# 微分積分II春課題

平成27年3月22日

**1**

(1)

$$\lim_{x \to -1} \frac{x^2+3x+2}{x^3+x^2+3x+3}$$
$$= \lim_{x \to -1} \frac{(x+2)(x+1)}{(x+1)(x^2+3)}$$
$$= \lim_{x \to -1} \frac{x+2}{x^2+3}$$
$$= \frac{1}{4}$$

(2)

$$\lim_{x \to \infty} \frac{2x^2+9x-5}{3x^2-x-2}$$
$$= \lim_{x \to \infty} \frac{2+\frac{9}{x}-\frac{5}{x^2}}{3-\frac{1}{x}-\frac{2}{x^2}}$$
$$= \frac{2+0-0}{3-0-0} = \frac{2}{3}$$

(3)

$$\lim_{x \to \infty} \frac{1-3^x}{3^{x+1}+2^x}$$
$$= \lim_{x \to \infty} \frac{3^{-x}-1}{3+\frac{2^x}{3^x}}$$
$$= \frac{0-1}{3+0} = -\frac{1}{3}$$

(4)

$$\lim_{x \to 3-0} \frac{|x-3|}{x^2-x+6}$$
$$= \lim_{x \to 3-0} \frac{1}{-x-2}$$
$$= -\frac{1}{5}$$

(5)

$$\lim_{x \to 0} \frac{\sin 5x}{4x}$$
$$= \lim_{x \to 0} (\frac{\sin 5x}{5x} \frac{5x}{4x})$$
$$= 1 \cdot \frac{5}{4} = \frac{5}{4}$$

(6)

$$\lim_{x \to 0} \frac{1-\cos x}{x^2}$$
$$= \lim_{x \to 0} \frac{\sin x^2}{x^2(1+\cos x)}$$
$$= \lim_{x \to 0} \frac{\sin x}{x} \lim_{x \to 0} \frac{\sin x}{x} \lim_{x \to 0} \frac{1}{1+\cos x}$$
$$= 1 \cdot 1 \cdot \frac{1}{2} = \frac{1}{2}$$

(7)

$$\lim_{x \to 2} \frac{x-2}{\sqrt{x^2+1}-\sqrt{5}}$$
$$= \lim_{x \to 2} \frac{(x-2)(\sqrt{x^2+1}+\sqrt{5})}{x^2-4}$$
$$= \lim_{x \to 2} \frac{\sqrt{x^2+1}+\sqrt{5}}{x+2}$$
$$= \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{4}$$

(8)

$$\lim_{h \to 0} (1+3h)^{\frac{1}{h}}$$
$$= \lim_{h \to 0} (1+3h)^{\frac{3}{3h}}$$
$$= e^3$$

**2**

（証明）$f(x) = x + \log_2 (x^2+1) - 1 = 0$ とすると，

$f(x)$ は $(-\infty.\infty)$ で連続であり

$f(0) = \log_2 1 - 1 = -1 < 0$

$f(1) = 1 + \log_2 2 - 1 = 1 > 0$

よって中間値の定理より

$f(x)$ は、0 と 1 の間に少なくとも 1 つの実数解を持つ

*Q.E.D.*

$\boxed{\mathbf{3}}$

$$
\begin{aligned}
f(x) &= \sqrt{x} \\
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \\
&= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} \\
&= \frac{1}{2\sqrt{x}}
\end{aligned}
$$

$\boxed{\mathbf{4}}$

(1)

$$
\begin{aligned}
y &= (5x-3)^7 \\
y' &= 35(5x-3)^6
\end{aligned}
$$

(2)

$$
\begin{aligned}
y &= \sqrt{6x+5} \\
y' &= \frac{3}{\sqrt{6x+5}}
\end{aligned}
$$

(3)

$$
\begin{aligned}
y &= \frac{4x-3}{x+2} \\
y' &= \frac{4(x+2)-4x+3}{(x+2)^2} \\
&= \frac{11}{(x+2)^2}
\end{aligned}
$$

(4)

$$
\begin{aligned}
y &= \frac{1}{3x^2-1} \\
y' &= -\frac{6x}{(3x^2-1)^2}
\end{aligned}
$$

(5)

$$
\begin{aligned}
y &= (x^2+1)^8 \\
y' &= 16x(x^2+1)^7
\end{aligned}
$$

(6)

$$
\begin{aligned}
y &= \frac{10}{3}\sqrt[5]{x^3} \\
y' &= \frac{10}{3}\frac{3}{5}\frac{1}{\sqrt[5]{x^2}} \\
&= \frac{2}{\sqrt[5]{x^2}}
\end{aligned}
$$

