

Forward Propagation of Neural Networks

Lecture.5
Conv Layers

Lecture.5 Conv Layers

- Image Tensors

$$X \in \mathbb{R}^{n_H \times n_W}$$

$$X = \begin{pmatrix} X[0, 0] & X[0, 1] & \dots & X[0, n_W - 1] \\ X[1, 0] & X[1, 1] & \dots & X[1, n_W - 1] \\ \vdots & \vdots & \ddots & \vdots \\ X[n_H - 1, 0] & X[n_H - 1, 1] & \dots & X[n_H - 1, n_W - 1] \end{pmatrix}$$

Lecture.5 Conv Layers

- Image Tensors

$$X \in \mathbb{R}^{n_H \times n_W}$$

$$X \in \mathbb{R}^{n_H \times n_W \times n_C}$$

$$X \in \mathbb{R}^{N \times n_H \times n_W \times n_C}$$

Lecture.5 Conv Layers

- Correlation

Classical Correlation

$$X \in \mathbb{R}^{n_H \times n_W}, \mathcal{F} \in \mathbb{R}^{n_H \times n_W}, z \in \mathbb{R}$$

$$z = X \otimes \mathcal{F} = \sum_{i=0}^{n_H-1} \sum_{j=0}^{n_W-1} X[i, j] \mathcal{F}[i, j]$$

$$X \otimes \mathcal{F} : \mathbb{R}^{n_H \times n_W} \times \mathbb{R}^{n_H \times n_W} \rightarrow \mathbb{R}$$

$$n_H = 3$$
$$n_W = 3$$

$X[0,0]$	$X[0,1]$	$X[0,2]$
$X[1,0]$	$X[1,1]$	$X[1,2]$
$X[2,0]$	$X[2,1]$	$X[2,2]$

$$X \in \mathbb{R}^{3 \times 3}$$

$\mathcal{F}[0,0]$	$\mathcal{F}[0,1]$	$\mathcal{F}[0,2]$
$\mathcal{F}[1,0]$	$\mathcal{F}[1,1]$	$\mathcal{F}[1,2]$
$\mathcal{F}[2,0]$	$\mathcal{F}[2,1]$	$\mathcal{F}[2,2]$

$$\mathcal{F} \in \mathbb{R}^{3 \times 3}$$

$$\begin{aligned} z &= X \otimes \mathcal{F} \\ &= \sum_{i=0}^2 \sum_{j=0}^2 X[i, j] \mathcal{F}[i, j] \\ &= X[0,0] \mathcal{F}[0,0] + X[0,1] \mathcal{F}[0,1] + X[0,2] \mathcal{F}[0,2] + \\ &\quad X[1,0] \mathcal{F}[1,0] + X[1,1] \mathcal{F}[1,1] + X[1,2] \mathcal{F}[1,2] + \\ &\quad X[2,0] \mathcal{F}[2,0] + X[2,1] \mathcal{F}[2,1] + X[2,2] \mathcal{F}[2,2] \end{aligned}$$

Lecture.5 Conv Layers

- Correlation

Correlation with Bias

$$X \in \mathbb{R}^{n_H \times n_W}, \mathcal{F} \in \mathbb{R}^{n_H \times n_W}, b \in \mathbb{R}, z \in \mathbb{R}$$

$$z = X \otimes \mathcal{F} + b = \sum_{i=0}^{n_H-1} \sum_{j=0}^{n_W-1} X[i, j] \mathcal{F}[i, j] + b$$

$$X \otimes \mathcal{F} + b : \mathbb{R}^{n_H \times n_W} \times \mathbb{R}^{n_H \times n_W} \times \mathbb{R} \rightarrow \mathbb{R}$$

$X[0,0]$	$X[0,1]$	$X[0,2]$
$X[1,0]$	$X[1,1]$	$X[1,2]$
$X[2,0]$	$X[2,1]$	$X[2,2]$

$$X \in \mathbb{R}^{3 \times 3}$$

$\mathcal{F}[0,0]$	$\mathcal{F}[0,1]$	$\mathcal{F}[0,2]$
$\mathcal{F}[1,0]$	$\mathcal{F}[1,1]$	$\mathcal{F}[1,2]$
$\mathcal{F}[2,0]$	$\mathcal{F}[2,1]$	$\mathcal{F}[2,2]$

$$\mathcal{F} \in \mathbb{R}^{3 \times 3}$$

$$b \in \mathbb{R}$$

$$z = X \otimes \mathcal{F} + b$$

$$= \sum_{i=0}^2 \sum_{j=0}^2 X[i, j] \mathcal{F}[i, j] + b$$

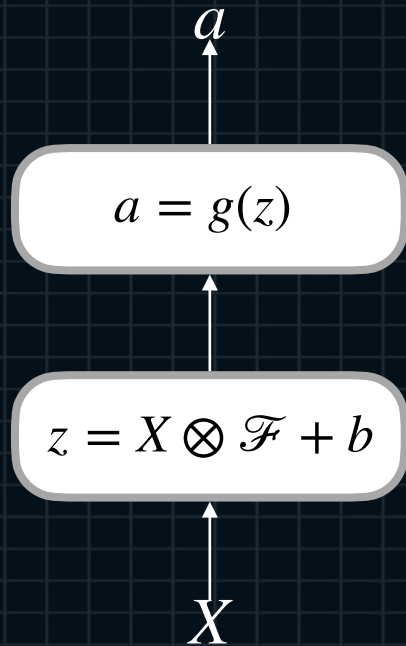
$$= X[0,0]\mathcal{F}[0,0] + X[0,1]\mathcal{F}[0,1] + X[0,2]\mathcal{F}[0,2] + \\ X[1,0]\mathcal{F}[1,0] + X[1,1]\mathcal{F}[1,1] + X[1,2]\mathcal{F}[1,2] + \\ X[2,0]\mathcal{F}[2,0] + X[2,1]\mathcal{F}[2,1] + X[2,2]\mathcal{F}[2,2] + b$$

Lecture.5 Conv Layers

- Correlation

Correlation with Activation

$$X \in \mathbb{R}^{n_H \times n_W}, \mathcal{F} \in \mathbb{R}^{n_H \times n_W}, b \in \mathbb{R}, z \in \mathbb{R}, a \in \mathbb{R}$$



$$z = X \otimes \mathcal{F} + b$$

$$= \sum_{i=0}^2 \sum_{j=0}^2 X[i, j] \mathcal{F}[i, j] + b$$

$$= X[0,0]\mathcal{F}[0,0] + X[0,1]\mathcal{F}[0,1] + X[0,2]\mathcal{F}[0,2] + \\ X[1,0]\mathcal{F}[1,0] + X[1,1]\mathcal{F}[1,1] + X[1,2]\mathcal{F}[1,2] + \\ X[2,0]\mathcal{F}[2,0] + X[2,1]\mathcal{F}[2,1] + X[2,2]\mathcal{F}[2,2] + b$$

$$a = g(z)$$

Lecture.5 Conv Layers

- Correlation

Correlation and Dot Product

$$\sum_{i=0}^{n_H-1} \sum_{j=0}^{n_W-1} X[i,j] \mathcal{F}[i,j] + b = (\text{flattn}(x))^T \text{flatten}(\mathcal{F}) + b$$

