Health Care and Game Theory - An application to liver transplantation

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Abstract

Even though nowadays medicine is necessarily linked with technology, it is still a service involving a lot of human interaction. Indeed, a medical consultation can be an extremely complex example of human interaction. This is where game theory may play a key role in helping to improve the results of medical processes. Game theory is widely used in an immense variety of study fields, but there is little application to healthcare issues, namely the doctor-patient relationship. This study uses game theory to model the liver transplantation consultation for patients suffering from Alcoholic Liver Disease. This is a very delicate disease, and patients at its end-stages require special dedication. They may try to deceive doctors, which may lead to bad outcomes. Strategic and extensive form games are analysed, and the equilibrium solutions computed. Results show some health policy implications on the parameters that should be managed to achieve better outcomes. The analysis of the strategic form game shows which parameters influence the cooperation rates, both for the doctor and the patient. The analysis of the extensive form game shows how to obtain a more cooperative outcome, depending on the player (patient or doctor) that moves first.

Keywords: Game Theory, Liver Transplantation, Alcoholic Liver Disease, Medical Consultation

1. Introduction

It is common knowledge that alcohol consumption is harmful for people's health, harming especially one vital organ: the liver. Alcohol abuse led to 3.8% of all deaths in the world in 2004 and 4.5% of the global disease burden which is measured in disability-adjusted life years lost. This is why alcohol abuse is often considered a public health problem (Marinho et al., 2015). This paper elaborates on Alcoholic Liver Disease and the process bearing its last resort treatment: liver transplantation. The main issue with liver transplantation is that there are not enough liver grafts to cover the demand, thus requiring a very complex and strictly ruled candidate selection process (Carrion et al., 2013). The selection criteria comprises requisites such as alcohol abstinence (minimum six months), assessment of the severity of patient's disease and existence of any other clinical conditions, as well as verification of the post-transplant prognostic. This last criterion is of utmost importance since 20%-25% of the patients lapse or relapse to heavy drinking postoperatively (Telles-Correia and Mega, 2015).

Although technological advances make medical practice change rapidly, healthcare is still a service for people offered by people, but the role of physi-cians has changed substantially (Henry, 2006). As stated in Elwyn (2004, page 415), "Medicine is a service delivered by a mix of episodic and repeated interactions between humans, medicated by the use of technologies such as tests, drugs and procedures". However, it is widely recognized that there exists an obvious communication gap between doctors and patients, which usually leads to a 40-50% less than optimal adherence to physicians' advice and treatment. Physicians are decision makers under extreme pressure, not only to offer the right treatment but also to make the best allocation of resources. Furthermore, technology has enabled physicians and patients to access a massive amount of information. Therefore, patients are nowadays more informed (for better or for worse) and are more aware of the options they have. This led to a scenario where empowered patients also assume the role of decision makers, being able to negotiate with physicians in order to defend their own interests. In an interaction like this, it is common to have conflict of interests of both players and the outcomes of the situation will be influenced by the decisions taken by both of them. This is where game theory appears as a suitable tool to study this interaction. The selection process for liver transplantation, is an in-

teraction engaging a tricky negotiation between the multidisciplinary medical team and the patients and their families. The physician must check if the patient has been following the recommendations, and at the same time assess the prognostic for the patient's liver transplantation. If the patient relapses into drinking after receiving a new healthy liver, it will be a big loss for all interested parties. At the same time, the patient may or may not comply with the doctor - since complying entails several sacrifices - and might try to deceive the doctor in order to not be excluded from the transplantation waiting list. Game theory can be used to obtain results that can help coming up with health policy implications targeted to increase patients' adherence to doctors' recommendations.

Game theory is used in several domains, such as economics, political science and psychology, logic and computer science, and even biology. Actually, game theory applied to a wide range of behavioral relations is considered a valuable tool for the science of logical decision making in humans, animals, and computers. However, studies applying game theory to medical consultations are still sparse. Most studies applied to health care are structured as a Prisoner's Dilemma game, putting in evidence the lack of variety in the studies in this area. Two studies are briefly described now.

