HW2.

Seongmin Lee (CID: 01247436)

(a) Indirect Method.

minimise $3x_1 + 5x_2 - x_3$

s.t. $x_1 + x_3 = 4$

$x_2 - 2x_3 \le 2$

$x_1, x_2 \ge 0$, x_3 unrestricted.

i) equality $\to$ inequality

$x_1 + x_3 = 4 \to x_1 + x_3 \ge 4$

$x_1 + x_3 \le 4$

ii) reverse the sign of inequality to $\ge$

$x_1 + x_3 \ge 4$ (ok)

$x_1 + x_3 \le 4 \xrightarrow{\times(-1)} -x_1 - x_3 \ge -4$

$x_2 - 2x_3 \le 2 \xrightarrow{\times(-1)} -x_2 + 2x_3 \ge -2$

iii) $x_3 = x_3^+ - x_3^-$ where $x_3^+, x_3^- \ge 0$

objective fn: minimise $3x_1 + 5x_2 - x_3^+ - x_3^-$

subject to
$x_1 + x_3^+ - x_3^- \ge 4$

※ Max $c^T x$ s.t. $Ax \le b$. $-x_1 - x_3^+ + x_3^- \ge -4$

Min $b^T y$ s.t. $A^T y \ge c$. $-x_2 + 2x_3^+ - 2x_3^- \ge -2$

$x_1, x_2, x_3^+, x_3^- \ge 0$

iv.) The Matrix form of the primal problem is as below:

$$\overset{b^T}{\begin{pmatrix} 3 \\ 5 \\ -1 \\ -1 \end{pmatrix}}^T \overset{y}{\begin{pmatrix} x_1 \\ x_2 \\ x_3^+ \\ x_3^- \end{pmatrix}} \quad \text{St.} \quad \overset{A^T}{\begin{pmatrix} 1 & 0 & 1 & -1 \\ -1 & 0 & -1 & 1 \\ 0 & -1 & 2 & -2 \end{pmatrix}}^T \overset{y}{\begin{pmatrix} x_1 \\ x_2 \\ x_3^+ \\ x_3^- \end{pmatrix}} \ge \overset{c}{\begin{pmatrix} 4 \\ -4 \\ -2 \end{pmatrix}}$$

v) change to the dual form.

$$\begin{pmatrix} 4 \\ -4 \\ -2 \end{pmatrix}^T \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \quad \text{subject to} \quad \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 2 \\ -1 & 1 & -2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \leq \begin{pmatrix} 3 \\ 5 \\ -1 \\ 1 \end{pmatrix}$$

(C^T, x, $A^T \to A$, x, b)

$\therefore$ maximise $4y_1 - 4y_2 - 2y_3$

subject to
$$y_1 - y_2 \leq 3$$
$$-y_3 \leq 5$$
$$y_1 - y_2 + 2y_3 \leq -1$$
$$-y_1 + y_2 - 2y_3 \leq 1$$
$$y_1, y_2, y_3 \geq 0.$$

(b) Direct Method.

maximise $x_1 - x_3$

subject to
$$x_1 + x_2 = 4$$
$$x_3 \leq 2$$
$x_1, x_2 \leq 0$ x_3 unrestricted.

i) change the expressions for the direct method.

maximise $x_1 + 0 \cdot x_2 - x_3$

s.t.
$x_1 + x_2 + 0 \cdot x_3 = 4 \to y_1$ (unrestricted)

$0x_1 + 0 \cdot x_2 + x_3 \leq 2 \to y_2 \geq 0$

checked by the SOB Table in the lecture note

1st con 2nd con 3rd con objective coef

minimise $4y_1 + 2y_2$ $x_1, x_2 \leq 0$ in the primal variables.

Subject to
$$y_1 + 0y_2 \leq 1$$
$$y_1 + 0y_2 \leq 0$$
$$0y_1 + y_2 = -1$$
x_3 is unrestricted.

$\therefore$ minimise $4y_1 + 2y_2$

subject to $y_1 \quad \le 1$

$y_1 \quad \le 0$

$y_2 = -1$

y_1 unrestricted, $y_2 \ge 0$.

$y_2 = -1$ & $y_2 \ge 0 \Rightarrow$ infeasible.

