Accurate Reconstruction of ECG Signals using Chebyshev Polynomials

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***Abstract*—Chebyshev approximation coefficients have been shown to be an accurate and compact representation of time-series datasets. However, for medical data, such as ECG beats, a clinician may require a time domain representation for medical intervention/diagnosis purposes. Thus, there is a need to accurately recreate these signals from the approximate Chebyshev coefficients - this is especially true in cases where the beat was captured using non-uniformly sampled signals. In this study, we show that a fast approach using an iterative algorithm can be used to reconstruct signals at any time point avoiding the use of interpolation in the reconstruction process resulting in improved accuracy. The average reconstruction error of our proposed algorithm compared to the direct approach is only 0.008%. We also show that this iterative approach can be implemented using a simple (two-pole) IIR filter and that this method can be applied to the reconstruction of any time-series data.**

***Index Terms*—Chebyshev polynomials, signal reconstruction, electrocardiogram, iterative method, IIR filters**

I. Introduction

In non-uniformly sampled ECG signals, randomly missing sample points can make it difficult for clinicians to draw accurate diagnoses on alarming ECG beats marked by an automated arrhythmia classifier. Often, the clinician may need to see the original arrhythmia beat at an arbitrary set of time points (uniform or non-uniform) or higher signal resolution for further medical intervention. In [1], the accurate representation of ECG beats using Chebyshev approximation features was presented. In this paper, we consider the problem of reconstruction of a signal at a set of N time domain points given the first K Chebyshev coefficients. An iterative approach based on the work in [2] is proposed using a two-pole IIR filter and a comparison with the direct method with interpolation is drawn. The application of this method is not limited to ECG signals and can be applied to the reconstruction of any time-domain signal, whether uniformly or non-uniformly sampled. The rest of the article is organized as follows: Section II presents the two reconstruction techniques and the iterative algorithm, Section III presents the experimentation and results, and Section IV draws conclusions.

II. Methdology

As shown in [1], a normalized time-domain signal, $y(x)$ can be represented by an infinite weighted sum of Chebyshev polynomials. In practice we approximate $y(x)$ with a finite weighted sum, $\tilde{y}(x)$, as follows:

$$y(x) \simeq \tilde{y}(x) \triangleq \sum_{k=0}^{K-1} c_k T_k(x) \tag{1}$$

where, $T_{k+1}(x) \triangleq 2xT_k(x) - T_{k-1}(x)$ are the Chebyshev polynomials, and the RMS error between $y(x)$ and $\tilde{y}(x)$ become smaller as more terms are added.

In [1] it is shown that the $\{c_k\}$ are generated by firstly interpolating the signal to an orthogonal set of time-domain points followed by a dot-product step as illustrated in Fig. 1. We now consider various methods for signal reconstruction at the original time points.

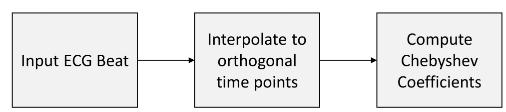

Fig. 1: Steps to generate approximation coefficients. To reconstruct the ECG beat, these steps are simply reversed.

A. Direct Method with Interpolation

Here we simply reverse the steps outlined in Fig. 1, i.e. we firstly employ (1) to compute the samples at the orthogonal time points and then use an interpolator to recreate the samples at the original time domain points. We note that we *could* use (1) to compute the signal directly at the original time domain points however this would require knowledge of the polynomials at each and every possible time point in advance which, certainly in the case of non-uniformly sampled data, is not feasible. Alternatively, we would require a method to dynamically compute the polynomials at run-time which would add significantly to the complexity.

B. Iterative Method

We propose a method based on [2] and works for the 1st, 2nd, 3rd or 4th kind Chebyshev polynomials. Using matrix notation:

$$\tilde{y}(x) = \boldsymbol{c}^T \boldsymbol{p}(x) \tag{2}$$

where $\boldsymbol{p}(x)$ and $\boldsymbol{c}$ are the $K \times 1$ vectors of polynomial samples at any *time* point, x, and the coefficients respectively, e.g.:

$$\boldsymbol{p}(x) \triangleq \begin{bmatrix} T_0(x) \\ T_1(x) \\ \vdots \\ T_{K-1}(x) \end{bmatrix} \quad \text{and} \quad \boldsymbol{c} = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{K-1} \end{bmatrix}$$

