# 기계학습원론 HW10

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1. We have one training sample, (1,1). The initial weights are $w1$ = 0.5, $w2$ = 0.5. The learning rate is $\eta$ = 0.1. The activation function is Sigmoid. Loss function is MSE.

① Update each of $w1$ and $w2$ once by gradient descent method.

1. ① $(x_t, y_t) = (1, 1)$, $w_1 = 0.5$, $w_2 = 0.5$, $\eta = 0.1$

1번째 퍼셉트론의 Input의 선형결합 결과를 $s_1$,

2번째 퍼셉트론의 Input의 선형결합 결과를 $s_2$라하자.

$$s_1 = w_1 \times x_t$$

$$h = \frac{1}{1+e^{-s_1}}$$

$$s_2 = w_2 \times h$$

$$y = \frac{1}{1+e^{-s_2}}$$

$$E(\text{Loss function}) = \frac{1}{2}(y_t - y)^2$$

variable dependency graph를 그려보면

$$x_t \rightarrow s_1 \rightarrow h \rightarrow s_2 \rightarrow y \rightarrow E$$

$$w_1 \nearrow \qquad w_2 \nearrow \qquad y_t \nearrow$$

이때 GDM을 적용시키기 위해서는 $\frac{\partial E}{\partial w_1}$, $\frac{\partial E}{\partial w_2}$가 필요하므로

$\frac{\partial E}{\partial w_1}$ $w_1 \rightarrow s_1 \rightarrow h \rightarrow s_2 \rightarrow y \rightarrow E$ / $\frac{\partial E}{\partial w_2}$ $w_2 \rightarrow s_2 \rightarrow y \rightarrow E$ 를 구하면된다.

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial h}\frac{\partial h}{\partial s_1}\frac{\partial s_1}{\partial w_1} = -(y_t - y)\cdot y(1-y)\cdot w_2 \cdot h(1-h)\cdot x_t$$

위변수들에 $x_t, y_t, w_1, w_2$ 대입하여 각 값을 구해 대입하면된다.

계산편의성 위해

$\rightarrow s_1 = 0.5$, $h = \frac{1}{1+e^{-0.5}} = 0.6225$ (소수점 4번째 자리까지 반올림)

$s_2 = 0.3113$, $y = \frac{1}{1+e^{-0.3113}} = 0.5772$ 이 값들을 위 식에 대입하면,

$$\frac{\partial E}{\partial w_1} = -(1-0.5772)\cdot 0.5772\cdot(1-0.5772)\cdot 0.5\cdot 0.6225(1-0.6225)\cdot 1 = -0.0121$$

$\therefore w_1' = w_1^{\circ} - \eta \times \frac{\partial E}{\partial w_1} = 0.5 + 0.1 \times 0.0121 = 0.5012$ (소수점 4번째 자리까지 반올림)

w2도 똑같이 구하면,

$$\frac{\partial E}{\partial w_2} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial w_2} = -(y_t - y)\cdot y(1-y)\cdot h$$
$$= -(1-0.5772)\cdot 0.5772\cdot(1-0.5772)\cdot 0.6225$$
$$= -0.0642$$
$$\therefore w_2^1 = w_2^0 - \eta \times \frac{\partial E}{\partial w_2} = 0.5 + 0.1 \times 0.0642 = 0.5064$$
$$\boxed{\therefore w_1^1 = 0.5012,\ w_2^1 = 0.5064}$$

따라서 한 번 update된 w1 = 0.5012, w2 = 0.5064이다.

② Update each of $w1$ and $w2$ once more by gradient descent method.

② 한번더 진행하면

$s_1 = w_1^1 \times x_t$

$h = \frac{1}{1+e^{-s_1}}$

$s_2 = w_2^1 \times h$

$y = \frac{1}{1+e^{-s_2}}$

$\xrightarrow[w_1^1,\ w_2^1\ \text{대입}]{x_t,\ y_t}$

$s_1 = 0.5012,\ h = \frac{1}{1+e^{-0.5012}} = 0.6227,$

$s_2 = 0.3153,\ y = \frac{1}{1+e^{-0.3153}} = 0.5782$

$E(\text{Loss function}) = \frac{1}{2}(y_t - y)^2$

0.4218

0.3773

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial h}\frac{\partial h}{\partial s_1}\frac{\partial s_1}{\partial w_1} = -(y_t - y)\cdot y(1-y)\cdot w_2 \cdot h(1-h)\cdot x_t$$
$$= -(1-0.5782)\cdot 0.5782\cdot(1-0.5782)\cdot 0.5064\cdot 0.6227(1-0.6227)\cdot 1$$
$$= -0.0122$$