Diamond et al. (1986) establish differences between game theory and decision theory. Contrasting with decision theory, game theory comprises more than one agent making a decision, and outcomes are interdependent on the actions taken by all agents. The clinical problem is based on a decision a doctor may have to take when dealing with liver diseased patients: patients may have one of two diseases, each one demanding different treatment paths. The hypothesis are: chronic progressive hepatitis and cirrhosis. The doctor has to decide whether to proceed with a treatment with steroids or ask for a biopsy. This latter decision will lead to a perfect diagnosis and appropriate treatment path but exposes the patient to a certain risk of dying. The patient may accept or reject the doctor's decision. Solving the problem with decision theory, *i.e.*, disregarding the patient's decision, the solution obtained was a deterministic strategy stating the doctor should always recommend a biopsy. In contrast, the equilibrium solution to this game obtained with game theory is in mixed strategies. It states that the doctor should recommend biopsy to five of every six patients, and only five out of every six patients should accept this recommendation.

The other study describes a different clinical situation. A doctor who follows a patient, does not know if the patient has a disease or not, and has to decide whether to recommend a specific treat-

ment assuming it is not possible to do more diagnostic testing. In turn, the patient may choose between accepting or rejecting the doctor's recommendation. Djulbegovic et al. (2015) solve the problem adopting the Prisoners' Dilemma structure. Interestingly, the authors add emotions to this game, namely trust, regret, guilt, and frustration. All of them account for the payoffs and have impact on any of diverse clinical outcomes. Since emotions were added to the game, players may change their payoff ordering depending on the values these emotions may be assigned with. For example, a patient who is not afraid of feeling regret may accept the treatment suggested without hesitation. Conversely, if a patient attaches more importance to avoiding regret, choosing to see other doctor instead of accepting the treatment may be considered a more valuable choice. This way, the equilibria solutions to this game depend on the how players value their emotions and the other parameters that constitute the payoffs. There can be equilibrium in dominant strategies or there can be an equilibrium in mixed strategies, depending on some conditions. Djulbegovic et al. (2015) also reinforces that it is not impossible to avoid a Prisoner's Dilemma game in a clinical interaction. For that it is imperative to reinforce trust between patients and physicians, for example by bringing closer their interests. Additionally, encouraging doctors to give clearer explanations and help patients managing the huge amounts of information they can access nowadays could be helpful to improve results.

The rest of the paper is structured as follows. Section 2 presents the model construction and the game structures to be analysed. Section 3 demonstrates the equilibria for the different game structures. The results are discussed in section 4. FInally, section 5 presents the conclusions and gives suggestions for future work.

2. Model

Every game theory model is built by defining the players, strategies, and the payoffs. The players in this game are the doctor - commonly a hepatologist - and the patient. The patient has two pure strategies: *Cooperation*, and *No Cooperation*. By cooperating, the patient follows the doctor's advice and complies with the requirements (most importantly staying abstinent). The patient may choose to not cooperate by drinking alcohol or stop following the recommendations in any other way (taking any forbidden risk behavior, missing appointments, among others). The doctor has three pure strategies: *Cooperation*, *No Cooperation T* (temporary), and *No Cooperation D* (definitive). The doctor's cooperation can be seen as a reward to the patient, keeping him in the liver transplantation waiting list (or introduce the patient's name if it was not there

yet). By not cooperating, the doctor excludes the patient from the list. This can be done temporarily, giving the patient a second chance. If by any reason the doctor permanently excludes the patient from the list, the game ends and the most likely outcome for the patient is death.