$A[0,0]$	$A[0,1]$	$A[0,2]$
$A[1,0]$	$A[1,1]$	$A[1,2]$
$A[2,0]$	$A[2,1]$	$A[2,2]$

$$X \in \mathbb{R}^{3 \times 3}$$

Flatten

$A[0,0]$	$A[0,1]$	$A[0,2]$	$A[1,0]$	$A[1,1]$	$A[1,2]$	$A[2,0]$	$A[2,1]$	$A[2,2]$
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$$\text{flatten}(X) \in \mathbb{R}^9$$

$\mathcal{F}[0,0]$	$\mathcal{F}[0,1]$	$\mathcal{F}[0,2]$
$\mathcal{F}[1,0]$	$\mathcal{F}[1,1]$	$\mathcal{F}[1,2]$
$\mathcal{F}[2,0]$	$\mathcal{F}[2,1]$	$\mathcal{F}[2,2]$

$$\mathcal{F} \in \mathbb{R}^{3 \times 3}$$

Flatten

$\mathcal{F}[0,0]$	$\mathcal{F}[0,1]$	$\mathcal{F}[0,2]$	$\mathcal{F}[1,0]$	$\mathcal{F}[1,1]$	$\mathcal{F}[1,2]$	$\mathcal{F}[2,0]$	$\mathcal{F}[2,1]$	$\mathcal{F}[2,2]$
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$$\text{flatten}(\mathcal{F}) \in \mathbb{R}^9$$

Lecture.5 Conv Layers

- Window Extraction

Windows(1D)

$$\vec{x} = (x_0 \ x_1 \ x_2 \ x_3 \ x_4 \ x_5)$$

$$\mathcal{W}_0 = X[0 : 3] = (x_0 \ x_1 \ x_2)$$

$$\mathcal{W}_1 = X[1 : 4] = (x_1 \ x_2 \ x_3)$$

$$\mathcal{W}_2 = X[2 : 5] = (x_2 \ x_3 \ x_4)$$

$$\mathcal{W}_3 = X[3 : 6] = (x_3 \ x_4 \ x_5)$$

$$\mathcal{W}_i = (\vec{x}_i \ \vec{x}_{i+1} \ \vec{x}_{i+2})$$

Lecture.5 Conv Layers

- Window Extraction

Windows(2D)

$$X = \begin{pmatrix} X[0, 0] & X[0, 1] & X[0, 2] & X[0, 3] \\ X[1, 0] & X[1, 1] & X[1, 2] & X[1, 3] \\ X[2, 0] & X[2, 1] & X[2, 2] & X[2, 3] \\ X[3, 0] & X[3, 1] & X[3, 2] & X[3, 3] \end{pmatrix}$$

$$\mathcal{W}_{i,j} = \begin{pmatrix} X[i, j] & X[i, j+1] & X[i, j+2] \\ X[i+1, j] & X[i+1, j+1] & X[i+1, j+2] \\ X[i+2, j] & X[i+2, j+1] & X[i+2, j+2] \end{pmatrix}$$

$$\mathcal{W}_{0,0} = X[0:3, 0:3] = \begin{pmatrix} X[0, 0] & X[0, 1] & X[0, 2] \\ X[1, 0] & X[1, 1] & X[1, 2] \\ X[2, 0] & X[2, 1] & X[2, 2] \end{pmatrix}$$

$$\mathcal{W}_{0,1} = X[0:3, 1:4] = \begin{pmatrix} X[0, 1] & X[0, 2] & X[0, 3] \\ X[1, 1] & X[1, 2] & X[1, 3] \\ X[2, 1] & X[2, 2] & X[2, 3] \end{pmatrix}$$

$$\mathcal{W}_{1,0} = X[1:4, 0:3] = \begin{pmatrix} X[1, 0] & X[1, 1] & X[1, 2] \\ X[2, 0] & X[2, 1] & X[2, 2] \\ X[3, 0] & X[3, 1] & X[3, 2] \end{pmatrix}$$

$$\mathcal{W}_{1,1} = X[1:4, 1:4] = \begin{pmatrix} X[1, 1] & X[1, 2] & X[1, 3] \\ X[2, 1] & X[2, 2] & X[2, 3] \\ X[3, 1] & X[3, 2] & X[3, 3] \end{pmatrix}$$

Lecture.5 Conv Layers

- Window Extraction

Window Formularization

$$X \in \mathbb{R}^{n_H \times n_W}, \mathcal{W}_{i,j} \in \mathbb{R}^{f \times f}$$

