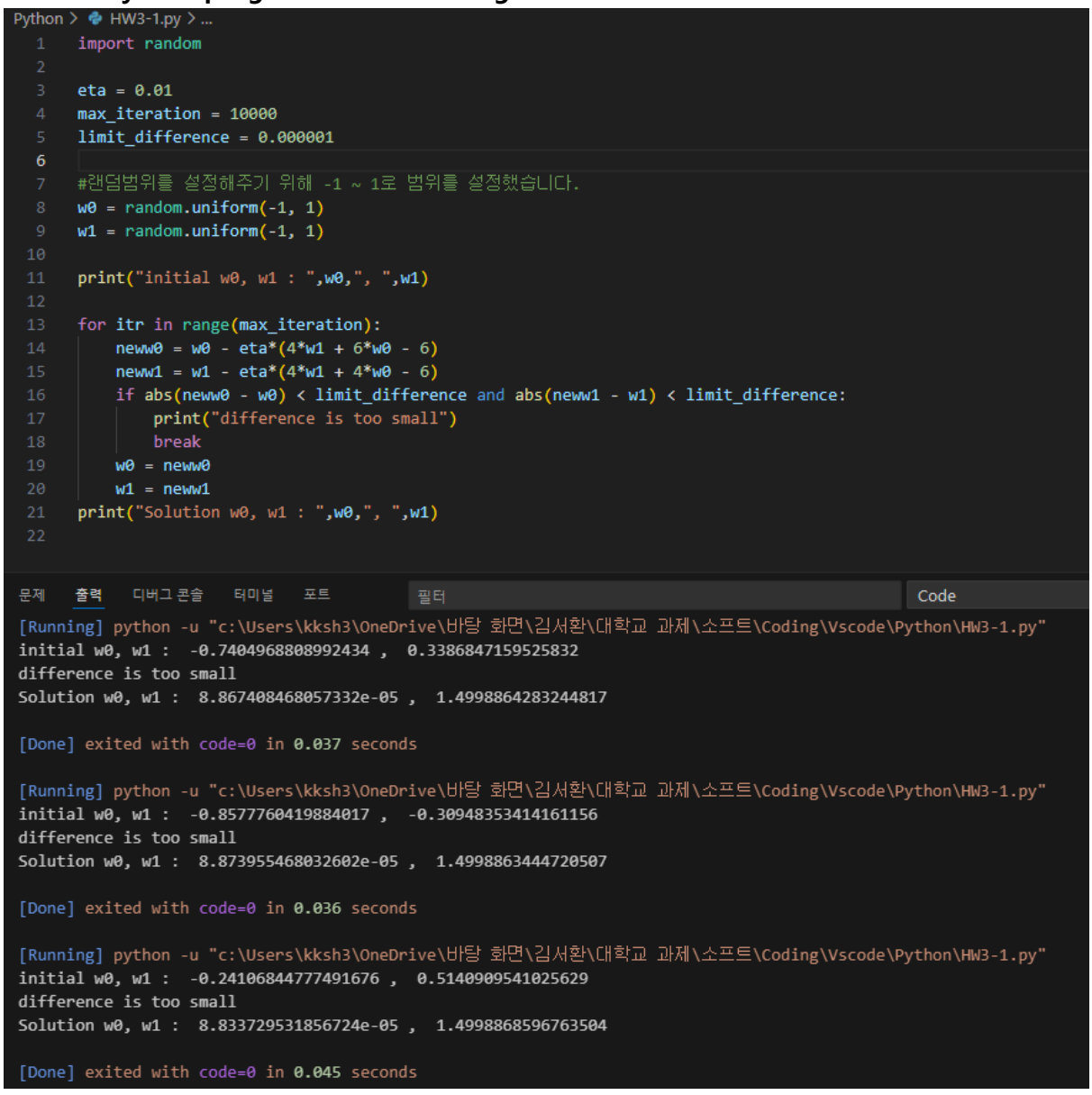
기계학습원론 HW3

2021312738 소프트웨어학과 김서환

1. Make a Python program of the final alg.



```
Python > HW3-1.py > ...
1  import random
2
3  eta = 0.01
4  max_iteration = 10000
5  limit_difference = 0.000001
6
7  #랜덤범위를 설정해주기 위해 -1 ~ 1로 범위를 설정했습니다.
8  w0 = random.uniform(-1, 1)
9  w1 = random.uniform(-1, 1)
10
11 print("initial w0, w1 : ",w0,", ",w1)
12
13 for itr in range(max_iteration):
14     neww0 = w0 - eta*(4*w1 + 6*w0 - 6)
15     neww1 = w1 - eta*(4*w1 + 4*w0 - 6)
16     if abs(neww0 - w0) < limit_difference and abs(neww1 - w1) < limit_difference:
17         print("difference is too small")
18         break
19     w0 = neww0
20     w1 = neww1
21 print("Solution w0, w1 : ",w0,", ",w1)
22
```

```
문제  출력  디버그 콘솔  터미널  포트        필터        Code
[Running] python -u "c:\Users\kksh3\OneDrive\바탕 화면\김서환\대학교 과제\소프트\Coding\Vscode\Python\HW3-1.py"
initial w0, w1 :  -0.7404968808992434 ,  0.3386847159525832
difference is too small
Solution w0, w1 :  8.867408468057332e-05 ,  1.4998864283244817

[Done] exited with code=0 in 0.037 seconds

[Running] python -u "c:\Users\kksh3\OneDrive\바탕 화면\김서환\대학교 과제\소프트\Coding\Vscode\Python\HW3-1.py"
