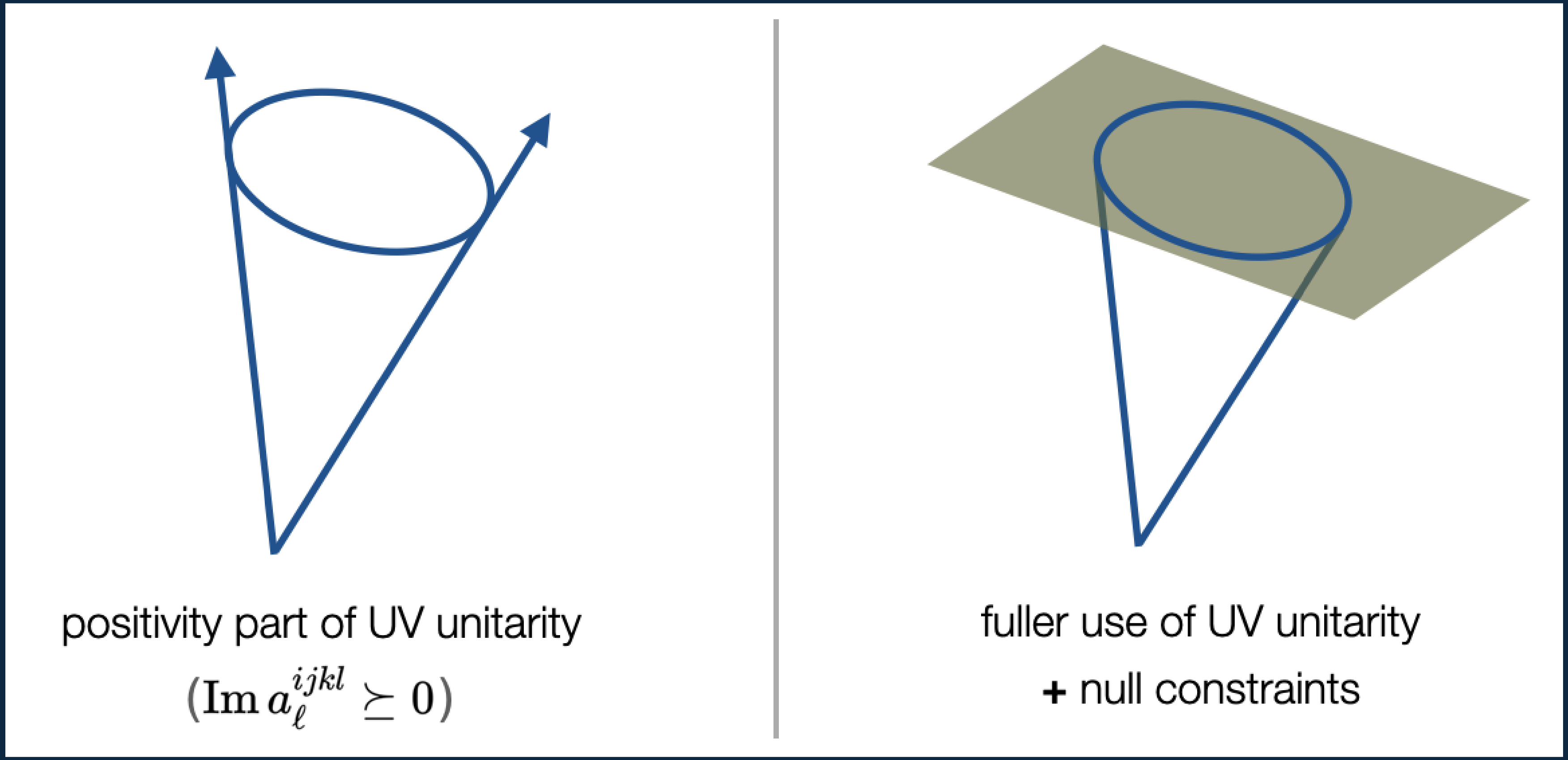
Positivity Bounds and SMEFT

- Recent Works

Capping the Positivity Cone

Chen, Qing, et al. "Capping the positivity cone: dimension-8 Higgs operators in the SMEFT." *JHEP* 2024.3: 1-47.