The payoffs are represented by the letter P if they refer to the patient, and D if they refer to the doctor. Furthermore, numbers are assigned regarding the actions taken. So, for the doctor 1 means cooperation, 2 means temporary non-cooperation, and 3 means definitive non-cooperation. For the patient, 1 means cooperation and 2 means non-cooperation. It is fundamental to sort the payoffs by order. Starting with the doctor's payoffs, one has $D_{11} > D_{22} > D_{32} > D_{21} > D_{31} > D_{12}$. From these, the only payoff considered positive with certainty is D_{11}. The negative payoffs are D_{21}, D_{31}, and D_{12}. Lastly, D_{22} and D_{32} are considered to be possibly negative or positive. For the patient, the payoffs order is $P_{11} > P_{12} > P_{21} > P_{22} > P_{31} > P_{32}$. The payoffs P_{11}, P_{12}, and P_{21} are considered to be positive, while P_{22}, P_{31}, and P_{32} are regarded as negative payoffs. The payoffs orders were obtained with the help of an experienced doctor, working in a reference hospital in Lisbon in the area of liver transplantation.

Similarly to what is done in Djulbegovic et al. (2015), the payoffs are expressed in terms of utilities and emotions. The letter U concerns the patient's utilities and the letter V refers to the doctor's utilities. U_1 and V_1 denote the utility the patient and the doctor respectively earn if the patient is kept on the list, eventually ending up with the transplantation being performed. U_2 and V_2 are utilities referring to the case when the patient is temporarily removed from the list. Finally, U_3 and V_3 pertain to the case when the patient is definitely excluded from the list. Naturally, $U_1 > U_2 > U_3$ and $V_1 > V_2 > V_3$. Both players may feel regret, R, frustration, F, and the doctor may also feel guilt G when cooperating with a non-cooperative patient. The emotions obtained in temporary decisions are distinguishable from the ones arising after definitive decisions using the superscript t or d, respectively. The patient additionally has two parameters representing the benefit (B) for staying alcohol abstinent and the harm (H) resulting from drinking alcohol. Lastly, parameters γ and β representing incentives to non-cooperation were added to the patient's and the doctor's payoffs, respectively. β represents how much the doctor values saving the liver graft to a patient with a better prognostic. Thus, this value will be added to the doctor's payoffs correspondent to the definitive non-cooperation. The value of γ represents the instant pleasure the patient gets by drinking (or not cooperating). Therefore, γ is added to the patient's

payoffs representing her/his non-cooperation. The payoffs are depicted in table 1.

Table 1: Representation of the payoffs in terms of utilities, for the doctor and for the patient.

Payoffs	
Doctor	$D_{11} = V_1$
	$D_{12} = V_1 - G \cdot (V_1 - V_2)$
	$D_{21} = V_2 - R_D^t \cdot (V_1 - V_2)$
	$D_{22} = V_2 - F_D^t \cdot (V_1 - V_2)$
	$D_{31} = V_3 - R_D^d \cdot (V_1 - V_3) + \beta$
	$D_{32} = V_3 - F_D^d \cdot (V_1 - V_3) + \beta$
Patient	$P_{11} = U_1 + B$
	$P_{12} = U_1 - H + \gamma$
	$P_{21} = U_2 - F_P^t \cdot (U_1 - U_2) + B$
	$P_{22} = U_2 - R_P^t \cdot (U_1 - U_2) - H + \gamma$
	$P_{31} = U_3 - F_P^d \cdot (U_1 - U_3) + B$
	$P_{32} = U_3 - R_P^d \cdot (U_1 - U_3) - H + \gamma$

To help understand the reasoning behind the payoffs, a few examples are now explained. The strategy pair (C, C) leads to the best payoffs, so it is associated with V_1 for the doctor and U_1 for the patient. The patient receives the additional health benefit (B) for cooperating. If the patient does not cooperate but the doctor does, the patient gets the utility of staying in the list (U_1) but with a subtracting element that represents the harm (H) that drinking carries. The pleasure of drinking γ is also added. Regret (R) is associated with the cases where the player did not choose the strategy that would be a best response to the other player's strategy. This can be seen as feeling regret for causing the other player to loose utility. As an example, in D_{21} the doctor regrets choosing *No Cooperation T* because the patient chose *Cooperation*, and it would have been better for both players if the doctor had chosen *Cooperation*. Note that the doctor's non-cooperation does not necessarily mean the doctor made a mistake, there might have been suspicions of a contraindication or any other cause. In turn, frustration is defined to represent a player's disappointment for not being able to persuade the other

