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- _____ **1.** (a) express $\partial w/\partial u$ and $\partial w/\partial v$ as functions of u and v both by using the Chain Rule and by expressing w directly in terms of u and v before differentiating. Then (b) evaluate $\partial w/\partial u$ and $\partial w/\partial v$ at the given point (u, v) .

$$w = \ln(x^2 + y^2 + z^2), \quad x = ue^v \sin u, \quad y = ue^v \cos u, \quad z = ue^v; \quad (u, v) = (-2, 0).$$

- _____ **2.** Assuming that the equation y as a differentiable function of x , use Theorem 8 (A Formula for Implicit Differentiation) to find the value of dy/dx at the given point.

$$xe^{x^2y} - ye^x = x - y + 2, \quad \text{Point : } (1, 1).$$

3. Suppose that we substitute polar coordinates $x = r \cos \theta$ and $y = r \sin \theta$ in a differentiable function $w = f(x, y)$.

a. Show that

$$\frac{\partial w}{\partial r} = f_x \cos \theta + f_y \sin \theta$$

and

$$\frac{1}{r} \frac{\partial w}{\partial \theta} = -f_x \sin \theta + f_y \cos \theta.$$

b. Solve the equations in part (a) to express f_x and f_y in terms of $\partial w / \partial r$ and $\partial w / \partial \theta$.

c. Show that

$$(f_x)^2 + (f_y)^2 = \left(\frac{\partial w}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial w}{\partial \theta} \right)^2.$$

4. Find ∇f at the given point.

$$f(x, y, z) = 2z^3 - 3(x^2 + y^2)z + \tan^{-1} xz, \quad \text{Point : } (1, 1, 1).$$

- _____ **5.** Find the derivative of the function at P_0 in the direction of \mathbf{u} .

$$f(x, y) = 2x^2 + y^2, \quad P_0(-1, 1), \quad \mathbf{u} = 3\mathbf{i} - 4\mathbf{j}.$$

- _____ **6.** Find the directions in which the functions increase and decrease most rapidly at P_0 . Then find the derivatives of the functions in these directions.

$$f(x, y) = x^2y + e^{xy} \sin y, \quad P_0(1, 0)$$

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- _____ **7.** Find equations for the **(a)** tangent plane and **(b)** normal line at the point P_0 on the given surface.

$$x^2 + y^2 - z^2 = 18, \quad P_0(3, 5, -4).$$

8. Find an equation for the plane that is tangent to the given surface at the given point.

$$z = 4x^2 + y^2, \quad \text{Point : } (1, 1, 5).$$

9. Find parametric equations for the line tangent to the curve of intersection of the surfaces at the given point.

$$\text{Surfaces : } x^2 + y^2 = 4, \quad x^2 + y^2 - z = 0, \quad \text{Point : } (\sqrt{2}, \sqrt{2}, 4).$$

10. Find the linearizations $L(x, y, z)$ of the functions at the given points.

$$f(x, t, z) = \tan^{-1}(xyz) \quad \text{at}$$

$$\text{a. } (1, 0, 0) \quad \text{b. } (1, 1, 0) \quad \text{c. } (1, 1, 1)$$