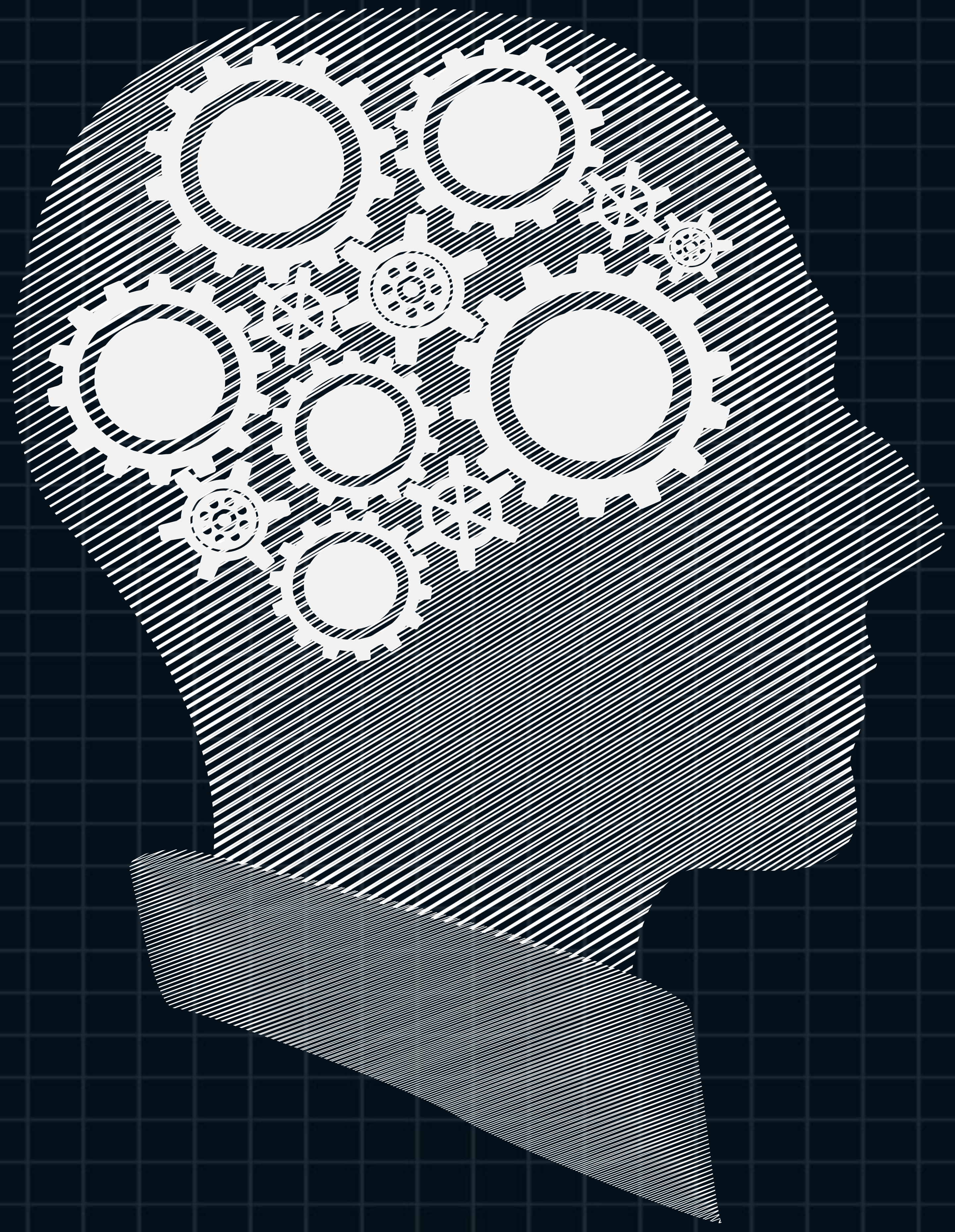
