SE - ETRx & EXTC
Sem - IV - AM-IV (CBGS)
Dt: - 23/05/14

QP Code : NP-19713

(3 Hours) **[Total Marks : 80**

N.B.: (1) Quesions No. **1** is **compulsory**.
(2) Solve any **three** from the **remaining.**

1. (a) Prove that Eigen values of a hermitian matrix are real. **5**

(b) Evaluate $\oint_c \frac{e^{kz}}{z} dz$ over the circle $|z|=1$ and k is real. Hence prove that $\int_0^{\pi} e^{k\cos\theta} \cos(k\sin\theta)\, d\theta = \pi$. **5**

(c) Find the extremal of $\int_{x_2}^{x_1} \left(16y^2 - (y'')^2 + x^2\right) dx$ **5**

(d) Find a vector orthogonal to both u = (–6, 4, 2) and v = (3, 1, 5). **5**

2. (a) Find the curve y = f(x) for which $\int_{x_1}^{x_2} y\sqrt{1+(y')^2}\, dx$ is minimum subject to the constraint $\int_{x_1}^{x_2} \sqrt{1+(y')^2}\, dx = \ell$. **6**

(b) Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$ **6**

(c) Obtain Taylor's series and two distinct Laurent's series expansion of $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ about z = 0, indicating region of covergence. **8**

3. (a) State Cayley-Hamilton Theroern, hence deduce that $A^8 = 625I$, where $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ **6**

(b) Using calculus of Residues, prove that $\int_0^{2\pi} e^{\cos\theta} \cos(\sin\theta - n\theta)\, d\theta = \frac{2\pi}{n!}$. **6**

(c) Find the plane curve of fixed perimeter and maximum area. **8**

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4. (a) State Cauchy-Schwartz inequality and hence show that 6

$\left(x^2+y^2+z^2\right)^{1/2} \geq \frac{1}{13}\left(3x+4y+12z\right)$, x, y, z are positive.

(b) Reduce the quadratic form $Q = x^2+y^2-2z^2-4xy-2yz+10xz$ to Canonical form using congruent transformation. 6

(c) (i) If $A=\begin{bmatrix} \pi/2 & 3\pi/2 \\ \pi & \pi \end{bmatrix}$, find Sin A. 4

(ii) Show that the matrix $A=\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is Derogatory. 4

5. (a) Using Rayleigh - Ritz method, find an appropriate solution for the extremal of the functional $I[y(x)]=\int_0^1 \left[xy+\frac{1}{2}\left(y^1\right)^2 \right] dx$ subject to $y(0) = y(1) = 0$. 6

(b) Find an orthonormal basis of the following subspace of R^3, $S = \{ [1, 2, 0]\ [0, 3, 1] \}$. 6

(c) Is the matrix $A=\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ diagonalizable. If so find diagonal form and transforming matrix. 8

6. (a) Find f(3), f ' (1 + i), f " (1 – i), if $f(a)=\oint_c \frac{3z^2+11z+7}{z-a}\,dz$, $c: |z| = 2$. 6

(b) Evaluate $\int_0^\infty \frac{x^3 \sin x}{\left(x^2+a^2\right)^2}$ using contour integration. 6

(c) Find the singular value decomposition of the matrix $A=\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix}$. 8
