Kinematics -the simplest types of motion

Uniform linear motion

$\vec{\mathbf{v}}(\mathbf{t}) = \mathbf{constant}$

On OX: $\vec{\mathbf{r}} = \mathbf{x}\,\vec{\mathbf{i}}$

$\vec{\mathbf{v}} = \mathbf{v_x}\vec{\mathbf{i}}$

$$x(t) = x_0 + v_x \cdot (t - t_0)$$

Uniformly accelerated linear motion

$\vec{a}(t) = \mathbf{constant}$

OY: $\vec{\mathbf{r}} = \mathbf{y}\,\vec{\mathbf{j}}$

$\vec{\mathbf{v}} = v_y\vec{\mathbf{j}}$

$\vec{\mathbf{a}} = \mathbf{a_y}\,\vec{\mathbf{j}}$

$$\mathbf{v_y(t) = v_{0y} + a_y \cdot (t - t_0)}$$

$$\mathbf{y(t) = y_0 + v_{oy} \cdot (t - t_0) + a_y \cdot \frac{(t - t_0)^2}{2}}$$

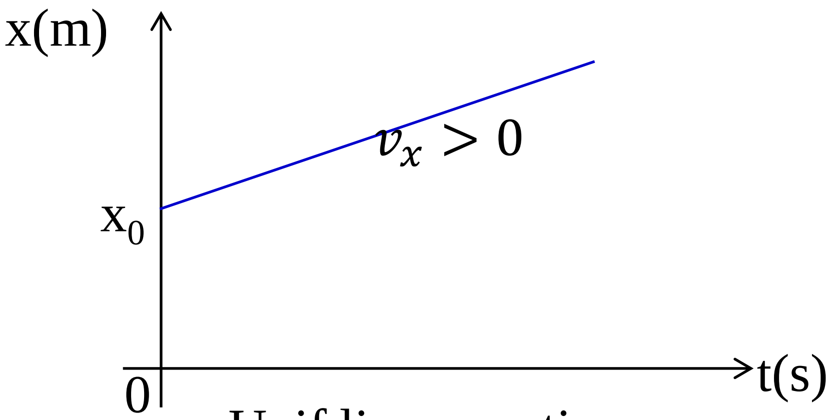
UNIFORM LINEAR MOTION $\vec{v}(t) = \text{constant}$

OX: $\vec{r} = x\,\vec{i}$

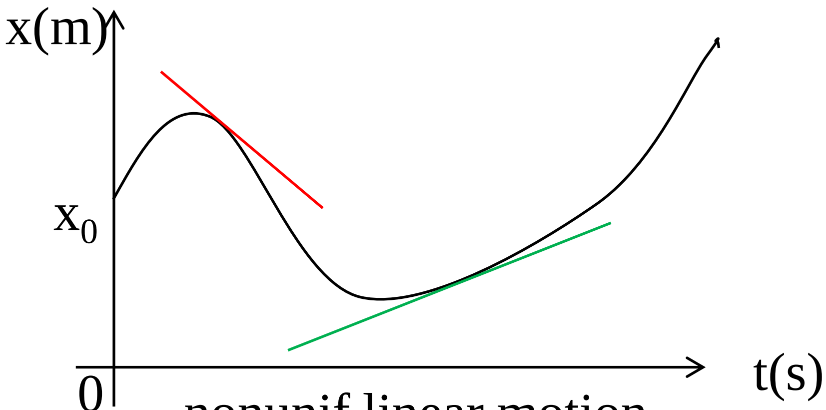
$\vec{F} = 0 \Longleftrightarrow \vec{v}(t) = \text{constant}$

$\Longrightarrow \mathbf{x(t) = x_0 + v_x \cdot (t - t_0)}$

Initial Conditions: initial position $\mathbf{x_0}$ at the initial moment of time $\mathbf{t_0}$



Unif linear motion



nonunif linear motion

UNIFORM LINEAR MOTION $\vec{v}(t) = \text{constant}$

- A man travels for 3.6 km with 3km/h, then 2.9 km in 66 min, and at the end, walks for 42min with 4km/h in the same direction. Find the man's average speed.