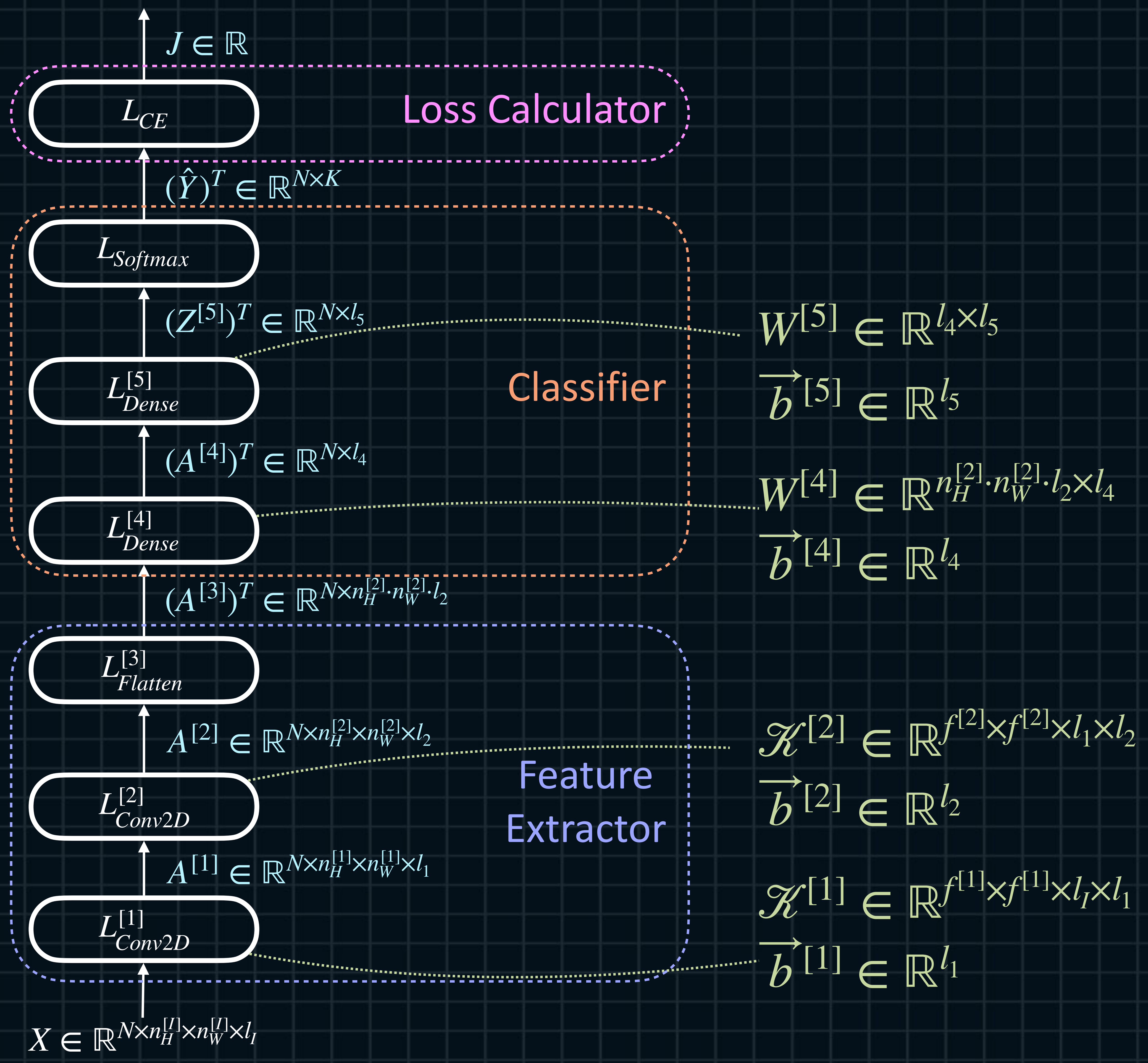


