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Problem 1. (5 points) Let R be a relation on a nonempty set X. Determine whether each proposition is true or false. You don't have to explain your answer.

(1) If R is symmetric, then R is not antisymmetric.

(2) If R is antisymmetric, then R is not symmetric.

(3) If R is not symmetric, then R is antisymmetric.

(4) If R is not antisymmetric, then R is symmetric.

(5) If R is symmetric and antisymmetric, then R satisfies the following condition: For every $x, y \in X$, if $(x, y) \in R$, then $x = y$.

Problem 2. (10 points) For each algorithm, answer the question. You don't have to explain your answer.

(1) Input : a, b, c (integers)
Output: k (integer)

Alice(a, b, c) {
    $k = a$
    if $(b < k)$
        $k = b + 2$
    if $(c > k)$
        $k = b + c$
    return k
}

What is the output of the algorithm **Alice** for the input $a = 5$, $b = 2$, $c = 8$?

(2) Input : s, n (s is a sequence of n numbers: $s = (s_1, s_2, \dots, s_n)$)
Output: k (integer)

Bob(s, n)
    $k = 0$
    $i = n$
    while $(i \geq 1)$
        for $j = 1$ **to** i
            $k = k + s_j$
        $i = \lfloor i/2 \rfloor$
    return k
}

What is the output of the algorithm **Bob** for the input $s = (1, 2, 3, 4, 5, 6, 7)$ and $n = 7$?

(3) Input : s, n (s is a sequence of n numbers: $s = (s_1, s_2, \dots, s_n)$)
Output: k (integer)

Chris(s, n)
    $k = 0$
    for $i = 1$ **to** $n - 1$
        if $(s_i > s_{i+1})$
            $k = k + i$
    return k
}

What is the output of the algorithm **Chris** for the input $s = (8, 9, 12, 5, 3, 5, 4, 7)$ and $n = 8$?

Problem 3. (15 points) Prove or disprove each statement.

1. $X \times (Y - Z) = (X \times Y) - (X \times Z)$ for any sets X, Y, Z.
2. $X - (Y \times Z) = (X - Y) \times (X - Z)$ for any sets X, Y, Z.
3. $X - (Y \times Z) = X$ for any sets X, Y, Z.
4. $X - (X \times X) = X$ for any set X.

Problem 4. (10 points) Prove that for every positive integer n we have

$$\sum_{i=1}^{n} (-1)^{i-1} i^2 = (-1)^{n-1} n(n+1)/2.$$

Problem 5. (15 points) Let $X = \{1, 2, \ldots, 2019\}$. Define a relation R on X^X, the set of functions from X to X, by $(f, g) \in R$ if $f(1) = g(1)$.

1. Prove that R is an equivalence relation on X^X.
2. Find the number of elements in the equivalence class $[f]$ for the function $f : X \to X$ given by $f(x) = x$ for all $x \in X$.
3. Find the number of equivalence classes in R.

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Problem 6. (15 points) Suppose that R is a relation on a nonempty set X. Let $R^2 = R \circ R$. Prove or disprove each statement.

1. If R is reflexive, then R^2 is reflexive.
2. If R^2 is reflexive, then R is reflexive.
3. If R is symmetric, then R^2 is symmetric.
4. If R^2 is symmetric, then R is symmetric.

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Problem 7. (15 points) Prove or disprove each statement.

1. If $f(n) = \Theta(g(n))$ then $f(n)^{2019} = \Theta(g(n)^{2019})$.
2. If $f(n) = \Theta(g(n))$ then $2019^{f(n)} = \Theta(2019^{g(n)})$.

Problem 8. (15 points) For a positive integer n let $\sigma(n)$ be the number of divisors of n. Find the number of positive integers n satisfying both of the following two conditions:

1. n is a divisor of 12!.
2. $\sigma(n)$ is not divisible by 3.

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