Solution:

$$v = \frac{\Delta x}{\Delta t}$$

$$v_{av} = \frac{\Delta x_{total}}{\Delta t_{total}} = \frac{\quad + \quad + \quad}{\frac{}{} + \frac{}{} + \frac{}{}} \frac{\text{km}}{\text{h}} = 3.1 \frac{\text{km}}{\text{h}} = \cdots \text{ m/s}$$

	Δx (km)	Δt (h)	v(km/h)
I	3.6	$= \frac{}{}$ h	3
II	2.9	66min = $\frac{}{}$ h	
III	$\cdots$	42min = $\frac{}{}$ h	4

- A man travels for 4,8km with 4km/h, then 1,5 km in 33min, and at the end, walks for 45min with 3,6km/h in the same direction. Find the man's average speed.

Uniformly accelerated linear motion $\vec{a}(t) = const.$

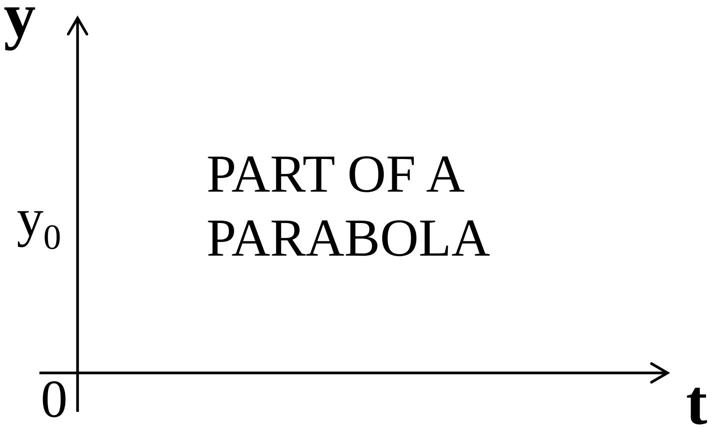
OY: $\vec{r} = y\,\vec{j}$

$\vec{F}$ =const ⟺ $\vec{a}(t) = \text{constant}$

$$\Rightarrow \quad v_y(t) = v_{0y} + a_y \cdot (t - t_0)$$

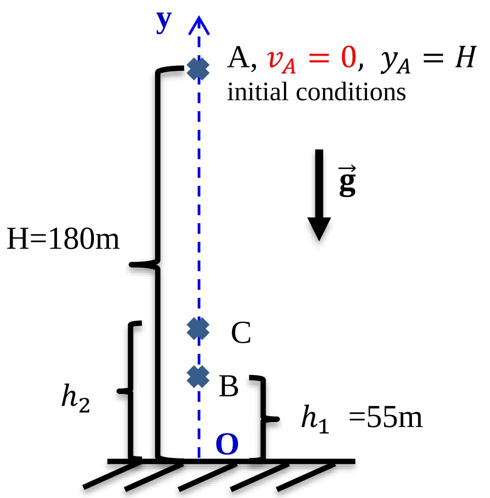
Initial Conditions : initial position y_0 and initial velocity v_{0y} at the initial moment of time $\mathbf{t_0}$

$$\Rightarrow \quad y(t) = y_0 + v_{oy} \cdot (t - t_0) + a_y \cdot \frac{(t - t_0)^2}{2}$$



Uniformly accelerated linear motion $\vec{a}(t) = const.$

•A small body is freely released from a platform located 180 meters above the ground. If g=10m/s^2 and we ignore air resistance find a) the time required to reach the ground b) the velocity at the ground c) the velocity at 55 m from the ground d) the distance traveled in the last 2 seconds of flight.



Velocity law $\mathbf{v(t) = v_A - g \cdot t}$

Motion law $\mathbf{y(t) = y_A + v_A \cdot t - g \cdot \frac{t^2}{2}}$ $\Longrightarrow$ a) $0 = H - g\frac{t_{AO}^2}{2}$ $\Longrightarrow$

