

ICSI431 Homework 3 Seoyeon Choi

Part 1 [28 pts.] Bayes Classifier. Consider the following dataset, classify the new point: (Age=23, Car=truck) via the full and naive Bayes approach, assuming that the domain of Car is given as sports, vintage, suv, truck.

Full Bayes Approach:

1. Calculate prior probabilities for classes L and H:

$$P(L) = \frac{2}{6}, \quad P(H) = \frac{4}{6}$$

2. Calculate likelihood $P(x|c)$ using the smoothing method:

$$P(\text{age} = 23, \text{car} = \text{truck} | L) = \frac{0 + 1}{2 + (4 \times 4)} = \frac{1}{18}$$

$$P(\text{age} = 23, \text{car} = \text{truck} | H) = \frac{0 + 1}{24 + (4 \times 4)} = \frac{1}{20}$$

3. Calculate posterior probabilities $P(c|x)$:

$$\begin{aligned} P(L | \text{age} = 23, \text{car} = \text{truck}) &= P(\text{age} = 23, \text{car} = \text{truck} | L) \times P(L) \\ &= \left(\frac{2}{6}\right) \times \left(\frac{1}{18}\right) = 0.0185 \end{aligned}$$

$$\begin{aligned} P(H | \text{age} = 23, \text{car} = \text{truck}) &= P(\text{age} = 23, \text{car} = \text{truck} | H) \times P(H) \\ &= \left(\frac{4}{6}\right) \times \left(\frac{1}{20}\right) = 0.0333 \end{aligned}$$

Naive Bayes Approach:

1. Calculate prior probabilities for classes L and H:

$$P(L) = \frac{2}{6}, \quad P(H) = \frac{4}{6}$$

2. Calculate likelihood $P(x|c)$ using normal distribution for the age attribute and the smoothing method for the categorical car attribute:

$$\begin{aligned}
 P(\text{age} = 23|L) &= \left(\frac{1}{\sqrt{2\pi} \cdot 0} \right) \cdot \exp \left(-\frac{(23 - 25)^2}{2 \cdot 0} \right) = 0 \\
 P(\text{age} = 23|H) &= \left(\frac{1}{\sqrt{2\pi} \cdot 10.31} \right) \cdot \exp \left(-\frac{(23 - 27.5)^2}{2 \cdot 106.25} \right) \\
 P(\text{car} = \text{truck}|L) &= \frac{0 + 1}{2 + 4} = \frac{1}{6} \\
 P(\text{car} = \text{truck}|H) &= \frac{0 + 1}{4 + 4} = \frac{1}{8}
 \end{aligned}$$

3. Calculate posterior probabilities $P(c|x)$:

$$\begin{aligned}
 P(L|\text{age} = 23, \text{car} = \text{truck}) &= P(\text{age} = 23, \text{car} = \text{truck}|L) \times P(L) \\
 &= \left(\frac{2}{6} \right) \times \left(\frac{1}{18} \right) = 0.0185 \\
 P(H|\text{age} = 23, \text{car} = \text{truck}) &= P(\text{age} = 23, \text{car} = \text{truck}|H) \times P(H) \\
 &= \left(\frac{4}{6} \right) \times \left(\frac{1}{20} \right) = 0.0333
 \end{aligned}$$

Naive Bayes Approach:

1. Calculate prior probabilities for classes L and H:

$$P(L) = \frac{2}{6}, \quad P(H) = \frac{4}{6}$$

2. Calculate likelihood $P(x|c_i)$ using normal distribution for the age attribute and the smoothing method for the categorical car attribute:

$$\begin{aligned} P(\text{age} = 23|L) &= \left(\frac{1}{\sqrt{2\pi} \cdot 0} \right) \cdot \exp \left(-\frac{(23 - 25)^2}{2 \cdot 0} \right) = 0 \\ P(\text{age} = 23|H) &= \left(\frac{1}{\sqrt{2\pi} \cdot 10.31} \right) \cdot \exp \left(-\frac{(23 - 27.5)^2}{2 \cdot 106.25} \right) \\ P(\text{car} = \text{truck}|L) &= \frac{0 + 1}{2 + 4} = \frac{1}{6} \\ P(\text{car} = \text{truck}|H) &= \frac{0 + 1}{4 + 4} = \frac{1}{8} \end{aligned}$$