Essentially infinite parameter space

(Cutoff-dependent) Upper bound

$$A_{ijkl}(s,t) = \frac{\lambda_{ijkl}}{-s} + \frac{\lambda_{ijkl}}{-t} + \frac{\lambda_{ijkl}}{-u} + a^{(0)}_{ijkl}(t) + a^{(1)}_{ijkl}(t)s$$
$$+ \int_{\Lambda^2}^{\infty} \frac{d\mu}{\pi(\mu+\frac{t}{2})^2} \left[\frac{(s+\frac{t}{2})^2}{\mu - s} \mathrm{Im}\, A_{ijkl}(\mu,t) + \frac{(u+\frac{t}{2})^2}{\mu - u} \mathrm{Im}\, A_{ilkj}(\mu,t) \right]$$

$$B_{ijkl}(s,t) := A_{ijkl}(s,t) - \frac{\lambda_{ijkl}}{-s} - \frac{\lambda_{ijkl}}{-t} - \frac{\lambda_{ijkl}}{-u}$$
$$= \tilde{a}^{(0)}_{ijkl}(t) + a^{(1)}_{ijkl}(t)v + \frac{v^2}{\pi} \int_{\Lambda^2}^{\infty} \frac{d\mu}{(\mu+\frac{t}{2})^2} \left[\frac{\mathrm{Im}\, A_{ijkl}(\mu,t)}{\mu - v + \frac{t}{2}} + \frac{\mathrm{Im}\, A_{ilkj}(\mu,t)}{\mu + v - \frac{t}{2}} \right]$$

$$B_{ijkl}(s,t) = \sum_{n\geq 0} c^{0,n}_{ijkl} t^n + \sum_{n \geq 0} c^{1,n}_{ijkl} v t^n + \sum_{m\geq 2} \sum_{n\geq 0} c^{m,n}_{ijkl} v^m t^n$$

Partial wave expansion

$$B_{ijkl}(s,t) = \tilde{a}^{(0)}_{ijkl}(t) + a^{(1)}_{ijkl}(t)v + \frac{v^2}{\pi}\int_{\Lambda^2}^{\infty}\frac{d\mu}{(\mu+\frac{t}{2})^2}\left[\frac{\mathrm{Im}\,A_{ijkl}(\mu,t)}{\mu-v+\frac{t}{2}} + \frac{\mathrm{Im}\,A_{ilkj}(\mu,t)}{\mu+v-\frac{t}{2}}\right]$$

$$A_{ijkl}(\mu,t) = 16\pi\sum_{\ell=0}^{\infty}(2\ell+1)P_\ell\left(1+\frac{2t}{\mu}\right)a_\ell^{ijkl}(\mu)$$

$$L_\ell^n := \frac{\Gamma(\ell+n+1)}{n!\Gamma(\ell-n+1)\Gamma(n+1)}$$

$$H_{m+1}^q := \frac{\Gamma(m+q+1)}{(-2)^q\Gamma(q+1)\Gamma(m+1)}$$

$$c_{ijkl}^{m,n} = \sum_\ell 16(2\ell+1)\int_{\Lambda^2}^{\infty}d\mu\,[\mathrm{Im}\,a_\ell^{ijkl}(\mu) + (-1)^m\mathrm{Im}\,a_\ell^{ilkj}(\mu)]\sum_{p=0}^{n}\frac{L_\ell^p H_{m+1}^{n-p}}{\mu^{m+n+1}}$$

Null constraints (crossing symmetry)

$$B_{ijkl}(s,t) = B_{ikjl}(t,s)$$

$$\implies \sum_{a=p}^{p+q} \frac{\Gamma(a+1) c_{ijkl}^{a,p+q-a}}{2^{a-p}\Gamma(p+1)\Gamma(a-p+1)} - \sum_{b=q}^{p+q} \frac{\Gamma(b+1) c_{ijkl}^{b,p+q-b}}{2^{b-p}\Gamma(p+1)\Gamma(b-p+1)} = 0$$

$$B_{ijkl}(s,t) = B_{ilkj}(u,t)$$

$$\implies c_{ijkl}^{1,n} + c_{ilkj}^{1,n} = 0$$

$$A_{ijkl}(\mu,t) = A_{jikl}(\mu,u) = A_{ijlk}(\mu,u)$$

$$\implies a_\ell^{ijkl}(\mu) = (-1)^\ell a_\ell^{jikl}(\mu) = (-1)^\ell a_\ell^{ijlk}(\mu)$$

