metric convention: $(+---)$

1 Toy Model: Photonless Weak Theory

We consider an SU(2) gauge theory, which is spontaneously broken to the trivial group. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} + (D_\mu H)^\dagger(D^\mu H) + m^2 H^\dagger H - \lambda(H^\dagger H)^2.$$

The field H is a complex SU(2)-doublet. The covariant derivative is

$$D_\mu H = \partial_\mu H - i\frac{g}{2}W^a_\mu t^a H,$$

where t^a are Pauli matrices. Near the vacuum, we may expand

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} -i\chi^1 - \chi^2 \\ v + h + i\chi^3 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix} - \frac{i}{\sqrt{2}}\chi^a t^a \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

where $v = \frac{m}{\sqrt{\lambda}}$.

The covariant derivative is now

$$D_\mu H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} - \frac{i}{\sqrt{2}}(\partial_\mu \chi^a)t^a \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \frac{ig}{2\sqrt{2}}W^a_\mu t^a \begin{pmatrix} 0 \\ v+h \end{pmatrix} - \frac{g}{2\sqrt{2}}W^a_\mu \chi^b t^a t^b \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

so the Higgs kinetic term becomes

$$\begin{aligned}(D_\mu H)^\dagger(D^\mu H) = &\frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{2}(\partial_\mu \chi^a)(\partial^\mu \chi^a) + \frac{g^2}{8}(v+h)^2 W^a_\mu W^{a\mu} + \frac{g^2}{8}W^a_\mu W^{a\mu}\chi^b\chi^b \\ &+ \frac{g}{2}(v+h)W^a_\mu \partial_\mu \chi^a - \frac{g}{2}(\partial_\mu h)W^a_\mu \chi^a - \frac{ig}{2}\epsilon_{abc}W^a_\mu(\partial_\mu \chi^b)\chi^c.\end{aligned}$$

Hence, the gauge fields become massive:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} + \frac{g^2v^2}{8}W^a_\mu W^{a\mu},$$

so the W boson mass is $m_W = gv/2$.

Next, the Higgs potential terms become

$$\begin{aligned}\mathcal{L}_{\text{pot}} &= m^2 H^\dagger H - \lambda(H^\dagger H)^2 \\ &= \frac{1}{2}m^2\left[(v+h)^2 + \chi^a\chi^a\right] - \frac{\lambda}{4}\left[(v+h)^2 + \chi^a\chi^a\right]^2 \\ &= \frac{1}{4}m^2v^2 - m^2h^2 - \frac{m^2}{v}h^3 - \frac{m^2}{4v^2}h^4 - \frac{m^2}{v}h\chi^a\chi^a - \frac{m^2}{2v^2}h^2\chi^a\chi^a - \frac{m^2}{4v^2}\chi^a\chi^a\chi^b\chi^b.\end{aligned}$$

So the physical Higgs mass is $m_h = \sqrt{2}m$.

In Feynman gauge, the gauge-fixing Lagrangian is

$$\begin{aligned}\mathcal{L}_{\text{gf}} &= -\frac{1}{2}\left(\partial^\mu W^a_\mu - \frac{gv}{2}\chi^a\right)\left(\partial^\nu W^a_\nu - \frac{gv}{2}\chi^a\right) \\ &= -\frac{1}{2}(\partial^\mu W^a_\mu)(\partial^\nu W^a_\nu) + \frac{gv}{2}(\partial^\mu W^a_\mu)\chi^a - \frac{g^2v^2}{8}\chi^a\chi^a,\end{aligned}$$

so the $W\chi$ vertex in the Higgs kinetic term vanishes (up to total derivative). The ghost Lagrangian is

$$\mathcal{L}_{\text{gh}} = -\bar{c}^a \partial^\mu D^{ab}_\mu c^b - \frac{g^2 v(v+h)}{4}\bar{c}^a c^a - \frac{g^2 v}{4}\epsilon_{abc}\bar{c}^a c^b \chi^c$$

There are left-handed fermions (q-type) which are SU(2)-doublet, and corresponded right-handed SU(2)-singlet fermions (u-type and d-type). The fermion sector Lagrangian is given as (in Dirac notation)

$$\mathcal{L}_{\text{fermion}} = i\bar{q}^i_L \not{D} q^i_L + i\bar{u}^i_R \not{\partial} u^i_R + i\bar{d}^i_R \not{\partial} d^i_R - (Y^{ij}_u \bar{q}^i_L \tilde{H} u^j_R + Y^{ij}_d \bar{q}^i_L H d^j_R + \text{h.c.}),$$

where

$$D_\mu q^i_L = \partial_\mu q^i_L - i\frac{g}{2} W^a_\mu t^a q^i_L.$$

Identifying $q^i_L = (u^i_L \; d^i_L)^T$, the fermion kinetic terms become

$$i\bar{q}^i_L \not{D} q^i_L + i\bar{u}^i_R \not{\partial} u^i_R + i\bar{d}^i_R \not{\partial} d^i_R = i\bar{u}^i \not{\partial} u^i + i\bar{d}^i \not{\partial} d^i + \frac{g}{2}\begin{pmatrix}\bar{u}^i & \bar{d}^i\end{pmatrix} \not{W}^a t^a P_L \begin{pmatrix} u^i \\ d^i \end{pmatrix},$$

and the Yukawa terms become

$$-Y^{ij}_u \bar{q}^i_L \tilde{H} u^j_R - Y^{ij}_d \bar{q}^i_L H d^j_R + \text{h.c.} = -\frac{v+h}{\sqrt{2}}(Y^{ij}_u \bar{u}^i_L u^j_R + Y^{ij}_d \bar{d}^i_L d^j_R) + \text{Goldstone Yukawa} + \text{h.c.}.$$

For simplicity, assume that Y_u and Y_d are real diagonal matrices, so that $Y^{ij}_u = \delta^{ij} Y^i_u$ and $Y^{ij}_d = \delta^{ij} Y^i_d$. Then the Yukawa terms give Dirac masses:

$$-\frac{v}{\sqrt{2}}(Y^{ij}_u \bar{u}^i_L u^j_R + Y^{ij}_d \bar{d}^i_L d^j_R + \text{h.c.}) = -\frac{v}{\sqrt{2}}(Y^i_u \bar{u}^i u^i + Y^i_d \bar{d}^i d^i).$$

Hence, $m^i_u = vY^i_u/\sqrt{2}$ and $m^i_d = vY^i_d/\sqrt{2}$.

1.1 Four-Fermion Scattering

Since there is no flavour mixing, the possible $ff \to ff$ procedures are elastic scattering and pair annihilation/creation. We look at the pair annihilation/creation process, because it manifestly shows the resonance structure.

1.2 EFT Matching

We want to obtain the effective field theory, where we only have light fermions and no Higgs or gauge bosons.