3. Calculate posterior probabilities:

$$\begin{aligned} P(H|\text{age} = 23, \text{car} = \text{truck}) &= P(\text{age} = 23|H) \times P(\text{car} = \text{truck}|H) \times \\ &= 0.035 \times \left(\frac{1}{8} \right) \times \left(\frac{4}{6} \right) = 0.002913 \\ P(L|\text{age} = 23, \text{car} = \text{truck}) &= P(\text{age} = 23|L) \times P(\text{car} = \text{truck}|L) \times \\ &= 0 \times \left(\frac{2}{6} \right) = 0 \end{aligned}$$

Therefore, both Full Bayes and Naive Bayes approaches classify the instance (age=23, car=truck) as belonging to class H.

Part 2 – (d)

Run the code on the cancer dataset with different values of ψ and ϵ . Check the change in cross-entropy values across iterations (in the plot)

and the average training and testing cross-entropy errors. What do you observe about the losses and number of iterations? What do you conclude?

First, we tried with learning rate $\psi = 0.1$, $\epsilon = 10$ (as stopping criterion), and max epochs = 5. This is the list of w values obtained from logisticRegression SGA with x_{train} , y_{train} , $\psi = 0.1$, $\epsilon = 10$, max epochs = 5.

```

(base) choeseoyeon@MacBook-Pro-3 HW3 % ls
HW3.pdf          cancer-data-train.csv  logisticRegression.py
cancer-data-test.csv  classifiers.py
(base) choeseoyeon@MacBook-Pro-3 HW3 % python3 logisticRegression.py
w :
[ 0.00656  0.007865  0.007055  0.0046935  0.005685  -0.006845
 -0.006795  0.00776  0.00811  -0.00644  -0.006175  0.00817
 -0.00652  -0.00559  -0.0048505  0.007095  -0.005125  -0.00763
 -0.005995  0.00827  -0.00611  -0.007335  0.00759  -0.00583
 -0.006675  0.00614  -0.00658  -0.005085  -0.006725  -0.00628
 0.006405  0.006325  0.005555  -0.00701  -0.00552  -0.00635
 0.005655  -0.007145  0.007005  0.00636  -0.005615  0.005485
 0.00897  -0.00718  -0.00662  -0.00595  -0.0061  -0.007135
 0.008885  -0.006815  0.008205  0.00787  -0.006355  0.01113
 -0.005715  -0.00572  0.00713  0.006275  0.005695  -0.00632
 -0.005365  0.007175  0.007055  -0.006845  -0.0047225  -0.00752
 -0.00608  0.006975  0.007665  0.00741  0.00687  0.00891
 -0.00603  0.006215  0.007655  0.009545  0.0075  -0.0073
 -0.00613  0.005185  -0.005855  -0.00532  0.007575  0.00825
 -0.00701  0.007245  -0.00568  0.005105  -0.007575  -0.0079
 -0.00692  -0.005485  -0.005475  -0.00669  -0.005225  -0.005005
 0.00722  -0.00558  -0.00564  0.00719  0.00732  -0.00687
 -0.006925  0.00939  -0.006745  -0.005385  -0.00798  -0.005365
 0.00661  -0.00641  -0.00701  0.00567  -0.00641  -0.00703
 -0.00515  -0.00554  -0.00559  -0.00805  -0.00611  -0.00609
 -0.00725  -0.005405  -0.004979  0.007455  -0.0047635  -0.005735
 -0.00571  0.006825  0.00708  -0.00656  -0.00713  -0.006945
 -0.0047285  0.007545  -0.006885  0.008195  0.008945  -0.00617
 -0.005975  -0.006745  0.00619  -0.00692  -0.00862  0.005995
 -0.006555  -0.00688  0.00749  0.00696  0.01092  -0.00678

