수학으로부터 인류를 자유롭게 하라

Free Humankind from Mathematics

Basic Algebra

Chap.6 Rational and Irrational Functions

Rational Expressions

두 다항식 A, B $(B \neq 0)$이 $\dfrac{A}{B}$로 나타낼 수 있는 식

$$\frac{1}{x}, \quad \frac{1}{3x+1}, \quad \frac{1}{(x+2)^2}, \quad \frac{3x}{2x+1}, \quad \frac{x-1}{(x-2)^2}$$

$$\frac{x(x+1)}{3x-1}, \quad \frac{x^2+3x+1}{2x}, \quad 4x+7, \quad 4x^2-2x+1$$

(Polynomials) $\subset$ (Rational Expressions)

Rational Expressions

Property of Rational Expressions

Polynomials A, B, C $(B \neq 0,\ C \neq 0)$

$$\frac{A}{B} = \frac{A \times C}{B \times C} = \frac{A \div C}{B \div C}$$

Examples

ex.1) $\dfrac{2x+1}{x} = \dfrac{(2x+1)(x+1)}{x(x+1)} = \dfrac{2x^2+3x+1}{x^2+x}$

ex.2) $\dfrac{x^3(x+1)}{x^2} = x(x+1)$

ex.3) $\dfrac{x^3(x+1)}{x(x+1)^3} = \dfrac{x^2}{(x+1)^2}$

ex.4) $\dfrac{2}{1-\frac{1}{x}} = \dfrac{2x}{x-1}$

ex.5) $\dfrac{x+\frac{1}{x}}{x-\frac{1}{x}} = \dfrac{x^2+1}{x^2-1} = \dfrac{x^2+1}{(x-1)(x+1)}$

Additions / Subtractions of Polynomials

Case.1) Same Denominators

$$\frac{A}{C} \pm \frac{B}{C} = \frac{A \pm B}{C}, \; C \neq 0$$

Examples

ex.1) $\frac{x+1}{x} + \frac{1}{x} = \frac{x+2}{x}$

ex.2) $\frac{x}{x+1} + \frac{1}{x+1} = \frac{x+1}{x+1} = 1$

ex.3) $\frac{x^2}{x+1} - \frac{1}{x+1} = \frac{x^2-1}{x+1} = \frac{(x-1)(x+1)}{x+1} = x-1$

Additions / Subtractions of Polynomials

Case.2) Different Denominators

$$\frac{A}{B} \pm \frac{C}{D} = \frac{AD}{BD} \pm \frac{BC}{BD} = \frac{AD \pm BC}{BD}, \ (B \neq 0, \ C \neq 0)$$

Examples

ex.1) $\frac{1}{x} + \frac{2}{x(x+1)} \quad \longrightarrow \frac{x+1}{x(x+1)} + \frac{2}{x(x+1)} = \frac{x+3}{x(x+1)}$

ex.2) $\frac{x+3}{x+1} + \frac{-x^2-x+4}{x^2-1} \quad \longrightarrow \frac{x^2+2x-3}{(x+1)(x-1)} + \frac{-x^2-x+4}{(x+1)(x-1)} = \frac{x+1}{(x+1)(x-1)} = \frac{1}{x-1}$

ex.3) $\frac{1}{x-1} + \frac{1}{x} \quad \longrightarrow \frac{x}{x(x-1)} + \frac{x-1}{x(x-1)} = \frac{2x-1}{x(x-1)}$

Additions / Subtractions of Polynomials

Examples

ex.4) $\dfrac{1}{x-1}-\dfrac{1}{x+1}-\dfrac{2}{x^2+1}-\dfrac{4}{x^4+1}$

$$=\frac{x+1}{x^2-1}-\frac{x-1}{x^2-1}-\frac{2}{x^2+1}-\frac{4}{x^4+1}=\frac{2}{x^2-1}-\frac{2}{x^2+1}-\frac{4}{x^4+1}$$

$$=\frac{2x^2+2}{x^4-1}-\frac{2x^2-2}{x^4-1}-\frac{4}{x^4+1}=\frac{4}{x^4-1}-\frac{4}{x^4+1}=\frac{8}{x^8-1}$$

ex.5) $1-\dfrac{1}{1-\dfrac{1}{1-\dfrac{1}{1-\frac{1}{x}}}}=1-\dfrac{1}{1-\frac{1}{1-\frac{x}{x-1}}}=1-\dfrac{1}{1+x-1}=\dfrac{x-1}{x}$

Additions / Subtractions of Polynomials

Case.3) Special Cases

$$\frac{1}{AB} = \frac{1}{B-A}\left(\frac{1}{A} - \frac{1}{B}\right)$$

Examples

ex.1) $\dfrac{1}{x(x+1)} = \dfrac{1}{x} - \dfrac{1}{x+1}$

ex.2) $\dfrac{1}{x(x+2)} = \dfrac{1}{2}\left(\dfrac{1}{x} - \dfrac{1}{x+2}\right)$

ex.3) $\dfrac{1}{x(x+3)} + \dfrac{1}{(x+3)(x+6)} + \dfrac{1}{(x+6)(x+9)}$

$= \dfrac{1}{3}\left(\dfrac{1}{x} - \dfrac{1}{x+3}\right) + \dfrac{1}{3}\left(\dfrac{1}{x+3} - \dfrac{1}{x+6}\right) + \dfrac{1}{3}\left(\dfrac{1}{x+6} - \dfrac{1}{x+9}\right)$

$= \dfrac{1}{3}\left(\dfrac{1}{x} - \dfrac{1}{x+9}\right) = \dfrac{3}{x(x+9)}$

Additions / Subtractions of Polynomials

Case.3) Special Cases

$$\frac{1}{AB} = \frac{1}{B-A}\left(\frac{1}{A} - \frac{1}{B}\right)$$

Examples

ex.4) $$\frac{1}{x(x+1)} + \frac{1}{(x+1)(x+2)} + \dots + \frac{1}{(x+n-2)(x+n-1)} + \frac{1}{(x+n-1)(x+n)}$$

$$= \left(\frac{1}{x} - \frac{1}{x+1}\right) + \left(\frac{1}{x+1} - \frac{1}{x+2}\right) + \dots + \left(\frac{1}{x+n-2} - \frac{1}{x+n-1}\right) + \left(\frac{1}{x+n-1} - \frac{1}{x+n}\right)$$

$$= \frac{1}{x} - \frac{1}{x+n} = \frac{n}{x(x+n)}$$

Additions / Subtractions of Polynomials

Case.4) Special Cases

$$\left(x^n \pm \frac{1}{x^n}\right)^2 = x^{2n} + \frac{1}{x^{2n}} \pm 2$$

Examples

ex.1) $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$ $\quad$ $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

ex.2) $\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2$ $\quad$ $\left(x^2 - \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} - 2$

ex.3) $x^2 + 4x + 1 = 0 \longrightarrow x^2 + x + \frac{1}{x} + \frac{1}{x^2}$

$$\begin{matrix} x + \frac{1}{x} = -4 \\ x^2 + \frac{1}{x^2} = 14 \end{matrix} \longrightarrow x^2 + x + \frac{1}{x} + \frac{1}{x^2} = \left(x + \frac{1}{x}\right) + \left(x^2 + \frac{1}{x^2}\right) = 10$$

Decompositions of Rational Expressions

$$\frac{A \pm B}{C} = \frac{A}{C} \pm \frac{B}{C} \ (C \neq 0)$$

Examples

ex.1) $\dfrac{(x-1)(x+1)}{x} = \dfrac{x-1}{x} + \dfrac{x+1}{x}$

ex.2) $\dfrac{x(x-1)(x+1)}{x^2-1} = \dfrac{x}{x^2-1} + \dfrac{x-1}{x^2-1} + \dfrac{x+1}{x^2-1} = \dfrac{x}{x^2-1} + \dfrac{1}{x+1} + \dfrac{1}{x-1}$