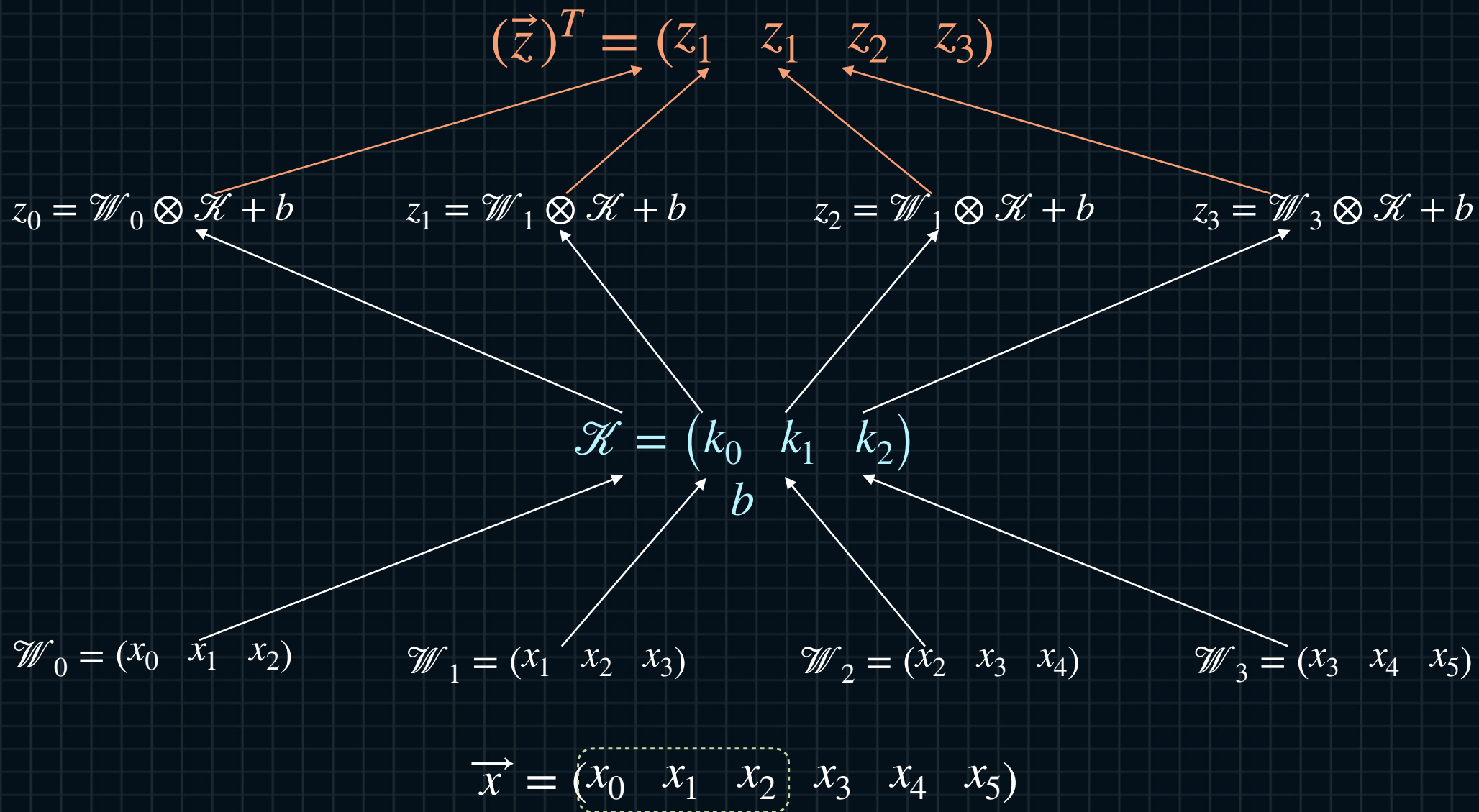
$$X = \begin{pmatrix} X[0, 0] & X[0, 1] & \dots & X[0, n_W - f] & \dots & X[0, n_W - 1] \\ X[1, 0] & X[1, 1] & \dots & X[1, n_W - f] & \dots & X[1, n_W - 1] \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ X[n_H - f, 0] & X[n_H - f, 1] & \dots & X[n_H - f, n_W - f] & \dots & X[n_H - f, n_W - 1] \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ X[n_H - 1, 0] & X[n_H - 1, 1] & \dots & X[n_H - 1, n_W - f] & \dots & X[n_H - 1, n_W - 1] \end{pmatrix}$$

$$\mathcal{W}_{i,j} = X[i : i + f, j : j + f] = \begin{pmatrix} X[i, j] & X[i, j + 1] & \dots & X[i, j + (f - 1)] \\ X[i + 1, j] & X[i + 1, j + 1] & \dots & X[i + 1, j + (f - 1)] \\ \vdots & \vdots & \ddots & \vdots \\ X[i + (f - 1), j] & X[i + (f - 1), j + 1] & \dots & X[i + (f - 1), j + (f - 1)] \end{pmatrix}$$