```

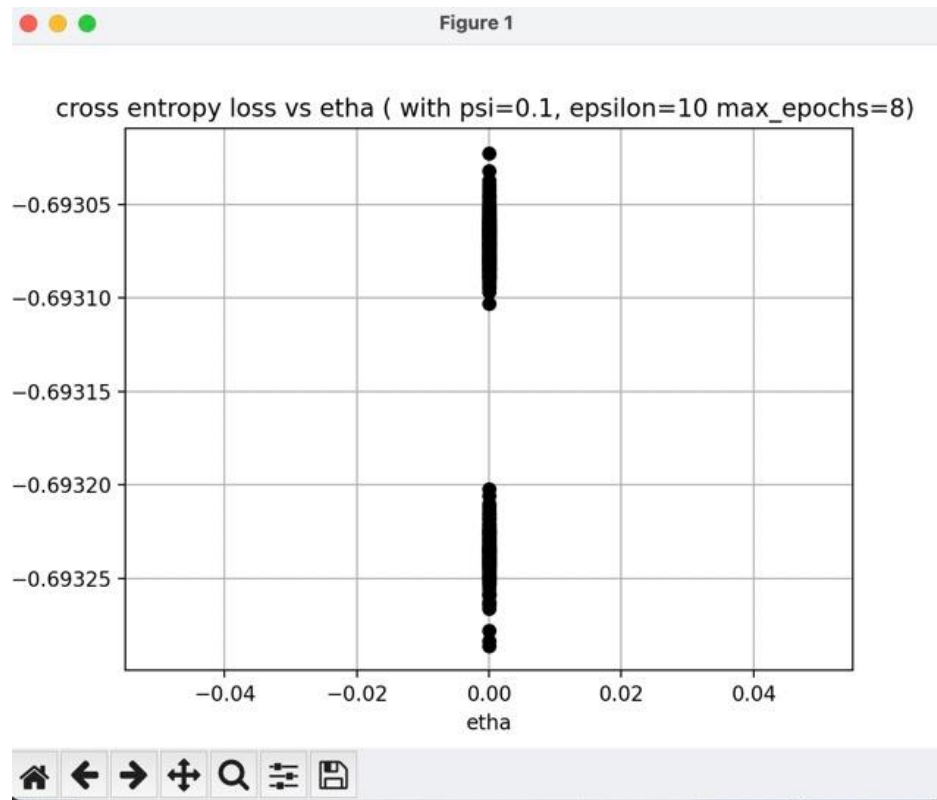
Figure 1: W values with $\psi = 0.1$, $\epsilon = 10$, max epochs = 5

This is the cross-entropy loss returned from the logisticRegression
SGA()

method.

```
0.00752 ]
cross ent :
[array([-0.69322899, -0.69324529, -0.69323517, -0.6932057 , -0.69321807,
        -0.69306202, -0.69306264, -0.69324398, -0.69324835, -0.69306705,
        -0.69307035, -0.6932491 , -0.69306606, -0.69307762, -0.69308681,
        -0.69323567, -0.6930834 , -0.69305227, -0.69307258, -0.69325035,
        -0.69307115, -0.69305593, -0.69324186, -0.69307463, -0.69306413,
        -0.69322375, -0.69306531, -0.6930839 , -0.69306351, -0.69306904,
        -0.69322706, -0.69322606, -0.69321645, -0.69305997, -0.69307849,
        -0.69306463, -0.6932177 , -0.69305829, -0.69323455, -0.6932265 ,
        -0.69307731, -0.69321557, -0.6932591 , -0.69305786, -0.69306482,
        -0.69307314, -0.69307128, -0.69305842, -0.69325804, -0.69306239,
        -0.69324954, -0.69324535, -0.69306811, -0.6932861 , -0.69307606,
        -0.693076 , -0.69323611, -0.69322544, -0.69321819, -0.69306854,
        -0.69308042, -0.69323667, -0.69323517, -0.69306202, -0.6930884 ,
        -0.69305363, -0.69307153, -0.69323418, -0.69324279, -0.69323961,
        -0.69323286, -0.69325835, -0.69307215, -0.69322469, -0.69324267,
        -0.69326629, -0.69324073, -0.69305637, -0.69307091, -0.69321183,
        -0.69307432, -0.69308097, -0.69324167, -0.6932501 , -0.69305997,
        -0.69323755, -0.6930765 , -0.69321083, -0.69305295, -0.69304891,
        -0.69306109, -0.69307892, -0.69307905, -0.69306395, -0.69308216,
        -0.69308489, -0.69323724, -0.69307774, -0.693077 , -0.69323686,
        -0.69323848, -0.69306171, -0.69306103, -0.69326435, -0.69306326,
        -0.69308017, -0.69304792, -0.69308042, -0.69322962, -0.69306743,
        -0.69305997, -0.69321788, -0.69306743, -0.69305972, -0.69308309,
        -0.69307824, -0.69307762, -0.69304705, -0.69307115, -0.6930714 ,
        -0.69305699, -0.69307992, -0.69308521, -0.69324017, -0.69308789,
        -0.69307582, -0.69307613, -0.6932323 , -0.69323549, -0.69306556,
        -0.69305848, -0.69306078, -0.69308833, -0.69324129, -0.69306152,
        -0.69324941, -0.69325879, -0.69307041, -0.69307283, -0.69306326,
        -0.69322437, -0.69306109, -0.69303997, -0.69322194, -0.69306562,
        -0.69306159, -0.69324061, -0.69323399, -0.69328348, -0.69306283,
        -0.69307625, -0.69323898, -0.69306314, -0.69304513, -0.69323324,
```