Rational Functions

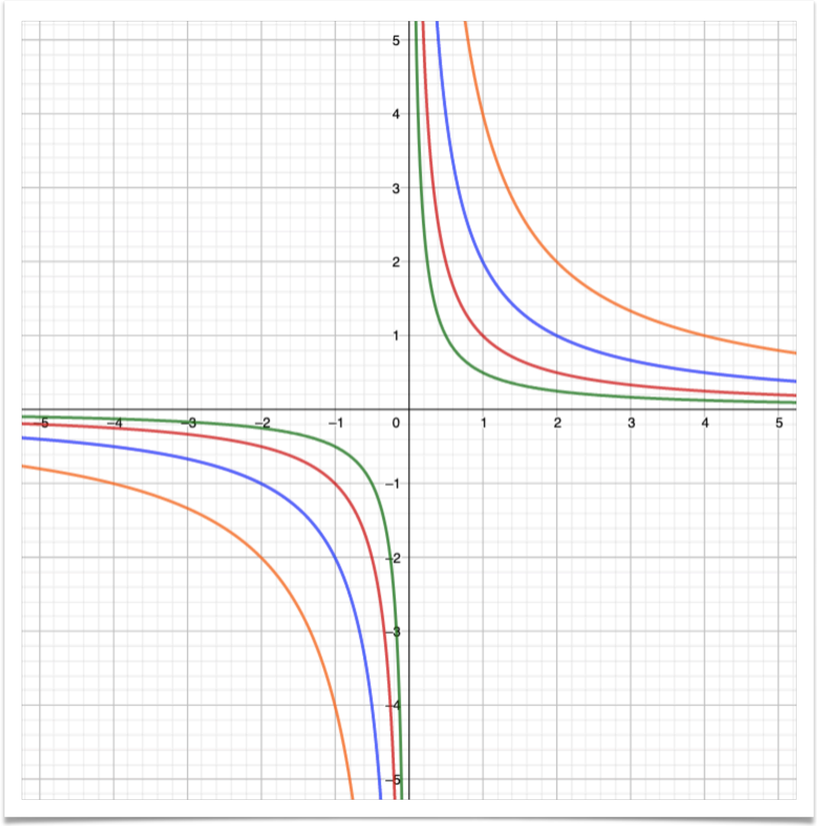
$$f(x) = \frac{A(x)}{B(x)}, B(x) \neq 0$$

Rational Functions

Basic Rational Functions

$$f(x) = \frac{a}{x},\ x \neq 0$$

x	$f(x) = \frac{1}{2x}$	$f(x) = \frac{1}{x}$	$f(x) = \frac{2}{x}$	$f(x) = \frac{4}{x}$
-2	-0.25	-0.5	-1	-2
-1	-0.5	-1	-2	-4
-0.5	-1	-2	-4	-8
-0.25	-2	-4	-8	-16
0.25	2	4	8	16
0.5	1	2	4	8
1	0.5	1	2	4
2	0.25	0.5	1	2



$$f_1(x) = \frac{1}{2x}$$

$$f_2(x) = \frac{1}{x}$$

$$f_3(x) = \frac{2}{x}$$

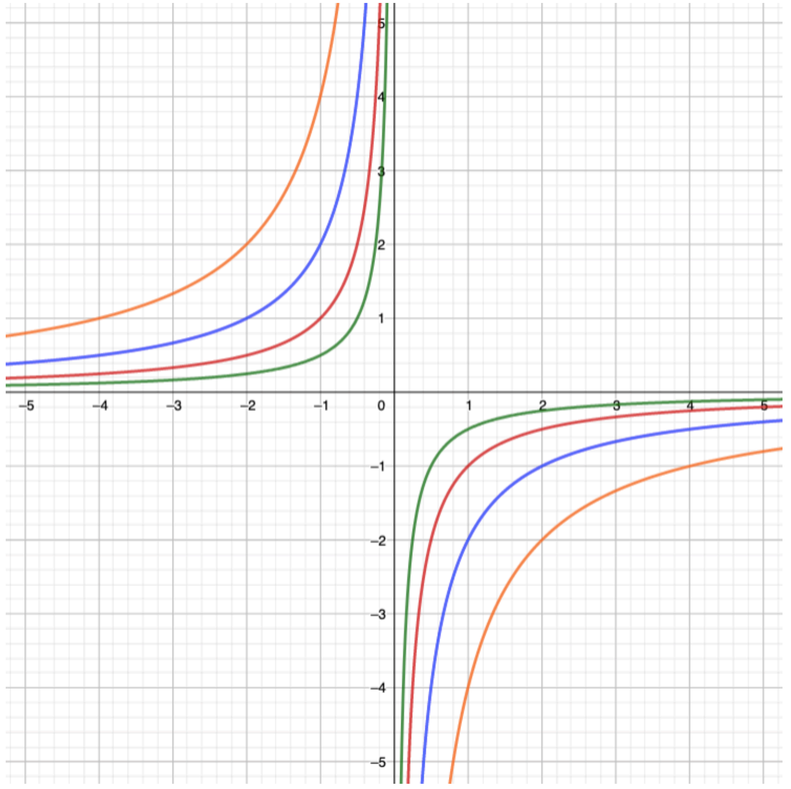
$$f_3(x) = \frac{2}{x}$$

Rational Functions

Basic Rational Functions

$$f(x) = \frac{a}{x},\ x \neq 0$$

x	$f(x) = -\frac{1}{2x}$	$f(x) = -\frac{1}{x}$	$f(x) = -\frac{2}{x}$	$f(x) = -\frac{4}{x}$
-2	0.25	0.5	1	2
-1	0.5	1	2	4
-0.5	1	2	4	8
-0.25	2	4	8	16
0.25	-2	-4	-8	-16
0.5	-1	-2	-4	-8
1	-0.5	-1	-2	-4
2	-0.25	-0.5	-1	-2



$$f_1(x) = -\frac{1}{2x}$$

$$f_2(x) = -\frac{1}{x}$$

$$f_3(x) = -\frac{2}{x}$$

$$f_3(x) = -\frac{2}{x}$$

Properties of Rational Functions

Odd Functions $f(x) = \frac{a}{x},\ x \neq 0 \longrightarrow f(-x) = -f(x)$

Asymptotes $f(x) = \frac{a}{x} \longrightarrow$ (asymptotes: x-axis, y-axis)

Domains, Ranges $f(x) = \frac{a}{x} \longrightarrow D = R = \mathbb{R} - \{0\}$

Translations of Rational Functions

$$f(x) = \frac{a}{x} \longrightarrow g(x) = \frac{a}{x-\alpha} + \beta$$

Asymptotes $x = \alpha,\ y = \beta$

Domains, Ranges $D = \mathbb{R} - \{\alpha\}$
$R = \mathbb{R} - \{\beta\}$

More General Rational Functions

Polynomial Denominators

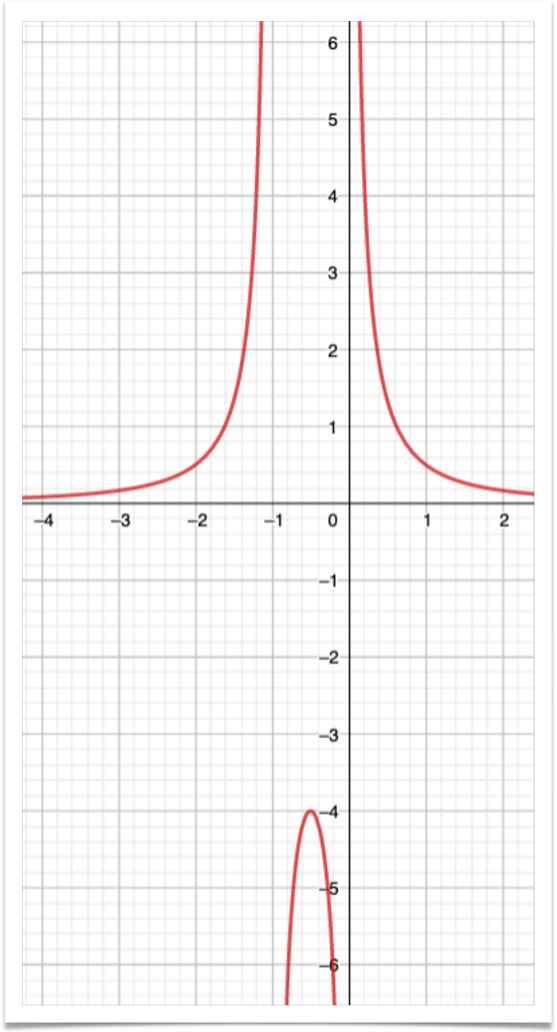
$$f(x) = \frac{a}{(x-\alpha)(x-\beta)}$$

