FEATURE IDENTIFICATION OF ECG WAVEFORMS VIA ORTHONORMAL FUNCTIONS

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Abstract

The linear predictor model (LPM), also known as autoregressive model (AR), has been used extensively by many authors for feature identification and classification. In this paper, simulations are presented showing that the LPM is inadequate for feature identification of ECG waveforms, because it can not discriminate waveforms with different morphologies. In addition, it needs very high order models for high resolution ECG. A new method, the orthonormal function model (OFM), is presented to identify the underlying features of ECG waveforms. This method is based on the approximation properties of orthonormal functions. Among the set of all orthonormal functions, the Chebyshev polynomial has been selected, because it can uniformly approximate a broad class of functions and it gives the strongest convergence among all ultraspherical polynomials. Simulation results from normal and abnormal (ST depression) ECG waveforms indicate that the OFM has the following properties: (1) it has a residual error that decays to zero faster than for the LPM, (2) it only needs a small model order for feature identification, (3) the model order for high resolution is smaller than for the LPM, and (4) most importantly the OFM can discriminate features from the ECG waveforms. The OFM can be successfully used for feature identification and as an aid in the classification and diagnosis of normal and abnormal patterns in the electrical activity of the heart.

Introduction

ECG waveform feature identification and classification are important tools that provide information on cardiac activity. It is desirable to have a method for identifying the underlying electrical activity of the heart in order to accurately diagnose cardiac dysfunctions. It is equally important that correct informative features be identified. Thus, it is essential to have a method to identify features with discriminatory information. Feature identification consists in identifying a set of parameters, called the *feature vector*, from a given waveform. The feature vector is assumed to belong to a *feature vector space* with Euclidean distance measure. Any vector that belongs to this space can be classified based on its distance from a given class. Therefore, waveform classification is reduced to identifying its feature vector and measuring its distance from a set of classes. The waveform is then said to belong to a class if its distance to that class is the smallest.

Various methods have been proposed to identify features from ECG waveforms such as the linear predictor model (LPM) [1, 2, 3] and exponential function model (EFM) [4, 5, 6, 7]. The LPM of order larger than two does not reduce significantly the approximation error of the ECG waveforms [1, 2]. LPM also has a dynamic range that is too small for feature discrimination [1]. For high accuracy, model orders of 20 to 40 are needed to represent the waveform [3]. The EFM has not been as widely used as the LPM mainly because it only applies to feature identification of normal QRS waveforms [6]. This method takes advantage of the morphology of the waveform that can be represented with functions of monophasic, biphasic, and triphasic shape.

We propose, a new method for ECG waveform feature identification based on the approximating properties of orthonormal functions. Among the set of all orthonormal functions, the Chebyshev polynomial has been selected, because it can uniformly approximate a broad class of functions and because it gives the strongest convergence among all ultraspherical polynomials. This method, named the orthonormal function model (OFM), is tested on normal and abnormal (ST depression) simulated ECG waveforms and compared with the LPM method.

Linear Predictor Model

LPM is used extensively in linear systems theory for a class of processes that fit the autoregressive (AR) model. Recently, it has been widely applied in speech signal processing [8] with relative success. The advantages of this method are that it is simple to compute and only requires few terms to approximate the signal. For a sampled ECG waveform the LPM approximates the sequence, $s(i)$, with

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another sequence, $\hat{s}(i)$, formed by a linear combination of linear predictor coefficients (LPC), a_k, and past values of the sampled waveform. A p-th order LPM of a signal $s(i)$ is expressed as:

$$\hat{s}(i) = \sum_{k=1}^{p} a_k s(i-k)$$

The LPM error, $e(i)$, is computed as the difference between the actual signal and the predicted signal:

$$e(i) = s(i) - \hat{s}(i) = s(i) - \sum_{k=1}^{p} a_k s(i-k)$$

There are several methods to compute the LPC a_k. One that is simple and efficient to compute is the *autocorrelation method* [8]. The LPC are computed such that the total approximation error:

$$E_L = \sum_{i=1}^{N+p} e^2(i)$$

is minimized. It is assumed that the signal $s(i)$ is zero for $1 > i > N$, where N is the number samples of the waveform. Then, the LPC are obtained from solving the linear matrix equation:

$$\mathbf{Ra} = \mathbf{b}$$

where:

$$\mathbf{R} = \begin{bmatrix} R(0) & R(1) & \cdots & R(p-1) \\ R(1) & R(0) & \cdots & R(p-2) \\ \vdots & \vdots & \ddots & \vdots \\ R(p-1) & R(p-2) & \cdots & R(0) \end{bmatrix}$$

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} R(1) \\ R(2) \\ \vdots \\ R(p) \end{bmatrix}$$

and $R(k) = \sum_{i=1}^{N-k} s(i)s(i+k), k = [0, p]$.

Orthonormal Function Model

The OFM is a general method for approximating functions with a finite linear combination of orthonormal basis functions, $\{\phi_i(t), i = 0, p\}$. Let the ECG waveform, $s(t)$, be a continuous function of time which is assumed to belong to a Hilbert space with L_2-norm. Then, $s(t) \in \mathbf{H_2}$ can be represented by a finite linear combination of orthonormal basis functions that span the finite dimensional subspace $\mathbf{S}_p$ defined as follows:

$$\mathbf{S}_p \equiv \{f(t) | f(t) = \sum_{i=0}^{p} a_i \phi_i(t);\ f(t), a_i \in \mathbf{R}\}$$

Since $\mathbf{S}_p \subset \mathbf{H_2}$, from approximation theory [9] there exists a unique best L_2-approximation $\hat{s}(t)$ of $s(t)$ such that:

$$||\hat{s}(t) - s(t)||_2 = \inf\{||f(t) - s(t)||_2 | f(t) \in \mathbf{S}_p\}$$

Therefore, a general ECG waveform $s(t)$ can be represented by a unique parameter vector (feature vector) $\mathbf{a} = [a_0, a_1, \ldots, a_p]^T$ in the orthogonal subspace $\mathbf{S}_p$. The properties of the approximation, error bound and convergence rate, can be determined once a specific basis function is selected. The Chebyshev polynomials of the first class are chosen as basis functions, since they can uniformly approximate a broader class of functions and give the strongest convergence rate among all ultraspherical polynomials [10]. The Chebyshev polynomials of the first class are defined as follows:

$$T_k(t) = \cos(k \cos^{-1}(t)),\ \ t \in [-1, 1]$$

Then, the approximation of an ECG waveform, $s(t)$, with Chebyshev polynomials of order p is:

$$\hat{s}(t) = \frac{a_0}{2} T_0(t) + \sum_{k=1}^{p} a_k T_k(t)$$

The following theorem gives an L_2 bound of the Chebyshev approximation error.

Theorem 1 (Chebyshev Approximation Error) *Let $s(t)$ be a function for $t \in [-1, 1]$ that is $p+1$ times differentiable with bounded $s^{(p+1)}(t) \leq M, \forall t \in [-1, 1]$. Then there exists a Chebyshev approximation $\hat{s}(t)$ such that:*

$$||s(t) - \hat{s}(t)||_2 \leq \frac{\sqrt{2}M}{2^p(p+1)!}$$

Proof: See reference [11].

□

The error bound of Theorem 1 indicates that the approximation error converges to zero very fast even for a small Chebyshev polynomial order.

