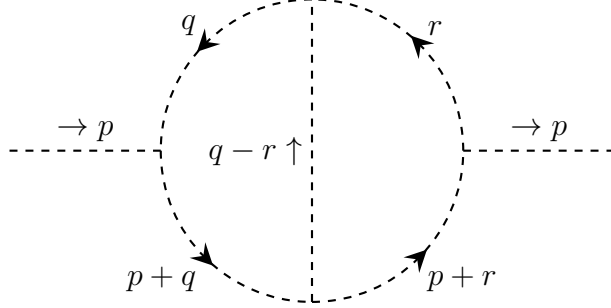


metric convention:  $(+, -, -, -)$

Lagrangian:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} M^2 \phi^2 + \partial_\mu \chi^\dagger \partial^\mu \chi - g \phi \chi^\dagger \chi.$$

## 1 Diagram 1



$$\Pi_1(p^2) = g^4 \int \frac{d^d q}{(2\pi)^d} \frac{d^d r}{(2\pi)^d} \frac{1}{q^2} \frac{1}{(p+q)^2} \frac{1}{r^2} \frac{1}{(p+r)^2} \frac{1}{(q-r)^2 - M^2}.$$

We first deal with the  $r$ -integral:

$$\begin{aligned} I_1 &:= \int \frac{d^d r}{(2\pi)^d} \frac{1}{r^2} \frac{1}{(p+r)^2} \frac{1}{(q-r)^2 - M^2} \\ &= 2 \int \frac{d^d r}{(2\pi)^d} \int_0^1 dx \int_0^{1-x} dy \frac{1}{[(1-x-y)r^2 + y(p+r)^2 + x((q-r)^2 - M^2)]^3} \\ &= 2 \int \frac{d^d r}{(2\pi)^d} \int_0^1 dx \int_0^{1-x} dy \frac{1}{(r'^2 - \Delta_r)^3} \\ &= \frac{-i\Gamma(3 - \frac{d}{2})}{(4\pi)^{d/2}} \int_0^1 dx \int_0^{1-x} dy \frac{1}{\Delta_r^{3-d/2}}, \end{aligned}$$

where  $r' = r - xq + yp$  and  $\Delta_r = -x(1-x)q^2 - y(1-y)p^2 - 2xyqp + xM^2$ . Now the total integral becomes

$$\Pi_1(p^2) = g^4 \frac{-i\Gamma(3 - \frac{d}{2})}{(4\pi)^{d/2}} \int_0^1 dx \int_0^{1-x} dy \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} \frac{1}{(p+q)^2} \frac{1}{\Delta_r^{3-d/2}}.$$

Let's look at the  $q$ -integral:

$$\begin{aligned} I_2 &:= \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} \frac{1}{(p+q)^2} \frac{1}{\Delta_r^{3-d/2}} \\ &= \frac{\Gamma(7 - \frac{d}{2})}{\Gamma(3 - \frac{d}{2})} \int_0^1 dz \int_0^{1-z} dw \int \frac{d^d q}{(2\pi)^d} \frac{z^{2-d/2} w(1-z-w)}{[z\Delta_r + w(p+q)^2 + (1-z-w)q^2]^{7-d/2}}. \end{aligned}$$

The denominator is

$$\begin{aligned} D &:= z\Delta_r + w(p+q)^2 + (1-z-w)q^2 \\ &= (1-z-xz+x^2z)q^2 + (-yz+y^2z+w)p^2 + 2(-xyz+w)pq + xzM^2 \\ &= (1-z-xz+x^2z) \left( q + \frac{-xyz+w}{1-z-xz+x^2z} p \right)^2 + \left( -\frac{(-xyz+w)^2}{1-z-xz+x^2z} - yz + y^2z + w \right) p^2 + xzM^2 \\ &=: (1-z-xz+x^2z)q'^2 - \Delta_q. \end{aligned}$$

Then

$$\int \frac{d^d q}{(2\pi)^d} \frac{1}{[(1-z-xz+x^2z)q'^2 - \Delta_q]^{7-d/2}} = \frac{-i(-1)^{-d/2} \Gamma(7-d)}{(4\pi)^{d/2} \Gamma(7-\frac{d}{2})} \frac{1}{(1-z-xz+x^2z)^{d/2}} \frac{1}{\Delta_q^{7-d}}.$$

Bringing all together,

$$\Pi_1(p^2) = g^4 \frac{(-1)^{-d/2}}{(4\pi)^d} \Gamma(7-d) \int_0^1 dx \int_0^{1-x} dy \int_0^1 dz \int_0^{1-z} dw \frac{z^{2-d/2} w (1-z-w)}{(1-z-xz+x^2z)^{d/2}} \frac{1}{\Delta_q^{7-d}}.$$