$t_{AO} = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \cdot 180}{10}} = 6s$

a) $v_O = -g \cdot t_{AO} = -60m/s$

b) $y(t) = y_A - g \cdot \frac{t^2}{2} \Longrightarrow$ a) $h_1 = H - g\frac{t_{AB}^2}{2}$ $\Longrightarrow t_{AB} = \sqrt{\frac{2(H-h_1)}{g}} = \sqrt{\frac{2 \cdot 125}{10}} =$ 5s

$$v_B = -g \cdot t_{AB} = -50m/s$$

a) Let h_2 -the distance traveled in the last 2 seconds of flight

$$t_{AC} = \cdots$$

$$h_2 = H - g\frac{t_{AC}^2}{2} = \cdots.$$

- A small body is freely released from a platform located 605 meters above the ground. If g=10m/s^2 and we ignore air resistance find: a) the time required to reach the ground; b) the velocity at the ground; c) the velocity at 425 m from the ground; d) the distance traveled in the firstt 4 seconds of flight; e) the distance traveled in the last 4 seconds of flight; f) the distance traveled in the third second of flight and in the seventh.

- A body is thrown vertically upward from the Earth ground with 10m/s. Neglecting air friction and considering g=10m/s^2 find: a) the maximum height of the body ; b) total time of flight; c) moments when the body has ¼ from its initial velocity.

Uniformly accelerated linear motion $\vec{a}(t) = const.$

•A car on a linear highway has an initial speed of 23 m/s. The car accelerates at a constant rate for 10 s to a final speed of 29 m/s. How far does the car travel during this time interval?

•An object moves along the x-axis with a constant acceleration of 6 m/s^2 and an initial velocity of –24 m/s. It is located at x = 6 m when t = 0 s. Write the motion law of the object. What is its position when its velocity is zero?

• A mass point (m=2kg) moves linearly onto Ox axes under the law: $x(t)= -10t^2 +20t - 8$ (m) (with the initial moment $t_0=0$, and the total time of motion 10s.).
The initial position is -8 m, the initial velocity is 20 m/s, time until stop is 1 s and the **distance** traveled until the stopping moment is 10 m.
After the stopping moment the body continues to move and returns in the initial position.

•A mass point (m=2kg) moves linearly onto Ox axes under the law: $x(t)= -10t^2 +20t - 8$ (m) (with the initial moment $t_0=0$, and the total time of motion 10s.).

The initial position is ……. m, the acceleration is ….. m/s^2, time until stop is ……s and the position at the stopping moment is …….m.
After the stopping moment the body ……….. Consequently the body passes …….. the reference origin at ………s .

Linear Momentum definition: $\vec{\mathbf{p}} = \mathbf{m} \cdot \vec{\mathbf{v}}$ (N·s)

The Kinetic Energy definition $E_K = \frac{m \cdot v^2}{2}$ **(JOULE)** $\left(\mathbf{E}_{\boldsymbol{K}} = \frac{\boldsymbol{p}^2}{\mathbf{2m}}\right)$

The Potential Energy *expression **depends** on the type of forces* that act upon the bodies of the system (exists *only for **conservative forces***).

Examples:

- Potential energy for near Earth gravity

  E_p=mgy (force G= -mg)

- Potential energy for a linear spring = elastic potential energy

  $E_p = \frac{kx^2}{2}$ (force F_{el}= –kx)

Work for $\vec{F}$**=const.** $\boldsymbol{L} = \vec{\boldsymbol{F}} \cdot \boldsymbol{\Delta}\vec{\boldsymbol{r}}$

THE LAW OF CONSERVATION OF LINEAR MOMENTUM

From Newton's second law if $\vec{\mathbf{F}} = \mathbf{0}$ $\Leftrightarrow$ $\vec{p}$=**constant**

The total linear momentum of an isolated physical system is conserved.

THE WORK- (KINETIC) ENERGY THEOREM

$$\mathbf{L= \Delta E_K} \qquad \mathrm{L} = \frac{\mathrm{m}{\mathrm{v}_2}^2}{2} - \frac{\mathrm{m}{\mathrm{v}_1}^2}{2}$$

For *conservative* forces

$$\mathbf{L= - \Delta E_p} \qquad \mathbf{L} = -\left(\mathbf{E_p(2) - E_p(1)}\right)$$

THE LAW OF CONSERVATION OF ENERGY

The total energy of an isolated system under the action of conservative forces is conserved.

$$\mathbf{E}_{\boldsymbol{K}}(\mathbf{1}) + \mathbf{E_P(1)} = \mathbf{E}_{\boldsymbol{K}}(\mathbf{2}) + \mathbf{E_P(2)} = \ \textbf{constant}$$