



# Introduction to regression





## Boston housing data

```
In [1]: boston = pd.read_csv('boston.csv')
In [2]: print(boston.head())
                  INDUS
      CRIM
                                  NX
                        CHAS
                                              AGE
                                                      DIS
                                                                  TAX \
                                                           RAD
   0.00632
            18.0
                  2.31
                              0.538
                                                   4.0900
                                                                296.0
                                      6.575
                                             65.2
   0.02731
            0.0
                  7.07
                                                               242.0
                              0.469
                                      6.421
                                             78.9
                                                  4.9671
   0.02729
                                                               242.0
            0.0
                  7.07
                              0.469
                                      7.185
                                             61.1
                                                  4.9671
   0.03237
            0.0
                              0.458
                                                               222.0
                  2.18
                                      6.998
                                            45.8
                                                   6.0622
                                                               222.0
  0.06905
             0.0
                  2.18
                              0.458
                                      7.147
                                             54.2
                                                  6.0622
                   LSTAT
   PTRATIO
                           MEDV
     15.3
           396.90
                    4.98
                          24.0
     17.8
           396.90
                    9.14
                          21.6
           392.83
     17.8
                    4.03
                          34.7
           394.63
                    2.94
     18.7
                          33.4
           396.90
                          36.2
                     5.33
     18.7
```





## Creating feature and target arrays

```
In [3]: X = boston.drop('MEDV', axis=1).values
In [4]: y = boston['MEDV'].values
```



#### Predicting house value from a single feature

```
In [5]: X_rooms = X[:,5]
In [6]: type(X_rooms), type(y)
Out[6]: (numpy.ndarray, numpy.ndarray)
In [7]: y = y.reshape(-1, 1)
In [8]: X_rooms = X_rooms.reshape(-1, 1)
```





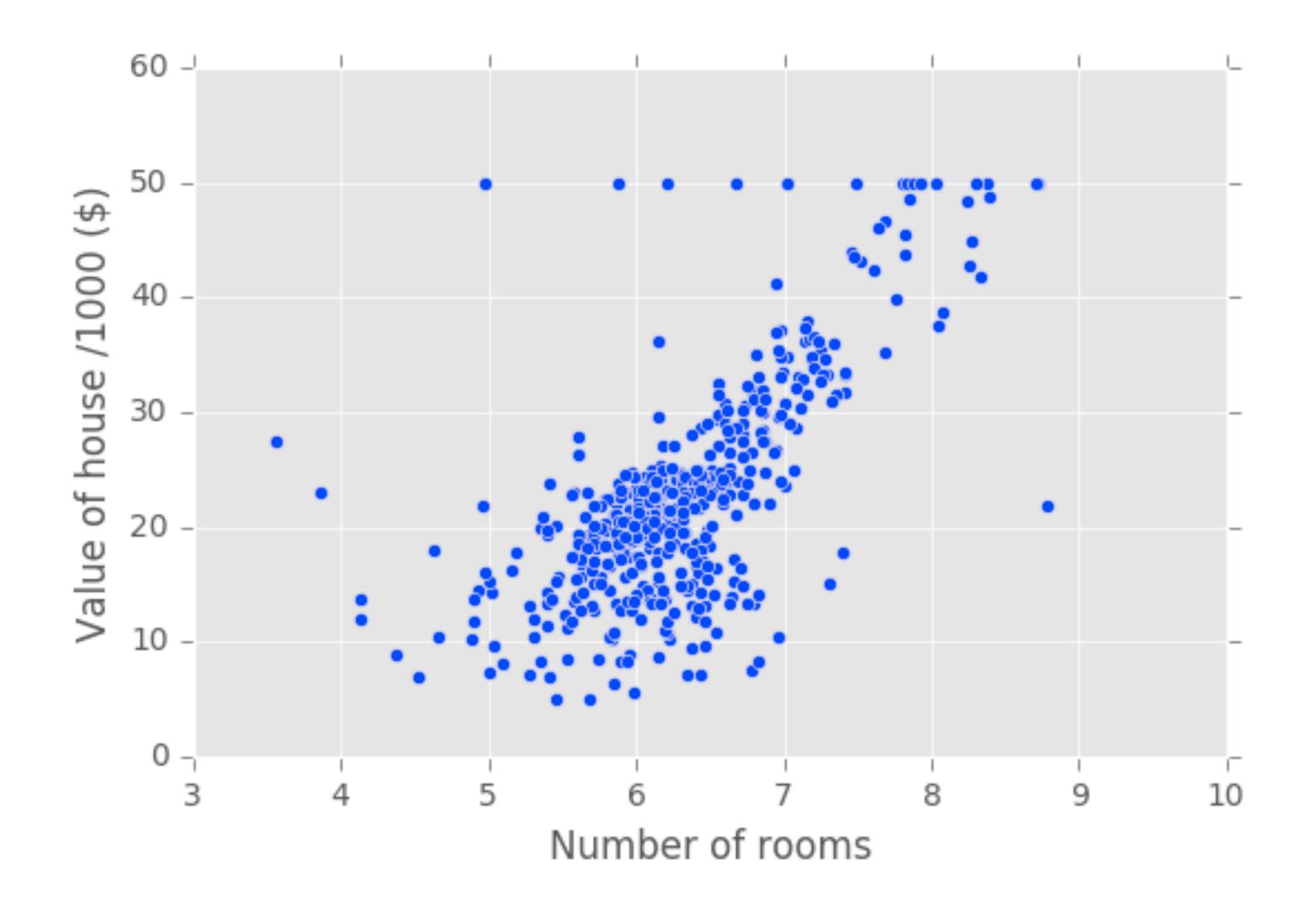
#### Plotting house value vs. number of rooms

```
In [9]: plt.scatter(X_rooms, y)
In [10]: plt.ylabel('Value of house /1000 ($)')
In [11]: plt.xlabel('Number of rooms')
In [12]: plt.show();
```