$$\frac{\partial E}{\partial w_2} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial w_2} = -(y_t - y)\cdot y(1-y)\cdot h$$
$$= -(1-0.5782)\cdot 0.5782\cdot(1-0.5782)\cdot 0.6227$$
$$= -0.0641$$

$$\therefore w_1^2 = w_1^1 - \eta \times \frac{\partial E}{\partial w_1} = 0.5012 + 0.1 \times 0.0122 = 0.5024$$
$$w_2^2 = w_2^1 - \eta \times \frac{\partial E}{\partial w_2} = 0.5064 + 0.1 \times 0.0641 = .5128$$
$$\boxed{\therefore w_1^2 = 0.5024,\ w_2^2 = 0.5128}$$

따라서 ①에서 한 번 더 update된 w1 = 0.5024, w2 = 0.5128이다.

2. We have two training samples, (1,1) and (0,0). The initial weights are $w1$ = 1, $w2$ = 1. The learning rate is $\eta$ = 0.1. The activation function is Sigmoid. Loss is MSE.
Update $w1$ once by gradient descent method.

2. 샘플이 2개 $\Rightarrow (x_1, t_1) = (1,1), (x_2, t_2) = (0,0), w_1 = 1, w_2 = 1$

1번째 퍼셉트론의 Input의 선형결합 결과를 $s_1$,

2번째 퍼셉트론의 Input의 선형결합 결과를 $s_2$라 하자.

$s_1 = w_1 \times x_i \ (i=1,2)$

$h = \frac{1}{1+e^{-s_1}}$

$s_2 = w_2 \times h$

$y = \frac{1}{1+e^{-s_2}}$

1번 문제의 풀이와 일관되게 1/2을 곱해 주겠습니다.

$$E\text{(Loss function)} = \frac{1}{2}\sum_{(x,t)\in Data}(t-y)^2 = \frac{1}{2}(t_1 - \overset{x_1\text{일때}}{y})^2 + \frac{1}{2}(t_2 - \overset{x_2\text{일때}}{y})^2$$

$E_1 = \frac{1}{2}(t_1 - y_{(x_1)})^2$, $E_2 = \frac{1}{2}(t_2 - y_{(x_2)})^2$ 이라 하자

$E = E_1 + E_2$이므로, $\frac{\partial E}{\partial w_1} = \frac{\partial E_1}{\partial w_1} + \frac{\partial E_2}{\partial w_1}$

$$\frac{\partial E_1}{\partial w_1} = \frac{\partial E_1}{\partial y}\frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial h}\frac{\partial h}{\partial s_1}\frac{\partial s_1}{\partial w_1} = -(t_1 - y)\cdot y(1-y)\cdot w_2 \cdot h(1-h)\cdot x_1$$

$x_1 = 1, t_1 = 1$, $w_1 = 1, w_2 = 1$ 대입 $\Rightarrow$ $s_1 = 1$, $h = \frac{1}{1+e^{-1}} = 0.7311$, $s_2 = 0.7311$, $y = \frac{1}{1+e^{-0.7311}} = 0.6750$

$$\frac{\partial E_1}{\partial w_1} = -(1-0.675)\cdot 0.675\cdot(1-0.675)\cdot 1 \cdot 0.7311\cdot(1-0.7311)\cdot 1$$
$$= -0.0140$$

$$\frac{\partial E_2}{\partial w_1} = \frac{\partial E_2}{\partial y}\frac{\partial y}{\partial s_2}\frac{\partial s_2}{\partial h}\frac{\partial h}{\partial s_1}\frac{\partial s_1}{\partial w_1} = -(t_2 - y)\cdot y(1-y)\cdot w_2 \cdot h(1-h)\cdot x_2$$

하지만 $x_2 = 0$이므로 $\frac{\partial E_2}{\partial w_1} = 0$이 된다

$\frac{\partial E}{\partial w_1} = \frac{\partial E_1}{\partial w_1} + \frac{\partial E_2}{\partial w_1}$이므로, $\frac{\partial E}{\partial w_1} = -0.0140 + 0 = -0.014$ 이다

$$w_1' = w_1^0 - \eta \times \frac{\partial E}{\partial w_1} = 1 + 0.1 \times 0.014 = 1.0014$$

$$\boxed{\therefore w_1' = 1.0014}$$

따라서 한 번 update된 w1 = 1.0014이다.