(4)

(a) Formulate the 1-norm regression problem

By the definition of the 1-norm regression,

minimise $\|y - X\beta\|_1$ where $\left\| \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \right\| = |x_1| + \cdots + |x_n|$

put $y = comp_i$

$X_{1i} = years_i$

$X_{2i} = change\_stock_i$

$X_{3i} = change\_sales_i$

$X_{4i} = mba_i$

$\Rightarrow comp_i = \beta_1 \cdot years_i + \beta_2 \cdot change\_stock_i + \beta_3 \cdot change\_sales + \beta_4 \cdot mba_i$

$\Rightarrow$ minimise $|comp_1 - (\beta_1 \cdot years_1 + \beta_3 \cdot change\_stock_1 + \beta_2 \cdot change\_sales_1 + \beta_4 \cdot mba_1)| +$

$\vdots$

$+ |comp_i - (\beta_1 years_i + \beta_2 \cdot chang\_stock_i + \beta_3 \, change\_sales_i + \beta_4 \cdot mba_i)|$

~~subject to~~ $\beta_1, \beta_2, \beta_3, \beta_4 \in \mathbb{R}$

reformulate as

minimise $\theta_1 + \theta_2 + \cdots + \theta_n$

subject to $\theta_1 = |comp_1 - (\beta_1 \cdot years_1 + \beta_2 \cdot change\_stock_1 + \beta_3 change\_sales_1 + \beta_4 \cdot mba_1)|$

$\vdots$

$\theta_n = |Comp_i - (\beta_1 years_i + \beta_2 chang\_stock_i + \beta_3 change\_sales_i + \beta_4 mba_i)|$

$\beta_1, \beta_2 \cdots, \beta_n \in \mathbb{R}$

To change equality to inequality ($\geq$) in minimisation problems

We can re-write ex. $\theta = |a-b|$ to $\theta \geq (a-b)$ and $\theta \geq -(a-b)$

Therefore, the 1 norm liner program is as blew;

Decision Variables : $\beta_1, \beta_2, \beta_3, \beta_4 \in \mathbb{R}$

Minimize $\theta_1 + \theta_2 + \cdots + \theta_n$

Subject to. $\theta_i \geq comp_i - (\beta_1 years_i + \beta_2 \cdot chang\_stock_i + \beta_3 \cdot chang\_sales_i + \beta_4 mba_i)$

$\theta_i \geq (\beta_1 years_i + \beta_2 change\_stock_i + \beta_3 chang\_sales_i + \beta_4 mba_i) - comp_i$

$\forall_i = 1, 2, \cdots n$

(b) The Ample file is attached in the Hub.

The objective value is 14652.83, which is the total residuals

at β_1 for years = 169.274

β_2 for change stock = 2.41

β_3 for change sales = -0.1425

β_4 for mba = 352.5

(c) Infinite norm regression

Consider the definition of norm, we can extend it to infinite level

$\|X\|_n = \sqrt[n]{|x_1|^n + |x_2|^n + \cdots + |x_n|^n}$ [normal form]

if $n \to \infty$, the largest number among $|x_1|^n \cdots |x_n|^n$ will be survived.

Therefore, we can generalize it as below.

ex) $\|x\|_\infty = \sqrt[\infty]{|1|^\infty + |10|^\infty + |100|^\infty}$

could be ignored by ∞

$= \sqrt[\infty]{|100|^\infty} = |100|$.

$\|X\|_\infty = \max(|x_1|, \cdots, |x_n|)$

If we apply the above method to the regression, we can re-write

minimise $\|y - X\beta\|_\infty \Leftrightarrow \max(|y_1 - x_1\beta|, \cdots |y_i - x_i\beta|)$.

So, we can put $\theta = |y - X\beta|$

$\theta = \max(\theta_1, \theta_2, \cdots, \theta_i)$

where $\theta_i = |Comp_i - (\beta_1 \cdot years_i + (\beta_2 \cdot change\_stock_i + \beta_3 \cdot change\_sales_i + (\beta_4 \cdot mba_i))|$

To re-summary

minimise θ

subject to $\theta = \max(\theta_1, \theta_2 \cdots, \theta_i)$, where $\theta_i = |y_i - x_i\beta|$

By relaxing the equality in that θ is greater or equal than $\theta_1 \cdots \theta_n$.

minimise θ

subject to $\theta \geq |y_i - x_i\beta|$, $\forall_i = \underbrace{1 \cdots 50}_{\text{the number of comp}}$.

Finally, we can remove the absolute value operator to re-formulate the LP as below

minimise θ

subject to $\theta \geq y_i - x_i\beta$

$\theta \geq -(y_i - x_i\beta)$ $\forall_i = 1 \cdots 50$.