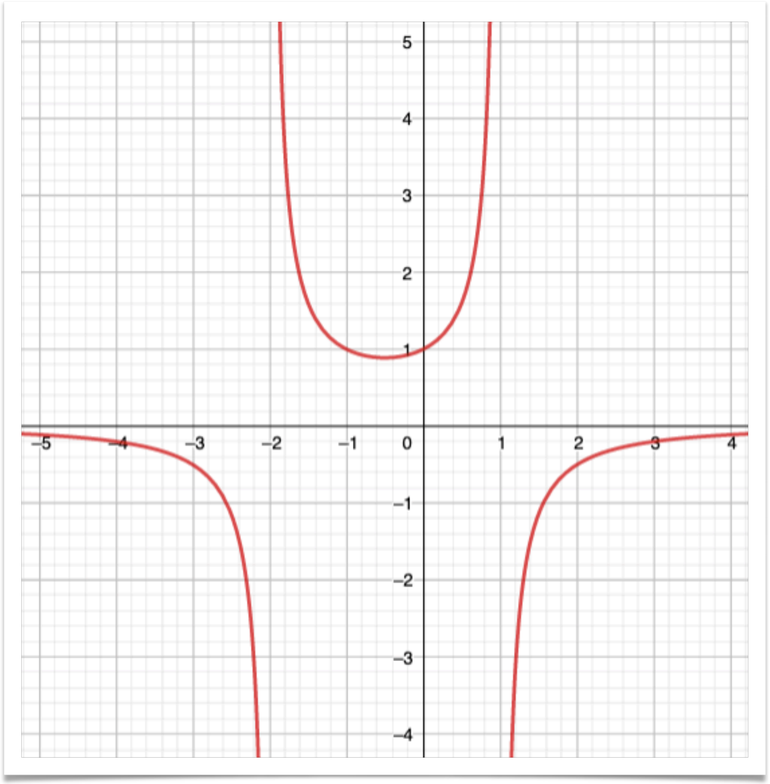
Asymptotes $x = \alpha$, $x = \beta$, $y = 0$