Backpropagation and Jacobian Matrices

Lecture.0 Orientation

Lecture.0 Orientation

- Previous Lecture



Backpropagation

for TRAINING neural networks

- Chain Rule and Backpropagation

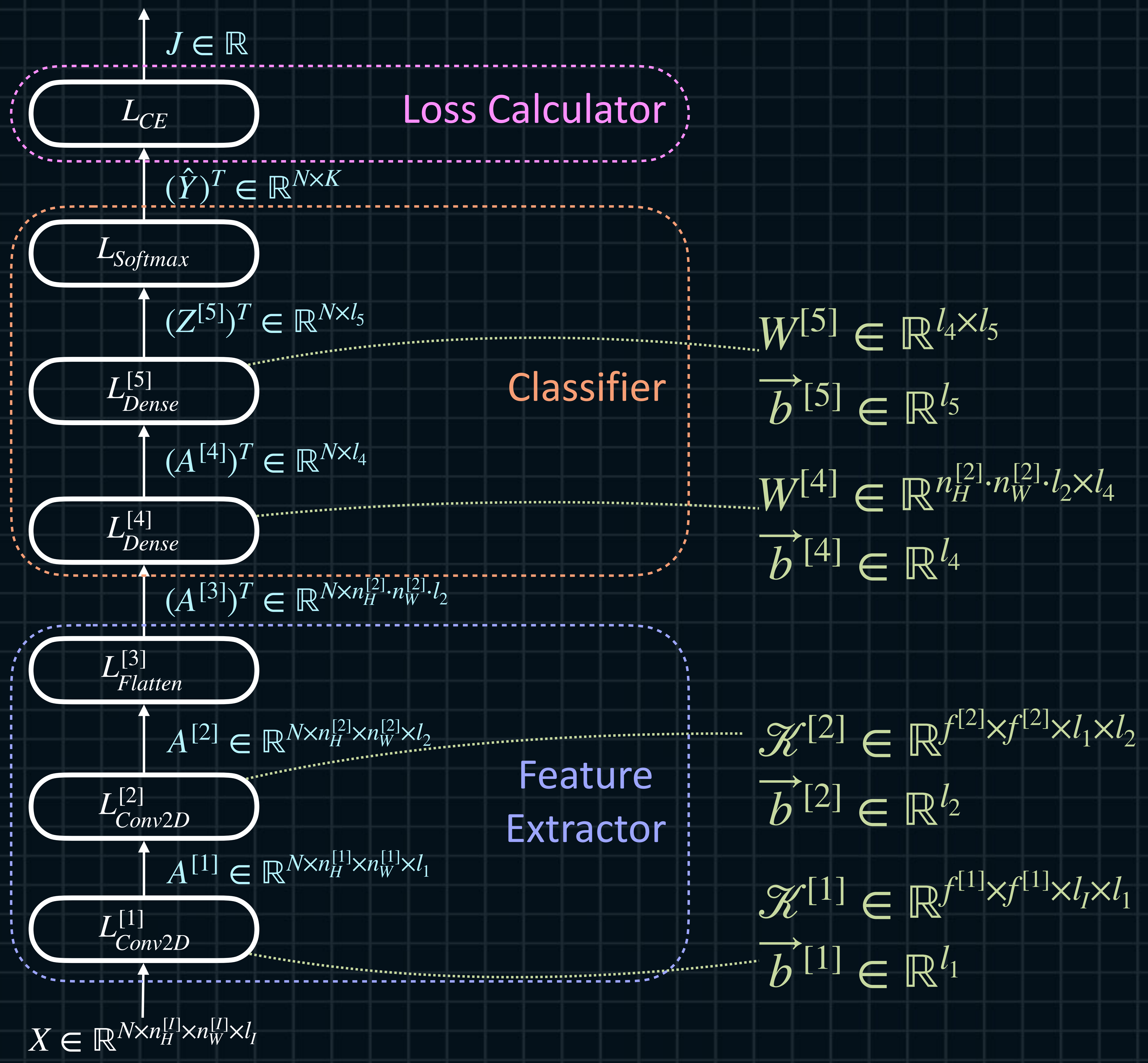
Chain Rule

↓

Backpropagation

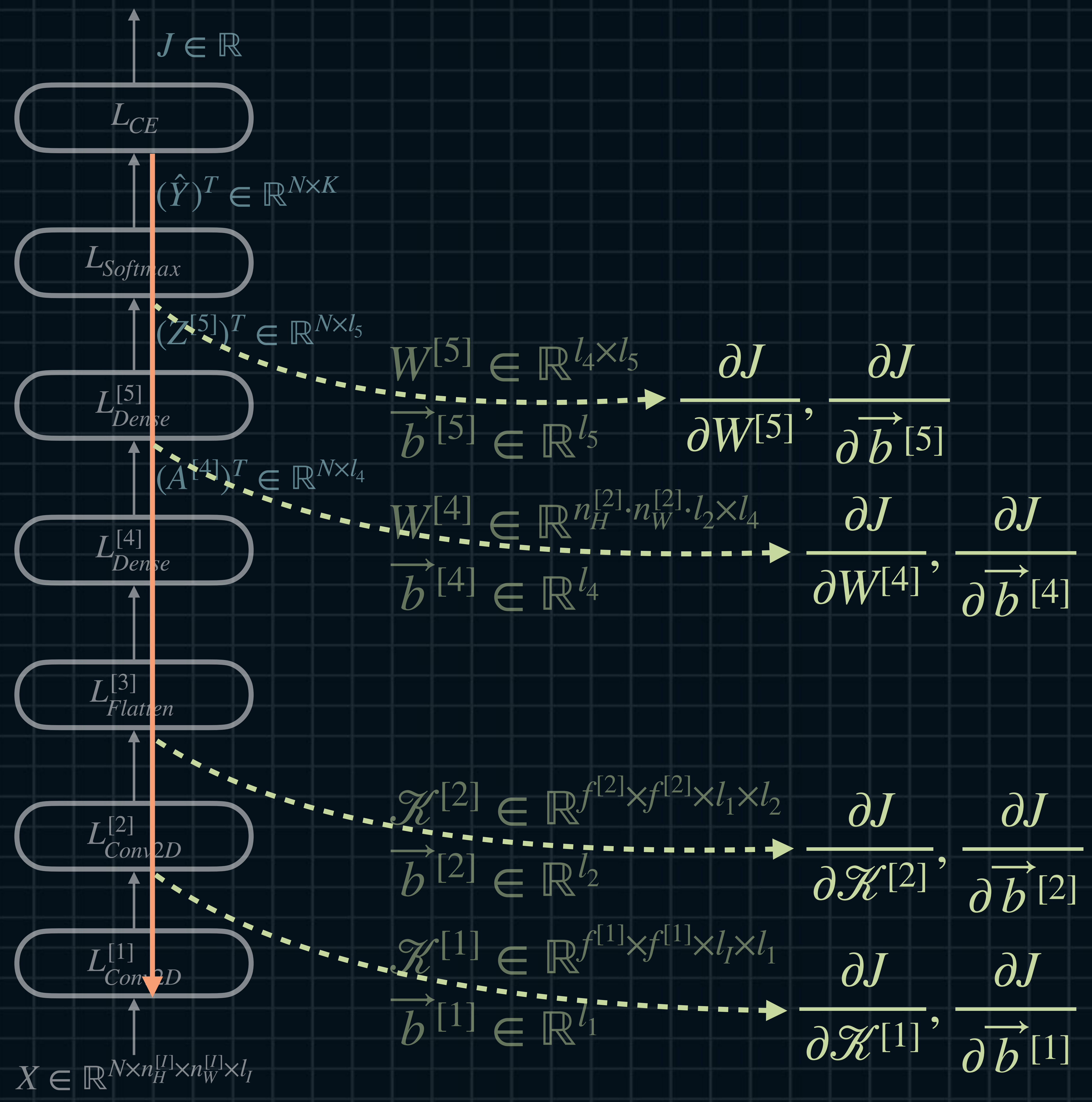
Lecture.0
Orientation

- Chain Rule and Backpropagation



Lecture.0 Orientation

- Chain Rule and Backpropagation



Lecture.0
Orientation

- Tensor Parameters

$$L_{Dense}$$

W

$\overrightarrow{b}$

$$\frac{\partial J}{\partial W}, \frac{\partial J}{\partial \overrightarrow{b}}$$

Lecture.0 Orientation

- Jacobian Matrices

$$\begin{array}{ccc} & x & \vec{x} \\ f & \boxed{\frac{\partial f}{\partial x}} & \boxed{\frac{\partial f}{\partial \vec{x}}} \\ \vec{f} & \boxed{\frac{\partial \vec{f}}{\partial x}} & \boxed{\frac{\partial \vec{f}}{\partial \vec{x}}} \end{array}$$

Lecture.0 Orientation

- Purpose of This Lecture

- Basic principle of learning