This plot shows the cross-entropy loss vs. η (with $\psi = 0.1$, $\epsilon = 10$, max epochs = 5).



These are the average cross-entropy errors for training and testing data.

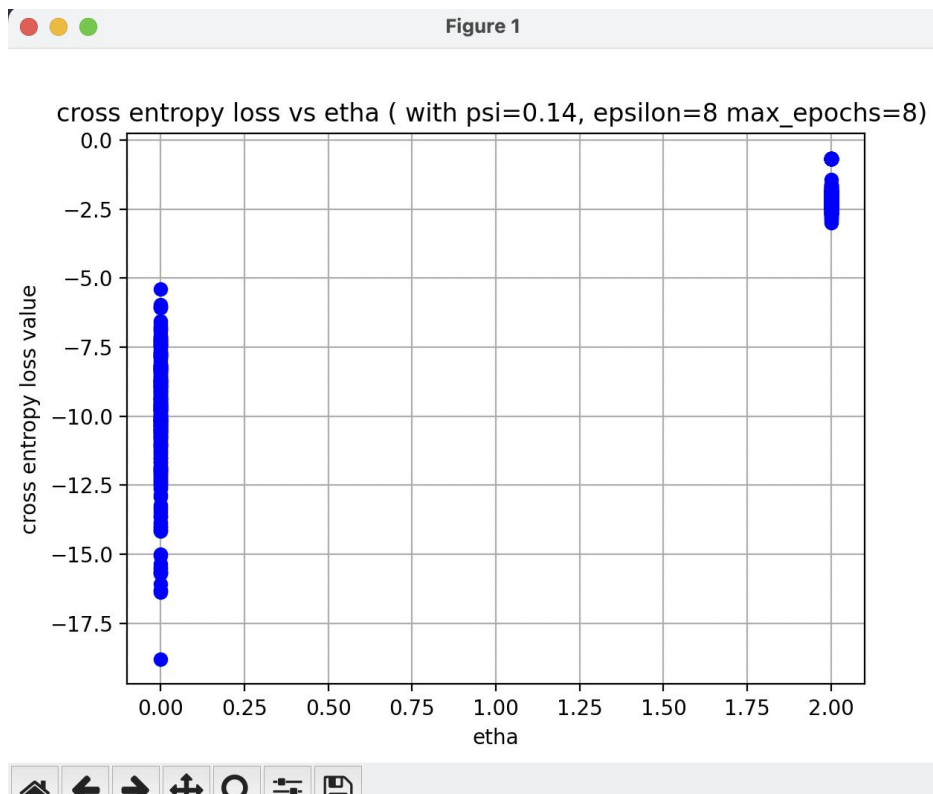
```

terminal  Shell  Edit  View  Window  Help  zoom  [icons]
HW3 — python3 logisticRegression.py — 127x33
Average cross-entropy error for training data: [[-0.69322899 -0.69324529 -0.69323517 ... -0.69305898 -0.69325104
-0.69324098]
[-0.70626752 -0.70937384 -0.70742645 ... -0.6836559 -0.7105079
-0.70853666]
[-1.9454793 -2.30717482 -2.08165776 ... -2.56840209 -2.43639884
-2.21076811]
...
[-2.32534568 -2.77184233 -2.49394527 ... -3.68843952 -2.93049521
-2.65321942]
[-0.69319805 -0.69320818 -0.69320189 ... -0.69309229 -0.69321175
-0.6932055 ]
[-0.69314559 -0.69314528 -0.69314547 ... -0.69314892 -0.69314517
-0.69314536]]
Average cross-entropy error for testing data: [[-0.69393976 -0.69409871 -0.694 ... -0.69230266 -0.69415484
-0.69405665]
[-0.69671895 -0.69771584 -0.69708591 ... -0.69199042 -0.69808984
-0.69744307]
[-0.70309952 -0.70559429 -0.70402576 ... -0.68725769 -0.70651424
-0.7049182 ]
...
[-0.69307645 -0.6930624 -0.69307112 ... -0.69322385 -0.69305745
-0.69306511]
[-0.69332211 -0.69335701 -0.69333534 ... -0.69295912 -0.69336933
-0.69334778]
[-0.69320259 -0.69321362 -0.69320677 ... -0.69308739 -0.69321751
-0.6932107 ]]
Finished First Task.

