기계학습원론 HW8

2021312738 소프트웨어학과 김서환

A. Find $\alpha 1$ and $\alpha 2$ in dual form.

A. Find α_1 and α_2 in Dual Form.

$$\underset{\alpha_1, \cdots, \alpha_n}{\text{argmax}} \left(\sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j y_i y_j K(x_i, x_j) \right)$$

subject to $\left\{ \begin{array}{l} \sum_{i=1}^{n} \alpha_i y_i = 0 \\[2mm] 0 \le \alpha_i \le C \quad i = 1, \cdots, n \end{array} \right.$

$C = \infty$, $K(x_i, x_j) = (x_i \cdot x_j + 1)^2$ 이므로

$\underset{x_1}{\underbrace{(0, 1,}} \underset{y_1}{\underbrace{1)}}$, $\underset{x_2}{\underbrace{(-1, 0,}} \underset{y_2}{\underbrace{-1)}}$ 이 대해 적용해보면

$\begin{aligned} x_1 \cdot x_1 &= (0,1) \cdot (0,1) = 0 \times 0 + 1 \times 1 = 1 \end{aligned}$

$\Rightarrow \underset{\alpha_1, \alpha_2}{\text{argmax}} \quad \alpha_1 + \alpha_2 - \frac{1}{2} \left(\begin{array}{l} \alpha_1 \times \alpha_1 \times |x| \times (1+1)^2 + \\ \alpha_1 \times \alpha_2 \times |x-1| \times (0+1)^2 + \\ \alpha_2 \times \alpha_1 \times -1 \times 1 \times (0+1)^2 + \\ \alpha_2 \times \alpha_2 \times -1 \times -1 \times (1+1)^2 \end{array} \right)$

$\left\{ \begin{array}{l} 0 \le \alpha_1 \le \infty, \quad 0 \le \alpha_2 \le \infty \\[2mm] \alpha_1 \times 1 + \alpha_2 \times (-1) = 0 \;\rightarrow\; \alpha_1 = \alpha_2 \end{array} \right.$

$\underset{\alpha_1, \alpha_2}{\text{argmax}} \quad \underline{\alpha_1 + \alpha_2 - \frac{1}{2}\left(4\alpha_1^2 + 4\alpha_2^2 - 2\alpha_1 \alpha_2 \right)}$

$\alpha_2 = \alpha_1$ 대입
$2\alpha_1 - 3\alpha_1^2 = -3\left(\alpha_1^2 - \frac{2}{3}\alpha_1 + \frac{1}{9} \right) + \frac{1}{3}$
$= -3\left(\alpha_1 - \frac{1}{3} \right)^2 + \frac{1}{3}$ $\boxed{\therefore \alpha_1 = \frac{1}{3}, \alpha_2 = \frac{1}{3} \text{일때 최대}}$

$\alpha 1 = 1/3,\ \alpha 2 = 1/3$

B. Which class does (1,1) belong to?

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$\alpha_1 = \frac{1}{3}$, $\alpha_2 = \frac{1}{3}$ 이고

unknown 기에 대한 다음과정을 통해 분류할 수있다.

$$y(x) = \sum_{i=1}^{n} \alpha_i y_i k(x_i, x) + b \implies \begin{array}{l} \text{If } y(x) \geq 0 \to \text{class of } x : +1 \\ y(x) < 0 \to \text{class of } x : -1 \end{array}$$

이 과정을 위해 b를 구해보면

$$b = y_u - \sum_{i=1}^{n} \alpha_i y_i k(x_i, x_u) \quad (\text{for any } k \text{ such that } 0 < \alpha_u < \infty)$$

$\alpha_1 = \frac{1}{3}$, $0 < \alpha_1 < \infty$ 를 만족하므로

$$b = y_1 - \sum_{i=1}^{n} \alpha_i y_i k(x_i, x_1)$$
$$= 1 - \left(\alpha_1 y_1 (x_1 \cdot x_1 + 1)^2 + \alpha_2 y_2 (x_2 \cdot x_1 + 1)^2 \right)$$
$$= 1 - \left(\frac{1}{3} \times 1 \times 4 + \frac{1}{3} \times (-1) \times 1 \right)$$
$$= 1 - 1 = 0 \qquad \therefore b = 0$$

$$\therefore y(x) = \sum_{i=1}^{n} \alpha_i y_i k(x_i, x)$$

unknown $x = (1,1)$ 이므로

$$\implies y(x) = \alpha_1 y_1 \underbrace{\left((0,1) \cdot (1,1) + 1 \right)^2}_{(x_1 \cdot x + 1)^2} + \alpha_2 y_2 \underbrace{\left((-1,0) \cdot (1,1) + 1 \right)^2}_{(x_2 \cdot x + 1)^2}$$
$$= \frac{1}{3} \times 1 \times 4 + \frac{1}{3} \times (-1) \times 0$$
$$= \frac{4}{3}$$

$\therefore y(x) \geq 0$ 이므로 x의 class는 $+1$ 이다.

$\boxed{(1,1) \text{은 } +1 \text{ class 에 속한다.}}$

(1,1) belong to +1 class.