Domains, Ranges $D = \mathbb{R} - \{\alpha, \beta\}$, $R = \mathbb{R} - \{0\}$

함수에 따라 달라질 수 있음

$$f(x) = \frac{1}{x(x+1)}$$

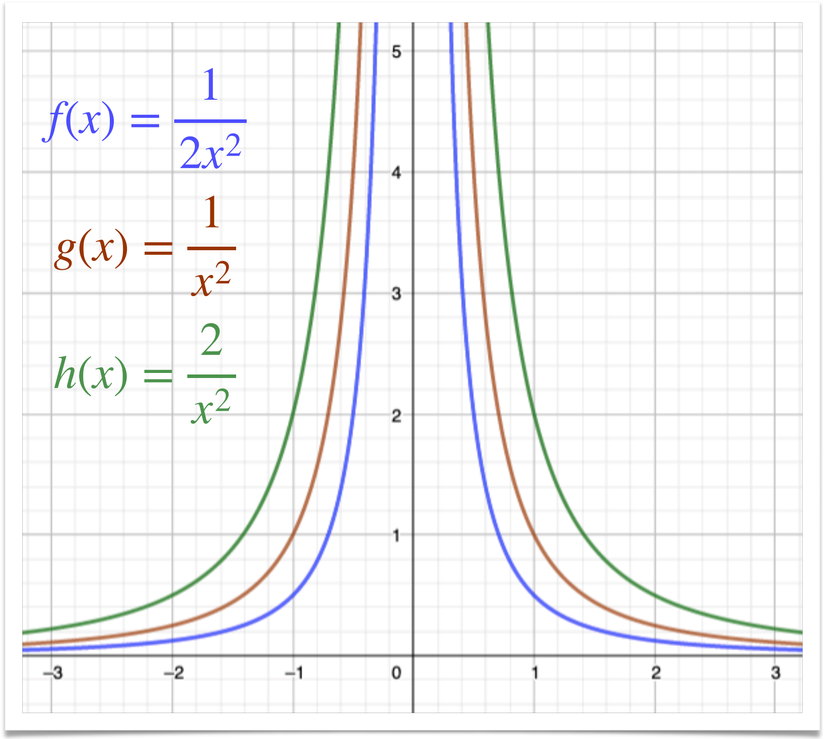


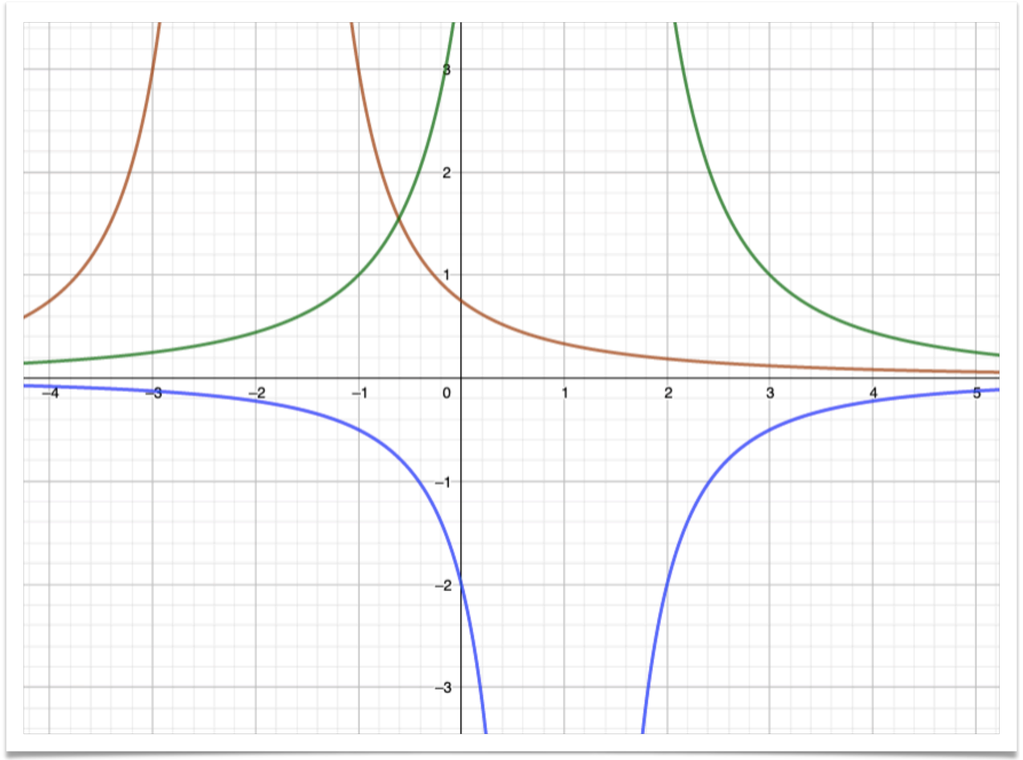


$$g(x) = -\frac{2}{(x+2)(x-1)}$$

More General Rational Functions

Polynomial Denominators





$$f(x) = -\frac{2}{(x-1)^2}$$

$$g(x) = \frac{3}{(x+2)^2}$$

$$h(x) = \frac{4}{(x-1)^2}$$

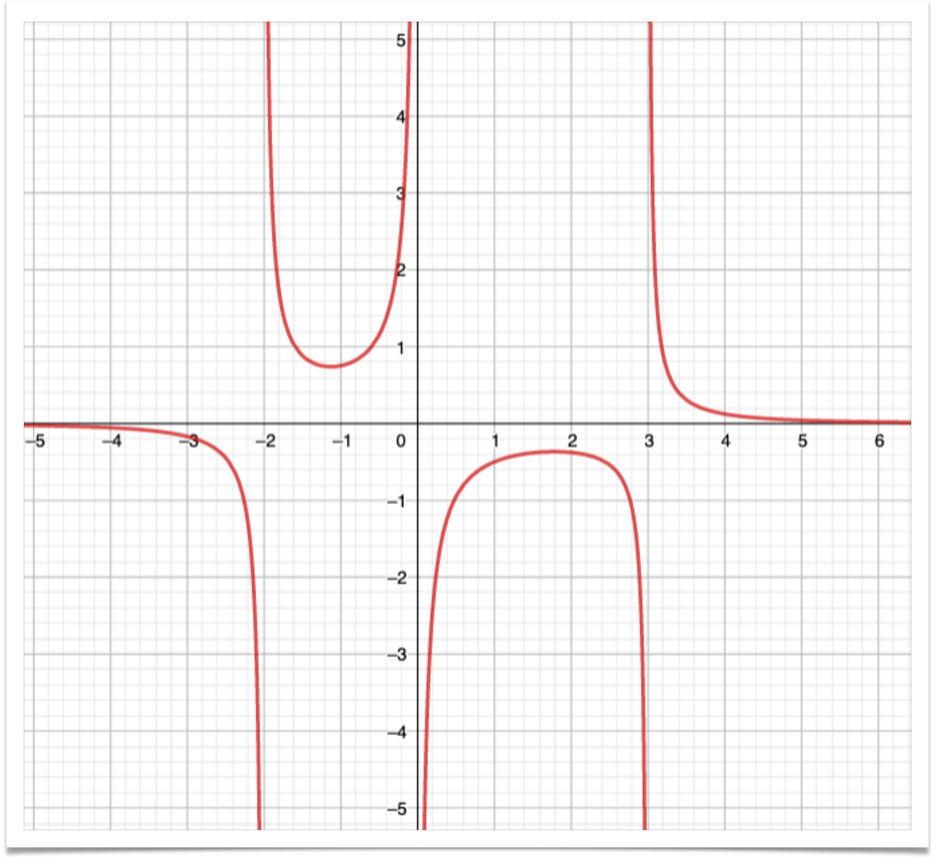
More General Rational Functions

Polynomial Denominators

$$f(x) = \frac{a}{(x-\alpha)(x-\beta)(x-\gamma)}$$

Asymptotes $x = \alpha$, $x = \beta$, $x = \gamma$, $y = 0$

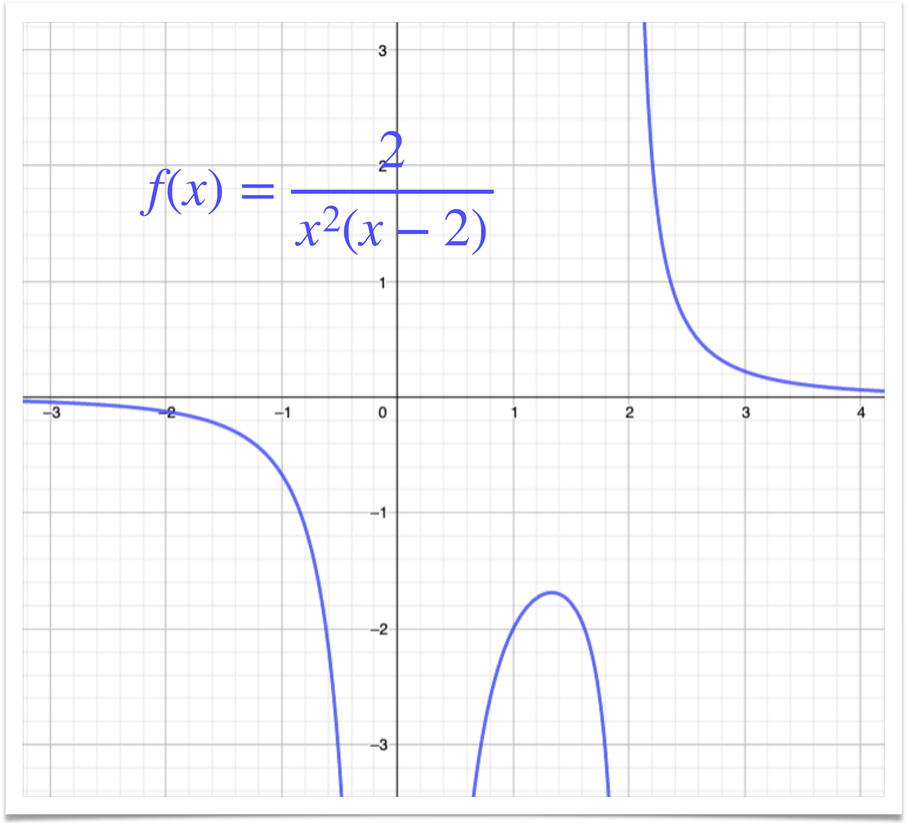
Domains, Ranges $D = \mathbb{R} - \{\alpha, \beta, \gamma\}$, $R = \mathbb{R} - \{0\}$



$$f(x) = \frac{3}{x(x+2)(x-3)}$$

More General Rational Functions

Polynomial Denominators



More General Rational Functions

Improper Rational Functions

분자의 차수가 더 높을 때, polynomial function과 rational function으로 나눌 수 있다.

$$P(x) = \frac{A(x)}{B(x)} = C(x) + \frac{A'(x)}{B(x)}$$

Examples

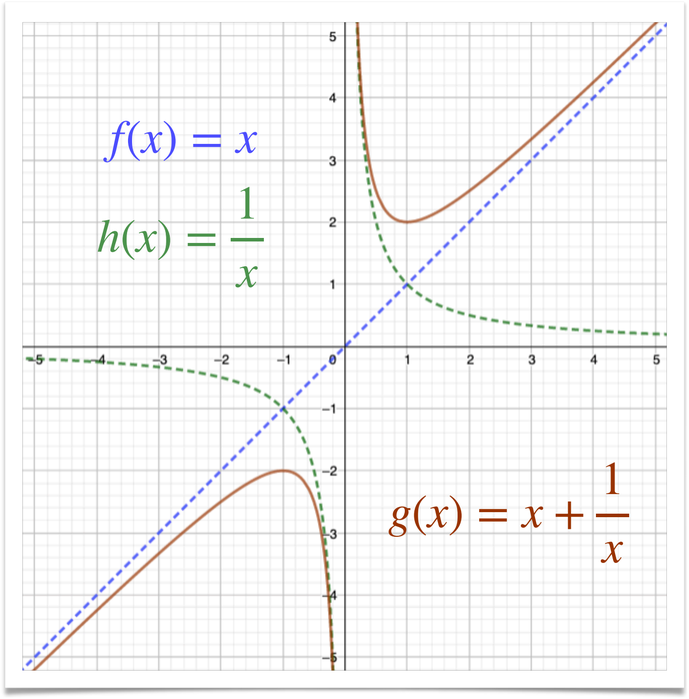
ex.1) $\dfrac{x^2+1}{x} = \dfrac{x^2}{x} + \dfrac{1}{x} = x + \dfrac{1}{x}$

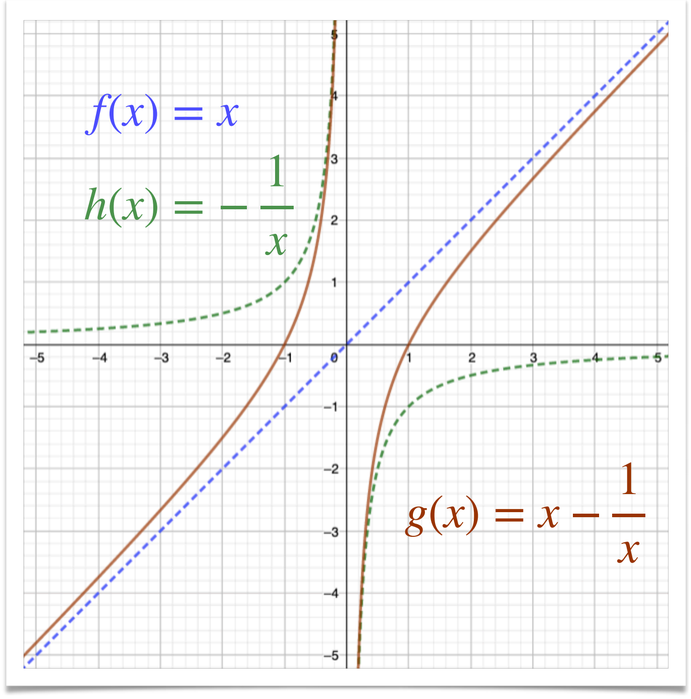
ex.2) $\dfrac{3x^3-2x^2+5}{x^2} = \dfrac{3x^3}{x^2} + \dfrac{-2x^2}{x^2} + \dfrac{5}{x^2} = (3x-2) + \dfrac{5}{x^2}$

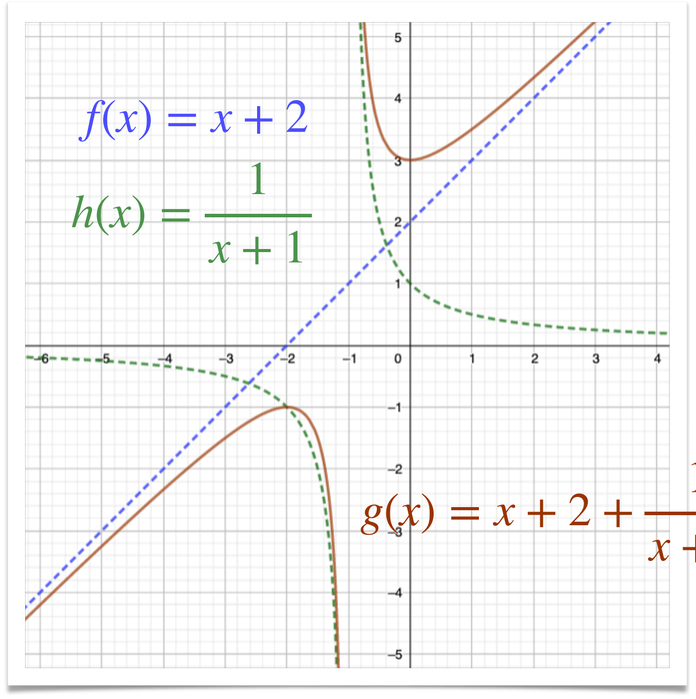
More General Rational Functions

Improper Rational Functions

$$P(x) = \frac{A(x)}{B(x)} = C(x) + \frac{A'(x)}{B(x)}$$







Multiplications / Divisions and Inequalities

$A(x)$, $B(x)$, $P(x)$, $Q(x)$: Polynomials

$$\frac{A(x)}{B(x)} \begin{matrix} > \\ \geq \end{matrix} \begin{matrix} < \\ \leq \end{matrix} 0 \quad \longleftrightarrow \quad A(x) \cdot B(x) \begin{matrix} > \\ \geq \end{matrix} \begin{matrix} < \\ \leq \end{matrix} 0,\ B(x) \neq 0$$