initial w0, w1 :  -0.8577760419884017 ,  -0.30948353414161156
difference is too small
Solution w0, w1 :  8.873955468032602e-05 ,  1.4998863444720507

[Done] exited with code=0 in 0.036 seconds

[Running] python -u "c:\Users\kksh3\OneDrive\바탕 화면\김서환\대학교 과제\소프트\Coding\Vscode\Python\HW3-1.py"
initial w0, w1 :  -0.24106844777491676 ,  0.5140909541025629
difference is too small
Solution w0, w1 :  8.833729531856724e-05 ,  1.4998868596763504

[Done] exited with code=0 in 0.045 seconds
```

<python code>

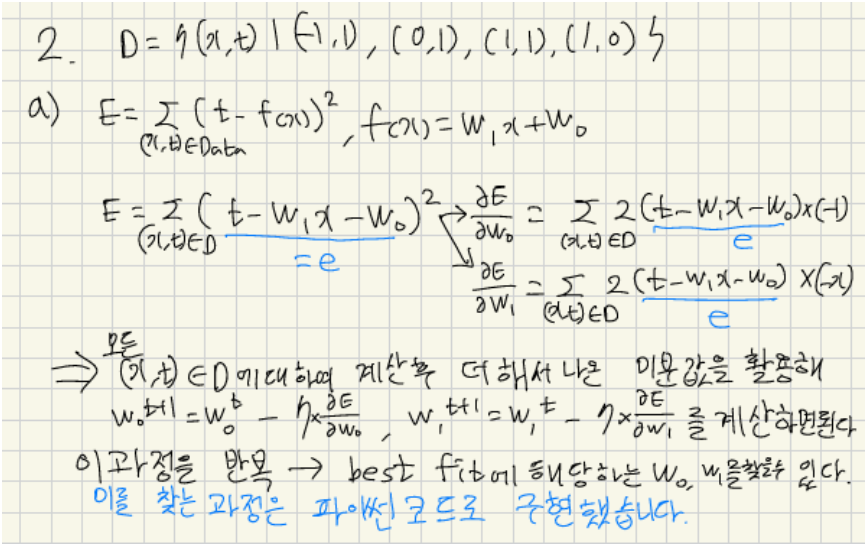
```
import random

eta = 0.01
max_iteration = 10000
```

```
limit_difference = 0.000001

#랜덤범위를 설정해주기 위해 -1 ~ 1로 범위를 설정했습니다.
w0 = random.uniform(-1, 1)
w1 = random.uniform(-1, 1)

print("initial w0, w1 : ",w0,", ",w1)

for itr in range(max_iteration):
    neww0 = w0 - eta*(4*w1 + 6*w0 - 6)
    neww1 = w1 - eta*(4*w1 + 4*w0 - 6)
    if abs(neww0 - w0) < limit_difference and abs(neww1 - w1) < limit_difference:
        print("difference is too small")
        break
    w0 = neww0
    w1 = neww1
print("Solution w0, w1 : ",w0,", ",w1)
```