- Basics for advanced techniques

- Improving math abilities

Lecture.0 Orientation

- The MOST Important Thing

$$\frac{\partial J}{\partial \vec{J}_0} = \begin{pmatrix} \frac{1}{N} & \frac{1}{N} & \dots & \frac{1}{N} \end{pmatrix}$$

$$\frac{\partial \vec{J}_0}{\partial \vec{\hat{y}}} = \begin{pmatrix} -2\left(y^{(1)} - \hat{y}^{(1)}\right) & 0 & \dots & 0 \\ 0 & -2\left(y^{(2)} - \hat{y}^{(2)}\right) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -2\left(y^{(N)} - \hat{y}^{(N)}\right) \end{pmatrix}$$

$$\frac{\partial J}{\partial \vec{\hat{y}}} = \frac{\partial J}{\partial \vec{J}_0}\frac{\partial \vec{J}_0}{\partial \vec{\hat{y}}} = \begin{pmatrix} \frac{1}{N} & \frac{1}{N} & \dots & \frac{1}{N} \end{pmatrix} \begin{pmatrix} -2\left(y^{(1)} - \hat{y}^{(1)}\right) & 0 & \dots & 0 \\ 0 & -2\left(y^{(2)} - \hat{y}^{(2)}\right) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -2\left(y^{(N)} - \hat{y}^{(N)}\right) \end{pmatrix}$$

