Discrete Mathematics Final Exam (GEDB007-46, Fall 2021)

Problem 1. Find the number of functions $f : \{1, 2, \ldots, 7\} \to \{1, 2, 3, 4\}$ such that the number of divisors of $f(1)f(2)\cdots f(7)$ is equal to 35.

Problem 2. Find the number of ways to order the letters MATHEMATICS with given condition.

(1) No conditions.

(2) T and H are not adjacent.

(3) The same letters are not adjacent.

Problem 3. Let S_n be the number of subsets of $\{1, 2, \ldots, n\}$ that do not contain consecutive integers. Let f_n be the nth Fibonacci number: $f_1 = 1$, $f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$. Show that $S_n = f_{n+2}$, $n = 1, 2, \ldots$.

Problem 4. Find a formula for the nth term a_n of the sequence $a_0, a_1, a_2, \ldots$, which satisfies the initial conditions $a_0 = 9$, $a_1 = -11$ and the recurrence relation given by

$$a_n = 2na_{n-1} + 15n^2a_{n-2} - 15na_{n-2}.$$

Problem 5. Let G be the simple graph whose vertices are $v_1, v_2, \ldots, v_{10}$ and distinct vertices v_i and v_j are adjacent if and only if $\gcd(i, j) > 1$.

(1) Find the number of edges of G.

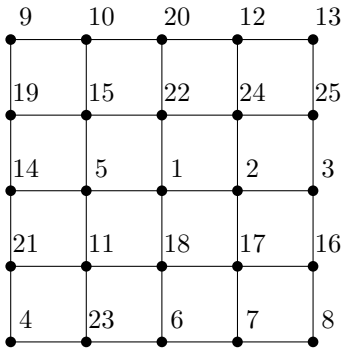
(2) Prove or disprove: G is planar.

Problem 6. Suppose that P is a polyhedron satisfying the following conditions.

- Every face is either a quadrilateral (4-gon) or a hexagon (6-gon).
- For each vertex, there are 2 hexagons and one quadrilateral meeting at this vertex.

Find the number of quadrilaterals and the number of hexagons of this polyhedron.

Problem 7. Let G be the following graph.



(1) Draw the spanning tree of G obtained by the depth-first search with the vertex ordering $1, 2, \ldots, 25$.

(2) Draw the spanning tree of G obtained by the breadth-first search with the vertex ordering $1, 2, \ldots, 25$.

Problem 8. Let X be the expression $((A + B)/(C - D)) - (E * (F + G))$.

(1) Write the expression X using the prefix form.

(2) Write the expression X using the postfix form.