2. Make Python programs

실제 오차 함수를 구하는 과정은 필기로 작성하였고, GDM을 통해 정답을 도출하는 과정은 파이썬 프로그래밍으로 구현해서 f(x)의 best fit을 찾았습니다.

a)



```python
import random

eta = 0.01
max_iteration = 10000
limit_difference = 0.000001

#랜덤범위를 설정해주기 위해 -1 ~ 1로 범위를 설정했습니다.
w0 = random.uniform(-1, 1)
w1 = random.uniform(-1, 1)

data = [(-1,1), (0,1), (1,1), (1,0)]

print("initial w0, w1 : ",w0,", ",w1)\

for itr in range(max_iteration):
    g0 = 0
    g1 = 0
    for x, t in data:
        fx = w1*x + w0
        e = t - fx
        derw0 = (-2)*e
        derw1 = 2*(-x)*e
        g0 = g0 + derw0
        g1 = g1 + derw1

    neww0 = w0 - eta*g0
    neww1 = w1 - eta*g1

    if abs(neww0 - w0) < limit_difference and abs(neww1 - w1) < limit_difference:
        print("difference is too small")
        break

    w0 = neww0
    w1 = neww1

print("Solution w0, w1 : ",w0,", ",w1)
```

```
문제  출력  디버그 콘솔  터미널  포트        필터        Code
[Running] python -u "c:\Users\kksh3\OneDrive\바탕 화면\김서환\대학교 과제\소프트\Coding\Vscode\Python\HW3-2.py"
initial w0, w1 :  0.26923915709414037 ,  0.30397398719612734
difference is too small
Solution w0, w1 :  0.8181693124235426 ,  -0.2727070383233932

[Done] exited with code=0 in 0.038 seconds

[Running] python -u "c:\Users\kksh3\OneDrive\바탕 화면\김서환\대학교 과제\소프트\Coding\Vscode\Python\HW3-2.py"
initial w0, w1 :  0.43762278751885897 ,  -0.48314899986467696
difference is too small
Solution w0, w1 :  0.818168086167206 ,  -0.2727063394728492

[Done] exited with code=0 in 0.037 seconds

[Running] python -u "c:\Users\kksh3\OneDrive\바탕 화면\김서환\대학교 과제\소프트\Coding\Vscode\Python\HW3-2.py"
initial w0, w1 :  -0.08772283341990383 ,  -0.9358540536602145
difference is too small
Solution w0, w1 :  0.818194414843605 ,  -0.27274783551822784

[Done] exited with code=0 in 0.038 seconds
```