player to cooperate and act as it would be best for herself/himself. To illustrate this, consider payoff D_{32}. Excluding the patient from the list results in V_3. Although the doctor probably took this decision to punish the patient's non-cooperation, there will still be a feeling of frustration for not being able to persuade the patient to cooperate. This is represented by F_D^d (the d refers to a definitive decision), multiplied by the loss in utility relative to V_1, $(V_1 - V_3)$. Apart from the frustration, the doctor receives an additional utility for preserving a liver graft and saving it to a patient with a better prognostic, represented by β.

To finish, it is important to describe D_{12}. This payoff refers to the case when the doctor is deceived by the patient. If a non-cooperative patient is rewarded with a liver transplantation there is a loss for society because the liver graft could have been allocated to a patient with a better prognostic. Thus, the doctor feels guilty (G). G is assumed to have the highest value of all the emotions. Regarding the other two emotions, and analogously for both players, regret (where $R^d > R^t$) is assumed to be greater than frustration (where $F^d > F^t$). This is considered to be a reasonable assumption since regret refers to an action taken by the player that affected negatively the other player. It is acceptable that one feels worse when the other is impaired by an action taken be her/him, than when she/he is not able to persuade the other to act according to her/his own interests. Naturally, definitive emotions are stronger than the temporary ones. Additionally, it seems reasonable to admit that $R^d - F^d > R^t - F^t$, that is, when the decision is definite the difference between regret and frustration is larger than when the decision is temporary. Concerning the benefits or harms that alcohol abstinence or consumption may bring to patient's health, it validated by an expert that the harm of drinking alcohol is greater in absolute value than the benefit that alcohol abstinence represents. In sum, $|H| > |B|$. This presumption is based on the fact that if the patient keeps drinking alcohol she/he will likely die, while staying alcohol abstinent is unlikely to lead to absolute healing or make the transplantation unnecessary. It may happen but only in a limited number of cases, while alcohol consumption at this stage is almost always fatal. In addition, H and B are lower than the values of the utilities U and V.

Let us now represent the strategic form game (table 2). Note that the first number represents the doctor's action and the second number the patient's action.

The strategic form game assumes decisions are taken simultaneously. This means that a player decides without knowing what the other player decided to do. Games can assume sequential dynam-

Table 2: Representation of the strategic form game.

		Doctor		
		C	NC_t	NC_d
Patient	C	P_{11}, D_{11}	P_{21}, D_{21}	P_{31}, D_{31}
	NC	P_{12}, D_{12}	P_{22}, D_{22}	P_{32}, D_{32}

ics, and be represented in their extensive form. Using an extensive form game is a more realistic approach to this problem. In such game, decisions are not taken simultaneously and this is closer to what happens in a medical consultation. In such games, there is a player deciding in the first place, and the other player replies afterwards. This gives us the chance to analyze the same game twice but with different playing orders, bearing different results. The possibility of having the doctor deciding before the patient raises a problem: if the doctor decides to definitively exclude the patient at first place, then the game does not even begin. For that situation, the payoffs are $D_{30} = \beta$, since the doctor saves a liver graft to other patient, and $P_{30} = 0$. This case may represent situations where patients' health status is already too serious for considering the possibility of being transplanted.

The extensive form games are represented in figures 1 and 2 in section 3. Figure 1 shows the game when the doctor decides first, and figure 2 shows the game when it is the patient taking the first step.

3. Model Equilibria
3.1. Simultaneous game
Let us start by analysing the strategic form game, and consider γ and β to take their lowest values so the previous payoffs order is respected.