```

Next, I tested with different values of ψ and ϵ . This time, I set $\psi = 0.14$ and $\epsilon = 8$. This plot shows the cross-entropy loss value vs.

η .



These are the average cross-entropy errors for training and testing data.

```

Average cross-entropy error for training data: [[ nan nan nan ... -87.80308916 nan
[ nan nan nan ... nan nan
[ nan
[ -0.67656645 -0.67887014 -0.6736766 ... -2.0374551 -0.67448763
-0.67709702]
...
[ -0.69760368 -0.69806751 -0.69616822 ... -0.67213281 -0.69698093
-0.69772247]
[ -2.10424752 -2.30147636 -1.50666874 ... -17.44275365 -1.84158759
-2.15466716]
[ nan nan nan ... nan nan
[ nan]]
Average cross-entropy error for testing data: [[ -1.20684096 -1.28548001 -0.98588477 ... -4.2012198 -1.10633593
-1.2266972 ]
[ nan nan -8.70937962 ... -76.16672294 nan
[ nan nan nan ... -94.58568574 nan
[ nan]]
...
[ -0.68157222 -0.68111604 -0.68382693 ... -1.05804028 -0.68239063
-0.6814431 ]
[ -0.76595776 -0.77628267 -0.73704828 ... -1.32111109 -0.75283492
-0.76855793]
[ -0.70974782 -0.71168305 -0.70399339 ... -0.70626976 -0.70720696
-0.71023998]]
(base) choesoyeon@MacBook-Pro-3 HW3 %

```

In conclusion, I didn't change the max epochs value, so the number of iterations for SGA was the same. The only thing I changed was the learning rate and epsilon for the stopping criterion. I increased the learning rate ψ from 0.1 to 0.14 and decreased epsilon from 10 to 8. As a result, we can see from the attached plots and the average cross-entropy error for training and testing data that increasing the learning step led to higher cross-entropy loss. This indicates that a learning rate that is too small is not optimal since it moves very slowly and requires more computation. However, increasing the learning rate from 0.1 to 0.14 was not a good choice, as it moved larger steps and increased the cross-entropy loss. Also, we can see that the average cross-entropy error value for training data increased up to -7.7906 , and the average cross-entropy error for testing data also slightly increased.

Therefore, with a higher learning rate (ψ), the algorithm takes larger steps towards finding the optimal point and may converge faster, but this could lead to higher cross-entropy loss. Decreasing the stopping criterion (ϵ) means that I changed the norm of the weight vector to be smaller, $\|W^{(t+1)} - W^*\| < \epsilon$, but this could result in fewer iterations for convergence. Therefore, to reduce cross-entropy loss, it is important to find a learning rate and stopping criterion that are neither too small nor too large.

Part 3 – (a)

Discuss your observations. Is a smaller or larger margin better for this dataset? (Need to explain which C values are likely to produce smaller versus larger values and then which end up being better in cross-validation.)

In linear SVM, there is a trade-off between maximizing margin width vs minimizing classification epsilon error on the training data. For example, if C is small, this indicates that we care more (give more weight) to have as large as possible margin width rather than focusing on minimizing classification error. With smaller C value and larger margin width, this model will be more generalized to unseen data (such as testing data) and have lower variance (robust).