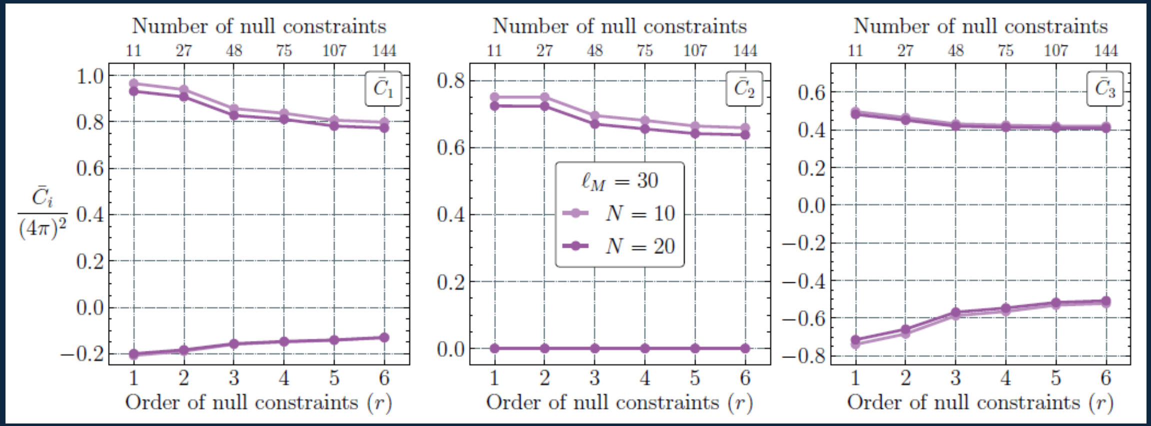
Partial wave unitarity, massless limit

$$\eta_{mn} = 1 - \frac{1}{2}\delta_{mn}$$

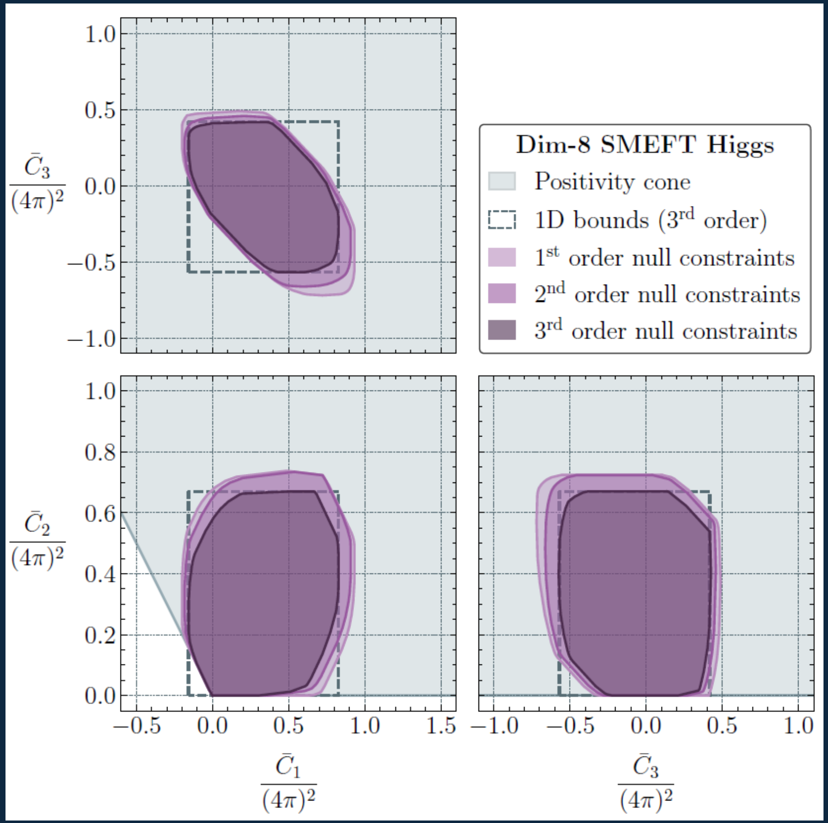
$$\mathrm{Im}\, a_\ell^{ijkl} = \sum_{mn} \eta_{mn} a_\ell^{ijmn} (a_\ell^{klmn})^* + \sum_{X \neq mn} a_\ell^{ij\to X} (a_\ell^{kl\to X})^*$$