f(x) best fit : -0.2727x + 0.8181

<python code>

import random

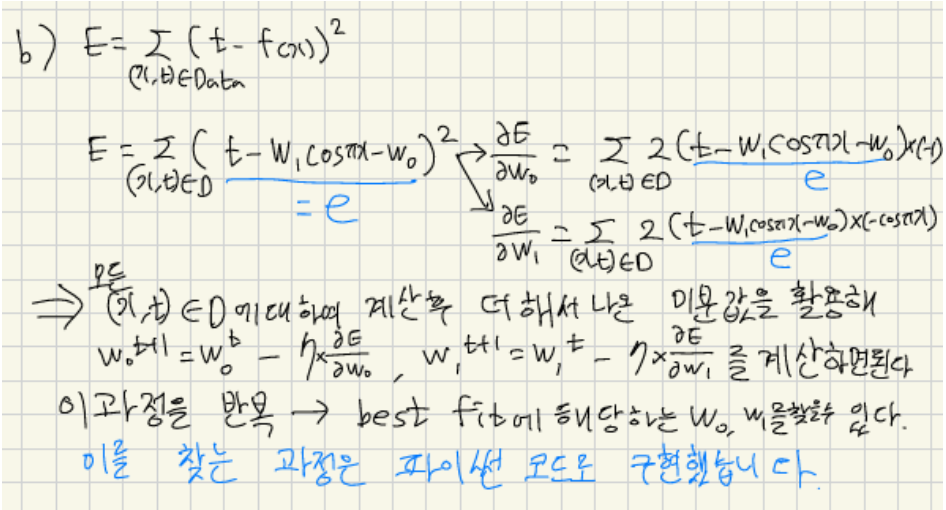
eta = 0.01

max_iteration = 10000

```
limit_difference = 0.000001

#랜덤범위를 설정해주기 위해 -1 ~ 1로 범위를 설정했습니다.
w0 = random.uniform(-1, 1)
w1 = random.uniform(-1, 1)

data = [(-1,1), (0,1), (1,1), (1,0)]

print("initial w0, w1 : ",w0,", ",w1)

for itr in range(max_iteration):
    g0 = 0
    g1 = 0
    for x, t in data:
        fx = w1*x + w0
        e = t - fx
        derw0 = (-2)*e
        derw1 = 2*(-x)*e
        g0 = g0 + derw0
        g1 = g1 + derw1

    neww0 = w0 - eta*g0
    neww1 = w1 - eta*g1

    if abs(neww0 - w0) < limit_difference and abs(neww1 - w1) < limit_difference:
```

```
            print("difference is too small")
            break

    w0 = neww0
    w1 = neww1

print("Solution w0, w1 : ",w0,", ",w1)
```

b)



```
Python > HW3-2.py > ...
 1  import random
 2  import math
 3
 4  eta = 0.01
 5  max_iteration = 10000
 6  limit_difference = 0.000001
 7
 8  #랜덤범위를 설정해주기 위해 -1 ~ 1로 범위를 설정했습니다.
 9  w0 = random.uniform(-1, 1)
10  w1 = random.uniform(-1, 1)
11
12  data = [(-1,1), (0,1), (1,1), (1,0)]
13
14  print("initial w0, w1 : ",w0,", ",w1)
15
16  for itr in range(max_iteration):
17      g0 = 0
18      g1 = 0
19      for x, t in data:
20          fx = w1*math.cos(math.pi*x) + w0
21          e = t - fx
22          derw0 = (-2)*e
23          derw1 = (-2)*math.cos(math.pi*x)*e
24          g0 = g0 + derw0
25          g1 = g1 + derw1
26
27      neww0 = w0 - eta*g0
28      neww1 = w1 - eta*g1
29
30      if abs(neww0 - w0) < limit_difference and abs(neww1 - w1) < limit_difference:
31          print("difference is too small")
32          break
33
34      w0 = neww0
35      w1 = neww1
36
37  print("Solution w0, w1 : ",w0,", ",w1)
```

```
문제  출력  디버그 콘솔  터미널  포트   필터   Code
[Running] python -u "c:\Users\kksh3\OneDrive\바탕 화면\김서환\대학교 과제\소프트\Coding\Vscode\Python\HW3-2.py"
initial w0, w1 :  0.5668960952964142 ,  0.03869016258058289
difference is too small
Solution w0, w1 :  0.8333085252442948 ,  0.16664185857771263

[Done] exited with code=0 in 0.038 seconds

[Running] python -u "c:\Users\kksh3\OneDrive\바탕 화면\김서환\대학교 과제\소프트\Coding\Vscode\Python\HW3-2.py"
initial w0, w1 :  0.7305429134643693 ,  -0.3905996578320461
difference is too small
Solution w0, w1 :  0.8333089131693995 ,  0.16664224650268

[Done] exited with code=0 in 0.037 seconds

[Running] python -u "c:\Users\kksh3\OneDrive\바탕 화면\김서환\대학교 과제\소프트\Coding\Vscode\Python\HW3-2.py"
initial w0, w1 :  0.9969413718064286 ,  -0.2617690772773895
difference is too small
Solution w0, w1 :  0.8333092806472594 ,  0.16664261397944932

[Done] exited with code=0 in 0.036 seconds
```

