기계학습원론 HW7

2021312738 소프트웨어학과 김서환

1. What is the maximum margin boundary for 0,1,1 and −1,0, −1. Solve with the original formulation.

1. $\underset{w,b}{\operatorname{argmin}} \frac{1}{2} w \cdot w$

subject to $y_i(w \cdot x_i + b) \geq 1 \quad i=1, \dots, n$

$(0,1,1)$, $(-1,0,-1)$ 이므로

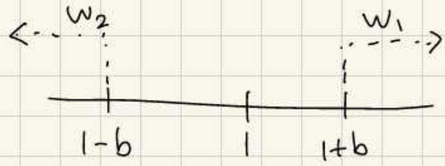
subject to ① $1(w_2 + b) \geq 1 \rightarrow w_2 \geq 1-b$

$w \cdot x_1 = w_1 \times 0 + w_2 \times 1 = w_2$

② $-1(-w_1 + b) \geq 1 \rightarrow w_1 \geq 1+b$

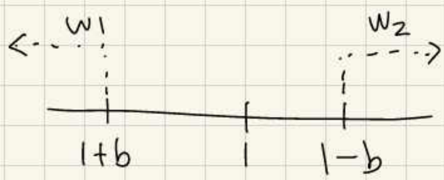
$w \cdot x_2 = w_1 \times -1 + w_2 \times 0$
$= -w_1$

$\frac{1}{2} w \cdot w = \frac{1}{2}(w_1^2 + w_2^2)$ 을 최소화하는 w_1, w_2를 찾아보자



$\boxed{b \geq 0 \text{ 이라 가정}}$

i) $1+b \leq 0 \rightarrow b \leq -1$ 이므로 모순
ii) $1-b \leq 0 \leq 1+b \rightarrow b \geq 1$
iii) $0 \leq 1-b \rightarrow 0 \leq b \leq 1$



$\boxed{b < 0 \text{ 이라 가정}}$

i) $1-b \leq 0 \rightarrow b \geq 1$ 이라 모순
ii) $1+b \leq 0 \leq 1-b \rightarrow b \leq -1$
iii) $0 \leq 1+b \rightarrow 0 > b \geq -1$

⇒ 모순을 제외한 4가지 경우에 대해 판단

i) $b \geq 1 \rightarrow w_1 = 1+b, w_2 = 1-b$ 일때 최소

$\frac{1}{2}(w_1^2 + w_2^2) = \frac{1}{2}(2b^2+2) = b^2+1 \rightarrow$ 최솟값 2 $(b=1)$

ii) $0 \le b \le 1 \rightarrow \begin{matrix} w_1 = 1+b \\ w_2 = 1-b \end{matrix} \rightarrow \begin{matrix} b=1 \rightarrow w_1 = 2, w_2 = 0 \\ b=0 \rightarrow w_1 = 1, w_2 = 1 \end{matrix}$ 이므로 $b=0$일 때 최소

$\frac{1}{2}(w_1^2 + w_2^2) = \frac{1}{2}((1-b)^2 + (1+b)^2) \rightarrow$ 최솟값 1 $(b=0)$

iii) $b \le -1 \rightarrow w_1 = 1+b,\ w_2 = 1-b$일 때 최소

$\frac{1}{2}(w_1^2 + w_2^2) = \frac{1}{2}(2b^2 + 2) = b^2 + 1 \rightarrow$ 최솟값 2 $(b=-1)$

iv) $-1 \le b \le 0 \rightarrow \begin{matrix} w_1 = 1+b \\ w_2 = 1-b \end{matrix} \rightarrow \begin{matrix} b=-1 \rightarrow w_1 = 0, w_2 = 2 \\ b=0 \rightarrow w_1 = 1, w_2 = 1 \end{matrix}$ 이므로 $b=0$일 때 최소

$\frac{1}{2}(w_1^2 + w_2^2) = \frac{1}{2}((1+b)^2 + (1+b)^2) \rightarrow$ 최솟값 1 $(b=0)$

$\therefore b=0$일 때 $\rightarrow w_1 = 1+b = 1,\ w_2 = 1-b = 1$일 때 $\frac{1}{2}w \cdot w$이 최솟값을 갖는다

$\therefore$ Maximum Margin Boundary : $w_1 x_1 + w_2 x_2 + b = 0$

$\Rightarrow \boxed{x_1 + x_2 = 0}$

Maximum Margin Boundary : x1 + x2 = 0

2. What is the maximum margin boundary for 0,1,1 and −1,0, −1. Solve with the dual form of the original formulation.

2. dual form을 이용

Data:
$(0,1,1),\ (-1,0,-1)$

$$\underset{\alpha}{\operatorname{argmax}} \sum_{i=1}^{n} \alpha_i - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} y_i y_j \alpha_i \alpha_j (x_i \cdot x_j)$$

subject to $\begin{cases} \alpha_i \ge 0 \text{ for } i = 1, \cdots, n \\ \sum_{i=1}^{n} \alpha_i y_i = 0 \end{cases}$

$(0,1) \cdot (0,1) = 1$

$$\Rightarrow \underset{\alpha}{\operatorname{argmax}}\ \alpha_1 + \alpha_2 - \frac{1}{2}\begin{pmatrix} 1 \times 1 \times \alpha_1 \times \alpha_1 \times 1 + \\ 1 \times -1 \times \alpha_1 \times \alpha_2 \times 0 + \\ -1 \times 1 \times \alpha_2 \times \alpha_1 \times 0 + \\ -1 \times -1 \times \alpha_2 \times \alpha_2 \times 1 \end{pmatrix}$$

$$\begin{cases} \alpha_1 \ge 0,\ \alpha_2 \ge 0 \\ 1 \times \alpha_1 + (-1) \times \alpha_2 = 0 \Rightarrow \alpha_1 = \alpha_2 \end{cases}$$

$$\operatorname{argmax}\ \alpha_1 + \alpha_2 - \frac{1}{2}(\alpha_1^2 + \alpha_2^2)$$

$\alpha_2 = \alpha_1$ 대입 : $2\alpha_1 - \frac{1}{2}(\alpha_1^2 + \alpha_1^2) = -(\alpha_1^2 - 2\alpha + 1) + 1$

$= -(\alpha_1 - 1)^2 + 1 \quad \therefore \alpha_1 = 1$일 때 최대

$\therefore \alpha_1 = \alpha_2 = 1$

$w = \sum_{i=1}^{n} \alpha_i y_i x_i$, $b = y_n - w \cdot x_n$ (for any x_n such that $a_n > 0$)

$\rightarrow w = 1 \times 1 \times (0,1) + 1 \times (-1) \times (-1,0) = (1,1)$

b에 $a_1 > 0$이므로 번 Data:(0,1,1) 에 대한 x,y 값 대입

$\rightarrow b = 1 - w \cdot (0,1) = 1 - (1,1) \cdot (0,1) = 1 - (1 \times 0 + 1 \times 1) = 1 - 1 = 0$

$\therefore$ Maximum Margin Boundary : $w_1 x_1 + w_2 x_2 + b = 0$

$\Rightarrow \boxed{x_1 + x_2 = 0}$

Maximum Margin Boundary : x1 + x2 = 0