Scanning the doctor's payoff order one notices that $D_{11} > D_{21} > D_{31}$ and that $D_{22} > D_{31} > D_{12}$, showing that the doctor never chooses *No Cooperation D*. Therefore, the doctor's strategy of permanently excluding the patient from the list is dominated. Now going through the patient's payoff order one has $P_{11} > P_{12}$, $P_{21} > P_{22}$, and $P_{31} > P_{32}$, so the patient always prefers to act cooperatively no matter what the doctor does. Hence, *Cooperation* is a dominant strategy for the patient. Given that the patient always cooperates, the doctor will also cooperate, and the strategy pair (C, C) is the only equilibrium in dominant strategies.

In order to determine the Nash equilibrium in pure strategies, it is necessary to find two strategies that are best replies to each other. The patient's best reply to a cooperative action from the doctor is to also cooperate (because $P_{11} > P_{12}$). The doctor's best reply to a cooperative action from the patient is also to cooperate (because $D_{11} > D_{21}$).

Therefore, the strategy pair (C, C) constitutes a Nash equilibrium. Checking for a non-cooperative move from the doctor, the patient will rationally choose to act cooperatively (because $P_{21} > P_{22}$ and $P_{31} > P_{32}$). Inversely, the doctor's best reply to a cooperative move from the patient is also a cooperative move (because $D_{11} > D_{21} > D_{31}$). If the patient chooses the strategy *No Cooperation*, the doctor will reply with a non-cooperative decision (because $D_{22} > D_{32} > D_{12}$). Thus, (C, C) is the only Nash equilibrium.

Varying β and γ may change this result. Let us define the boundary conditions for β and γ. Starting with γ, one has to check the necessary conditions to make the patient prefer to not cooperate. It is necessary to find the intervals of γ that change the payoffs order - $P_{11} > P_{12}$, $P_{21} > P_{22}$, and $P_{31} > P_{32}$. The resulting conditions are shown in table 3. It is easy to see that $\gamma_{NC_d} > \gamma_{NC_t} > \gamma_C$.

Table 3: Conditions for the payoffs order variation as a function of γ.

Boundary conditions for γ
$P_{12} > P_{11}$; $\gamma > B + H = \gamma_C$
$P_{22} > P_{21}$; $\gamma > B + H + (U_1 - U_2) \cdot (R_P^t - F_P^t) = \gamma_{NC_t}$
$P_{32} > P_{31}$; $\gamma > B + H + (U_1 - U_3) \cdot (R_P^d - F_P^d) = \gamma_{NC_d}$

The same reasoning can be applied to the doctor's payoffs. The parameter β is added to payoffs D_{31} and D_{32}, since they are the ones referring to a definitive exclusion of the patient. Once again, it is necessary to find the value intervals of β that will change the doctor's payoff order. When replying to a cooperative patient, the current payoff order is $D_{11} > D_{21} > D_{31}$. When dealing with a non-cooperative patient, the order is $D_{22} > D_{32} > D_{12}$. Therefore, one must find the values of β that will lead to $D_{31} > D_{21}$, $D_{31} > D_{11}$, and $D_{32} > D_{22}$. It is important to stress that even if $D_{31} > D_{21}$ the doctor will still rationally choose to cooperate. The resulting conditions for β are shown in table 4. It is possible to verify that $\beta_{NC} < \beta_C < \beta_{CC}$.

Table 4: Conditions for the payoffs order variation as a function of β.

Boundary conditions for β
$D_{32} > D_{22}$; $\beta > (V_2 - V_3) + F_D^d \cdot (V_1 - V_3) - F_D^t \cdot (V_1 - V_2) = \beta_{NC}$
$D_{31} > D_{21}$; $\beta > (V_2 - V_3) + R_D^d \cdot (V_1 - V_3) - R_D^t \cdot (V_1 - V_2) = \beta_C$
$D_{31} > D_{11}$; $\beta > (V_1 - V_3) + R_D^d \cdot (V_1 - V_3) = \beta_{CC}$

The Nash equilibria obtained are described now.