Next, a method to compute the parameter vector $\mathbf{a}$ is presented. For a sampled signal $s(i)$ corresponding to time $\Delta T i$, where ΔT is the uniform sampling time of the signal. The Chebyshev polynomial approximation, $\hat{s}(i)$, at this instance can be written as:

$$\hat{s}(i) = \frac{a_0}{2} T_0(\Delta T i) + \sum_{k=1}^{p} a_k T_k(\Delta T i)$$

If the sampled signal consists of N points, the parameter vector $\mathbf{a}$ can be computed such that the error:

$$E_T = \sum_{i=1}^{N} (s(i) - \hat{s}(i))^2$$

is minimized. This is a classical least-squares problem with solution:

$$\mathbf{Sa} = \mathbf{b}$$

where:

$$\mathbf{S} = \begin{bmatrix} \sigma(0,0) & \sigma(0,1) & \cdots & \sigma(0,p) \\ \sigma(0,1) & \sigma(1,1) & \cdots & \sigma(1,p) \\ \vdots & \vdots & \ddots & \vdots \\ \sigma(0,p) & \sigma(1,p) & \cdots & \sigma(p,p) \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_p \end{bmatrix}$$

$$\mathbf{a} = \begin{bmatrix} a_0/2 \\ a_1 \\ \vdots \\ a_p \end{bmatrix}$$

and $\sigma(j,k) = \sigma(k,j) = \sum_{i=1}^{N} T_j(\Delta Ti)T_k(\Delta Ti)$, $b_j = \sum_{i=1}^{N} s(i)T_j(\Delta Ti)$ for $j,k = [0,p]$.

Euclidean Distance Measure

Given two ECG waveforms, $s_1(t)$ and $s_2(t)$, a way to determine how close (or far) they are from each other is using their feature vector Euclidean distance, defined as:

$$d^2 \equiv |\mathbf{a}_1 - \mathbf{a}_2|^2 = (\mathbf{a}_1 - \mathbf{a}_2)^T(\mathbf{a}_1 - \mathbf{a}_2)$$

where $\mathbf{a}_1$ and $\mathbf{a}_2$ are the feature vectors of $s_1(t)$ and $s_2(t)$ respectively. Therefore, a small distance d means that both signals belong to the same class. This is only true if the feature vector has been able to identify the actual morphology of the waveform.

Although we are only interested in the ECG waveform morphology, the feature vector can be affected by amplitude, time scale, and time shift of the waveform. In order to avoid these effects, the ECG signal is normalized. Two waveforms $s_1(t)$ and $s_2(t)$ are equal if [6]:

$$s_1(t) = \alpha s_2(\beta t + \tau)$$

for some τ and $\alpha, \beta > 0$. Therefore, the signals $s_1(t)$ and $s_2(t)$ are said to be normalized if $\alpha = 1$, $\beta = 1$, and $\tau = 0$. In addition, the feature vector is normalized such that $|\mathbf{a}| = 1$. This implies that the feature vector lies on a hypersphere of radius 1 and the morphology of the ECG waveform is determined only by its location on the surface of this hypersphere.

Simulation Results

Simulations were performed with the ECG waveforms shown in Figure 1. These signals were generated using a digital simulator device, that has actual ECG signals stored in ROM (Dynathech Nevada Inc., Medsim 300). One is a normal waveform and the second is an abnormal ECG with ST depression. Both signals contain 200 sample points, sampled at 300 Hz with 16 bit amplitude resolution. The signals have been normalized i.e. they are aligned in time relative to the R wave and scaled so that this peak is equal to one.