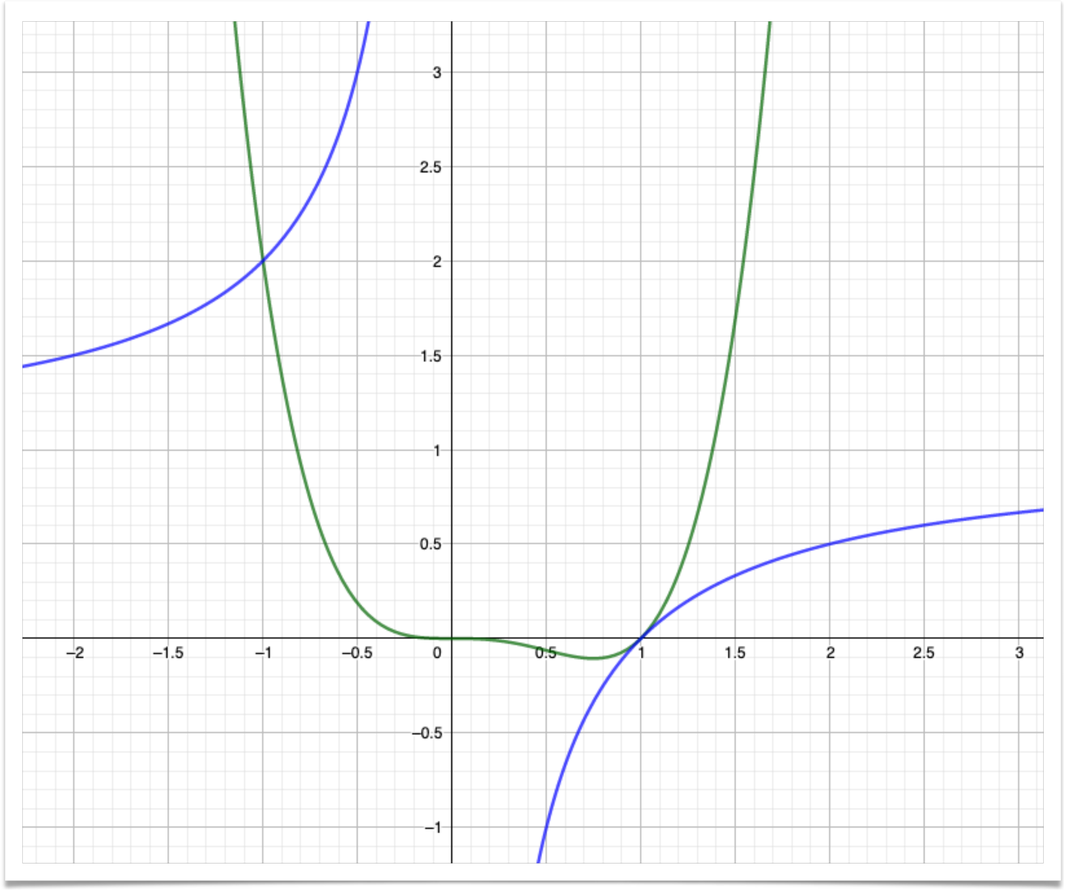
Multiplications / Divisions and Inequalities

ex.1) $A(x) = x(x-1), \quad B(x) = x^2$일 때, $A(x)B(x) > 0$의 solution set을 구하세요.

$$A(x) \cdot B(x) > 0$$
$$\longrightarrow \quad x^3(x-1) > 0$$
$$\longrightarrow \quad S = (-\infty, 0) \cup (1, \infty)$$

ex.2) $A(x) = x(x-1), \quad B(x) = x^2$일 때, $\dfrac{A(x)}{B(x)} > 0$의 solution set을 구하세요.

$$\frac{A(x)}{B(x)} > 0$$
$$\longrightarrow \quad A(x) \cdot B(x) > 0$$
$$\longrightarrow \quad x^3(x-1) > 0$$
$$\longrightarrow \quad S = (-\infty, 0) \cup (1, \infty)$$



Additions / Subtractions and Inequalities

$$\frac{A(x)}{B(x)} \pm \frac{P(x)}{Q(x)} \begin{matrix} > \\ \geq \end{matrix} \begin{matrix} < \\ \leq \end{matrix} 0$$

$$\longrightarrow \quad \frac{A(x)Q(x) \ \pm \ B(x)P(x)}{B(x)Q(x)} \begin{matrix} > \\ \geq \end{matrix} \begin{matrix} < \\ \leq \end{matrix} 0$$

$$\longrightarrow \quad B(x)Q(x)\left[A(x)Q(x) \ \pm \ B(x)P(x)\right] \begin{matrix} > \\ \geq \end{matrix} \begin{matrix} < \\ \leq \end{matrix} 0$$

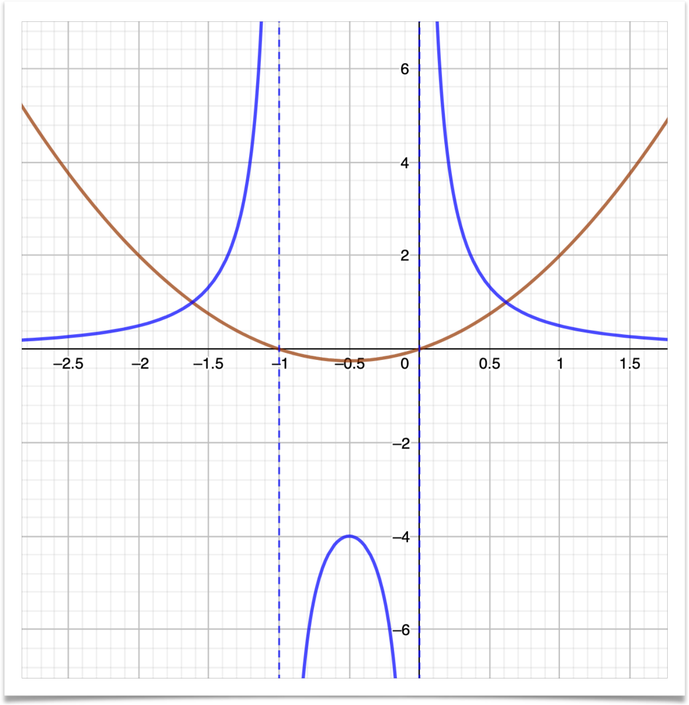
$$B(x) \neq 0, \ \ Q(x) \neq 0$$

Additions / Subtractions and Inequalities

ex.1) $\frac{A(x)}{B(x)} = \frac{1}{x}, \quad \frac{P(x)}{Q(x)} = \frac{1}{x+1}$일 때, $\frac{A(x)}{B(x)} - \frac{P(x)}{Q(x)} \le 0$의 solution set을 구하세요.