#### Plotting house value vs. number of rooms







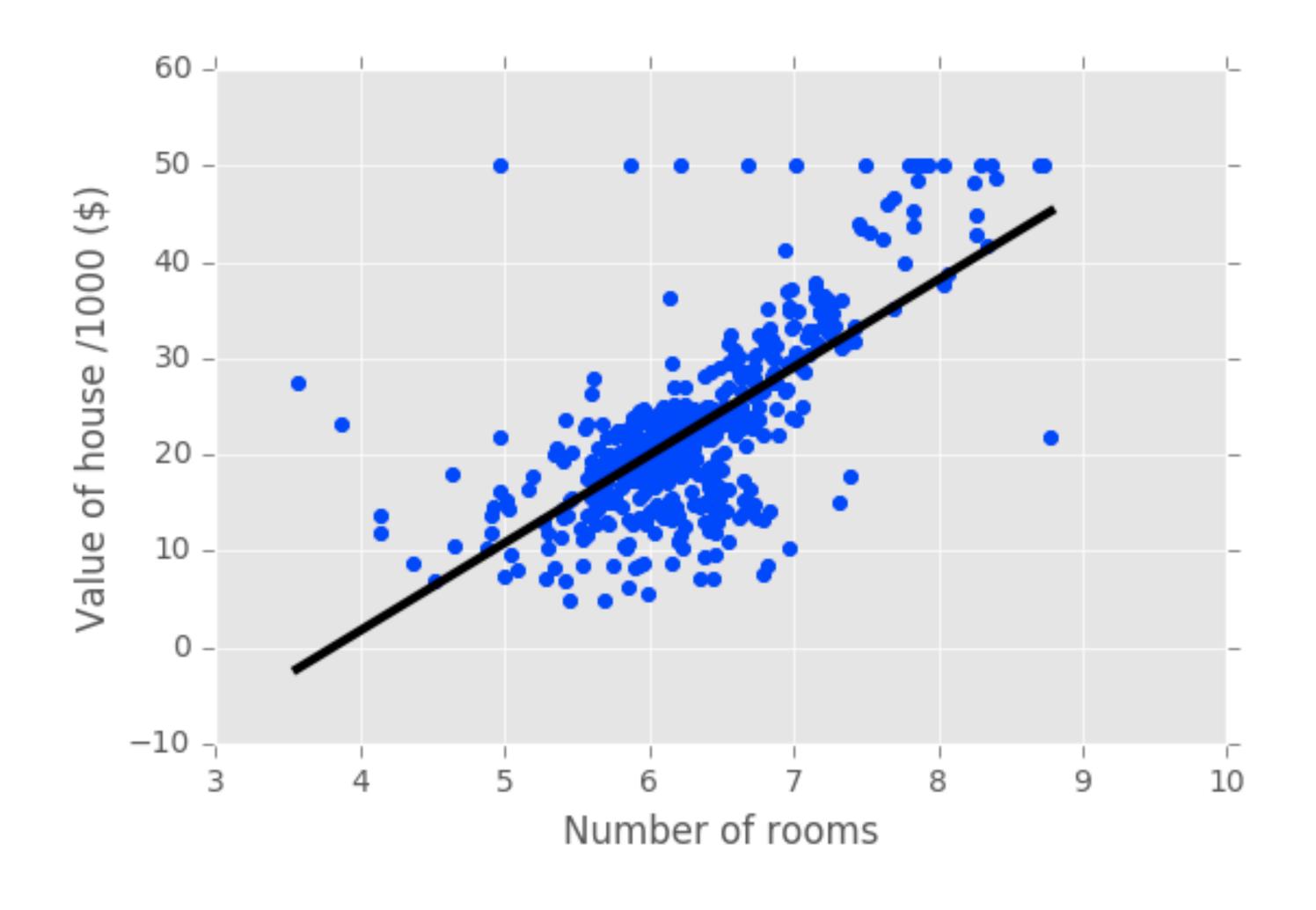
## Fitting a regression model

```
In [13]: import numpy as np
In [14]: from sklearn import linear_model
In [15]: reg = linear_model.LinearRegression()
In [16]: reg.fit(X_rooms, y)
In [17]: prediction_space = np.linspace(min(X_rooms),
                                         max(X_{rooms})).reshape(-1, 1)
   • • • •
In [18]: plt.scatter(X_rooms, y, color='blue')
In [19]: plt.plot(X_rooms, reg.predict(prediction_space),
                 color='black', linewidth=3)
In [20]: plt.show()
```