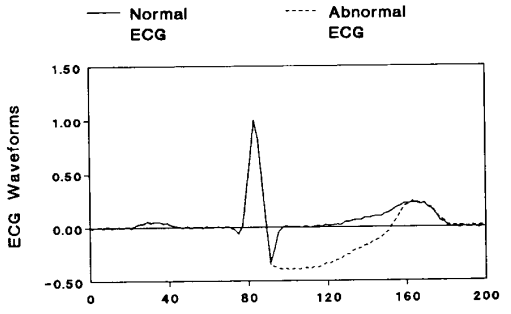

Figure 1: Normal and Abnormal (ST depression) ECG Waveforms

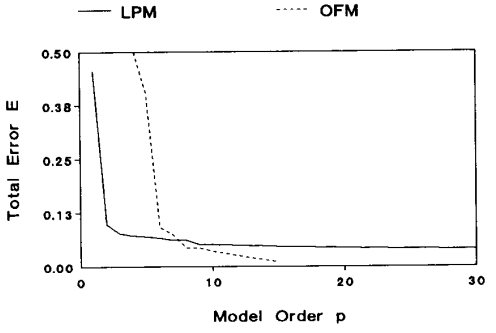

Figure 2: Total Error E_L and E_T for various model orders

Now, the LPM and the OFM were applied to both ECG waveforms for model orders $p \in [1,30]$ for the LPM and $p \in [2,15]$ for the OFM. In the OFM the waveforms were partitioned in three segments corresponding to the normal P, QRS, and T waves. Then, the numerical algorithm for computing the parameter vector **a** was applied to each segment. This has the advange of reducing the computational effort by solving three smaller dimension linear matrix equations. The total approximation error for the normal ECG waveform was computed for each model order and plotted as shown in Figure 2. The LPM error (E_L) with order $p > 2$ does not decrease significantly. However, the OFM for $p > 8$ has an error (E_T) smaller than the LPM error which becomes very small for $p \geq 15$. This result agrees well with Theorem 1.

Finally for each model order, the normalized distance was computed between the normal and abnormal ECG. Figure 3 shows this distance for the LPM. It is unclear from this graph whether the two waveforms are close or not. For some model orders the distance is small for others it is larger. In addition, increasing the model order does not resolve the problem. In Figure 4 the normalized distance for the OFM is plotted for each segment. The distance measure for each

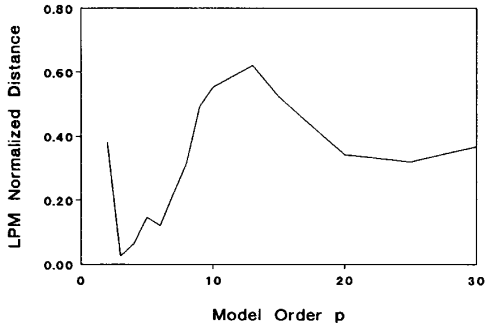

Figure 3: LPM Normalized Distance between Normal and Abnormal ECG

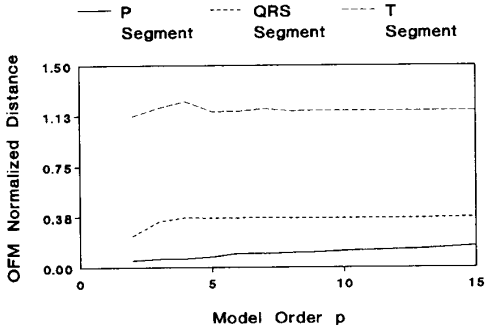

Figure 4: OFM Normalized Distance between Normal and Abnormal ECG

segment does not change significantly for $p \geq 4$ indicating that the feature vector for $p = 4$ has most of the information needed to discriminate one waveform from the other. It is clear from this graph that for the abnormal ECG the T segment is at significant distance from the normal one, the QRS segment shows some distance from the normal one, and the P segment has a minimal distance (the maximum distance is 2). From Figure 1, it can be observed that the abnormal ECG differs from the normal ECG: (1) significantly at the T segment, (2) moderately at the QRS segment, and (3) minimal at the P segment.

Conclusions

A new method called the OFM has been presented for feature identification of normal and abnormal ECG waveforms. The approach has been shown to be superior to the LPM: (1) the total error converges to zero fast and with a small model order, (2) the model order needed to identify the features of the waveform is small (typically 4 or 5), (3) the number of terms for high resolution ECG is smaller than for the LPM, and (4) more importantly the OFM is able to identify the significant features of the waveform even for small model orders. The OFM has the appropriate elements for feature identification and for aiding in the classification and diagnosis of normal and abnormal patterns in the electrical activity of the heart.

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