2022학년도 1학기

미분적분학(1) 7차 QUIZ

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부정행위시 누적 퀴즈점수 모두 0점 부여 답만 적을 시 0점 부여 (풀이과정 상세히)
※7.1 7.2 7.3

1. $\int_1^{e^2} \frac{\ln x}{x^2} dx$ 를 계산하시오. (1점)

$$\int_1^{e^2} \frac{1}{x^2} \ln x \, dx$$

$$= \left[-\frac{1}{x}\ln x\right]_1^{e^2} - \int_1^{e^2} \left(-\frac{1}{x}\right)\cdot\frac{1}{x}\,dx$$

0.5

$$= -\frac{1}{e^2}\ln e^2 + \int_1^{e^2} \frac{1}{x^2}\,dx$$

$$= -\frac{2}{e^2} + \left[-\frac{1}{x}\right]_1^{e^2}$$

$$= -\frac{2}{e^2} + \left(-\frac{1}{e^2} + 1\right)$$

$$= \left(1 - \frac{3}{e^2}\right)$$

0.5

2. $\int_1^{e^2} (\ln x)^2 dx$ 를 계산하시오. (1점)

$$\int_1^{e^2} 1\cdot(\ln x)^2\,dx$$

$$= \left[x(\ln x)^2\right]_1^{e^2} - \int_1^{e^2} x \cdot \frac{2\ln x}{x}\,dx$$

0.5

$$= 4e^2 - 2\int_1^{e^2} \ln x\,dx$$

$$= 4e^2 - 2\left[x\ln x - x\right]_1^{e^2}$$

$$= 4e^2 - 2\left(2e^2 - e^2 + 1\right)$$

$$= \boxed{2e^2 - 2}$$

0.5

3. $\int_0^{\frac{\pi}{3}} \sin x \ln(\cos x) dx$ 를 계산하시오. (1.5점)

$$\left[-\cos x \ln(\cos x)\right]_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} (-\cos x)\frac{(-\sin x)}{\cos x}\,dx$$

$$= -\frac{1}{2}\ln\frac{1}{2} - \int_0^{\frac{\pi}{3}} \sin x\,dx$$

$$= \frac{1}{2}\ln 2 - \left[-\cos x\right]_0^{\frac{\pi}{3}}$$

$$= \frac{1}{2}\ln 2 - \left(-\frac{1}{2} + 1\right)$$

$$= \boxed{\frac{1}{2}\ln 2 - \frac{1}{2}}$$

0.5

4. $\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$ 를 계산하시오. (1점)

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

0.5

$$\int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{2}\,dx - \int_0^{\frac{\pi}{2}} \frac{\cos 2x}{2}\,dx$$

$$= \left[\frac{1}{2}x\right]_0^{\frac{\pi}{2}} - \frac{1}{4}\cancel{\left[\sin 2x\right]_0^{\frac{\pi}{2}}}$$

$$= \boxed{\frac{\pi}{4}}$$

0.5

5. $\int_0^{\frac{\pi}{4}} \sec^3 x\, dx$ 를 계산하시오. (1.5점)

$$I = \int_0^{\frac{\pi}{4}} \sec^2 x \cdot \sec x\, dx$$

$$= \left[\tan x \cdot \sec x\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan x (\sec x \tan x)\, dx$$

0.5

$$= \sqrt{2} - \int_0^{\frac{\pi}{4}} \tan^2 x \sec x\, dx$$

$\tan^2 x = \sec^2 x - 1$

$$= \sqrt{2} - \int_0^{\frac{\pi}{4}} (\sec^3 x - \sec x)\, dx$$

$$= \sqrt{2} - \underbrace{\int_0^{\frac{\pi}{4}} \sec^3 x\, dx}_{= I} + \int_0^{\frac{\pi}{4}} \sec x\, dx$$

0.5

$\therefore$

$$2I = \sqrt{2} + \int_0^{\frac{\pi}{4}} \sec x\, dx$$

뒤쪽 참고(공식)

$$= \sqrt{2} + \Big[\ln|\sec x + \tan x|\Big]_0^{\frac{\pi}{4}}$$

$$= \sqrt{2} + \ln|\sqrt{2}+1|$$

$$\therefore \int_0^{\frac{\pi}{4}} \sec^3 x\, dx = I = \boxed{\frac{\sqrt{2}}{2} + \frac{1}{2}\ln(\sqrt{2}+1)}$$

