sqrt{2}}$$

12

(1)

$$y=\frac{1}{x}$$
$$y'=-\frac{1}{x^2}$$
$$y-\frac{1}{a}=-\frac{1}{a^2}(x-a)$$
$$y=-\frac{x}{a^2}+\frac{2}{a}$$

(2)

この直線と x 軸の交点は

$$0=-\frac{x}{a^2}+\frac{2}{a}\ \therefore x=2a$$

この直線と y 軸の交点は

$$y=-\frac{0}{a^2}+\frac{2}{a}\therefore y=\frac{2}{a}$$

よって三角形の面積は

$2a\cdot\frac{2}{a}\cdot\frac{1}{2}=2$ となり、常に一定である.

13

$$f(h)\fallingdotseq f(0)+f'(0)h$$
$$f(h)=\sqrt{1+h}\ f(0)=1$$
$$f'(h)=\frac{1}{2\sqrt{1+h}}\ f'(0)=\frac{1}{2}$$
$$\therefore 1+\frac{1}{2}h$$

$\boxed{\textbf{14}}$

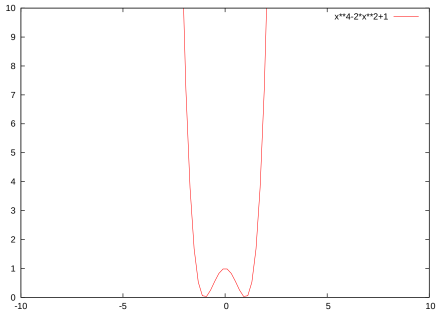
$$y = x^4 - 2x^2 + 1$$

$y' = 4x^3 - 4x$ $y' = 0$ を解くと $x = 0, \pm 1$

$y'' = 12x^2 - 4$ $y'' = 0$ を解くと $x = \pm\frac{1}{\sqrt{3}}$

x	$\cdots$	-1	$\cdots$	$-\frac{1}{\sqrt{3}}$	$\cdots$	0	$\cdots$	$\frac{1}{\sqrt{3}}$	$\cdots$	1	$\cdots$
y'	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$
y''	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$
y	↘	0	↗	$\frac{4}{9}$	↗	1	↘	$\frac{4}{9}$	↘	0	↗

$\therefore$ 変曲点 $(\pm\frac{1}{\sqrt{3}}, \frac{4}{9})$



$\boxed{\textbf{15}}$

$$y = x\log x$$

$y' = \log x + 1$ $y' = 0$ を解くと $x = 0, \pm 1$

x	0	$\cdots$	$\frac{1}{e}$	$\cdots$
y'	/	$-$	0	$+$
y	/	↘	0	↗

$\therefore x = \frac{1}{e}$のとき極小値 $-\frac{1}{e}$, 極大値なし

$\boxed{\textbf{16}}$

$$y = 2\sin x - x$$

$y' = 2\cos x - 1$ $y' = 0$ を解くと $x = \frac{\pi}{3}, \frac{5\pi}{3}$

x	0	$\cdots$	$\frac{\pi}{3}$	$\cdots$	$\frac{5\pi}{3}$	$\cdots$	2π
y'	1	$+$	0	$-$	0	$+$	1
y	0	↗	$\sqrt{3} - \frac{\pi}{3}$	↘	$-\sqrt{3} - \frac{5\pi}{3}$	↗	-2π

$\therefore x = \frac{\pi}{3}$のとき最大値$\sqrt{3} - \frac{\pi}{3}$

$x = \frac{5\pi}{3}$のとき最小値 $-\sqrt{3} - \frac{5\pi}{3}$

$\boxed{\textbf{17}}$

（証明）$y = e^x - 1 - x \qquad (x > 0)$

$y' = e^x - 1$ $\qquad y' = 0$ を解くと $x = 0$

$x > 0$ のとき $y > 0$

$\therefore y > 0$

$\therefore e^x > 1 + x$

Q.E.D.

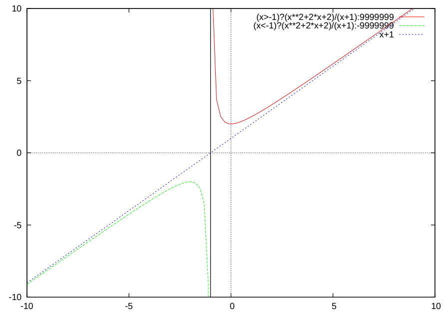
$\boxed{\textbf{18}}$

$$y = \frac{x^2 + 2x + 2}{x + 1} = x + 1 + \frac{1}{x + 1}$$

$$y' = 1 - \frac{1}{(x+1)^2} = \frac{x^2 + 2}{(x+1)^2}$$

$y' = 0$ を解くと $x = 0, 2, x \neq -1$

x	$\cdots$	-2	$\cdots$	-1	$\cdots$	0	$\cdots$
y'	$+$	0	$-$	/	$-$	0	$+$
y	↗	-2	↘	/	↘	2	↗



$\boxed{\textbf{19}}$

$\boxed{\textbf{20}}$

$\boxed{\textbf{21}}$

$\boxed{\textbf{22}}$

$\boxed{\textbf{23}}$

$\boxed{\textbf{24}}$

(1)

(2)

(3)

(4)

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