$$0 \leq i \leq n_H - f, \quad 0 \leq j \leq n_W - f$$

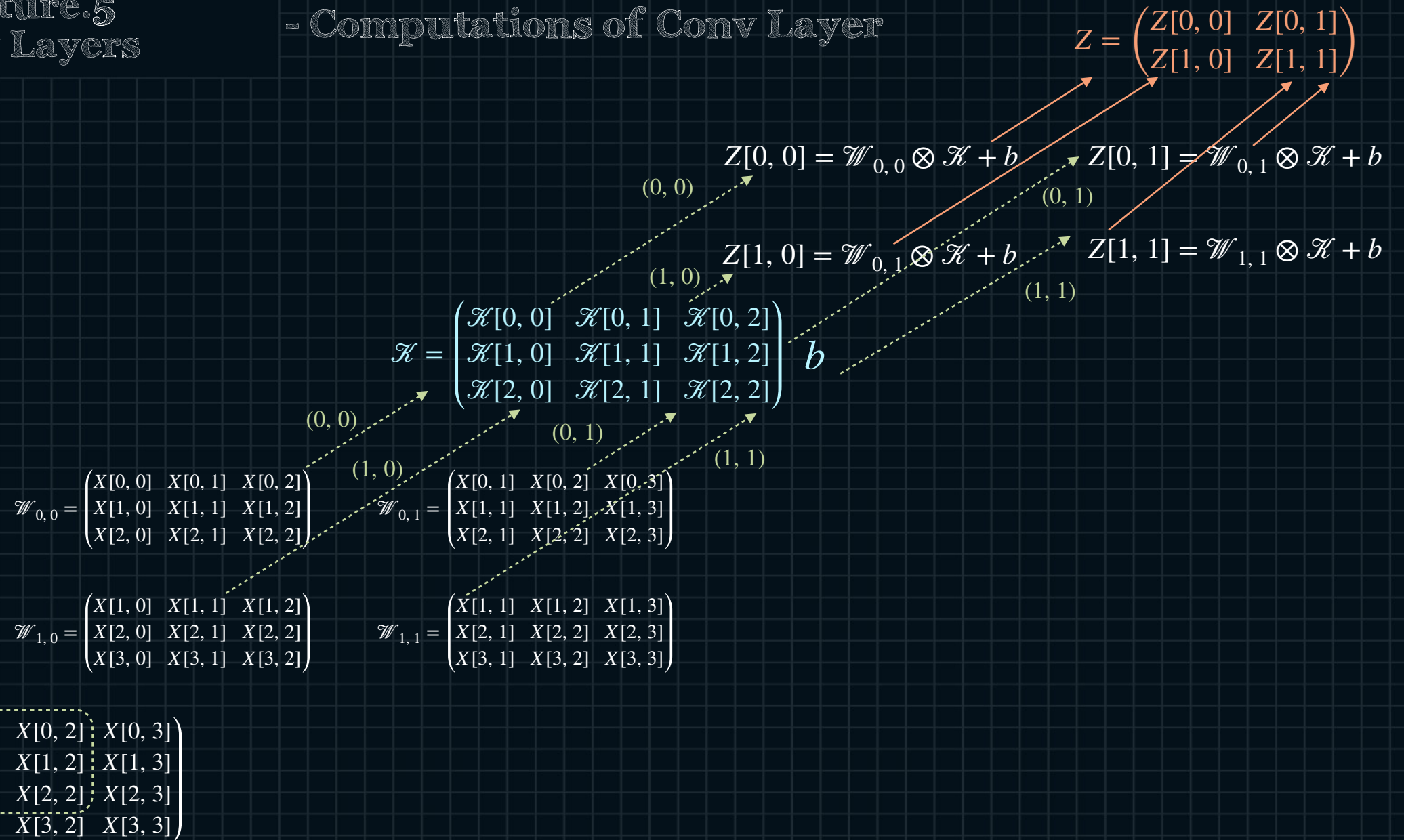
Lecture.5 Conv Layers

- Computations of Conv Layer



Lecture.5 Conv Layers

- Computations of Conv Layer



Lecture.5 Conv Layers

- Computations of Conv Layer

$$\begin{aligned} X &\in \mathbb{R}^{n_H \times n_W} \\ \mathcal{W}_{i,j} &\in \mathbb{R}^{f \times f} \quad \mathcal{K} \in \mathbb{R}^{f \times f}, b \in \mathbb{R} \end{aligned}$$

$$Z[i, j] = \mathcal{W}_{i,j} \otimes \mathcal{K} + b$$

$$Z = \text{Conv2D}(X; \mathcal{K}, b) = \begin{pmatrix} \mathcal{W}_{0,0} \otimes \mathcal{K} + b & \mathcal{W}_{0,1} \otimes \mathcal{K} + b & \dots & \mathcal{W}_{0,n_W-f} \otimes \mathcal{K} + b \\ \mathcal{W}_{1,0} \otimes \mathcal{K} + b & \mathcal{W}_{1,1} \otimes \mathcal{K} + b & \dots & \mathcal{W}_{1,n_W-f} \otimes \mathcal{K} + b \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{W}_{n_H-f,0} \otimes \mathcal{K} + b & \mathcal{W}_{n_H-f,1} \otimes \mathcal{K} + b & \dots & \mathcal{W}_{n_H-f,n_W-f} \otimes \mathcal{K} + b \end{pmatrix}$$

$$n'_H = n_H - f + 1$$

$$n'_W = n_W - f + 1$$

Lecture.5 Conv Layers

- Computations of Conv Layer

$$Z[i, j] = \mathcal{W}_{i, j} \otimes \mathcal{K} + b$$

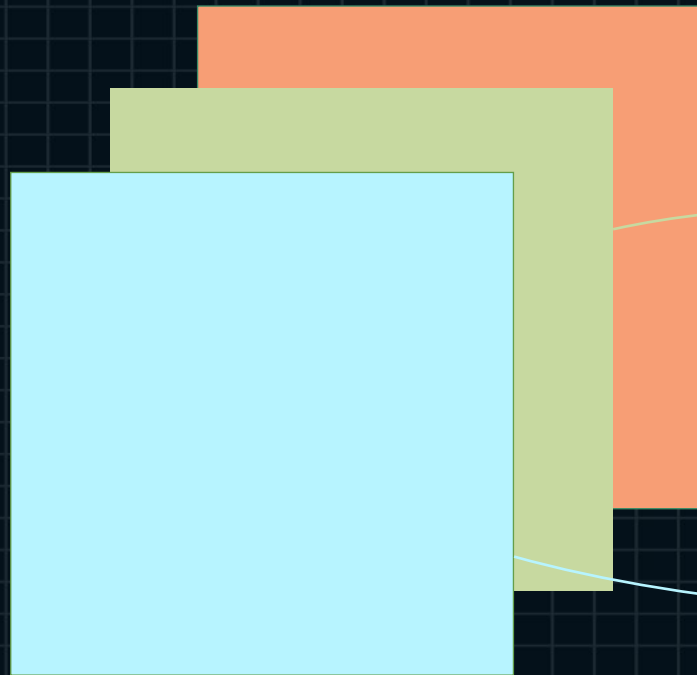
$$A[i, j] = \sigma(Z[i, j])$$

$$A = \text{Conv2D}(X; \mathcal{K}, b; \sigma) = \begin{pmatrix} \sigma(\mathcal{W}_{0,0} \otimes \mathcal{K} + b) & \sigma(\mathcal{W}_{0,1} \otimes \mathcal{K} + b) & \dots & \sigma(\mathcal{W}_{0,n_W-f} \otimes \mathcal{K} + b) \\ \sigma(\mathcal{W}_{1,0} \otimes \mathcal{K} + b) & \sigma(\mathcal{W}_{1,1} \otimes \mathcal{K} + b) & \dots & \sigma(\mathcal{W}_{1,n_W-f} \otimes \mathcal{K} + b) \\ \vdots & \vdots & \ddots & \vdots \\ \sigma(\mathcal{W}_{n_H-f,0} \otimes \mathcal{K} + b) & \sigma(\mathcal{W}_{n_H-f,1} \otimes \mathcal{K} + b) & \dots & \sigma(\mathcal{W}_{n_H-f,n_W-f} \otimes \mathcal{K} + b) \end{pmatrix}$$

Lecture.5 Conv Layers

- n-Channel Input

$$X \in \mathbb{R}^{n_H \times n_W \times n_C}$$



$$X[:, :, 0] = \begin{pmatrix} X[0, 0, 0] & X[0, 1, 0] & \dots & X[0, n_W - 1, 0] \\ X[1, 0, 0] & X[1, 1, 0] & \dots & X[1, n_W - 1, 0] \\ \vdots & \vdots & \ddots & \vdots \\ X[n_H - 1, 0, 0] & X[n_H - 1, 1, 0] & \dots & X[n_H - 1, n_W - 1, 0] \end{pmatrix}$$

$$X[:, :, 1] = \begin{pmatrix} X[0, 0, 1] & X[0, 1, 1] & \dots & X[0, n_W - 1, 1] \\ X[1, 0, 1] & X[1, 1, 1] & \dots & X[1, n_W - 1, 1] \\ \vdots & \vdots & \ddots & \vdots \\ X[n_H - 1, 0, 1] & X[n_H - 1, 1, 1] & \dots & X[n_H - 1, n_W - 1, 1] \end{pmatrix}$$

$$X[:, :, 2] = \begin{pmatrix} X[0, 0, 2] & X[0, 1, 2] & \dots & X[0, n_W - 1, 2] \\ X[1, 0, 2] & X[1, 1, 2] & \dots & X[1, n_W - 1, 2] \\ \vdots & \vdots & \ddots & \vdots \\ X[n_H - 1, 0, 2] & X[n_H - 1, 1, 2] & \dots & X[n_H - 1, n_W - 1, 2] \end{pmatrix}$$