$$\implies \mathrm{Im}\, a_\ell^{ijkl} \text{ cannot be arbitrarily large}$$

Dim-8 SMEFT Higgs operators



$$\mathcal{O}^{(1)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$$
$$\mathcal{O}^{(2)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$$
$$\mathcal{O}^{(3)}_{H^4} = (D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$$



vs. Experimental Sensitivity

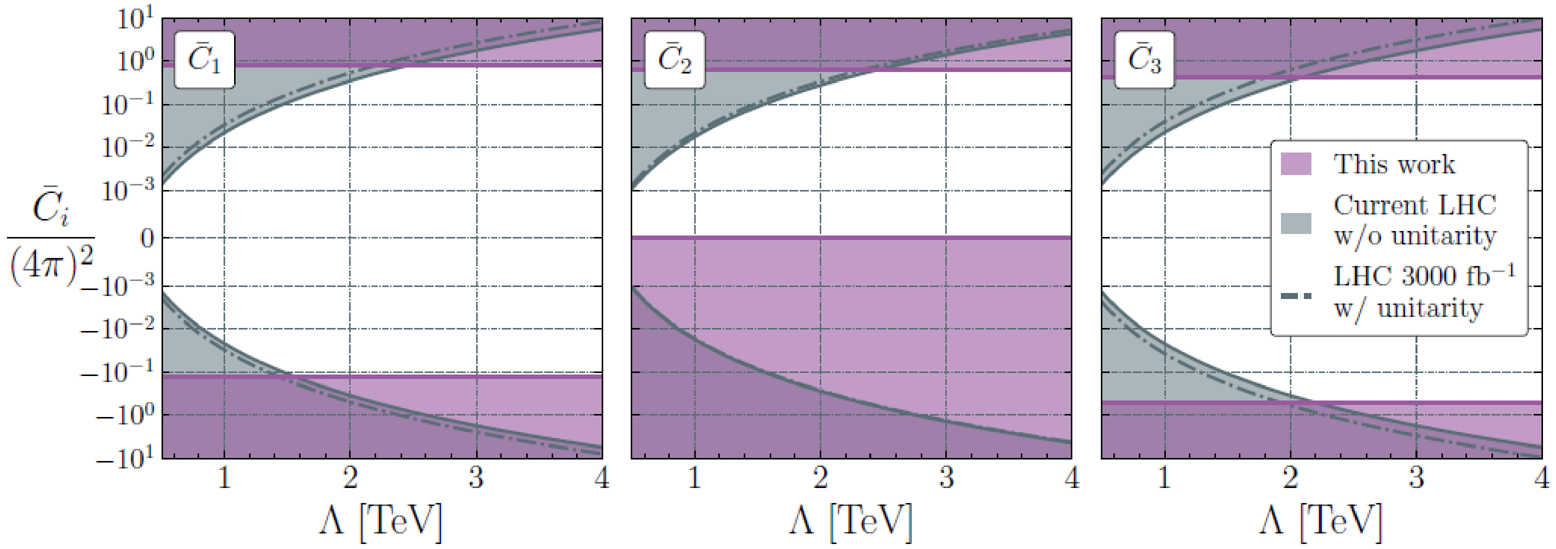


Figure 7. Upper and lower positivity bounds on the Higgs scattering Wilson coefficients obtained in this work (purple shaded) compared to the current [97] (grey shaded) and projected [98] (grey dashed) LHC exclusion limits from VBS measurements as a function of the EFT cutoff, Λ.