f(x) best fit : 0.1666x + 0.8333

<python code>

import random

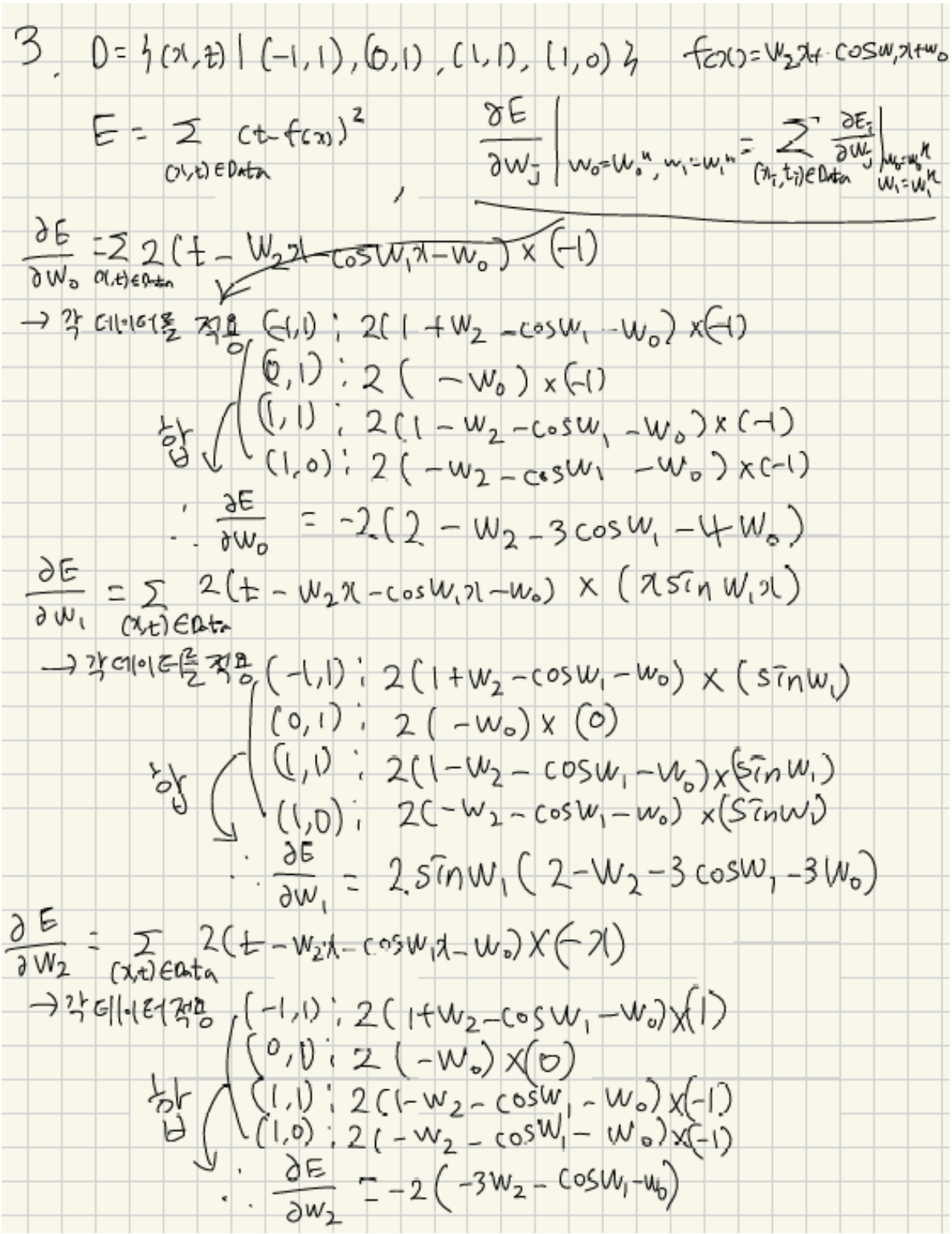
import math

eta = 0.01

```
max_iteration = 10000
limit_difference = 0.000001

#랜덤범위를 설정해주기 위해 -1 ~ 1로 범위를 설정했습니다.
w0 = random.uniform(-1, 1)
w1 = random.uniform(-1, 1)

data = [(-1,1), (0,1), (1,1), (1,0)]

print("initial w0, w1 : ",w0,", ",w1)

for itr in range(max_iteration):
    g0 = 0
    g1 = 0
    for x, t in data:
        fx = w1*math.cos(math.pi*x) + w0
        e = t - fx
        derw0 = (-2)*e
        derw1 = (-2)*math.cos(math.pi*x)*e
        g0 = g0 + derw0
        g1 = g1 + derw1

    neww0 = w0 - eta*g0
    neww1 = w1 - eta*g1
```

```
    if abs(neww0 - w0) < limit_difference and abs(neww1 - w1) < limit_difference:
        print("difference is too small")
        break

    w0 = neww0
    w1 = neww1

print("Solution w0, w1 : ",w0,", ",w1)
```

3. Gradient