For $\gamma < \gamma_C < \gamma_{NC_t} < \gamma_{NC_d}$, the patient always cooperates. Evaluating the doctor's choices, one observes that there are also no incentives to not cooperate, except for when $\beta > \beta_{CC}$. So, (C, C) is a Nash equilibrium if $\gamma < \gamma_C$ and $\beta < \beta_{CC}$. For a high value of β, one may have $D_{31} > D_{11}$. This result can be due to the appearance of an absolute contraindication, for example the development of hepatocellular carcinoma. Therefore, (NC_d, C) is a Nash equilibrium.

Increasing γ to $\gamma_C < \gamma < \gamma_{NC_t} < \gamma_{NC_d}$, the patient experiences more pleasure by drinking, and feels tempted to not cooperate if it is known that the doctor chose to cooperate. This is why the previous Nash equilibrium (C, C) does not exist anymore. The only existent equilibrium for such γ is (NC_d, C) for $\beta > \beta_{CC}$.

If $\gamma_C < \gamma_{NC_t} < \gamma < \gamma_{NC_d}$, the patient prefers to not cooperate even if the doctor does not cooperate temporarily. The doctor does not cooperate temporarily if the patient does not cooperate for $\beta < \beta_{NC}$ (because $D_{22} > D_{32}$). This way, and if these parameter conditions are verified, (NC_t, NC) is a Nash equilibrium. Furthermore, analogously to the previous cases, if $\beta > \beta_{CC}$ the Nash equilibrium (NC_d, C) is once again obtained. For $\beta_{NC} < \beta < \beta_{CC}$ there is no Nash equilibrium.

Lastly, if γ takes the highest possible values $(\gamma_{NC_d} < \gamma)$, the patient never cooperates because alcohol provides a high level of pleasure. For $\beta < \beta_{NC}$, the doctor still prefers to exclude the patient temporarily. So, (NC_t, NC) is a Nash equilibrium. For doctors associated with higher $\beta_{NC} < \beta$ (making $D_{32} > D_{22}$), the preferred action to respond to the patient's non-cooperation is *No Cooperation D*. As the patient never cooperates, *No Cooperation* will also be the best response to the doctor's definitive non-cooperation, and (NC_d, NC) is a Nash equilibrium.

By restraining β to its lower values and vary γ to verify how the equilibria change along the different values of pleasure patients feel when drinking, may lead to simpler conclusions. This would allow to make the problem computationally more tractable. Additionally, it seems reasonable to exclude very high β other than very high γ, since it is very common to have patients struggling to keep the abstention. If this is done, then the doctor has a dominated strategy: *No Cooperation D*.

The results found meet the expectations, since $\gamma < \gamma_C$ implies that the patient is more prone to cooperate, and so is the doctor. The Nash equilibrium is then (C, C). For $\gamma_{NC_t} < \gamma$, it works exactly the opposite way and the Nash equilibrium is the strategy pair (NC_t, NC). However, there is an interval for γ where there is no Nash equilibrium:

$\gamma_C < \gamma < \gamma_{NC_t} < \gamma_{NC_d}$. This happens because, for such γ, $P_{12} > P_{11}$ and $P_{21} > P_{22}$, meaning that the patient feels enough pleasure when drinking to try to deceive a cooperative doctor. Yet, the patient cooperates if the doctor decides for a temporary exclusion. This temporary exclusion can then be seen as a warning, to which the patient responds cooperatively (for $\gamma < \gamma_{NC_t}$).

Let us now determine the conditions for the players' indifference regarding their strategies, thus accepting to randomize their decisions. In other words, compute the equilibrium in mixed strategies.