$$\frac{A(x)}{B(x)} - \frac{P(x)}{Q(x)} \le 0 \quad \longrightarrow \quad \frac{1}{x} - \frac{1}{x+1} \le 0 \quad \longrightarrow \quad \frac{1}{x(x+1)} \le 0 \quad \longrightarrow \quad x(x+1) \le 0$$

$$\longrightarrow \quad S = (-1, 0)$$

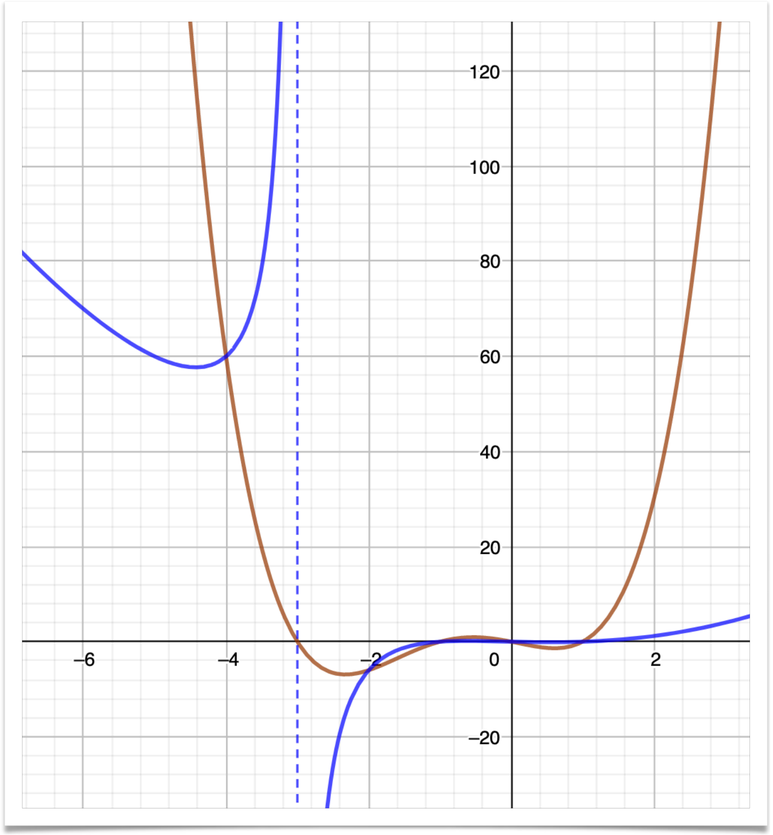


Additions / Subtractions and Inequalities

ex.2) $\frac{A(x)}{B(x)} = x^2 - 3x + 8, \quad \frac{P(x)}{Q(x)} = \frac{24}{x+3}$일 때, $\frac{A(x)}{B(x)} - \frac{P(x)}{Q(x)} > 0$의 solution set을 구하세요.

$$\frac{A(x)}{B(x)} - \frac{P(x)}{Q(x)} > 0 \quad \longrightarrow \quad x^2 - 3x + 8 - \frac{24}{x+3} > 0 \quad \longrightarrow \quad \frac{x(x+1)(x-1)}{x+3} > 0$$

$$\longrightarrow \quad x(x+3)(x+1)(x-1) > 0 \quad \longrightarrow \quad S = (-\infty, -3) \cup (-1,0) \cup (1,\infty)$$



Irrational Expressions

root안에 변수가 포함된 식

$$\sqrt{x},\quad \sqrt{x+2},\quad -2\sqrt{x-1},\quad \sqrt{x+3}+3$$

$$\sqrt{-x},\quad \sqrt{-x+12},\quad -2\sqrt{-x^2+1},\quad \sqrt{-x^2+2x-1}+3x$$

$$\sqrt[3]{x^2-2}-2,\quad \sqrt[4]{x+5},\quad \sqrt[5]{x^2-5}$$

Square Roots

$$a^2 = b \longrightarrow \begin{cases} \sqrt{b} = a, \text{ if } a > 0 \\ \sqrt{b} = -a, \text{ if } a < 0 \end{cases}$$

Examples

ex.1) $2^2 = 4 \longrightarrow \sqrt{4} = 2$ $\quad (-2)^2 = 4 \longrightarrow \sqrt{4} = -(-2) = 2$

ex.2) $3^2 = 9 \longrightarrow \sqrt{9} = 3$ $\quad (-3)^2 = 9 \longrightarrow \sqrt{9} = -(-3) = 3$

ex.3) $4^2 = 16 \longrightarrow \sqrt{16} = 4$ $\quad (-4)^2 = 16 \longrightarrow \sqrt{16} = -(-4) = 4$

ex.4) $\left(\frac{1}{2}\right)^2 = \frac{1}{4} \longrightarrow \sqrt{\frac{1}{4}} = \frac{1}{2}$ $\quad \left(-\frac{1}{2}\right)^2 = \frac{1}{4} \longrightarrow \sqrt{\frac{1}{4}} = -\left(-\frac{1}{2}\right) = \frac{1}{2}$

ex.5) $\left(\frac{1}{5}\right)^2 = \frac{1}{25} \longrightarrow \sqrt{\frac{1}{25}} = \frac{1}{5}$ $\quad \left(-\frac{1}{5}\right)^2 = \frac{1}{25} \longrightarrow \sqrt{\frac{1}{25}} = -\left(-\frac{1}{5}\right) = \frac{1}{5}$

Square Roots

Examples

ex.6) $\sqrt{49} = 7$

$-\sqrt{4} = -2$

$\sqrt{36} = 6$

$-\sqrt{100} = -10$

$\sqrt{64} = 8$

$-\sqrt{400} = -20$

$\sqrt{121} = 11$

$-\sqrt{169} = -13$

$\sqrt{(-4)^4} = 16$

$\sqrt{(-3)^2} = 3$

$-\sqrt{(-11)^2} = -11$

$\sqrt{\frac{1}{49}} = \frac{1}{7}$

$-\sqrt{\frac{1}{9}} = -\frac{1}{3}$

$-\sqrt{\left(-\frac{1}{3}\right)^4} = -\frac{1}{9}$

$\sqrt{\frac{4}{9}} = \frac{2}{3}$

$\sqrt{\frac{49}{25}} = \frac{7}{5}$

$-\sqrt{\frac{9}{64}} = -\frac{3}{8}$

$-\sqrt{\left(-\frac{4}{3}\right)^2} = -\frac{4}{3}$

$\sqrt{\left(-\frac{2}{3}\right)^6} = \frac{8}{27}$

Cube Roots

$$a^3 = b \longrightarrow \sqrt[3]{b} = a$$

Examples

ex.1) $2^3 = 8 \longrightarrow \sqrt[3]{8} = 2$ $\quad (-2)^3 = -8 \longrightarrow \sqrt[3]{-8} = -2$

ex.2) $3^3 = 27 \longrightarrow \sqrt[3]{27} = 3$ $\quad (-3)^3 = -27 \longrightarrow \sqrt[3]{-27} = -3$

ex.3) $\left(\frac{1}{2}\right)^3 = \frac{1}{8} \longrightarrow \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$ $\quad \left(-\frac{1}{2}\right)^3 = -\frac{1}{8} \longrightarrow \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$

ex.4) $\left(\frac{1}{5}\right)^3 = \frac{1}{125} \longrightarrow \sqrt[3]{\frac{1}{125}} = \frac{1}{5}$ $\quad \left(-\frac{1}{5}\right)^3 = -\frac{1}{125} \longrightarrow \sqrt[3]{-\frac{1}{125}} = -\frac{1}{5}$

nth Roots(Radicals)

$$a^{2m} = b \longrightarrow \begin{cases} \sqrt[2m]{b} = a, \text{ if } a > 0 \\ \sqrt[2m]{b} = -a, \text{ if } a < 0 \end{cases}$$

$$a^{2m-1} = b \longrightarrow \sqrt[2m-1]{b} = a$$

Examples

ex.1) $2^4 = 16 \longrightarrow \sqrt[4]{16} = 2$

ex.2) $\left(-\frac{1}{3}\right)^5 = -\frac{1}{243} \longrightarrow \sqrt[5]{\frac{1}{-243}} = -\frac{1}{3}$

Properties of Roots

- $(\sqrt{a})^2 = a$
- $\sqrt{a^2} = |a|$

  $\sqrt{3^2} = \sqrt{9} = 3 = |3|, \quad \sqrt{(-3)^2} = \sqrt{9} = 3 = |-3|$

  $\sqrt{\left(\frac{1}{4}\right)^2} = \sqrt{\frac{1}{16}} = \frac{1}{4} = \left|\frac{1}{4}\right|, \quad \sqrt{\left(-\frac{1}{4}\right)^2} = \sqrt{\frac{1}{16}} = \frac{1}{4} = \left|-\frac{1}{4}\right|$

- $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$

  $\sqrt{4} \cdot \sqrt{9} = \sqrt{36}$

  $\sqrt{4} \cdot \sqrt{\frac{1}{9}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

Rationalization of Denominators

Case.1)

$$\frac{a}{\sqrt{b}} = \frac{a\sqrt{b}}{\sqrt{b}\cdot\sqrt{b}} = \frac{a\sqrt{b}}{b}$$

ex.1) $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

ex.2) $\frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

Case.2)

$$\frac{c}{\sqrt{a}+\sqrt{b}} = \frac{c(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})} = \frac{c(\sqrt{a}-\sqrt{b})}{a-b}$$

ex.1) $\frac{1}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{5}-\sqrt{3}}{5-3} = \frac{1}{2}(\sqrt{5}-\sqrt{3})$

ex.2) $\frac{1}{\sqrt{5}+2} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$

ex.3) $\frac{\sqrt{5}-1}{\sqrt{5}+1} - \frac{\sqrt{5}+1}{\sqrt{5}-1} = \frac{(\sqrt{5}-1)^2}{4} - \frac{(\sqrt{5}+1)^2}{4} = -\sqrt{5}$

