

# 微分積分Ⅱ春課題

平成 27 年 3 月 24 日

**1**

(1)

$$\begin{aligned} \lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x^3 + x^2 + 3x + 3} \\ &= \lim_{x \rightarrow -1} \frac{(x+2)(x+1)}{(x+1)(x^2+3)} \\ &= \lim_{x \rightarrow -1} \frac{x+2}{x^2+3} \\ &= \frac{1}{4} \end{aligned}$$

(2)

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{2x^2 + 9x - 5}{3x^2 - x - 2} \\ &= \lim_{x \rightarrow \infty} \frac{2 + \frac{9}{x} - \frac{5}{x^2}}{3 - \frac{1}{x} - \frac{2}{x^2}} \\ &= \frac{2 + 0 - 0}{3 - 0 - 0} = \frac{2}{3} \end{aligned}$$

(3)

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{1 - 3^x}{3^{x+1} + 2^x} \\ &= \lim_{x \rightarrow \infty} \frac{3^{-x} - 1}{3 + \frac{2^x}{3^x}} \\ &= \frac{0 - 1}{3 + 0} = -\frac{1}{3} \end{aligned}$$

(4)

$$\begin{aligned} \lim_{x \rightarrow 3-0} \frac{|x-3|}{x^2 - x + 6} \\ &= \lim_{x \rightarrow 3-0} \frac{1}{-x - 2} \\ &= -\frac{1}{5} \end{aligned}$$

(5)

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin 5x}{4x} \\ &= \lim_{x \rightarrow 0} \left( \frac{\sin 5x}{5x} \cdot \frac{5x}{4x} \right) \\ &= 1 \cdot \frac{5}{4} = \frac{5}{4} \end{aligned}$$

(6)

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \\ &= \lim_{x \rightarrow 0} \frac{\sin x^2}{x^2(1 + \cos x)} \\ &= \lim_{x \rightarrow 0} \frac{\sin x}{x} \lim_{x \rightarrow 0} \frac{\sin x}{x} \lim_{x \rightarrow 0} \frac{1}{1 + \cos x} \\ &= 1 \cdot 1 \cdot \frac{1}{2} = \frac{1}{2} \end{aligned}$$

(7)

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x^2+1} - \sqrt{5}} \\ &= \lim_{x \rightarrow 2} \frac{(x-2)(\sqrt{x^2+1} + \sqrt{5})}{x^2 - 4} \\ &= \lim_{x \rightarrow 2} \frac{\sqrt{x^2+1} + \sqrt{5}}{x+2} \\ &= \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{4} \end{aligned}$$

(8)

$$\begin{aligned} \lim_{h \rightarrow 0} (1 + 3h)^{\frac{1}{h}} \\ &= \lim_{h \rightarrow 0} (1 + 3h)^{\frac{3}{3h}} \\ &= e^3 \end{aligned}$$

**2**

(証明)  $f(x) = x + \log_2(x^2 + 1) - 1 = 0$  とすると,

$f(x)$  は  $(-\infty, \infty)$  で連続であり

$$f(0) = \log_2 1 - 1 = -1 < 0$$

$$f(1) = 1 + \log_2 2 - 1 = 1 > 0$$

よって中間値の定理より

$f(x)$  は、0 と 1 の間に少なくとも 1 つの実数解を持つ

*Q.E.D.*

**3**

$$\begin{aligned}
f(x) &= \sqrt{x} \\
f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \\
&= \lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} \\
&= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \\
&= \frac{1}{2\sqrt{x}}
\end{aligned}$$

**4**

(1)

$$\begin{aligned}
y &= (5x-3)^7 \\
y' &= 35(5x-3)^6
\end{aligned}$$

(2)

$$\begin{aligned}
y &= \sqrt{6x+5} \\
y' &= \frac{3}{\sqrt{6x+5}}
\end{aligned}$$

(3)

$$\begin{aligned}
y &= \frac{4x-3}{x+2} \\
y' &= \frac{4(x+2) - 4x+3}{(x+2)^2} \\
&= \frac{11}{(x+2)^2}
\end{aligned}$$

(4)

$$\begin{aligned}
y &= \frac{1}{3x^2-1} \\
y' &= -\frac{6x}{(3x^2-1)^2}
\end{aligned}$$

(5)

$$\begin{aligned}
y &= (x^2+1)^8 \\
y' &= 16x(x^2+1)^7
\end{aligned}$$

(6)

$$\begin{aligned}
y &= \frac{10}{3} \sqrt[5]{x^3} \\
y' &= \frac{10}{3} \cdot \frac{3}{5} \cdot \frac{1}{\sqrt[5]{x^2}} \\
&= \frac{2}{\sqrt[5]{x^2}}
\end{aligned}$$