Starting with the doctor, one has to find the probability p, with which the patient cooperates and leads to the doctor's indifference, *i.e.*, to find p such that equation 1 is respected. The strategy *No Cooperation D* is omitted here because β is being restricted to its lowest values ($\beta < \beta_{NC}$). This way, $E_D[NC_d]$ is lower than $E_D[C] = E_D[NC_t]$ for such value of p, and the strategy is dominated.

$$E_D[C] = E_D[NC_t] \qquad (1)$$

Assuming the patient cooperates p percent of the time and acts non-cooperatively $(1-p)$ per cent of the time, and by multiplying the payoffs the doctor gets when the patient cooperates by p and the others by $(1-p)$, one obtains p, presented in equation 2.

$$p = \frac{G - F_D^t - 1}{G - F_D^t + R_D^t} \qquad (2)$$

Doing the same for the patient, if the doctor chooses *Cooperation* q per cent of the time and *No Cooperation T* $(1-q)$ per cent of the time, the patient's indifference is obtained when the expected payoffs for the patient of both strategies are equal, *i.e.*, if equation 3 is verified.

$$E_P[C] = E_P[NC] \qquad (3)$$

The value of q is then presented in equation 4.

$$q = 1 - \frac{\gamma - (B + H)}{(R_P^t - F_P^t) \cdot (U_1 - U_2)} \qquad (4)$$

In conclusion, the doctor randomizes between playing *Cooperation* and *No Cooperation T* if the patient plays *Cooperation* in p percent of the time. The patient randomizes between playing *Cooperation* and *No Cooperation* if the doctor plays *Cooperation* in q percent of the time. Obviously, the probabilities p and q must respect $0 < p < 1$ and $0 < q < 1$. From the initial payoffs order, it is possible to infer some conditions for the parameters that immediately show $0 < p$ (namely, $G > F_D^t + 1$). Moreover, it is obvious to conclude that $p < 1$. For the probability q, the condition is only respected if $\gamma_C < \gamma < \gamma_{NC_t}$. This is a reasonable result, since for lower γ the patient always cooperates and there is no equilibrium in mixed strategies. For higher γ,

the patient never cooperates, and once again there is no equilibrium in mixed strategies.

It is interesting to see which of the two players is more cooperative under the mixed strategies equilibrium, by comparing the values of p and q. The relation between these two probabilities is dependent on the value of γ, which, as already seen, is bounded by $\gamma_C < \gamma < \gamma_{NC_t}$. The results are:

- If $\gamma_C < \gamma < \frac{(R_D^t + 1) \cdot (R_P^t - F_P^t) \cdot (U_1 - U_2)}{G - F_D^t + R_D^t} + B + H$, then $q > p$ and the doctor will cooperate with more probability than the patient;

- If $\frac{(R_D^t + 1) \cdot (R_P^t - F_P^t) \cdot (U_1 - U_2)}{G - F_D^t + R_D^t} + B + H < \gamma < \gamma_{NC_t}$, then $p > q$ and the patient will cooperate with more probability than the doctor.

In short, for lower values of γ the doctor cooperates with higher probability, and for higher values of γ the patient cooperates with higher probability. Though this result may seem counterintuitive, recall that in a mixed strategies equilibrium the players must be indifferent between their strategies. For higher values of γ, the patient must cooperate with higher probability to maintain the doctor's indifference between *Cooperation* and *No Cooperation T*. Otherwise, the doctor tends to adopt the non-cooperative strategy with higher probability (because q decreases with the increase of γ). It is also possible to assess how the other parameters contribute to the probability with which players cooperate. This is done in the comparative statics analysis below.