3. We have one training sample, (1,1) . The initial weight for all the weight is 1. The learning rate is $\eta$ = 0.1. The activation function is ReLU. Loss is MSE.

Update $w1$ once by gradient descent method.

3. $(x, t) = (1, 1)$, 모든 w값 $= 1$, $\eta = 0.1$

왼쪽 위를 기준으로 차례대로 1, 2, 3, 4번째 퍼셉트론이라고 생각하고

n 번째 퍼셉트론의 Input의 선형결합 결과를 $s_n$ 이라고 하고, ($n = 1, 2, 3, 4$)

n 번째 퍼셉트론의 output을 $h_n$ 이라 하자 ($h_4 = y$ 이므로 $h_1, h_2, h_3$ 만 정의)

$s_1 = w_1 \times x$

$h_1 = ReLU(s_1) \longleftarrow ReLU(s) = \begin{cases} s & \text{if } s \geq 0 \\ 0 & \text{if } s < 0 \end{cases}$

$s_2 = w_2 \times h_1 \qquad \Rightarrow \frac{\partial}{\partial s}(ReLU(s)) = \begin{cases} 1 & \text{if } s \geq 0 \\ 0 & \text{if } s < 0 \end{cases}$

$h_2 = ReLU(s_2)$

$s_3 = w_3 \times h_1$

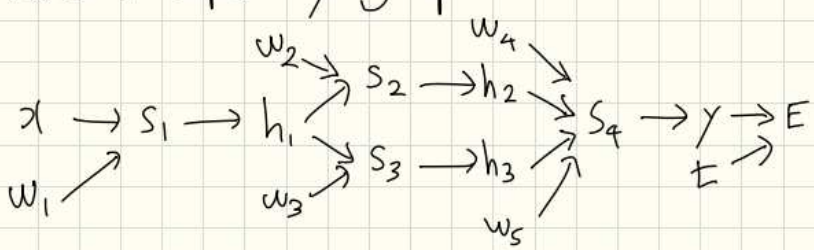
$h_3 = ReLU(s_3)$

$s_4 = w_4 \times h_2 + w_5 \times h_3$

$y = ReLU(s_4)$

$E$ (Loss function) $= \frac{1}{2}(t - y)^2$

variable dependency graph를 그려보면




따라서 $\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial s_4}\frac{\partial s_4}{\partial h_2}\frac{\partial h_2}{\partial s_2}\frac{\partial s_2}{\partial h_1}\frac{\partial h_1}{\partial s_1}\frac{\partial s_1}{\partial w_1} + \frac{\partial E}{\partial y}\frac{\partial y}{\partial s_4}\frac{\partial s_4}{\partial h_3}\frac{\partial h_3}{\partial s_3}\frac{\partial s_3}{\partial h_1}\frac{\partial h_1}{\partial s_1}\frac{\partial s_1}{\partial w_1}$

$= -(t-y)\cdot\frac{\partial y}{\partial s_4}\cdot w_4\cdot\frac{\partial h_2}{\partial s_2}\cdot w_2\cdot\frac{\partial h_1}{\partial s_1}\cdot x + (-1)\cdot(t-y)\cdot\frac{\partial y}{\partial s_4}\cdot w_5\cdot\frac{\partial h_3}{\partial s_3}\cdot w_3\cdot\frac{\partial h_1}{\partial s_1}\cdot x$

$x=1, t=1$, 모든 $w=1$ 대입 $\Rightarrow s_1 = 1, h_1 = 1, s_2 = 1, h_2 = 1, s_3 = 1, h_3 = 1, s_4 = 2, y = 2$

$\frac{\partial E}{\partial w_1} = -(1-2)\cdot\overset{s_4 \geq 0}{1}\cdot 1\cdot\overset{s_2 \geq 0}{1}\cdot 1\cdot\overset{s_1 \geq 0}{1}\cdot 1 + (-1)\cdot(1-2)\cdot 1\cdot 1\cdot 1\cdot 1\cdot 1\cdot 1$

$= 1 + 1 = 2$

$\therefore w_1^1 = w_1^0 - \eta \times \frac{\partial E}{\partial w_1} = 1 - 0.1 \times 2 = 0.8$

$\boxed{\therefore w_1^1 = 0.8}$

따라서 한 번 update된 w1 = 0.8이다.