$$= -\frac{2}{N}\begin{pmatrix} \left(y^{(1)} - \hat{y}^{(1)}\right) & \left(y^{(2)} - \hat{y}^{(2)}\right) & \dots & \left(y^{(N)} - \hat{y}^{(N)}\right) \end{pmatrix}$$

$$\frac{\partial J}{\partial w} = \frac{\partial J}{\partial \vec{\hat{y}}}\frac{\partial \vec{\hat{y}}}{\partial w} = -\frac{2}{N}\begin{pmatrix} \left(y^{(1)} - \hat{y}^{(1)}\right) & \left(y^{(2)} - \hat{y}^{(2)}\right) & \dots & \left(y^{(N)} - \hat{y}^{(N)}\right) \end{pmatrix}\begin{pmatrix} x^{(1)} \\ x^{(2)} \\ \vdots \\ x^{(N)} \end{pmatrix}$$

$$= -\frac{2}{N}\sum_{i=1}^{N} x^{(i)}\left(y^{(i)} - \hat{y}^{(i)}\right)$$

$$\frac{\partial J}{\partial b} = \frac{\partial J}{\partial \vec{\hat{y}}}\frac{\partial \vec{\hat{y}}}{\partial b} = -\frac{2}{N}\begin{pmatrix} \left(y^{(1)} - \hat{y}^{(1)}\right) & \left(y^{(2)} - \hat{y}^{(2)}\right) & \dots & \left(y^{(N)} - \hat{y}^{(N)}\right) \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