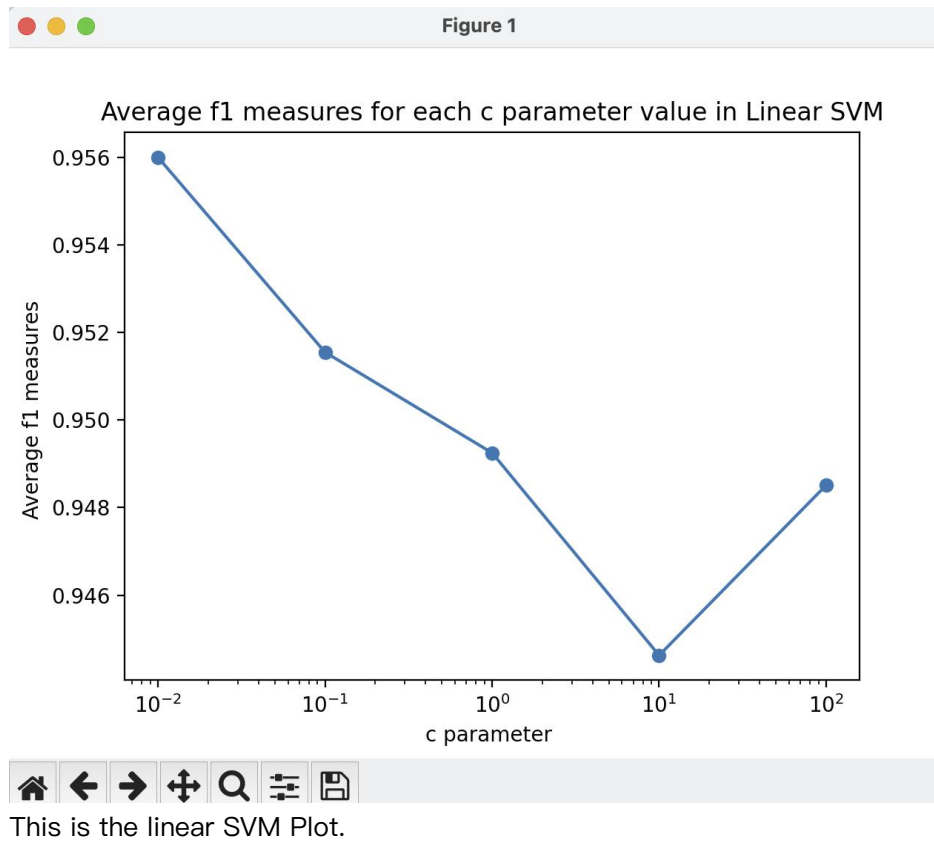
If C value is larger, this indicates that we care more (give more weight) to minimizing classification error, and this results in reducing margin width. With larger C value, we can reduce classification error, but since

we end up with smaller margin width, this model may poorly work on unseen data such as testing data (higher variance and not a generalized model).

Based on the implemented SVM function for Part 3 – (a), the best C value from the linear SVM was 0.01, which gives the largest margin width, and corresponding $C = 0.01$ had an average F-measure of around 0.956. Therefore, the smallest C value with the largest margin width was better to use.