## Fitting a regression model







## Let's practice!





# The basics of linear regression



## Regression mechanics

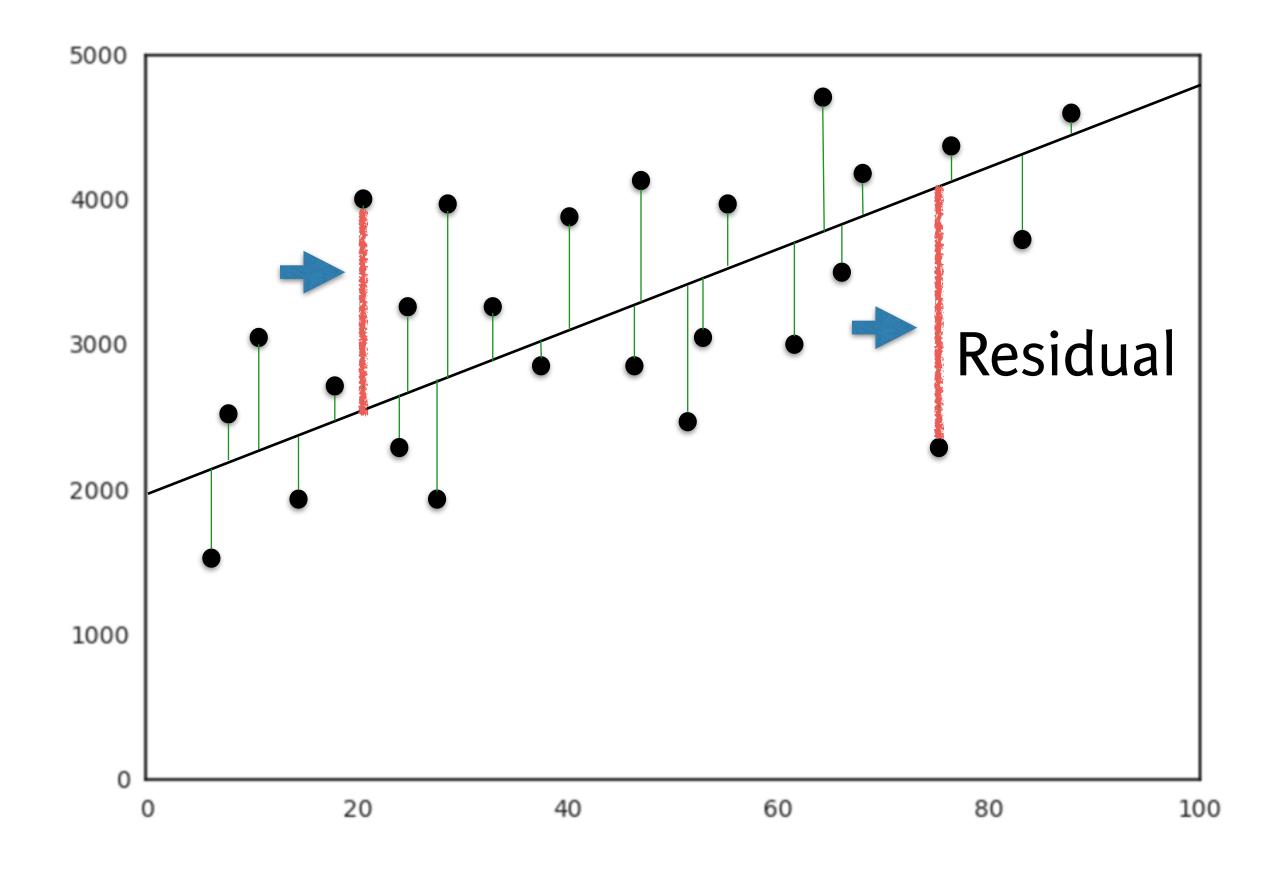
- y = ax + b
  - y = target
  - x = single feature
  - a, b = parameters of model
- How do we choose a and b?
- Define an error function for any given line
  - Choose the line that minimizes the error function