$$= -\frac{2}{N}\sum_{i=1}^{N}\left(y^{(i)} - \hat{y}^{(i)}\right)$$

$$\frac{\partial J}{\partial l_j^{(i)}} = \frac{\partial J}{\partial \hat{y}_1^{(i)}}\frac{\partial \hat{y}_1^{(i)}}{\partial l_j^{(i)}} + \frac{\partial J}{\partial \hat{y}_2^{(i)}}\frac{\partial \hat{y}_2^{(i)}}{\partial l_j^{(i)}} + \dots + \frac{\partial J}{\partial \hat{y}_j^{(i)}}\frac{\partial \hat{y}_j^{(i)}}{\partial l_j^{(i)}} + \dots + \frac{\partial J}{\partial \hat{y}_K^{(i)}}\frac{\partial \hat{y}_K^{(i)}}{\partial l_j^{(i)}}$$

$$= d\hat{y}_1^{(i)}\frac{\partial \hat{y}_1^{(i)}}{\partial l_j^{(i)}} + d\hat{y}_2^{(i)}\frac{\partial \hat{y}_2^{(i)}}{\partial l_j^{(i)}} + \dots + d\hat{y}_j^{(i)}\frac{\partial \hat{y}_j^{(i)}}{\partial l_j^{(i)}} + \dots + d\hat{y}_K^{(i)}\frac{\partial \hat{y}_K^{(i)}}{\partial l_j^{(i)}}$$

$$= d\hat{y}_1^{(i)}\cdot\left(-\hat{y}_1^{(i)}\hat{y}_j^{(i)}\right) + d\hat{y}_2^{(i)}\cdot\left(-\hat{y}_2^{(i)}\hat{y}_j^{(i)}\right) + \dots + d\hat{y}_j^{(i)}\cdot\left(\hat{y}_j^{(i)}\left(1 - \hat{y}_j^{(i)}\right)\right) + \dots + d\hat{y}_K^{(i)}\cdot\left(-\hat{y}_K^{(i)}\hat{y}_j^{(i)}\right)$$

$$= d\hat{y}_j^{(i)}\hat{y}_j^{(i)} + d\hat{y}_1^{(i)}\cdot\left(-\hat{y}_1^{(i)}\hat{y}_j^{(i)}\right) + d\hat{y}_2^{(i)}\cdot\left(-\hat{y}_2^{(i)}\hat{y}_j^{(i)}\right) + \dots + d\hat{y}_j^{(i)}\cdot\left(-\hat{y}_j^{(i)}\hat{y}_j^{(i)}\right) + \dots + d\hat{y}_K^{(i)}\cdot\left(-\hat{y}_K^{(i)}\hat{y}_j^{(i)}\right)$$