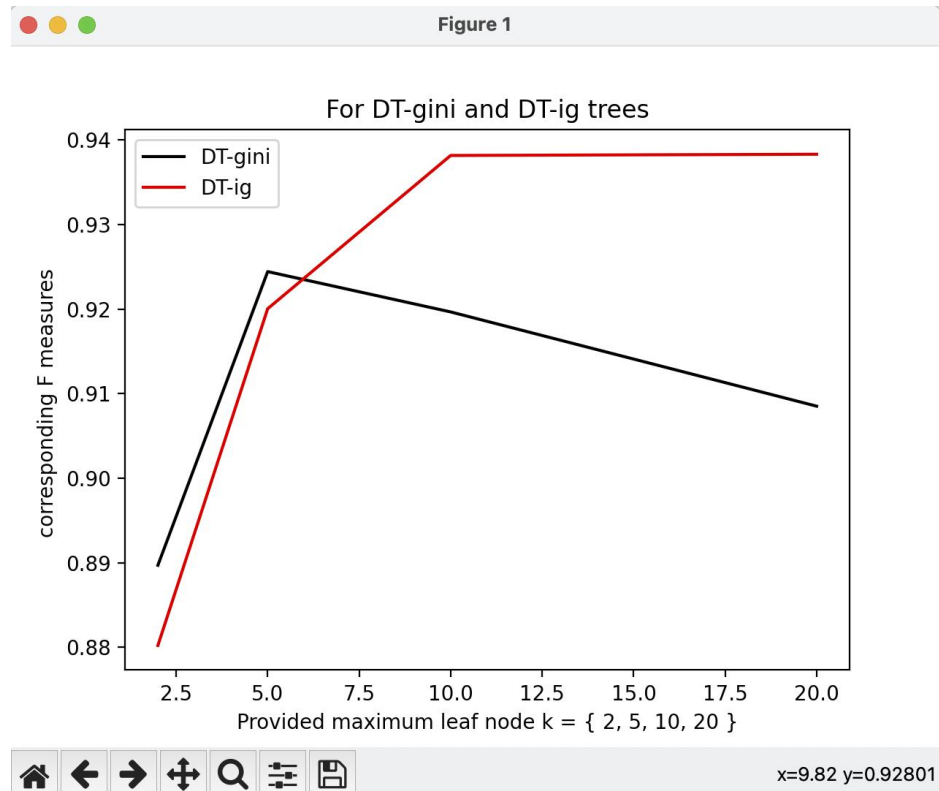
[illegible]

```
HW3 — python3 classifiers.py — 116x26
c : 10 average f1 measure value : 0.9446236976982847
c : 100 average f1 measure value : 0.9485107016562948
best c value from linear SVM part(a) : 0.01
corresponding average f measure value : 0.9560028022421697
c : 0.01 average f1 measure value : 0.9560028022421697
c : 0.1 average f1 measure value : 0.9515503116762674
c : 1 average f1 measure value : 0.9492508815920256
c : 10 average f1 measure value : 0.9446236976982847
c : 100 average f1 measure value : 0.9485107016562948
best c value from linear SVM part(a) : 0.01
corresponding average f measure value : 0.9560028022421697
c : 0.01 average f1 measure value : 0.9560028022421697
c : 0.1 average f1 measure value : 0.9515503116762674
c : 1 average f1 measure value : 0.9492508815920256
c : 10 average f1 measure value : 0.9446236976982847
c : 100 average f1 measure value : 0.9485107016562948
best c value from linear SVM part(a) : 0.01
corresponding average f measure value : 0.9560028022421697
c : 0.01 average f1 measure value : 0.9560028022421697
c : 0.1 average f1 measure value : 0.9515503116762674
c : 1 average f1 measure value : 0.9492508815920256
c : 10 average f1 measure value : 0.9446236976982847
c : 100 average f1 measure value : 0.9485107016562948
best c value from linear SVM part(a) : 0.01
corresponding average f measure value : 0.9560028022421697
```



Part 3 – (b)

Discuss your observations from the figures. Does a larger tree mean a better F-measure? Which criterion is better?



This is the plot for DT-gini and DT-ig tree. The x-axis represents the maximum leaf node k , and the y-axis represents the corresponding F-measures.

For DT-gini: F-measure started increasing from maximum leaf node size $k = 2$ to 5, and after $k = 5$, F-measure started to decrease (this indicates overfitting). For DT-ig: F-measure started to increase from maximum leaf

node size $k = 2$ to around 11, after $k = 11$, F-measure maintained around 0.94.

Does a larger tree mean a better F-measure value?

No. Increasing maximum leaf node size k increases the F-measure value at the beginning, but too much big size k causes overfitting and decreases the F-measure value as we see in the DT-gini plot.

Which criterion is better?

It depends on various factors and dataset characteristics, but based on the output that I got, the output plot of DT-gini and DT-ig shows that DT-ig was able to achieve a higher F-measure value at $k = 20$ compared to DT-gini, so choosing DT-gini could be a better criterion since it had a higher F-measure. On the side note, it is also important to consider

generalizability, robustness, and computational efficiency.