(7)

$$
\begin{aligned}
y &= \sqrt[4]{(2x^2+4x+3)^3} \\
y' &= \frac{3}{4}(2x^2+4x+3)^{-\frac{1}{4}}(4x+4) \\
&= \frac{3(x+1)}{\sqrt[4]{2x^2+4x+3}}
\end{aligned}
$$

(8)

$$
\begin{aligned}
y &= \frac{x}{(1-x)^2} \\
y' &= \frac{(1-x)^2+2x(1-x)}{(1-x)^4} \\
&= \frac{1+x}{(1-x)^3}
\end{aligned}
$$

(9)

$$
\begin{aligned}
y &= (x+1)\sqrt{2x-1} \\
y' &= \sqrt{2x-1}+\frac{x+1}{\sqrt{2x-1}} \\
&= \frac{3x}{\sqrt{2x-1}}
\end{aligned}
$$

$\boxed{\mathbf{5}}$

(1)

$$
\begin{aligned}
y &= \cos\frac{x}{4} \\
y' &= -\frac{1}{4}\sin\frac{x}{4}
\end{aligned}
$$

(2)

$$
\begin{aligned}
y &= \sin^4 x \\
y' &= 4\sin^3 x\cos x
\end{aligned}
$$

(3)

$$
\begin{aligned}
y &= \cos^3 5x \\
y' &= -15\cos^2 5x\sin 5x
\end{aligned}
$$

(4)

$$
\begin{aligned}
y &= x^2\cos\frac{1}{x} \\
y' &= 2x\cos\frac{1}{x}+x^2\sin\frac{1}{x}\frac{1}{x^2} \\
&= 2x\cos\frac{1}{x}+\sin\frac{1}{x}
\end{aligned}
$$

(5)

$$
\begin{aligned}
y &= \frac{\cos x}{1-\sin x} \\
y' &= \frac{-\sin x(1-\sin x)+cos^2 x}{(1-\sin x)^2} \\
&= \frac{-\sin x+\sin^2 x+cos^2 x}{(1-\sin x)^2} \\
&= \frac{1}{1-\sin x}
\end{aligned}
$$

(6)

$$
\begin{aligned}
y &= \sin^2 \frac{1}{\sqrt{x}} \\
y' &= 2\sin \frac{1}{\sqrt{x}} \cos \frac{1}{\sqrt{x}} (-\frac{1}{2x\sqrt{x}}) \\
&= -\frac{1}{x\sqrt{x}} \sin \frac{1}{\sqrt{x}} \cos \frac{1}{\sqrt{x}}
\end{aligned}
$$

(7)

$$
\begin{aligned}
y &= x^2 \tan x \\
y' &= 2x \tan x + x^2 \sec^2 x \\
&= x(2\tan x + x\sec^2 x)
\end{aligned}
$$

(8)

$$
\begin{aligned}
y &= \frac{1+\tan x}{1-\tan x} \\
y' &= \frac{\sec^2 x(1-\tan x) + \sec^2 x(1+\tan x)}{(1-\tan x)^2} \\
&= \frac{2\sec^2 x}{(1-\tan x)^2}
\end{aligned}
$$

(9)

$$
\begin{aligned}
y &= \arcsin \frac{x}{2} \\
y' &= \frac{\frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}} \\
&= \frac{1}{\sqrt{4-x^2}}
\end{aligned}
$$

(10)

$$
\begin{aligned}
y &= \arctan \frac{2}{x} \\
y' &= \frac{-\frac{2}{x^2}}{1+\frac{4}{x^2}} \\
&= -\frac{2}{x^2+4}
\end{aligned}
$$

(11)

$$
\begin{aligned}
y &= \log(x^2+3x+1) \\
y' &= \frac{2x+3}{x^2+3x+1}
\end{aligned}
$$

(12)

$$
\begin{aligned}
y &= \log \left| \frac{1+x}{1-x} \right| \\
y' &= \frac{\frac{1-x+1+x}{(1-x)^2}}{\frac{1+x}{1-x}} \\
&= \frac{2}{1-x^2}
\end{aligned}
$$

(13)

$$
\begin{aligned}
x &= (e^{3t} + e^{-3t})^5 \\
x' &= 15(e^{3t} + e^{-5t})^4 (e^{3t} - e^{-3t})
\end{aligned}
$$

## 6

(1)

$$\arcsin \frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

(2)

$$\arccos -1 = \pi$$

(3)

$$\arctan -\sqrt{3} = -\frac{\pi}{3}$$

## 7

(1)

$$
\begin{aligned}
y &= \frac{(x+2)^5}{(x+1)^4} \\
\frac{y'}{y} &= \left( \log \frac{(x+2)^5}{(x+1)^4} \right)' \\
&= (5\log(x+2) - 4\log(x+1))' \\
&= \frac{5}{x+2} - \frac{4}{x+1} \\
y' &= \frac{(x+2)^4}{(x+1)^4} \left( 5 - \frac{4(x+2)}{(x+1)} \right) \\
&= \frac{(x+2)^4(x+3)}{(x+1)^5}
\end{aligned}
$$

