book-lm

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Preface

This is a Quarto book.

To learn more about Quarto books visit https://quarto.org/docs/books.

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1 Introduction

This is a book created from markdown and executable code.

See Knuth (1984) for additional discussion of literate programming.

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2 Asymptotic Theory for Least Squares

2.1 Introduction

Theorem 2.1 (Random sampling assumption (@Hansen2022 Definition 1.2))**.** *The variables* (Y_i, X_i) *are a* ***random sample*** *if they are mutually independent and identically distributed (i.i.d.) across* $i = 1, \dots, n$*.*

Theorem 2.2 (Best linear predictor assumption (@Hansen2022 Assumption 2.1))**.**

1. $E[Y^2] < \infty$
2. $E\|X\|^2 < \infty$
3. $Q_{XX} = E[XX^T]$ *is positive definite*

X, Y , . Q_{XX} *column linearly independent* .

(Q_{XX} *positive definite linearly independence)*

random sampling finite second moment assumption least squares estimation assumption . (@Hansen2022 Assumption 7.1)

1. The variables (Y_i, X_i), $i = 1, \dots, n$ are i.i.d.
2. $E[Y^2] < \infty$.
3. $E\|X\|^2 < \infty$.
4. $Q_{XX} = E[XX^T]$ is positive definite.

2.2 Consistency of Least Squares Estimator

$\hat{\beta}$ β consistent

1. weak law of large numbers (WLLN)
2. continuous mapping theorem (CMT)

. (@Hansen2022 7.2)

Derivation .

1. OLS estimatior sample moment .
2. WLLN sample moments population moments converge in probability .
3. CMT converges in probability

OLS estimator sample moments $\hat{Q}_{XX} = \frac{1}{n}\sum_{i=1}^{n} X_i X_i^T$ $\hat{Q}_{XX} = \frac{1}{n}\sum_{i=1}^{n} X_i Y_i$.

$$\hat{\beta} = \left(\frac{1}{n}\sum_{i=1}^{n} X_i X_i^T\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} X_i Y_i\right) = \hat{Q}_{XX}^{-1}\hat{Q}_{XY}$$

(Y_i, X_i) mutually i.i.d. (Y_i, X_i) , $X_i X_i^T$ $X_i Y_i$ i.i.d. . Assumption 7.1 finite expectation . , $n \to \infty$ WLLN

$$\hat{Q}_{XX} = \frac{1}{n}\sum_{i=1}^{n} X_i X_i^T \xrightarrow{p} E[XX^T] = Q_{XX}, \quad \hat{Q}_{XY} = \frac{1}{n}\sum_{i=1}^{n} X_i Y_i \xrightarrow{p} E[XY] = Q_{XY}.$$

continuous mapping theorem $\hat{\beta} \to \beta$. $n \to \infty$,

$$\hat{\beta} = \hat{Q}_{XX}^{-1}\hat{Q}_{XY} \xrightarrow{p} Q_{XX}^{-1}Q_{XY} = \beta.$$

Stochastic order notation .

$$\hat{\beta} = \beta + o_p(1).$$

2.3 Asymptotic Normality

Asymptotic normality

1. estimator sample moment .
2. zero-mean random vector sum CLT .

$\hat{\beta} - \beta = \hat{Q}_{XX}^{-1}\hat{Q}_{Xe}$. $\sqrt{n}$.

$$\sqrt{n}(\hat{\beta} - \beta) = \left(\frac{1}{n}\sum_{i=1}^{n} X_i X_i^T\right)^{-1}\left(\frac{1}{\sqrt{n}}\sum_{i=1}^{n} X_i e_i\right).$$

normalized and centered estimator $\sqrt{n}(\hat{\beta} - \beta)$ (1) sample average $\left(\frac{1}{n}\sum_{i=1}^{n} X_i X_i^T\right)^{-1}$ normalized sample average $\left(\frac{1}{\sqrt{n}}\sum_{i=1}^{n} X_i e_i\right)$.

$E[Xe] = 0$ $k \times k$.

$$\Omega = E[(Xe)(Xe)^T] = E[XX^T e^2].$$

$\Omega < \infty$ $X_i e_i$ i.i.d. mean zero, CLT

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} X_i e_i \xrightarrow{d} \mathcal{N}(0, \Omega).$$

(@Hansen2022 Assumption 7.2)

1. The variables $(Y_i, X_i), i = 1, \dots, n$ are i.i.d.
2. $E[Y^4] < \infty$.
3. $E\|X\|^4 < \infty$.
4. $Q_{XX} = E[XX^T]$ is positive definite.

$\Omega < \infty$ jl $E[X_j X_l e^2]$. Properties of Linear Projection Model (@Hansen2022 Theorem 2.9.6) (If $E|Y|^r < \infty$ and $E|X|^r < \infty$ for $r \geq 2$, then $E|e|^r < \infty$) 2, 3 $E[e^4] < \infty$. expectation inequality Ω jl bounded .

$$|E[X_j X_l e^2]| \leq E|X_j X_l e^2| = E[|X_j||X_l|e^2].$$

Stochastic order notation .

$$\hat{\beta} = \beta + O_p(n^{-1/2}).$$

3 Summary

In summary, this book has no content whatsoever.

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References

Knuth, Donald E. 1984. "Literate Programming." *Comput. J.* 27 (2): 97–111. https://doi.org/10.1093/comjnl/27.2.97.