COMPUTER EXERCISES

Several exercises will make use of the following three-dimensional data sampled from three categories, denoted ω_i.

	ω_1			ω_2			ω_3		
sample	x_1	x_2	x_3	x_1	x_2	x_3	x_1	x_2	x_3
1	0.42	−0.087	0.58	−0.4	0.58	0.089	0.83	1.6	−0.014
2	−0.2	−3.3	−3.4	−0.31	0.27	−0.04	1.1	1.6	0.48
3	1.3	−0.32	1.7	0.38	0.055	−0.035	−0.44	−0.41	0.32
4	0.39	0.71	0.23	−0.15	0.53	0.011	0.047	−0.45	1.4
5	−1.6	−5.3	−0.15	−0.35	0.47	0.034	0.28	0.35	3.1
6	−0.029	0.89	−4.7	0.17	0.69	0.1	−0.39	−0.48	0.11
7	−0.23	1.9	2.2	−0.011	0.55	−0.18	0.34	−0.079	0.14
8	0.27	−0.3	−0.87	−0.27	0.61	0.12	−0.3	−0.22	2.2
9	−1.9	0.76	−2.1	−0.065	0.49	0.0012	1.1	1.2	−0.46
10	0.87	−1.0	−2.6	−0.12	0.054	−0.063	0.18	−0.11	−0.49

Section 3.2

1. Consider Gaussian density models in different dimensions.
   - **(a)** Write a program to find the maximum-likelihood values $\hat{\mu}$ and $\hat{\sigma}^2$. Apply your program individually to each of the three features x_i of category ω_1 in the table above.
   - **(b)** Modify your program to apply to two-dimensional Gaussian data $p(\mathbf{x}) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Apply your data to each of the three possible pairings of two features for ω_1.
   - **(c)** Modify your program to apply to three-dimensional Gaussian data. Apply your data to the full three-dimensional data for ω_1.
   - **(d)** Assume your three-dimensional model is separable, so that

     $$\boldsymbol{\Sigma} = diag(\sigma_1^2, \sigma_2^2, \sigma_3^2).$$

     Write a program to estimate the mean and the diagonal components of $\boldsymbol{\Sigma}$. Apply your program to the data in ω_2.
   - **(e)** Compare your results for the mean of each feature μ_i calculated in the above ways. Explain why they are the same or different.
   - **(f)** Compare your results for the variance of each feature σ_i^2 calculated in the above ways. Explain why they are the same or different.