Irrational Functions

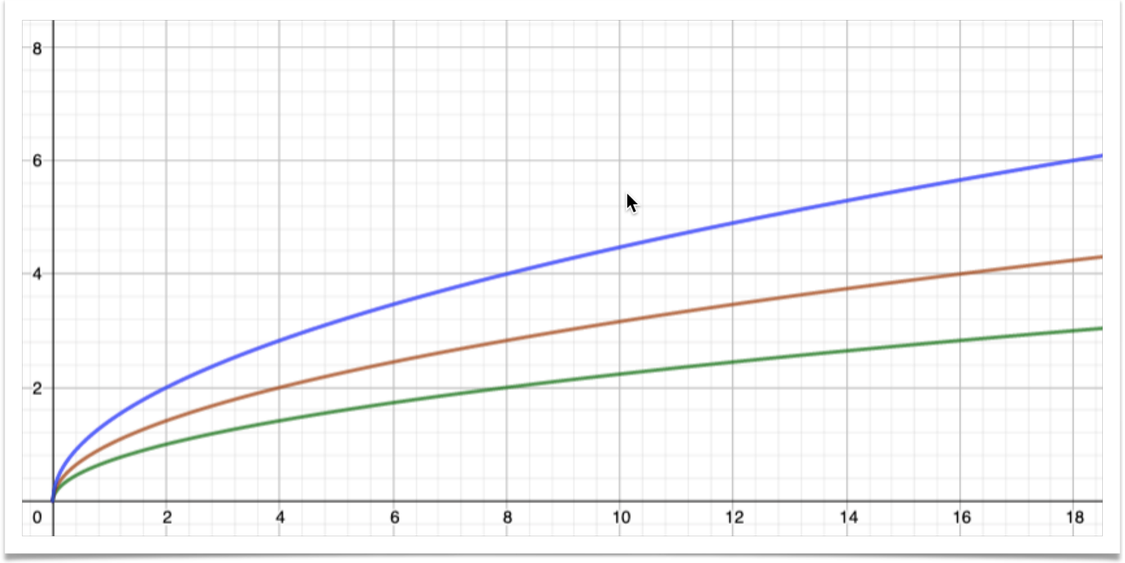
$$f(x) = \sqrt{A(x)}, \;\; A(x) \geq 0$$

$$f(x) = \sqrt[3]{A(x)}$$

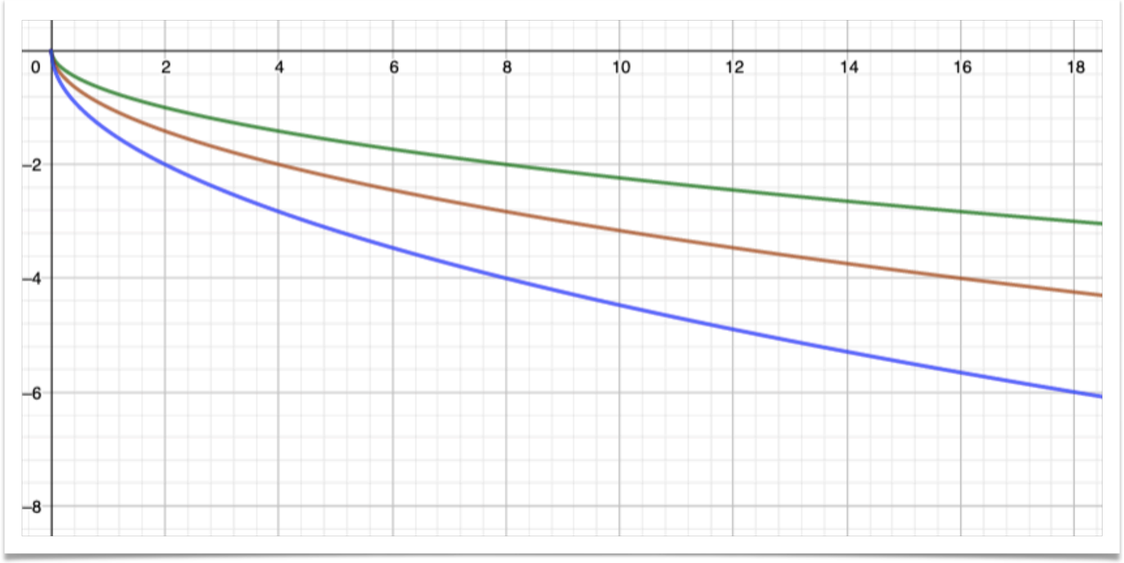
Irrational Functions

Basic Irrational Functions

$$f(x) = \pm\sqrt{ax},\ a > 0,\ x \geq 0$$



$$f(x) = \sqrt{\frac{1}{2}x} \quad g(x) = \sqrt{x} \quad h(x) = \sqrt{2x}$$

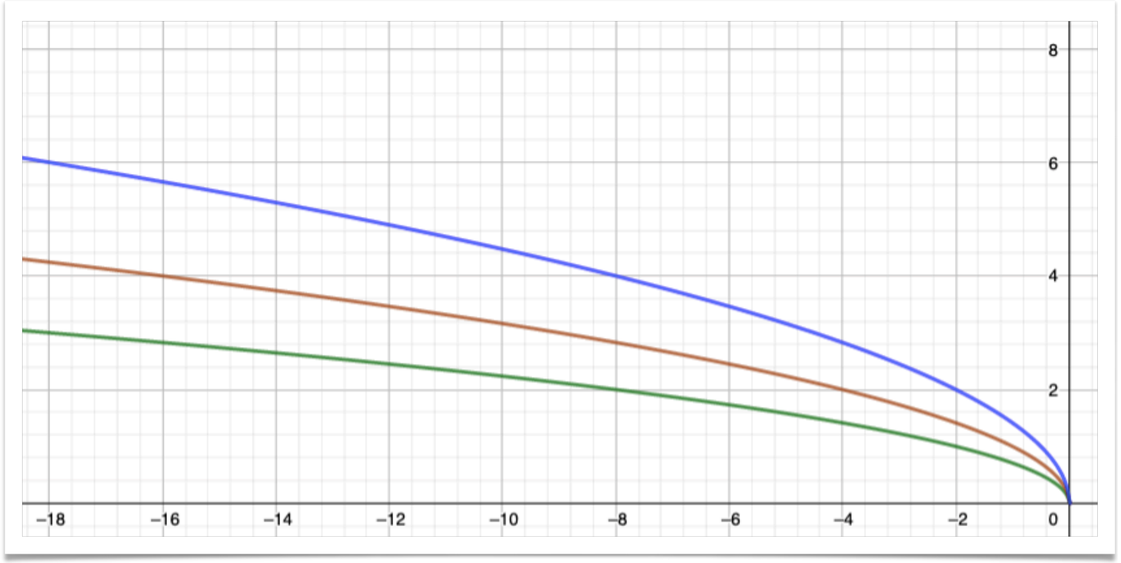


$$f(x) = -\sqrt{\frac{1}{2}x} \quad g(x) = -\sqrt{x} \quad h(x) = -\sqrt{2x}$$

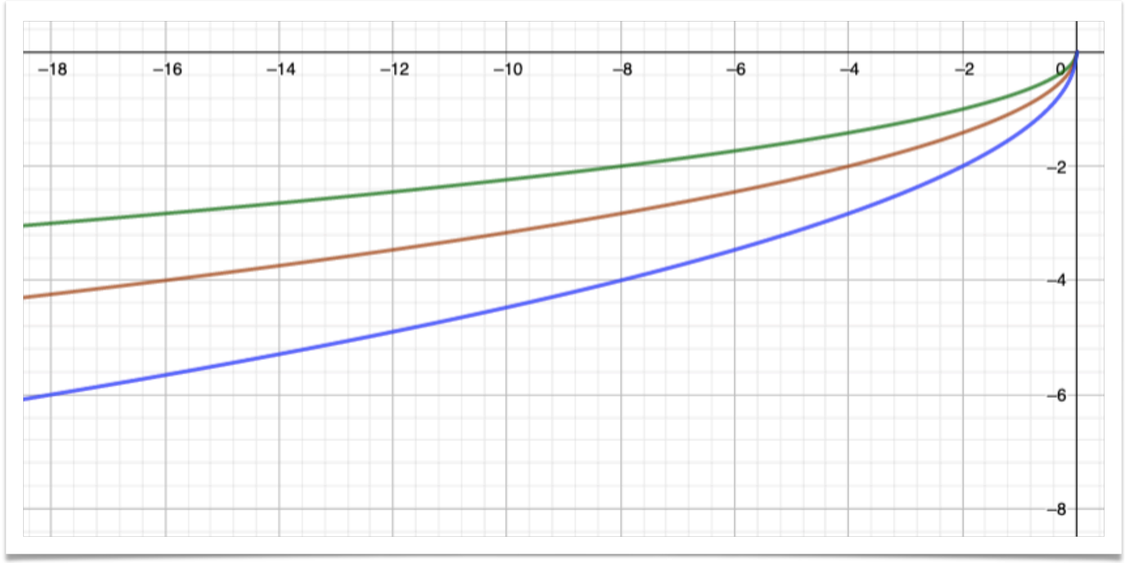
Irrational Functions

Basic Irrational Functions

$$f(x) = \pm\sqrt{ax},\ a < 0,\ x \le 0$$



$$f(x) = \sqrt{-\frac{1}{2}x} \quad g(x) = \sqrt{-x} \quad h(x) = \sqrt{-2x}$$



$$f(x) = -\sqrt{-\frac{1}{2}x} \quad g(x) = -\sqrt{-x} \quad h(x) = -\sqrt{-2x}$$