Lecture.5 Conv Layers

- n-Channel Input

$$f = 3, n_C = 3$$

$$\mathcal{W}_{i,j} \in \mathbb{R}^{3 \times 3 \times 3}$$

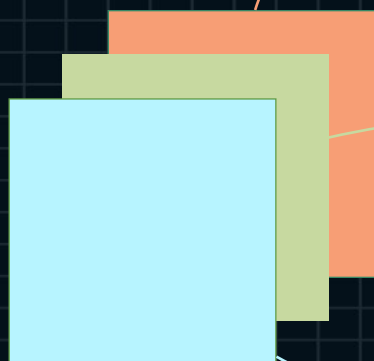


Diagram illustrating the input volumes for a 3D convolution operation with kernel size $f=3$ and $n_C=3$ channels. The input volumes are represented by three overlapping 3x3x3 blocks (cyan, green, and orange).

For the orange volume (top channel), the input is defined as:

$$\mathcal{W}_{i,j} = X[i : i + f, j : j + f, 0]$$
$$= \begin{pmatrix} X[i, j, 0] & X[i, j + 1, 0] & X[i, j + 2, 0] \\ X[i + 1, j, 0] & X[i + 1, j + 1, 0] & X[i + 1, j + 2, 0] \\ X[i + 2, j, 0] & X[i + 2, j + 1, 0] & X[i + 2, j + 2, 0] \end{pmatrix}$$

For the green volume (middle channel), the input is defined as:

$$\mathcal{W}_{i,j} = X[i : i + f, j : j + f, 1]$$
$$= \begin{pmatrix} X[i, j, 1] & X[i, j + 1, 1] & X[i, j + 2, 1] \\ X[i + 1, j, 1] & X[i + 1, j + 1, 1] & X[i + 1, j + 2, 1] \\ X[i + 2, j, 1] & X[i + 2, j + 1, 1] & X[i + 2, j + 2, 1] \end{pmatrix}$$

For the cyan volume (bottom channel), the input is defined as:

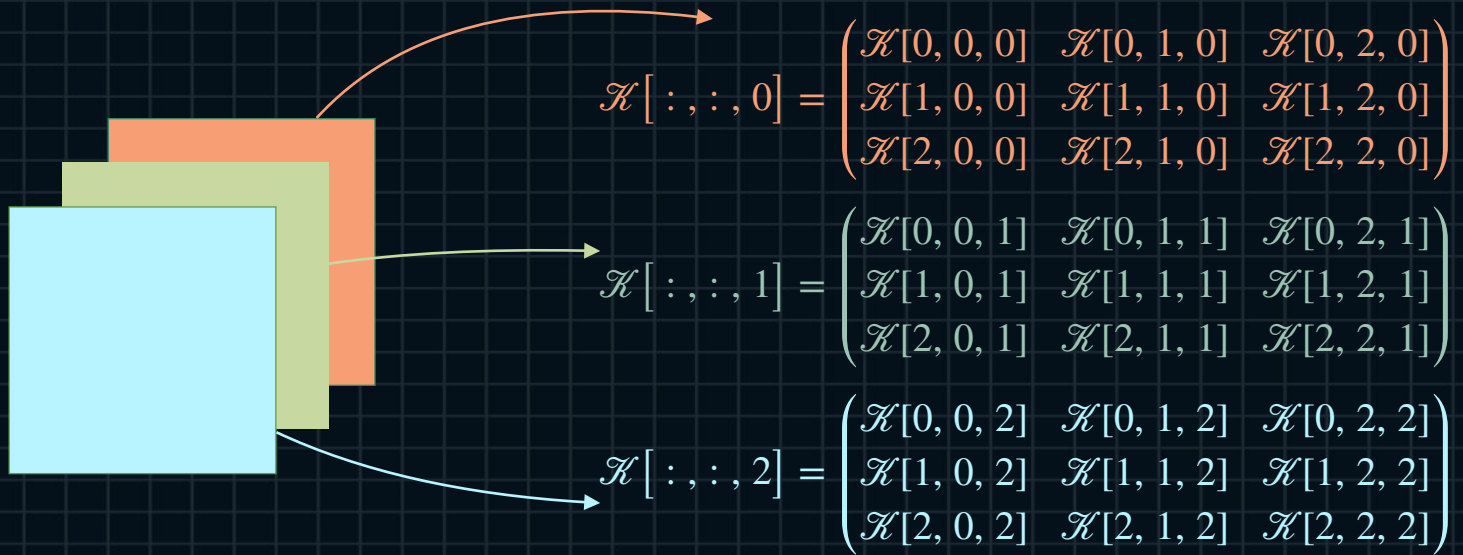
$$\mathcal{W}_{i,j} = X[i : i + f, j : j + f, 2]$$
$$= \begin{pmatrix} X[i, j, 2] & X[i, j + 1, 2] & X[i, j + 2, 2] \\ X[i + 1, j, 2] & X[i + 1, j + 1, 2] & X[i + 1, j + 2, 2] \\ X[i + 2, j, 2] & X[i + 2, j + 1, 2] & X[i + 2, j + 2, 2] \end{pmatrix}$$

Lecture.5 Conv Layers

- n-Channel Input

$$f = 3, n_C = 3$$

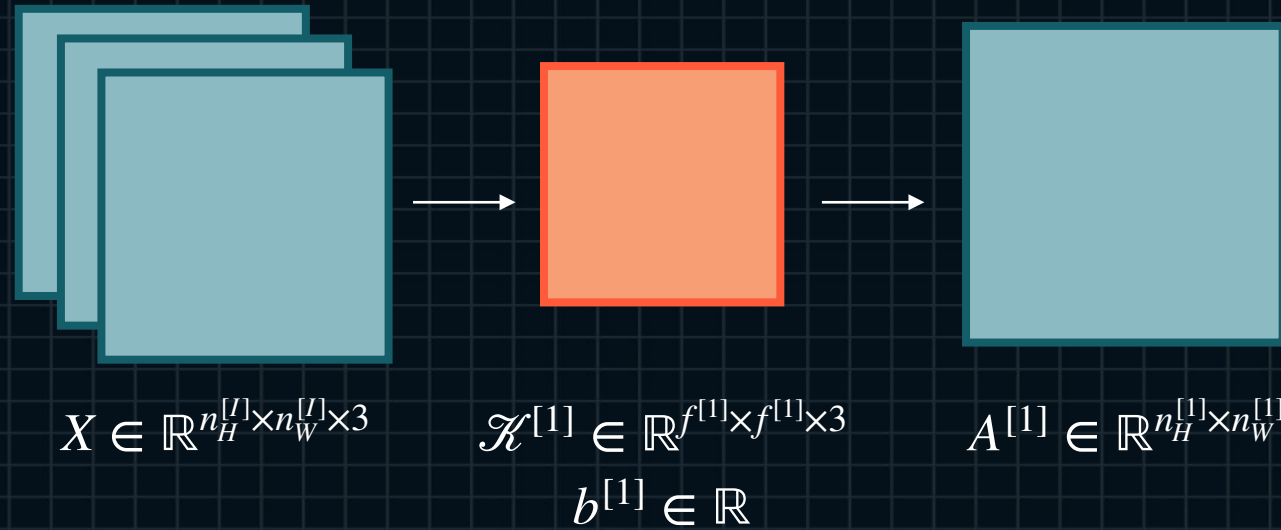
$$\mathcal{K}_{i,j} \in \mathbb{R}^{3 \times 3 \times 3}$$