(7)

$$\begin{aligned}
y &= \sqrt[4]{(2x^2+4x+3)^3} \\
y' &= \frac{3}{4}(2x^2+4x+3)^{-\frac{1}{4}}(4x+4) \\
&= \frac{3(x+1)}{\sqrt[4]{2x^2+4x+3}}
\end{aligned}$$

(8)

$$\begin{aligned}
y &= \frac{x}{(1-x)^2} \\
y' &= \frac{(1-x)^2 + 2x(1-x)}{(1-x)^4} \\
&= \frac{1+x}{(1-x)^3}
\end{aligned}$$

(9)

$$\begin{aligned}
y &= (x+1)\sqrt{2x-1} \\
y' &= \sqrt{2x-1} + \frac{x+1}{\sqrt{2x-1}} \\
&= \frac{3x}{\sqrt{2x-1}}
\end{aligned}$$

**5**

(1)

$$\begin{aligned}
y &= \cos \frac{x}{4} \\
y' &= -\frac{1}{4} \sin \frac{x}{4}
\end{aligned}$$

(2)

$$\begin{aligned}
y &= \sin^4 x \\
y' &= 4 \sin^3 x \cos x
\end{aligned}$$

(3)

$$\begin{aligned}
y &= \cos^3 5x \\
y' &= -15 \cos^2 5x \sin 5x
\end{aligned}$$

(4)

$$\begin{aligned}
y &= x^2 \cos \frac{1}{x} \\
y' &= 2x \cos \frac{1}{x} + x^2 \sin \frac{1}{x} \cdot \frac{1}{x^2} \\
&= 2x \cos \frac{1}{x} + \sin \frac{1}{x}
\end{aligned}$$

(5)

$$\begin{aligned}
y &= \frac{\cos x}{1-\sin x} \\
y' &= \frac{-\sin x(1-\sin x) + \cos^2 x}{(1-\sin x)^2} \\
&= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2} \\
&= \frac{1}{1-\sin x}
\end{aligned}$$

(6)

$$\begin{aligned}
y &= \sin^2 \frac{1}{\sqrt{x}} \\
y' &= 2 \sin \frac{1}{\sqrt{x}} \cos \frac{1}{\sqrt{x}} \left( -\frac{1}{2x\sqrt{x}} \right) \\
&= -\frac{1}{x\sqrt{x}} \sin \frac{1}{\sqrt{x}} \cos \frac{1}{\sqrt{x}}
\end{aligned}$$

(7)

$$\begin{aligned}
y &= x^2 \tan x \\
y' &= 2x \tan x + x^2 \sec^2 x \\
&= x(2 \tan x + x \sec^2 x)
\end{aligned}$$

(8)

$$\begin{aligned}
y &= \frac{1 + \tan x}{1 - \tan x} \\
y' &= \frac{\sec^2 x (1 - \tan x) + \sec^2 x (1 + \tan x)}{(1 - \tan x)^2} \\
&= \frac{2 \sec^2 x}{(1 - \tan x)^2}
\end{aligned}$$

(9)

$$\begin{aligned}
y &= \arcsin \frac{x}{2} \\
y' &= \frac{\frac{1}{2}}{\sqrt{1 - \frac{x^2}{4}}} \\
&= \frac{1}{\sqrt{4 - x^2}}
\end{aligned}$$

(10)

$$\begin{aligned}
y &= \arctan \frac{2}{x} \\
y' &= \frac{-\frac{2}{x^2}}{1 + \frac{4}{x^2}} \\
&= -\frac{2}{x^2 + 4}
\end{aligned}$$

(11)

$$\begin{aligned}
y &= \log(x^2 + 3x + 1) \\
y' &= \frac{2x + 3}{x^2 + 3x + 1}
\end{aligned}$$

(12)

$$\begin{aligned}
y &= \log \left| \frac{1+x}{1-x} \right| \\
y' &= \frac{\frac{1-x+1+x}{(1-x)^2}}{\frac{1+x}{1-x}} \\
&= \frac{2}{1-x^2}
\end{aligned}$$

(13)

$$\begin{aligned}
x &= (e^{3t} + e^{-3t})^5 \\
x' &= 15(e^{3t} + e^{-5t})^4 (e^{3t} - e^{-3t})
\end{aligned}$$

**6**

(1)

$$\arcsin \frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

(2)

$$\arccos -1 = \pi$$

(3)

$$\arctan -\sqrt{3} = -\frac{\pi}{3}$$

**7**

(1)

$$\begin{aligned}
y &= \frac{(x+2)^5}{(x+1)^4} \\
\frac{y'}{y} &= \left( \log \frac{(x+2)^5}{(x+1)^4} \right)' \\
&= (5 \log(x+2) - 4 \log(x+1))' \\
&= \frac{5}{x+2} - \frac{4}{x+1} \\
y' &= \frac{(x+2)^4}{(x+1)^4} \left( 5 - \frac{4(x+2)}{(x+1)} \right) \\
&= \frac{(x+2)^4(x+3)}{(x+1)^5}
\end{aligned}$$

