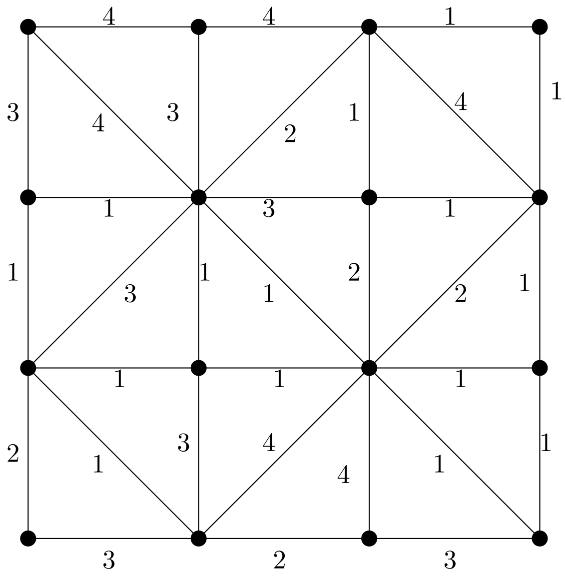
Student ID		Name		Instructor	Jang Soo Kim

Problem 1 (10 points)**.** Find a minimal spanning tree and compute the weight of it.



Problem 2 (10 points)**.** Answer the following questions.

1. Construct an optimal Huffman code for the set of letters in the table.

Letter	Frequency
A	12
C	5
E	6
H	4
I	13
M	9
S	1
T	2

2. How many binary digits are needed to encode "MATHEMATICS" using the Huffman code that you obtained above?

Problem 3 (10 points)**.** Let A be the adjacency matrix of a simple graph G.

1. Prove or disprove: If A^2 has a diagonal entry equal to 0, then G is disconnected.
2. Prove or disprove: If G is disconnected, A^2 has a diagonal entry equal to 0.

Problem 4 (10 points)**.** Suppose that T is a binary tree with vertices $A, B, C, D, E, F, G, H, I, J, K, L$ such that

- the postorder listing of the vertices of T is $LCHBGKIJDEAF$, and
- the inorder listing of the vertices of T is $CLGHBFAKIDJE$.

Draw the binary tree T.

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Problem 5 (10 points)**.** Find the number of sequences $(a_1, a_2, \dots, a_{2019})$ satisfying the following three conditions:

- $a_i \in \{1, 2, 3, 4, 5\}$ for all $1 \le i \le 2019$,
- the number of integers $1 \le i \le 2019$ such that $a_i \le 2$ is 1000, and
- there is no integer $1 \le i \le 2018$ such that $a_i \le 2$ and $a_{i+1} \le 2$.

(Your answer must be as simple as possible without summation.)

Problem 6 (20 points)**.** Let $\{a_n\}_{n\ge0}$ be the sequence given by $a_0 = 0$, $a_1 = 1$ and for $n \ge 2$,

$$a_n = 4(a_0 + a_1 + \cdots + a_{n-2}) + a_{n-1}.$$

Find a general formula for a_n for $n \ge 1$.

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Problem 7 (15 points)**.** Suppose that P is a polyhedron satisfying the following conditions:

- At every vertex, there are 3 or 4 faces meeting at this vertex.
- Every face is a quadrilateral (4-gon).

Find the number of vertices of degree 3 in this polyhedron. Prove your answer.

Problem 8 (15 points)**.** Let X be a collection of subsets of $\{1, 2, \dots, n\}$ such that for any two elements A, B in X we have $A \cap B \ne \emptyset$. What is the maximum size of X? Prove your answer.