Lecture.5

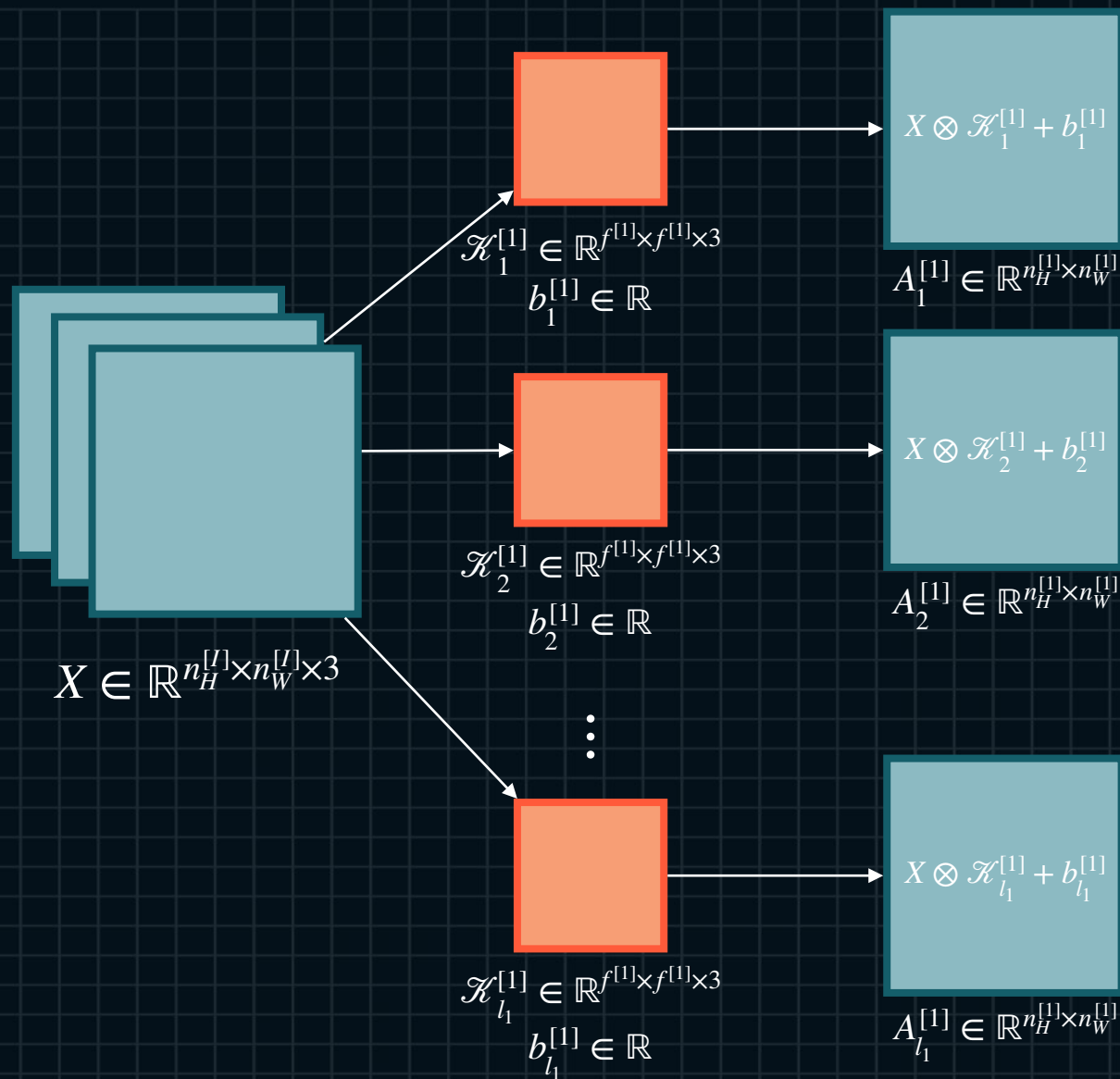
Conv Layers

- Conv Layers



Lecture.5 Conv Layers

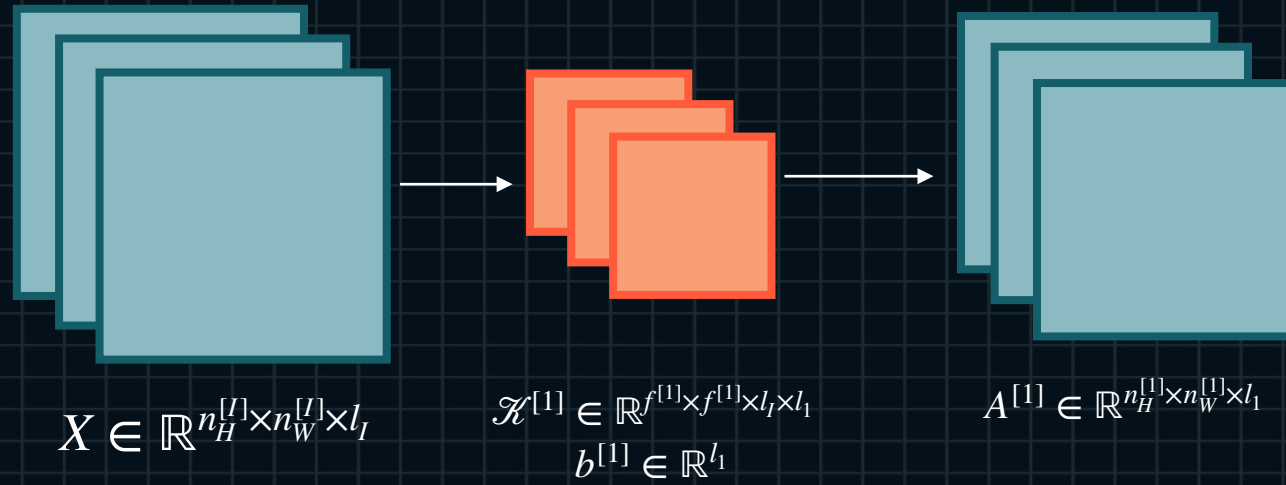
- Conv Layers



Lecture.5

Conv Layers

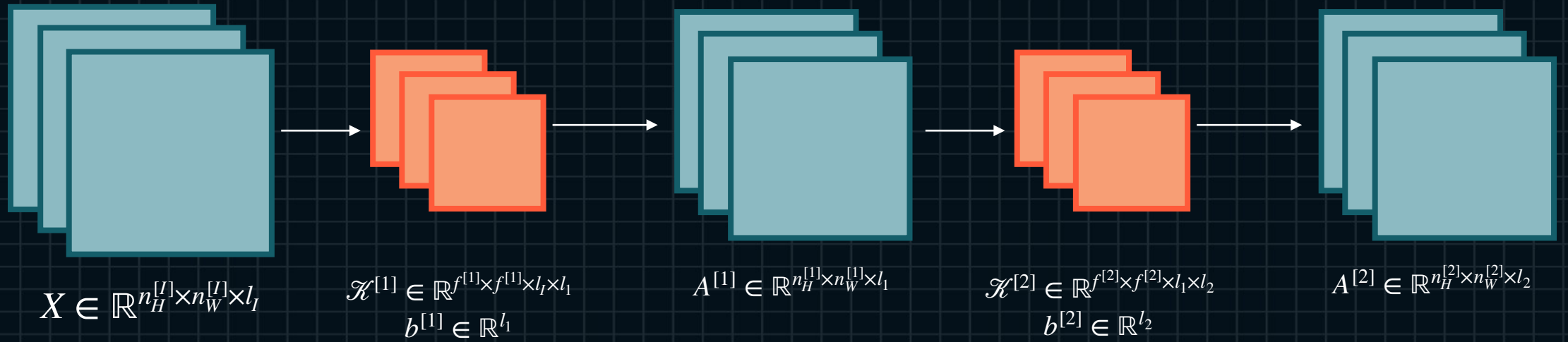
- Conv Layers



Lecture.5

Conv Layers