0.5

6. $\int_0^1 \frac{x}{\sqrt{3-2x-x^2}}\, dx$ 을 계산하시오. (1.5점)

$$3 - 2x - x^2$$

$$= 4 - (x+1)^2 \quad (\text{완전제곱식 꼴})$$

$$\int_0^1 \frac{(x+1)-1}{\sqrt{4-(x+1)^2}}\, dx$$

$t = x+1$
$dt = dx$

$$= \int_1^2 \frac{t-1}{\sqrt{4-t^2}}\, dt$$

0.5

$$= \int_1^2 \frac{t}{\sqrt{4-t^2}}\, dt - \int_1^2 \frac{1}{\sqrt{4-t^2}}\, dt$$

$4 - t^2 = k$
$(-2t)dt = dk$

$t = 2\sin\theta$
$dt = 2\cos\theta\, d\theta$

$$= -\frac{1}{2}\int_3^0 \frac{1}{\sqrt{k}}\, dk - \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1}{\cancel{2\cos\theta}} \cancel{(2\cos\theta)}\, d\theta$$

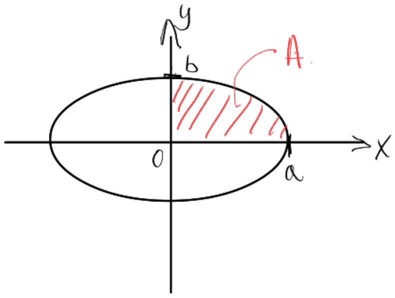
0.5

$$= \frac{1}{2}\int_0^3 \frac{1}{\sqrt{k}}\, dk - \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 1 \cdot d\theta$$

$$= \frac{1}{2}\left[2\sqrt{k}\right]_0^3 - \left[\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \boxed{\sqrt{3} - \frac{\pi}{3}}$$

0.5

7. 다음의 타원에 의해 둘러싸인 부분의 넓이를 구하여라. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (1.5점)



타원은 두 축에 대하여 대칭이므로 전체넓이는 4A와 같다.

제1사분면에 놓인 타원곡선은 $y = b\sqrt{1 - \frac{x^2}{a^2}}$ $(0 \le x \le a)$

$$\therefore \int_0^a b\sqrt{1 - \frac{x^2}{a^2}}\, dx$$

$$= \frac{b}{a}\int_0^a \sqrt{a^2 - x^2}\, dx$$

$(x = a\sin\theta$로 치환,
$dx = a\cos\theta\, d\theta$)

0.5

$$= \frac{b}{a}\int_0^{\frac{\pi}{2}} (a\cos\theta)\cdot(a\cos\theta)\, d\theta$$

$$= \frac{b}{a}\cdot a^2 \int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$

$$= ab\int_0^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2}\, d\theta$$

0.5

$$= ab\left[\frac{\theta}{2}\right]_0^{\frac{\pi}{2}} + ab\int_0^{\frac{\pi}{2}} \frac{\cos 2\theta}{2}\, d\theta$$

$$= \frac{\pi ab}{4} + \cancel{\frac{ab}{4}\Big[\sin 2\theta\Big]_0^{\frac{\pi}{2}}} \to 0$$

$$= \frac{\pi ab}{4} \quad (= A)$$

$\therefore$ 전체넓이 $= \boxed{\pi ab}$

0.5

점수 확인: https://math.seoultech.ac.kr

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$ 증명

$$\int \sec x\,dx = \int \frac{1}{\cos x}dx$$

$$= \int \frac{\cos x}{\cos^2 x}dx$$

$$= \int \frac{\cos x}{1-\sin^2 x}dx$$

$t = \sin x$

$dt = \cos x\,dx$

$$= \int \frac{1}{1-t^2}dt$$

$$= \int \frac{-1}{(t-1)(t+1)}dt$$

부분분수 $\frac{1}{AB} = \frac{1}{B-A}\left(\frac{1}{A} - \frac{1}{B}\right)$

$$= -\frac{1}{2}\int \left(\frac{1}{t-1} - \frac{1}{t+1}\right)dt$$

$$= -\frac{1}{2}\left(\ln|t-1| - \ln|t+1|\right) + C$$

$$= \frac{1}{2}\ln\left|\frac{t+1}{t-1}\right| + C$$

$$= \frac{1}{2}\ln\left|\frac{(t+1)^2}{t^2-1}\right| + C$$

$$= \ln\left|\sqrt{\frac{(t+1)^2}{1-t^2}}\right| + C$$

$\left(\because -1 \le \overset{\sin x}{\overset{\parallel}{t}} \le 1\right.$

$1-t^2 \ge 0$

단, $t \ne \pm 1$

$\left.\therefore 1-t^2 > 0\right)$

$$= \ln\left|\frac{(t+1)}{\sqrt{1-t^2}}\right| + C$$

$$= \ln\left|\frac{\sin x+1}{\cos x}\right| + C$$

$$= \ln|\tan x + \sec x| + C$$