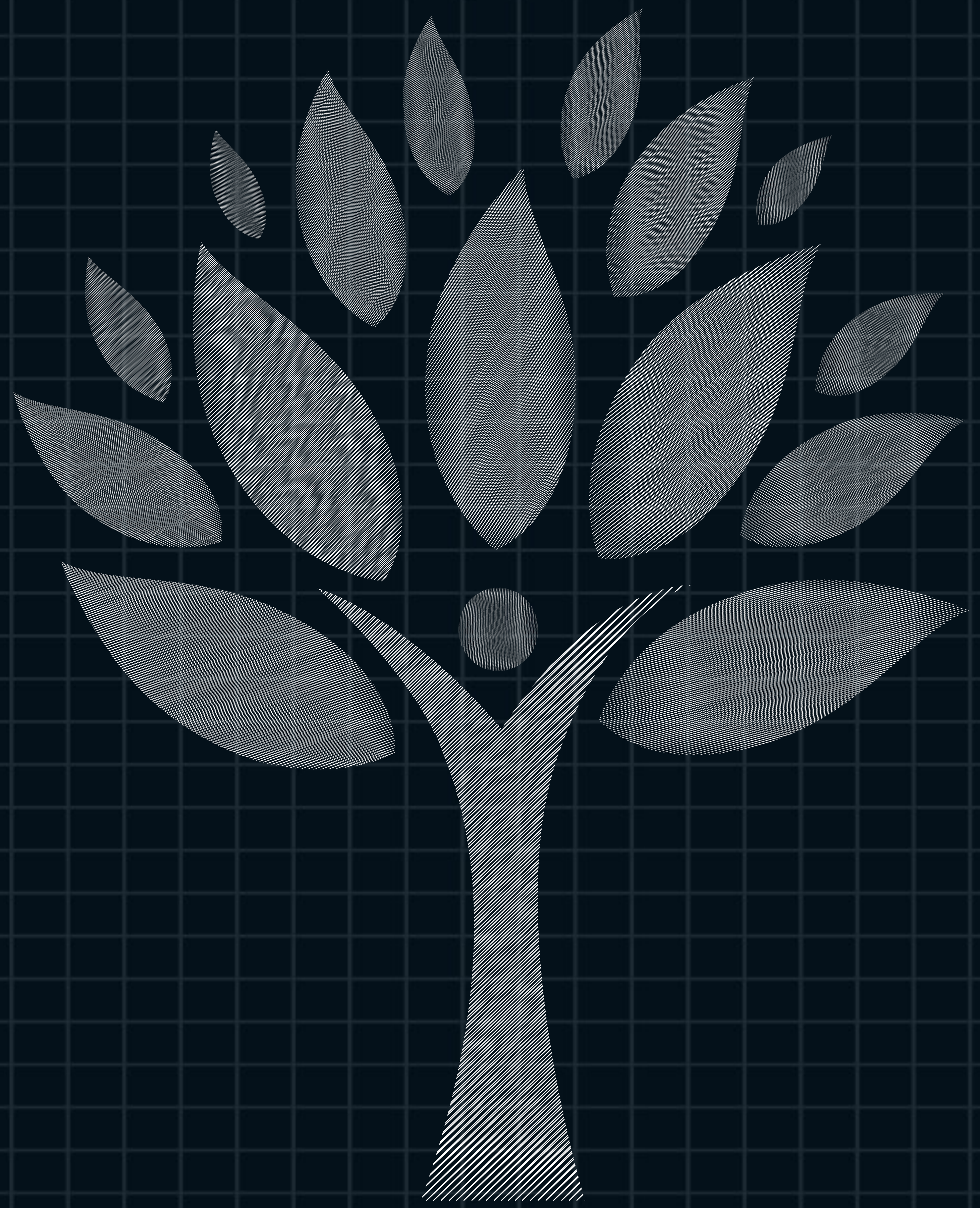
$$= d\hat{y}_j^{(i)}\hat{y}_j^{(i)} - \hat{y}_j^{(i)}\sum_{k=1}^{K} d\hat{y}_k^{(i)}\cdot\hat{y}_k^{(i)}$$

$$= -\frac{1}{N}\left[\frac{y_j^{(i)}}{\hat{y}_j^{(i)}}\cdot\hat{y}_j^{(i)} - \hat{y}_j^{(i)}\sum_{k=1}^{K}\frac{y_k^{(i)}}{\hat{y}_k^{(i)}}\cdot\hat{y}_k^{(i)}\right]$$

$$= -\frac{1}{N}\left[y_j^{(i)} - \hat{y}_j^{(i)}\sum_{k=1}^{K} y_k^{(i)}\right]$$

$$= -\frac{1}{N}\left[y_j^{(i)} - \hat{y}_j^{(i)}\right]$$

$$\frac{\partial J}{\partial L} = -\frac{1}{N}(Y - \hat{Y})$$



Backpropagation and Jacobian Matrices

Lecture.0 Orientation