Irrational Functions

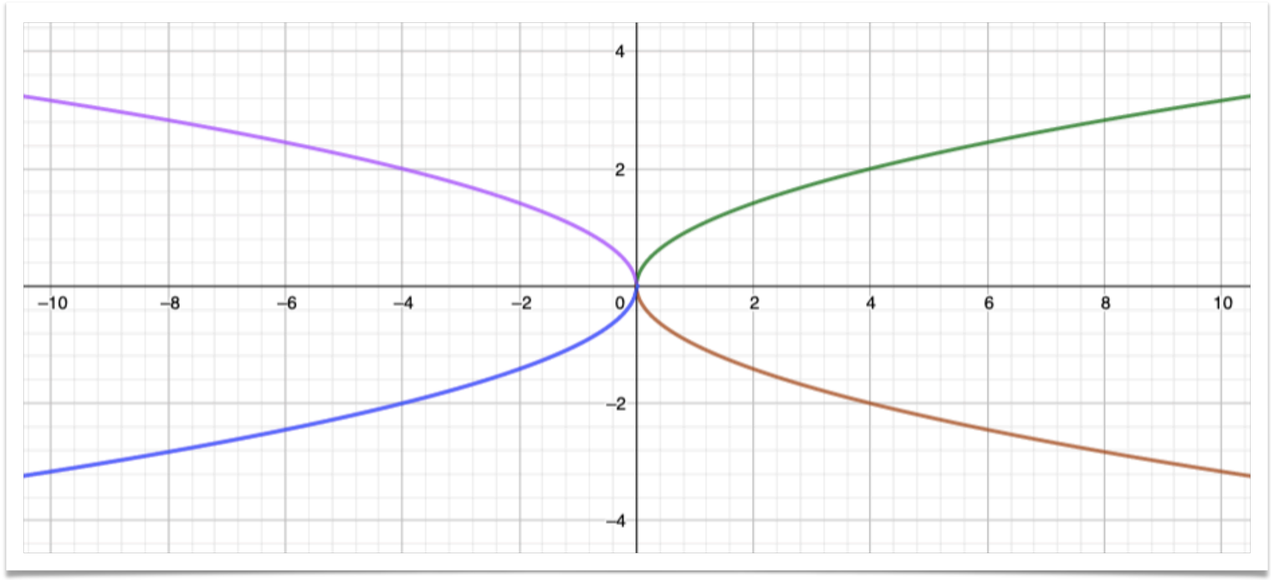
Basic Irrational Functions

$$f(x) = \pm\sqrt{ax},\ x \geq 0$$

$y = \sqrt{ax},\ a < 0$

$D = (-\infty, 0]$

$R = [0, \infty)$



$y = \sqrt{ax},\ a > 0$

$D = [0, \infty)$

$R = [0, \infty)$

$y = -\sqrt{ax},\ a < 0$

$D = (-\infty, 0]$

$R = (-\infty, 0]$

$y = -\sqrt{x},\ a > 0$

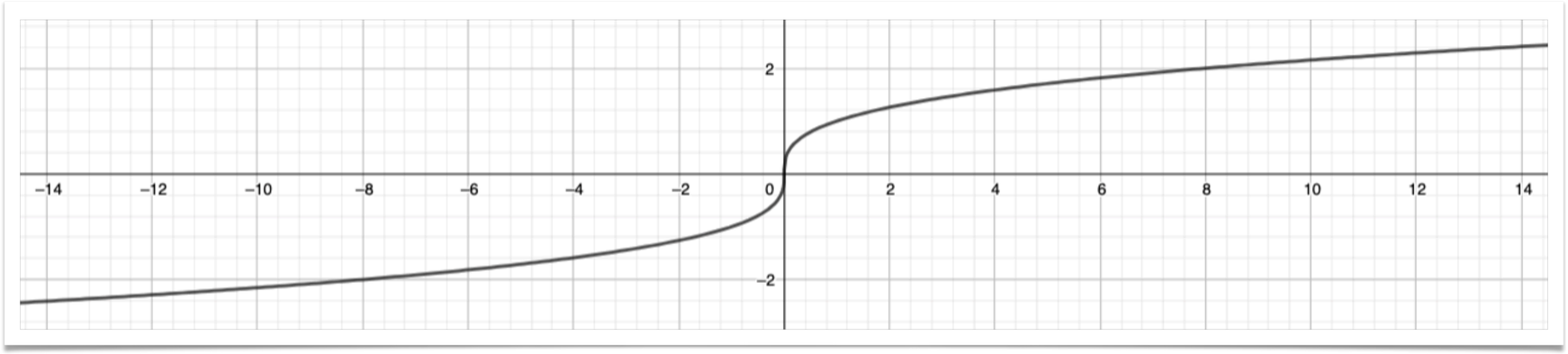
$D = [0, \infty)$

$R = (-\infty, 0]$

Irrational Functions

Cube Roots

$$f(x) = \sqrt[3]{x}$$
$$D = R = \mathbb{R}$$



Translations of Irrational Functions

$$f(x) = \sqrt{a(x-\alpha)} + \beta$$

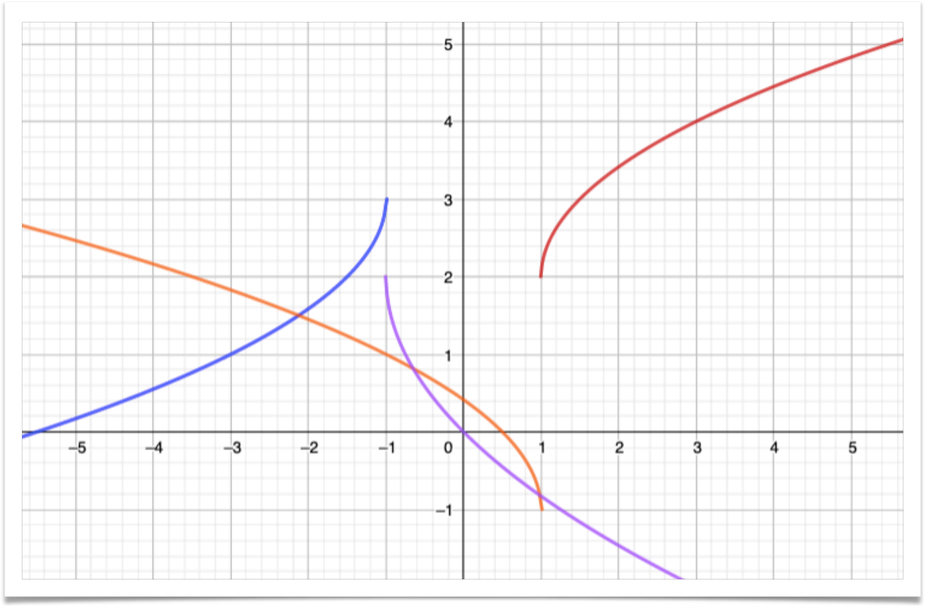
Examples 다음 irrational function들의 그래프를 그리고, domain과 codomain을 구하세요.

ex.1) $f_1(x) = \sqrt{2(x-1)} + 2$

ex.2) $f_2(x) = -\sqrt{-2(x+1)} + 3$

ex.3) $f_3(x) = \sqrt{-2(x-1)} - 1$

ex.4) $f_4(x) = -2\sqrt{x+1} + 2$



CLOSING

Basic Algebra

Chap.6 Rational and Irrational Functions