### The loss function

Ordinary least squares (OLS): Minimize sum of squares of residuals





#### Linear regression in higher dimensions

$$y = a_1 x_1 + a_2 x_2 + b$$

- To fit a linear regression model here:
  - Need to specify 3 variables
- In higher dimensions:

$$y = a_1x_1 + a_2x_2 + a_3x_3 + a_nx_n + b$$

- Must specify coefficient for each feature and the variable b
- Scikit-learn API works exactly the same way:
  - Pass two arrays: Features, and target





## Linear regression on all features

```
In [1]: from sklearn.model_selection import train_test_split
In [2]: X_train, X_test, y_train, y_test = train_test_split(X, y,
   \dots: test_size = 0.3, random_state=42)
In [3]: reg_all = linear_model.LinearRegression()
In [4]: reg_all.fit(X_train, y_train)
In [5]: y_pred = reg_all.predict(X_test)
In [6]: reg_all.score(X_test, y_test)
Out[6]: 0.71122600574849526
```





## Let's practice!





### Cross-validation



#### Cross-validation motivation

- Model performance is dependent on way the data is split
- Not representative of the model's ability to generalize
- Solution: Cross-validation!





#### Cross-validation basics

Split 1	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Metric 1
Split 2	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Metric 2
Split 3	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Metric 3
Split 4	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Metric 4
Split 5	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5	Metric 5

Training data

Test data





#### Cross-validation and model performance

- 5 folds = 5 -fold CV
- 10 folds = 10 -fold CV
- k folds = k -fold CV
- More folds = More computationally expensive





#### Cross-validation in scikit-learn

```
In [1]: from sklearn.model_selection import cross_val_score
In [2]: reg = linear_model.LinearRegression()
In [3]: cv_results = cross_val_score(reg, X, y, cv=5)
In [4]: print(cv_results)
[ 0.63919994   0.71386698   0.58702344   0.07923081 -0.25294154]
In [5]: np.mean(cv_results)
Out[5]: 0.35327592439587058
```





## Let's practice!





# Regularized regression



## Why regularize?

- Recall: Linear regression minimizes a loss function
- It chooses a coefficient for each feature variable
- Large coefficients can lead to overfitting
- Penalizing large coefficients: Regularization



## Ridge regression

- Loss function = OLS loss function +  $\alpha * \sum_{i=1}^{n} a_i^2$
- Alpha: Parameter we need to choose
- Picking alpha here is similar to picking k in k-NN
- Hyperparameter tuning (More in Chapter 3)
- Alpha controls model complexity
  - Alpha = o: We get back OLS (Can lead to overfitting)
  - Very high alpha: Can lead to underfitting





## Ridge regression in scikit-learn

```
In [1]: from sklearn.linear_model import Ridge
In [2]: X_train, X_test, y_train, y_test = train_test_split(X, y,
   \dots: test_size = 0.3, random_state=42)
In [3]: ridge = Ridge(alpha=0.1, normalize=True)
In [4]: ridge.fit(X_train, y_train)
In [5]: ridge_pred = ridge.predict(X_test)
In [6]: ridge.score(X_test, y_test)
Out[6]: 0.69969382751273179
```



## Lasso regression

• Loss function = OLS loss function +  $\alpha * \sum_{i=1}^{\infty} |a_i|$ 





## Lasso regression in scikit-learn

```
In [1]: from sklearn.linear_model import Lasso
In [2]: X_train, X_test, y_train, y_test = train_test_split(X, y,
   \dots: test_size = 0.3, random_state=42)
In [3]: lasso = Lasso(alpha=0.1, normalize=True)
In [4]: lasso.fit(X_train, y_train)
In [5]: lasso_pred = lasso.predict(X_test)
In [6]: lasso.score(X_test, y_test)
Out[6]: 0.59502295353285506
```





### Lasso regression for feature selection

- Can be used to select important features of a dataset
- Shrinks the coefficients of less important features to exactly o