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For the 2nd, 3rd or 4th kind Chebyshev polynomials the vector $\boldsymbol{p}(x)$ could be computed by exploiting the inherent recursion property in the definition of the Chebyshev polynomials [3]. This can be embodied in the following matrix equation:

$$\boldsymbol{A}(x)\boldsymbol{p}(x) = \boldsymbol{d}(x)$$

where $\boldsymbol{A}(x)$ is a $K \times K$ matrix and $\boldsymbol{d}(x)$ a $K \times 1$ vector defined as::

$$\boldsymbol{A}(x) \triangleq \begin{bmatrix} 1 & & & & \\ -2x & 1 & & & \\ 1 & -2x & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & -2x & 1 \end{bmatrix} \quad \text{and} \quad \boldsymbol{d}(x) \triangleq \begin{bmatrix} 1 \\ \zeta \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

and ζ is either $-x$, 0, -1, or 1 for the 1st, 2nd, 3rd or 4th kind Chebyshev polynomials respectively. Now if we define a $K \times 1$ vector $\boldsymbol{b}(x)$ such that it is the solution to

$$\boldsymbol{c}^T = \boldsymbol{b}^T(x)\boldsymbol{A}(x) \tag{3}$$

we can rewrite the direct reconstruction formula (2) as:

$$\begin{aligned} \tilde{y}(x) &= \boldsymbol{b}^T(x)\boldsymbol{A}(x)\boldsymbol{p}(x) \\ &= \boldsymbol{b}^T(x)\boldsymbol{d}(x) = b_0 + \zeta b_1 \end{aligned} \tag{4}$$

So only the first two entries in the vector $\boldsymbol{b}(x)$, i.e. the scalars b_0 and b_1 (both dependant on x), need be computed then the reconstruction via (4) is trivial. The entire procedure has complexity $O(K)$ and is summarized in Algorithm 1.

Let $b_K = b_{K+1} = 0$
for $r = K-1 \rightarrow 0$ **do**
| $b_r = c_r + 2xb_{r+1} - b_{r+2}$
end
$\tilde{y}(x) = b_0 + \zeta b_1$

Algorithm 1: Procedure for computing $\tilde{y}(x)$ given $\boldsymbol{c}$

An important advantage of this method over the direct method is that there is no need to decide on a discrete time-grid in advance as this process can be performed for any point x on the continuous range $[-1, +1]$.

1) IIR filter approach: Note that the loop in algorithm 1 is that of a difference equation which, after some manipulation of the input and output orderings, can be implemented using a two-pole IIR filter structure with zero initial state. This observation is useful as it permits fast implementations in many standard programming languages, e.g. the MATLAB code is as follows:

```
bb = filter(1,[1,-2*x,1],c(end:-1:1));
y  = bb(end) + zeta*bb(end-1);
```

III. Experiment and Results

The two methods were evaluated using all beats in the Record 234 of the MIT-BIH Arrhythmia dataset [4] using 200 Chebyshev coefficients. Table I shows a comparison of the two signal reconstruction methods. The direct method, i.e. evaluation of the signal at the Chebyshev orthogonal time points followed by an interpolation process, had an average reconstruction error of 4.82%, while our proposed iterative method (which avoids interpolation) had an average reconstruction error of 0.008%. The main advantages of our proposed method are that we no longer need an evaluation of polynomials to reconstruct the signal and we can reconstruct the signal at any set of time points without the use of a separate interpolation process. Fig. 2 shows a reconstructed ECG beat using the iterative method with an RMS error of 0.009%.

TABLE I: Comparison of signal reconstruction techniques

Method	RMS error	Available time domain points
Direct & interpolation	4.821%	the polynomials need to be known at said time points in advance or we need the capability of computing them at run-time (expensive).
Iterative	0.008%	any set of N points.

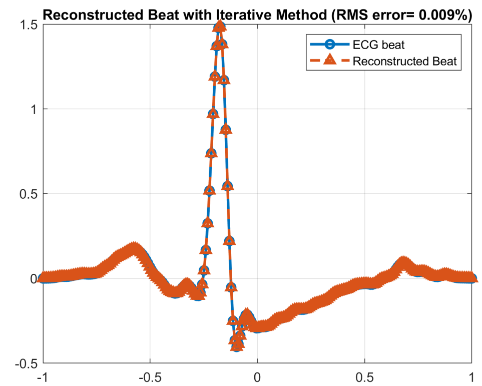

Fig. 2: Reconstructed ECG beat using the iterative method.

IV. Conclusion

In this paper, we briefly showed that an iterative algorithm based on the work in [2] can be used for the accurate reconstruction of ECG beats from approximate Chebyshev coefficients. This approach which avoids interpolation resulted in only 0.008% average reconstruction error in comparison with the direct method (which includes interpolation), which had an average error of 4.821%. Moreover, the iterative method allowed for the reconstruction to any set of arbitrary time points. This method can also be applied to the reconstruction of any time-series data.

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