Positivity Bounds and VEV

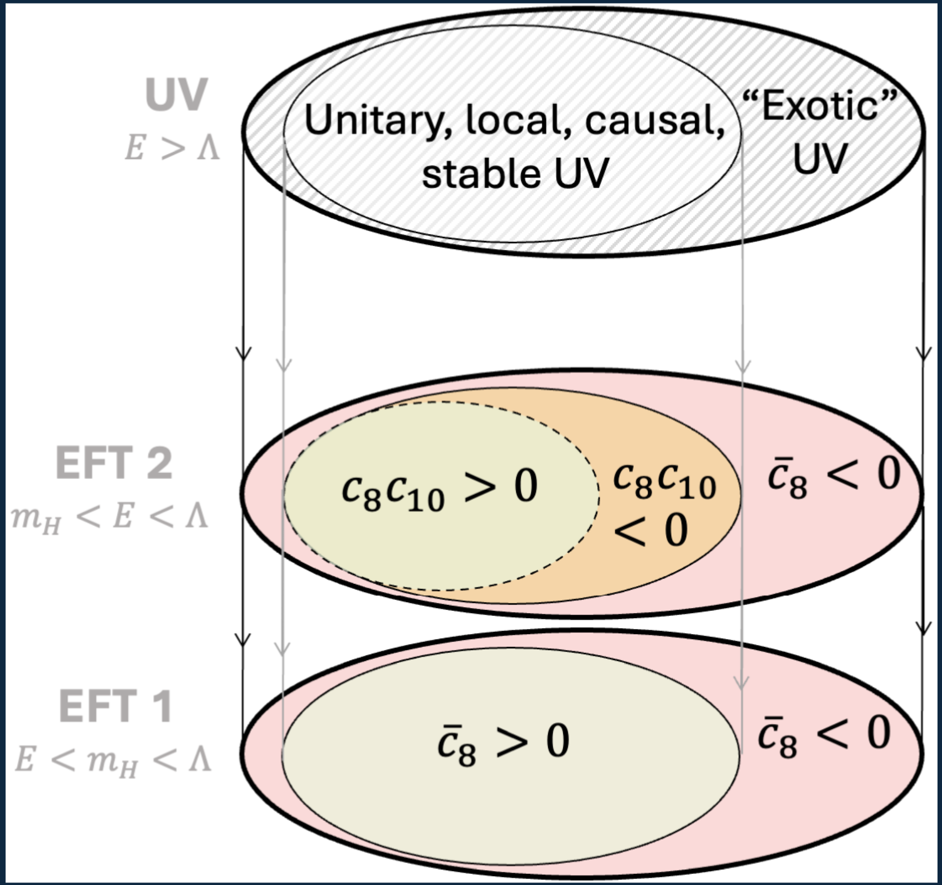
Davighi, Joe, et al. "Positivity and the electroweak hierarchy." *Physical Review D* 109.3 (2024): 033009.

$$\mathcal{L}_{\mathrm{EFT}}[H] = c_8 \frac{\mathcal{O}_8}{\Lambda^4} + c_{10} \frac{|H|^2 \mathcal{O}_8}{\Lambda^6} \quad \xRightarrow{\text{Integrate out } H} \quad \mathcal{L}_{\mathrm{EFT}}[v] = \bar{c}_8 \frac{\mathcal{O}_8}{\Lambda^4}$$

$$\bar{c}_8 = c_8 + \frac{v^2}{\Lambda^2} c_{10}$$

Positivity holds regardless of UV details: $c_8 + \dfrac{v^2}{\Lambda^2} c_{10} \geq 0$

If $c_8 > 0,\ c_{10} < 0$ and $|c_8| \ll |c_{10}|$ then $\dfrac{v^2}{\Lambda^2} \leq \dfrac{|c_8|}{|c_{10}|} \ll 1$ Light Higgs!



UV
$E > \Lambda$
Unitary, local, causal, stable UV
"Exotic" UV
EFT 2
$m_H < E < \Lambda$
$c_8 c_{10} > 0$
$c_8 c_{10} < 0$
$\bar{c}_8 < 0$
EFT 1
$E < m_H < \Lambda$
$\bar{c}_8 > 0$
$\bar{c}_8 < 0$

Rough Reviews of Several Papers

(Because I didn't fully understand the background)

Causality: UV vs IR

Gonzalez, Mariana Carrillo, et al. "Causal effective field theories." *Physical Review D* 106.10 (2022): 105018.

Carrillo González, Mariana, et al. "Positivity-causality competition: a road to ultimate EFT consistency constraints." *JHEP* 2024.6: 1-56.

S-matrix Bootstrap and Positivity Bounds

Miró, Joan Elias, Andrea Guerrieri, and Mehmet Asım Gümüş. "Bridging positivity and S-matrix bootstrap bounds." *JHEP* 2023.5: 1-62.