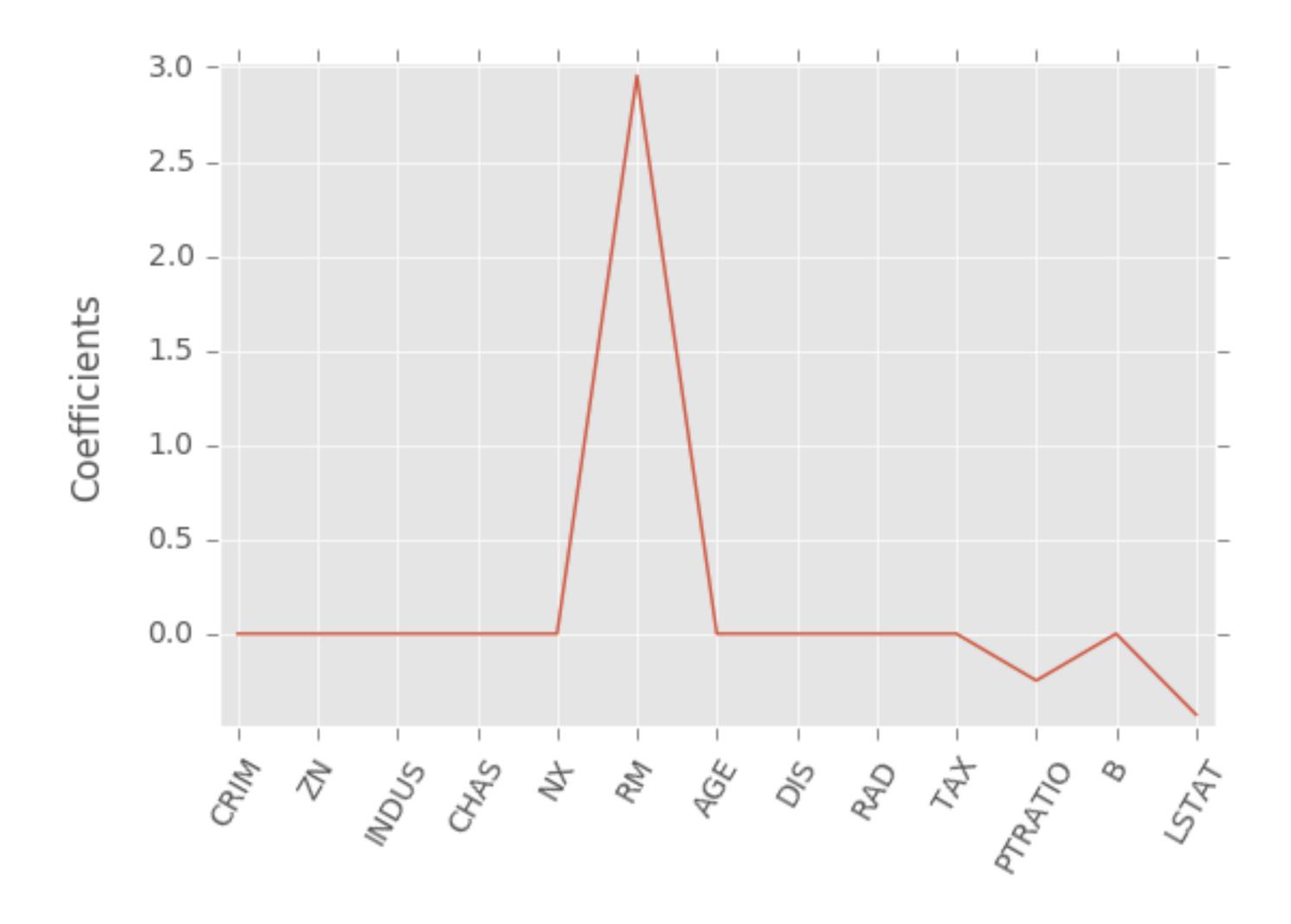
#### Lasso for feature selection in scikit-learn

```
In [1]: from sklearn.linear_model import Lasso
In [2]: names = boston.drop('MEDV', axis=1).columns
In [3]: lasso = Lasso(alpha=0.1)
In [4]: lasso_coef = lasso.fit(X, y).coef_
In [5]: _ = plt.plot(range(len(names)), lasso_coef)
In [6]: _ = plt.xticks(range(len(names)), names, rotation=60)
In [7]: _ = plt.ylabel('Coefficients')
In [8]: plt.show()
```





#### Lasso for feature selection in scikit-learn







## Let's practice!