3. D = {(x,t) | (-1,1), (0,1), (1,1), (1,0)}
f(x) = w2x + cos(w1x + w0)
E = Σ (t - f(x))^2
(x,t)∈Data
∂E/∂wj |w0=w0^k, w1=w1^k = Σ ∂Ei/∂wj |w0=w0^k, w1=w1^k
(xi,ti)∈Data
∂E/∂w0 = Σ 2(t - w2x - cos(w1x - w0)) x (-1)
(x,t)∈Data
→ 각 데이터를 적용
(-1,1): 2(1 + w2 - cos(w1 - w0)) x (-1)
(0,1): 2(-w0) x (-1)
(1,1): 2(1 - w2 - cos(w1 - w0)) x (-1)
(1,0): 2(-w2 - cos(w1 - w0)) x (-1)
합
∴ ∂E/∂w0 = -2(2 - w2 - 3cos(w1 - 4w0))
∂E/∂w1 = Σ 2(t - w2x - cos(w1x - w0)) x (x sin(w1x))
(x,t)∈Data
→ 각 데이터를 적용
(-1,1): 2(1 + w2 - cos(w1 - w0)) x (sin(w1))
(0,1): 2(-w0) x (0)
(1,1): 2(1 - w2 - cos(w1 - w0)) x (sin(w1))
(1,0): 2(-w2 - cos(w1 - w0)) x (sin(w1))
합
∴ ∂E/∂w1 = 2 sin(w1)(2 - w2 - 3cos(w1 - 3w0))
∂E/∂w2 = Σ 2(t - w2x - cos(w1x - w0)) x (-x)
(x,t)∈Data
→ 각 데이터를 적용
(-1,1): 2(1 + w2 - cos(w1 - w0)) x (1)
(0,1): 2(-w0) x (0)
(1,1): 2(1 - w2 - cos(w1 - w0)) x (-1)
(1,0): 2(-w2 - cos(w1 - w0)) x (-1)
합
∴ ∂E/∂w2 = -2(-3w2 - cos(w1 - w0))

