Positivity of Dim-6 Operators

Azatov, Aleksandr, Diptimoy Ghosh, and Amartya Harsh Singh. "Four-fermion operators at dimension 6: Dispersion relations and UV completions." *Physical Review D* 105.11 (2022): 115019.

Dim-8 operators → Straightforward positivity bounds

Some useful applications (e.g. positivity cone)

But hard to measure!

How can we include dim-6 operators into our picture?

$$\left.\frac{\partial}{\partial s}A_{ab\to ab}(s,0)\right|_{s=\mu^2} = \frac{1}{2\pi i}\oint ds\,\frac{A_{ab\to ab}(s,0)}{(s-\mu^2)^2} - \cancel{(\text{IR pole residues})}$$

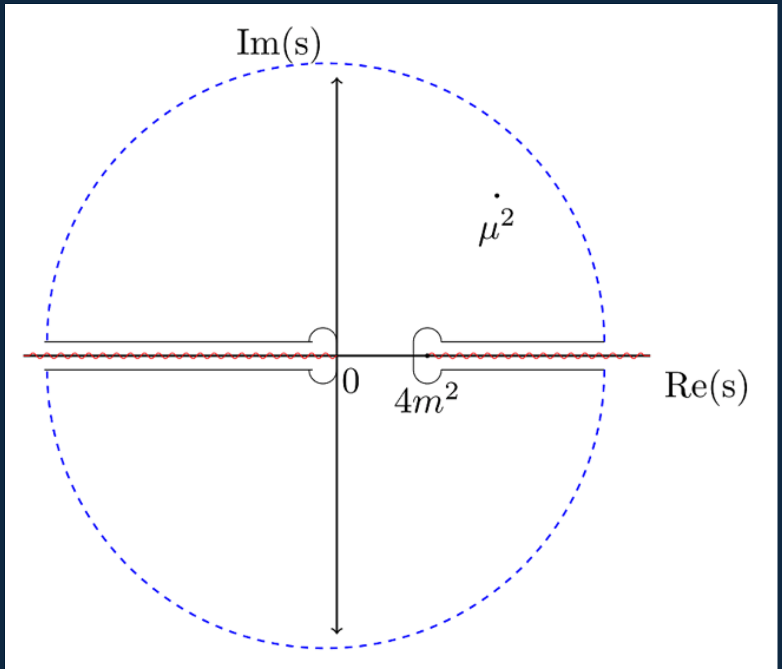
$$= \int \frac{ds}{\pi s}\left(\sigma_{ab\to ab} - \sigma_{a\bar{b}\to a\bar{b}}\right) + \boxed{\int_0^{2\pi}\frac{d\theta}{2\pi}\,\frac{A_{ab\to ab}(|s_\Lambda|e^{i\theta},0)}{(|s_\Lambda|e^{i\theta}-\mu^2)^2}|s_\Lambda|e^{i\theta}}$$

$$\cancel{+\,O(m_{\text{IR}}^2/\mu^2,\mu^2/\Lambda^2)}$$

C_∞

Not necessarily zero → subtleties

$\Lambda^2 \gg \mu^2 \gg m_{\text{IR}}^2$



Example: Four-fermi operators in SMEFT

$$c_{RR}(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R)$$

$$-8c_{RR} = \int \frac{ds}{\pi s}\left(\sigma_{e_R\bar{e}_R} - \sigma_{e_R e_R}\right) + C_\infty$$

Sum rules

Possible UV completions – negative sign (1)

$$\mathcal{L}_{Z'} = \lambda Z'_\mu \bar{e}_R \gamma^\mu e_R$$

$$c_{RR} = -\frac{\lambda^2}{2M_{Z'}^2} \quad \text{(tree-level)}$$

$$\begin{aligned} A_{\bar{e}_R e_R \to \bar{e}_R e_R}(s,t) &= -2\lambda^2 [23]\langle 14\rangle \left(\frac{1}{s - M_{Z'}^2} + \frac{1}{t - M_{Z'}^2}\right) \\ &\overset{t\to 0}{=} -2\lambda^2 s \left(\frac{1}{s - M_{Z'}^2} - \frac{1}{M_{Z'}^2}\right) \end{aligned}$$

$$\implies C_\infty^{(Z')} = \frac{2\lambda^2}{M_{Z'}^2} = \left[\int \frac{ds}{\pi s}\left(\sigma_{e_R \bar{e}_R} - \sigma_{e_R e_R}\right)\right]^{(Z')}$$

Possible UV completions – negative sign (2)

$$\mathrm{EFT}: c_{e\mu}(\bar{e}_R\gamma_\mu e_R)(\bar{\mu}_R\gamma^\mu\mu_R)$$

- $\mathcal{L}_{\mathrm{UV}}^{(1)} = \lambda Z_{(1)}^{\mu}(\bar{e}_R\gamma_\mu\mu_R + h.c.)$

$$c_{e\mu} = -\frac{1}{2}\left[\int\frac{ds}{\pi s}(\sigma_{e_R\bar{\mu}_R} - \sigma_{e_R\mu_R})\right] = -\frac{|\lambda|^2}{M_{(1)}^2}, \quad \boxed{C_\infty = 0}$$

No t-channel Z(1)

- $\mathcal{L}_{\mathrm{UV}}^{(2)} = (\lambda_1 Z_{(2)}^{\mu}\bar{e}_R\gamma_\mu e_R + \lambda_2 Z_{(2)}^{\mu}\bar{\mu}_R\gamma_\mu\mu_R)$

$$\boxed{\sigma_{e_R\bar{e}_R} = \sigma_{e_R e_R} = 0, \quad c_{e\mu} = -\frac{C_\infty}{2} = -\frac{\lambda_1\lambda_2}{M_{(2)}^2}}$$

No UV cross-section,
but infinity pole saturates the dispersion relation

Possible UV completions – positive sign

$$\mathcal{L} = \kappa\phi e_R^T \mathcal{C} e_R + h.c.$$

$$c_{RR} = \frac{|\kappa|^2}{2M_\phi^2}, \quad C_\infty = 0$$

Both signs of the Wilson coefficient are possible,
but they suggest different UV models.

Additional idea: utilising t-derivative together

Remmen, Grant N., and Nicholas L. Rodd. "Signs, spin, SMEFT: Sum rules at dimension six." *Physical Review D* 105.3 (2022): 036006.