Optimisation Assignment 1

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Q1)

(a) Formulate a linear program for this problem.

- Decision Variables.
  - Sales quantity of Special risk: r
  - Sales quantity of Mortgage: m

- Objective function.
  - maximise $5 \times r + 2 \times m$

- constraints
  - Underwriting: $3 \times r + 2 \times m \le 2400$
  - Administration: $0 \times r + 1 \times m \le 800 \iff m \le 800$
  - Claims: $2 \times r + 0 \times m \le 1200 \iff r \le 600$
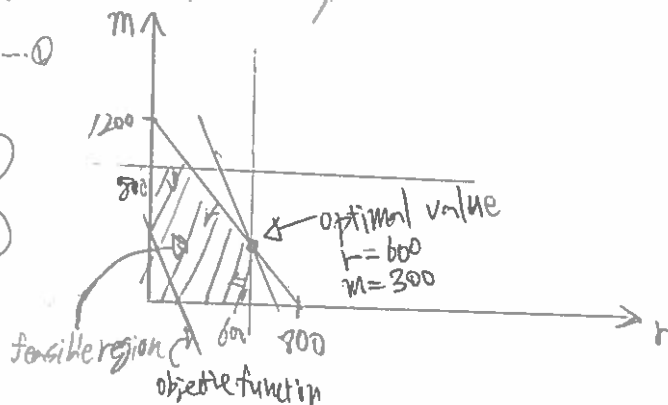 - non negative 1: $m \ge 0$
  - non negative 2: $r \ge 0$

(b) Use the graphical method to solve this problem.

$\rightarrow m \le -\frac{3}{2} r + 1200 \cdots ①$

$\rightarrow 0 \le m \le 800 \cdots ②$

$\rightarrow 0 \le r \le 600 \quad ③$

$\rightarrow m = -\frac{5}{2} r \cdots ④$

optimal value
$r = 600$
$m = 300$

feasible region

objective function

- optimal solution
  - $r = 600$
  - $m = 300$

- objective value
  - $5 \times 600 + 2 \times 300 = £3600$

- binding
  - ① [underwriting], ③ [claims] are binding.
    $3 \times r + 2 \times m \le 2400$ $r \le 600$

(c) From the constraints, we could

	1. $m=0, r=0$	2. $m=800$ $r=\frac{800}{3}$	3. $r=600, m=300$.
optimal value	0	$5 \times \frac{800}{3} + 2 \times 800$ $= \frac{95}{3} + \frac{4800}{3} = \frac{4895}{3}$ $= 1631\frac{2}{3}$	$5 \times 600 + 2 \times 300$ $= 3600$.

∴ $r=600, m=300$ are the optimal solution with the optimal value of 3600.

Q2)

(a) Formulate a LP.

- Decision variables
  - number of Stir Fry: S
  - number of Barbecue: b
  - number of Hearty Mushrooms: h
  - number of Veggie Crunch: V

- objective function
  - Maximise $0.22 \times S + 0.2 \times b + 0.18 \times h + 0.18 \times V$

- constraints
  - Carrots: $62.5 \times S + 50 \times b + \quad 62.5 \times V \leq 3750000$
  - Mushrooms: $75 \times S + \quad 100 \times h \quad \leq 2000000$
  - Green peppers: $62.5 \times S + 36 \times b + 75 \times h + 62.5 \times V \leq 3375000$
  - Broccoli: $50 \times S + 75 \times b + 75 \times h + 62.5 \times V \leq 3500000$
  - Corn: $\quad 75 \times b + \quad 62.5 \times V \leq 3750000$

(b) AMPL.

- optimal solution
  - Stir fry (S) = 26666.7 Bags
  - Barbecue (b) = 18333.3 Bags
  - Hearty Mushrooms (h) = 0 Bags
  - Veggie Crunch (V) = 12666.7 Bags

- objective value
  - $0.22 \times 26666.7 + 0.2 \times 18333.3 + 0.18 \times 0 + 0.18 \times 12666.7 = 11813.35$

- Binding constraints. (by using .slack)
  - Mushrooms, Green peppers, Broccoli, Hearty Mushrooms

(c) The value of an extra $100kg (=100000g)$ of green peppers is $11877.33 - 11813.33 = £64$, which indicates that the purchasing cost for extra green peppers should be lower than $£0.64/kg (=£64/100kg)$, the shadow price of green pepper, in order to make a profit.