(2)

$$
\begin{aligned}
y &= x^{\sin x} \\
\frac{y'}{y} &= (\log x^{\sin x})' \\
&= (\sin x \log x)' \\
&= \cos x \log x + \frac{1}{x} \sin x \\
y' &= x^{\sin x} \left( \cos x \log x + \frac{1}{x} \sin x \right)
\end{aligned}
$$

## 8

$$
\begin{aligned}
x^2 + 2xy - 3y^2 + 6x - 1 &= 0 \\
2x + 2y + 2xy' - 6yy' + 6 &= 0 \\
y'(6y - 2x) &= 2x + 2y + 6 \\
y' &= \frac{x+y+3}{3y-x}
\end{aligned}
$$

## 9

$$y=(x+1)^2\ (x<-1)$$
$$x+1=\sqrt{y}$$
$$x=\sqrt{y}-1$$
$$\therefore y=\sqrt{x}-1$$
$$y'=-\frac{1}{2\sqrt{x}}$$

## 10

(1)

$y=x^3-2x+1$ 上に点 $(2,1)$ は存在しない

$$f(x)=x^3-2x+1$$
$$f'(x)3x^2-2$$
$$f'(2)=10$$
$$y-1=10(x-2)$$
$$y=10x-19$$

$$f(x)=x^2-2x+1$$
$$f'(x)=2x-2$$
$$f'(2)=2$$
$$y-1=2(x-2)$$
$$y=2x-3$$

(2)

$$f(x)=\log(1+x^2)$$
$$f'(x)=\frac{2x}{1+x^2}$$
$$f'(2)=\frac{4}{5}$$
$$y-\log 5=\frac{4}{5}(x-2)$$
$$y=\frac{4}{5}x-\frac{8}{5}+\log 5$$

(3)

$$f(x)=x^2-2x+1$$
$$f(a)=a^2-2a+1$$
$$f'(x)=2x-2$$
$$f'(a)=2a-2$$
$$y-f(a)=f'(a)(x-a)$$

この直線は点 $(1,-1)$ 上を通るから

$$-1-a^2+2a-1=2a-2(1-a)$$
$$a^2+2a=0$$
$$\therefore a=0,2$$

$a=0$ のとき $y-1=-2(x-0)\ \therefore y=-2x+1$

$a=2$ のとき $y-1=2(x-2)\ \therefore y=2x-3$

## 11

(1)

$$\frac{dy}{dx}=-\frac{3\sin^2 t\cos t}{3\cos^2 t\sin t}=-\tan t$$

(2)

$$x=\cos^3 t=\frac{1}{2\sqrt{2}}$$
$$y=\sin^3 t=\frac{1}{2\sqrt{2}}$$
$$y-\frac{1}{2\sqrt{2}}=-\left(x-\frac{1}{2\sqrt{2}}\right)$$
$$y=-x+\frac{1}{\sqrt{2}}$$

## 12

(1)

$$y=\frac{1}{x}$$
$$y'=-\frac{1}{x^2}$$
$$y-\frac{1}{a}=-\frac{1}{a^2}(x-a)$$
$$y=-\frac{x}{a^2}+\frac{2}{a}$$

(2)

この直線と x 軸の交点は

$$0=-\frac{x}{a^2}+\frac{2}{a}\ \ \therefore x=2a$$

この直線と y 軸の交点は

$$y=-\frac{0}{a^2}+\frac{2}{a}\ \therefore y=\frac{2}{a}$$

よって三角形の面積は

$2a\cdot\frac{2}{a}\cdot\frac{1}{2}=2$ となり、常に一定である.

## 13

$$f(h)\fallingdotseq f(0)+f'(0)h$$
$$f(h)=\sqrt{1+h}\ \ f(0)=1$$
$$f'(h)=\frac{1}{2\sqrt{1+h}}\ \ f'(0)=\frac{1}{2}$$
$$\therefore 1+\frac{1}{2}h$$

## 14

$\boxed{\textbf{15}}$

$\boxed{\textbf{16}}$

$\boxed{\textbf{17}}$

$\boxed{\textbf{18}}$

$\boxed{\textbf{19}}$

$\boxed{\textbf{20}}$

$\boxed{\textbf{21}}$

$\boxed{\textbf{22}}$

$\boxed{\textbf{23}}$

$\boxed{\textbf{24}}$

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)