a) $W_0=1, W_1=1, W_2=1$

$\therefore \frac{\partial E}{\partial W_0} = 6+6\cos 1$

$\therefore \frac{\partial E}{\partial W_1} = -4\sin 1 - 6\sin 1\cdot\cos 1$

$\therefore \frac{\partial E}{\partial W_2} = 8+2\cos 1$

b) $W_0=2, W_1=2, W_2=2$

$\therefore \frac{\partial E}{\partial W_0} = 16+6\cos 2$

$\therefore \frac{\partial E}{\partial W_1} = -12\sin 2 - 6\sin 2\cdot\cos 2$

$\therefore \frac{\partial E}{\partial W_2} = 16+2\cos 2$

4. Python program for new algorithm

```python
import random
import math

eta = 0.01
max_iteration = 10000
limit_difference = 0.000001

#랜덤범위를 설정해주기 위해 -1 ~ 1로 범위를 설정했습니다.
w0 = random.uniform(-1, 1)
w1 = random.uniform(-1, 1)
w2 = random.uniform(-1, 1)

data = [(-1,1), (0,1), (1,1), (1,0)]

print("initial w0, w1, w2 : ",w0,", ",w1,", ",w2)\

for itr in range(max_iteration):
    g0 = 0
    g1 = 0
    g2 = 0
    for x, t in data:
        fx = w2*x + math.cos(w1*x) + w0
        e = t - fx
        derw0 = (-2)*e
        derw1 = 2*x*math.sin(w1*x)*e
        derw2 = (-2)*x*e
        g0 = g0 + derw0
        g1 = g1 + derw1
        g2 = g2 + derw2

    neww0 = w0 - eta*g0
    neww1 = w1 - eta*g1
    neww2 = w2 - eta*g2

    if abs(neww0 - w0) < limit_difference and abs(neww1 - w1) < limit_difference and abs(neww2 - w2) < limit_difference:
        print("difference is too small")
        break

    w0 = neww0
    w1 = neww1
    w2 = neww2
print("Solution w0, w1, w2: ",w0,", ",w1,", ",w2)
```

```
문제  출력  디버그 콘솔  터미널  포트   필터   Code
[Running] python -u "c:\Users\kksh3\OneDrive\바탕 화면\김서환\대학교 과제\소프트\Coding\Vscode\Python\1.py"
initial w0, w1, w2 :  -0.4395406298338458 ,  -0.6125878139575578 ,  0.4843746891402636
difference is too small
Solution w0, w1, w2:  -0.00010101461040679245 ,  -0.7225382945790537 ,  -0.25001040941810615

[Done] exited with code=0 in 0.041 seconds

[Running] python -u "c:\Users\kksh3\OneDrive\바탕 화면\김서환\대학교 과제\소프트\Coding\Vscode\Python\1.py"
initial w0, w1, w2 :  0.01316327399191608 ,  -0.4859837662058555 ,  0.43297707584917
difference is too small
Solution w0, w1, w2:  -0.00010133332093401645 ,  -0.7225376762425331 ,  -0.25001044226401253

[Done] exited with code=0 in 0.044 seconds

[Running] python -u "c:\Users\kksh3\OneDrive\바탕 화면\김서환\대학교 과제\소프트\Coding\Vscode\Python\1.py"
initial w0, w1, w2 :  -0.9640079698007584 ,  -0.06285835846486543 ,  0.9669451641523306
difference is too small
Solution w0, w1, w2:  -0.00010117104688505812 ,  -0.7225379910736534 ,  -0.25001042554025044

[Done] exited with code=0 in 0.042 seconds
```

따라서 f(x)의 best fit은 다음과 같다.

f(x) = -0.250010*x + cos(-0.722538*x) – 0.000101

w1은 삼각함수와 관련되어 w를 어떻게 잡느냐에 따라 달라질 수도 있음.

<python code>

```
import random
import math

eta = 0.01
max_iteration = 10000
limit_difference = 0.000001

#랜덤범위를 설정해주기 위해 -1 ~ 1로 범위를 설정했습니다.
w0 = random.uniform(-1, 1)
w1 = random.uniform(-1, 1)
w2 = random.uniform(-1, 1)

data = [(-1,1), (0,1), (1,1), (1,0)]

print("initial w0, w1, w2 : ",w0,", ",w1,", ",w2)

for itr in range(max_iteration):
    g0 = 0
    g1 = 0
    g2 = 0
    for x, t in data:
        fx = w2*x + math.cos(w1*x) + w0
        e = t - fx
```

```
            derw0 = (-2)*e
            derw1 = 2*x*math.sin(w1*x)*e
            derw2 = (-2)*x*e
            g0 = g0 + derw0
            g1 = g1 + derw1
            g2 = g2 + derw2

        neww0 = w0 - eta*g0
        neww1 = w1 - eta*g1
        neww2 = w2 - eta*g2

        if abs(neww0 - w0) < limit_difference and abs(neww1 - w1) < limit_difference and
abs(neww2 - w2) < limit_difference:
            print("difference is too small")
            break

        w0 = neww0
        w1 = neww1
        w2 = neww2
print("Solution w0, w1, w2: ",w0,", ",w1,", ",w2)
```