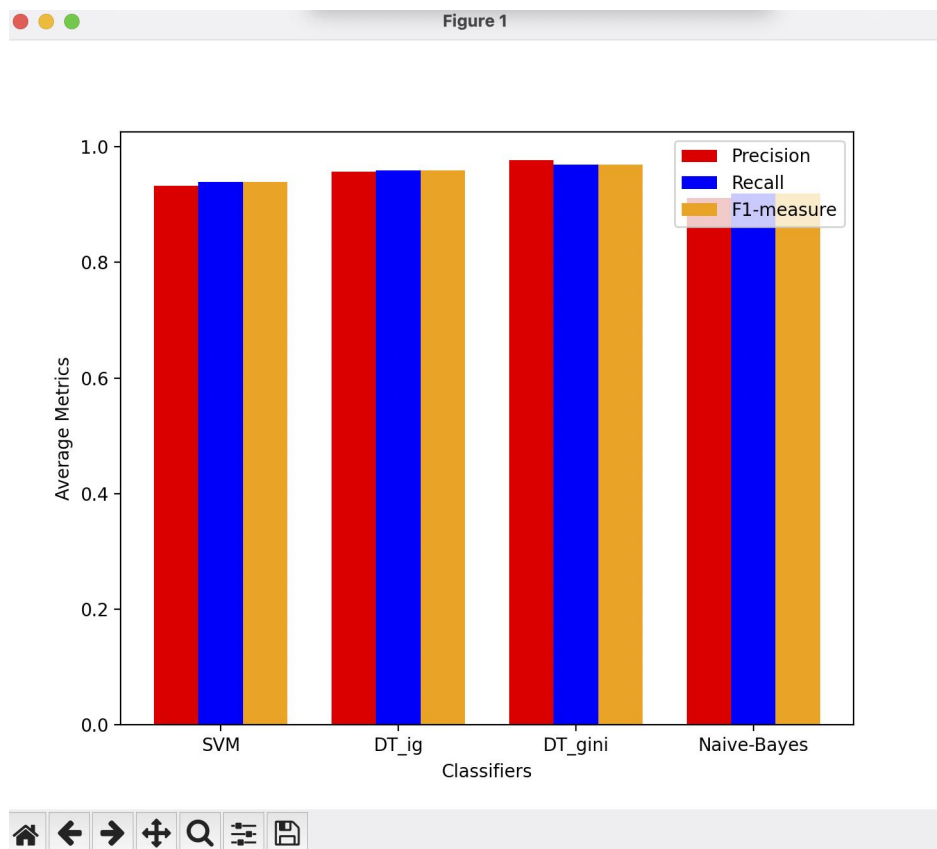
```
HW3 — python3 classifiers.py — 116x40
c : 10 average f1 measure value : 0.9446236976982847
c : 100 average f1 measure value : 0.9485107016562948
best c value from linear SVM part(a) : 0.01
corresponding average f measure value : 0.9560028022421697
c : 0.01 average f1 measure value : 0.9560028022421697
c : 0.1 average f1 measure value : 0.9515503116762674
c : 1 average f1 measure value : 0.9492508815920256
c : 10 average f1 measure value : 0.9446236976982847
c : 100 average f1 measure value : 0.9485107016562948
best c value from linear SVM part(a) : 0.01
corresponding average f measure value : 0.9560028022421697
c : 0.01 average f1 measure value : 0.9560028022421697
c : 0.1 average f1 measure value : 0.9515503116762674
c : 1 average f1 measure value : 0.9492508815920256
c : 10 average f1 measure value : 0.9446236976982847
c : 100 average f1 measure value : 0.9485107016562948
best c value from linear SVM part(a) : 0.01
corresponding average f measure value : 0.9560028022421697
c : 0.01 average f1 measure value : 0.9560028022421697
c : 0.1 average f1 measure value : 0.9515503116762674
c : 1 average f1 measure value : 0.9492508815920256
c : 10 average f1 measure value : 0.9446236976982847
c : 100 average f1 measure value : 0.9485107016562948
best c value from linear SVM part(a) : 0.01
corresponding average f measure value : 0.9560028022421697
c : 0.01 average f1 measure value : 0.9560028022421697
c : 0.1 average f1 measure value : 0.9515503116762674
c : 1 average f1 measure value : 0.9492508815920256
c : 10 average f1 measure value : 0.9446236976982847
c : 100 average f1 measure value : 0.9485107016562948
best c value from linear SVM part(a) : 0.01
corresponding average f measure value : 0.9560028022421697

Start part3 - b task
Best size of tree for DT-ig: 20
Corresponding average f1 measure for DT-ig: 0.9186032006776454
Best size of tree for DT-gini: 20
Corresponding average f1 measure for DT-gini: 0.9145582482104638
```

This is best size of the trees for DT-ig and DT-gini from part(b). I will use this $k = 20$ values in part c.

Part 3 – (c)

Discuss your findings. Which are the best classifiers when you consider the different metrics? Is there a single winner for this dataset?



This is output box plot, where red color represent precision, blue color represent recall, and yellow color represent f1-measure values. Total there are 4 box plot, each of them is SVM, DT-ig, DT-gini, and Naïve Bayes classifier.