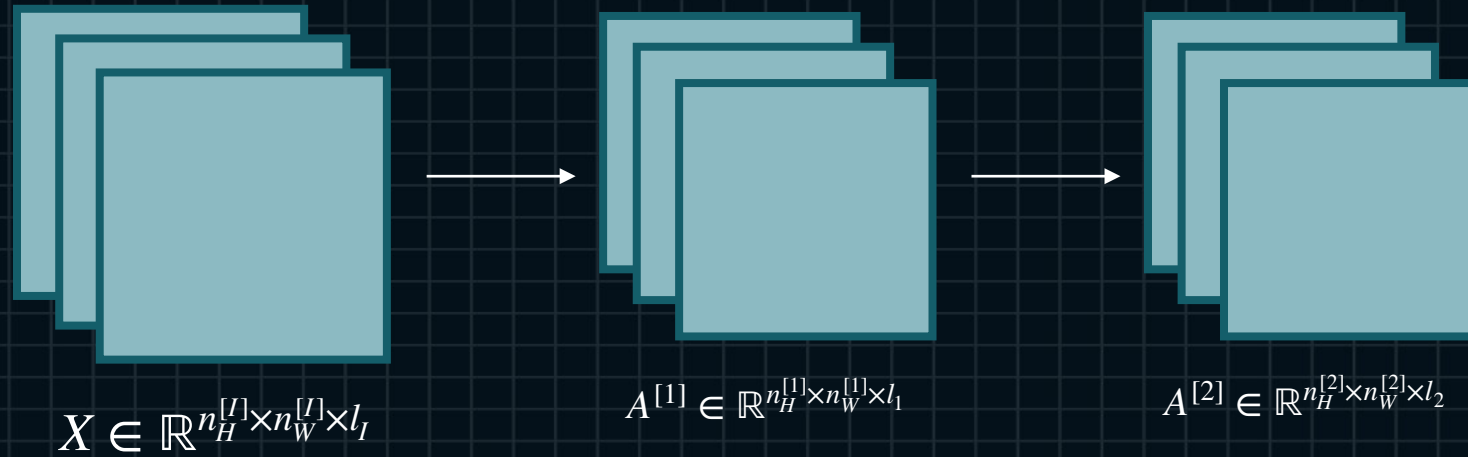
- Conv Layers



Lecture.5

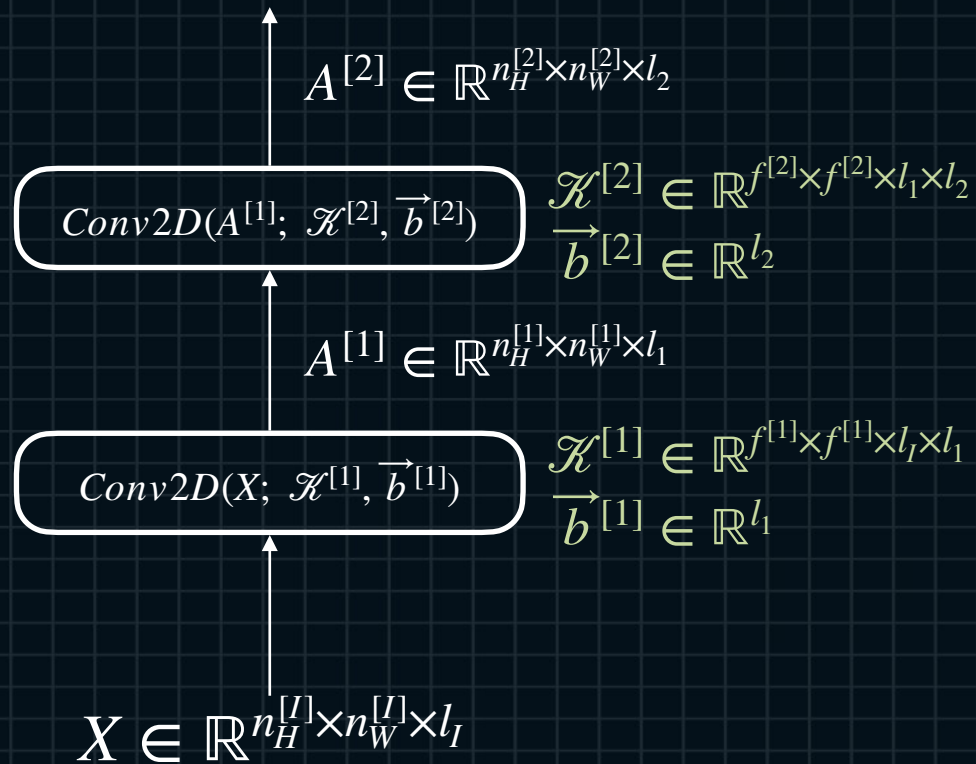
Conv Layers

- Conv Layers



Lecture.5 Conv Layers

- Cascaded Conv Layers

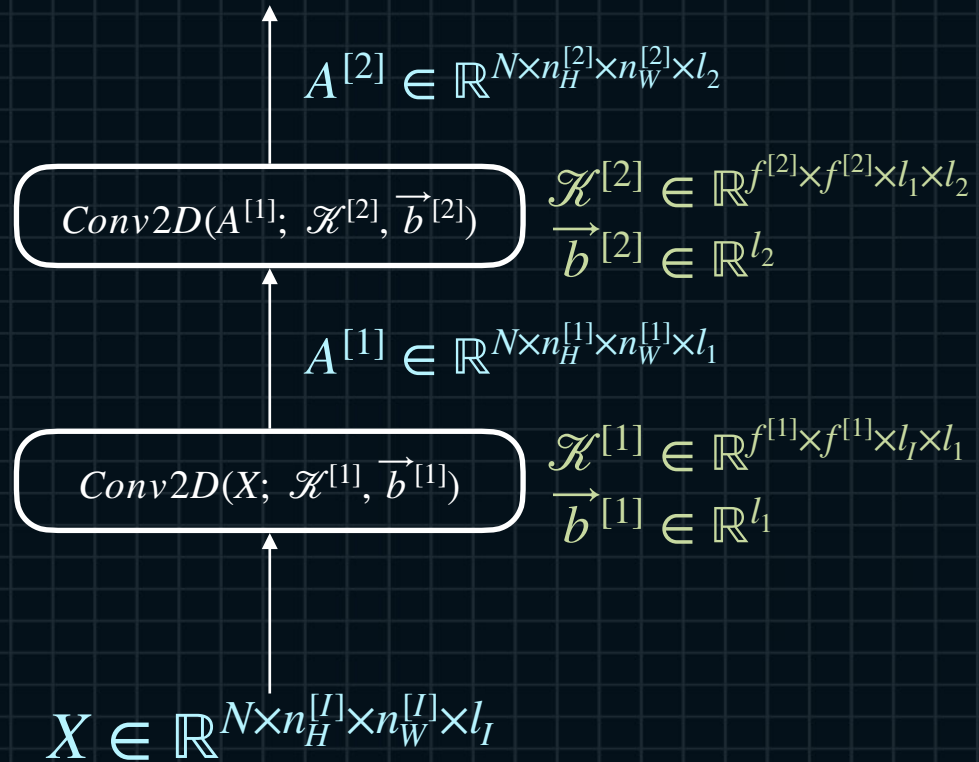


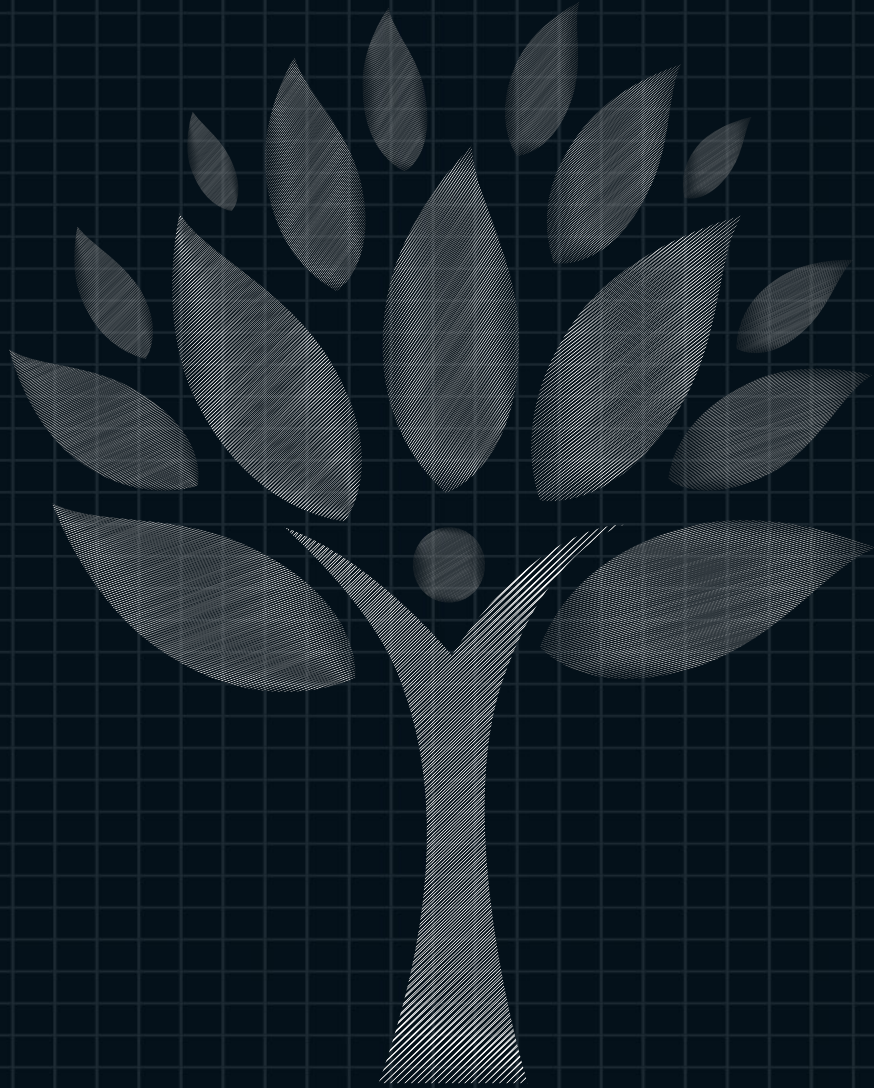
$$n_H^{[2]} = n_H^{[1]} - f^{[2]} + 1$$
$$n_W^{[2]} = n_W^{[1]} - f^{[2]} + 1$$

$$n_H^{[1]} = n_H^{[I]} - f^{[1]} + 1$$
$$n_W^{[1]} = n_W^{[I]} - f^{[1]} + 1$$

Lecture.5 Conv Layers

- Minibatch in Conv Layers





Forward Propagation of Neural Networks

Lecture.5
Conv Layers