(2)

$$\begin{aligned}
y &= x^{\sin x} \\
\frac{y'}{y} &= (\log x^{\sin x})' \\
&= (\sin x \log x)' \\
&= \cos x \log x + \frac{1}{x} \sin x \\
y' &= x^{\sin x} \left( \cos x \log x + \frac{1}{x} \sin x \right)
\end{aligned}$$

**8**

$$\begin{aligned}
x^2 + 2xy - 3y^2 + 6x - 1 &= 0 \\
2x + 2y + 2xy' - 6yy' + 6 &= 0 \\
y'(6y - 2x) &= 2x + 2y + 6 \\
y' &= \frac{x + y + 3}{3y - x}
\end{aligned}$$

**9**

$$y = (x+1)^2 \quad (x < -1)$$

$$x+1 = \sqrt{y}$$

$$x = \sqrt{y} - 1$$

$$\therefore y = \sqrt{x} - 1$$

$$y' = -\frac{1}{2\sqrt{x}}$$

**10**

(1)

$y = x^3 - 2x + 1$  上に点  $(2, 1)$  は存在しない

$$f(x) = x^3 - 2x + 1$$

$$f'(x) = 3x^2 - 2$$

$$f'(2) = 10$$

$$y - 1 = 10(x - 2)$$

$$y = 10x - 19$$

$$f(x) = x^2 - 2x + 1$$

$$f'(x) = 2x - 2$$

$$f'(2) = 2$$

$$y - 1 = 2(x - 2)$$

$$y = 2x - 3$$

(2)

$$f(x) = \log(1+x^2)$$

$$f'(x) = \frac{2x}{1+x^2}$$

$$f'(2) = \frac{4}{5}$$

$$y - \log 5 = \frac{4}{5}(x - 2)$$

$$y = \frac{4}{5}x - \frac{8}{5} + \log 5$$

(3)

$$f(x) = x^2 - 2x + 1$$

$$f(a) = a^2 - 2a + 1$$

$$f'(x) = 2x - 2$$

$$f'(a) = 2a - 2$$

$$y - f(a) = f'(a)(x - a)$$

この直線は点  $(1, -1)$  上を通るから

$$-1 - a^2 + 2a - 1 = 2a - 2(1 - a)$$

$$a^2 + 2a = 0$$

$$\therefore a = 0, 2$$

$$a = 0 \text{ のとき } y - 1 = -2(x - 0) \therefore y = -2x + 1$$

$$a = 2 \text{ のとき } y - 1 = 2(x - 2) \therefore y = 2x - 3$$

**11**

(1)

$$\frac{dy}{dx} = -\frac{3\sin^2 t \cos t}{3\cos^2 t \sin t} = -\tan t$$

(2)

$$x = \cos^3 t = \frac{1}{2\sqrt{2}}$$

$$y = \sin^3 t = \frac{1}{2\sqrt{2}}$$

$$y - \frac{1}{2\sqrt{2}} = -\left(x - \frac{1}{2\sqrt{2}}\right)$$

$$y = -x + \frac{1}{\sqrt{2}}$$

**12**

(1)

$$y = \frac{1}{x}$$

$$y' = -\frac{1}{x^2}$$

$$y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$$

$$y = -\frac{x}{a^2} + \frac{2}{a}$$

(2)

この直線と  $x$  軸の交点は

$$0 = -\frac{x}{a^2} + \frac{2}{a} \therefore x = 2a$$

この直線と  $y$  軸の交点は

$$y = -\frac{0}{a^2} + \frac{2}{a} \therefore y = \frac{2}{a}$$

よって三角形の面積は

$$2a \cdot \frac{2}{a} \cdot \frac{1}{2} = 2 \text{ となり、常に一定である。}$$

**13**

$$f(h) \doteq f(0) + f'(0)h$$

$$f(h) = \sqrt{1+h} \quad f(0) = 1$$

$$f'(h) = \frac{1}{2\sqrt{1+h}} \quad f'(0) = \frac{1}{2}$$

$$\therefore 1 + \frac{1}{2}h$$

14

$$y = x^4 - 2x^2 + 1$$

$$y' = 4x^3 - 4x \quad y' = 0 \text{ を解くと } x = 0, 1$$

$$y'' = 12x^2 - 4 \quad y'' = 0 \text{ を解くと } x = \pm \frac{1}{\sqrt{3}}$$

$\therefore$  変曲点  $(0, 0), (1, -1)$

15

16

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21

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24

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)