The probability q is positively related with B and H. An increase in B and H work as incentives to cooperation for the patient, so it is likely that the doctor also cooperates with higher probability. Nevertheless, since we are dealing with mixed strategies, players' indifference must subsist. So, if the patient has incentives to cooperate, the doctor must act in order to counterbalance those incentives. Since $\gamma_C < \gamma < \gamma_{NC_t}$, $P_{12} > P_{11}$ which means that the patient does not cooperate if the doctor does so. Therefore, the doctor cooperating with higher probability leads the patient to feel tempted to not cooperate, keeping the indifference. The probability q decreases with γ, that works as an incentive for the patient to not cooperate. Once again, it makes sense that a doctor hesitates to cooperate with patients associated with higher γ, thus cooperating with less probability (lower q). In terms of indifference, the way the doctor has to counterbalance this incentive to non-cooperation is to not cooperate and make the patient cooperate (because $P_{21} > P_{22}$ for $\gamma < \gamma_{NC_t}$). The probability q is also positively related with $(R_P^t - F_P^t)$ and with $(U_1 - U_2)$. If the difference between regret and frustration increases, the patient values more cooperating. The same happens

if the difference between utilities increases. Analogously to the previous cases, the doctor will cooperate with higher probability to reverse the incentives for cooperation the patient gets. These effects are more important for patients with higher propensity to drink (increases with γ).

The same comparative statics may be performed for the probability of the patient's cooperation, p. An increase in G acts as an incentive for the doctor's non-cooperation, since it is an emotion that the doctor wants to avoid. To offset this pattern, the patient cooperates with higher probability p. If the patient feels the doctor will easily exclude him from the list, he gets more cooperative. Higher levels of R_D^t or F_D^t will promote the doctor's cooperation, hence unbalancing the equilibrium in mixed strategies and leading to a decrease in p. This patient's non-cooperative trend will boost the doctor to not cooperate, maintaining the indifference between the strategies.

3.2. Sequential game

Let us now compute the equilibria of the extensive form game. Figure 1 represents the extensive form game when the doctor plays first. The subgame perfect Nash equilibrium is obtained by backward induction.
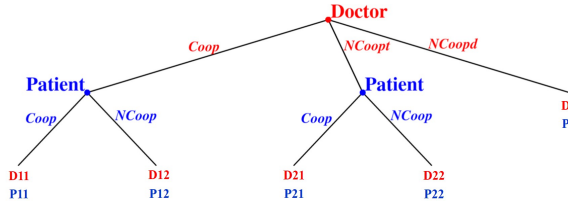

Figure 1: Tree representation of the extensive form game, illustrating the interaction between the doctor and the patient in a consultation, with the doctor acting first.

Let us begin by restricting β to its lowest values, so that the branches containing the doctor's definitive non-cooperative strategy can be ignored again.

Beginning with the game from figure 1, let us analyze how the patient replies to each doctor's decision. For $\gamma < \gamma_C$, the patient always cooperates, because $P_{11} > P_{12}$ and $P_{21} > P_{22}$. Given that $D_{11} > D_{21}$, the doctor replies with cooperation. Therefore, the equilibrium is the strategy pair (C, C). When γ exceeds γ_C but not γ_{NC_t}, the patient has incentives to try to deceive the doctor if he cooperates, which means $P_{12} > P_{11}$. When the doctor does not cooperate, the patient still prefers to cooperate (because $P_{21} > P_{22}$). Hence, the doctor chooses not to cooperate, given that $D_{21} > D_{12}$. The subgame perfect Nash equilibrium is then (NC_t, C). For any γ greater than γ_{NC_t}, the patient never cooperates ($P_{12} > P_{11}$ and $P_{22} > P_{21}$), so the doctor

chooses again to not cooperate ($D_{22} > D_{12}$). So, for $\gamma_{NC_t} < \gamma$, the subgame perfect Nash equilibrium is (NC_t, NC).

Let us now consider the tree referring to the game starting with the patient's decision. This tree is represented in figure 2.
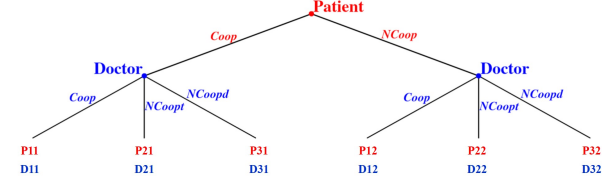

Figure 2: Tree representation of the extensive form game, illustrating the interaction between the doctor and the patient in a consultation, with the patient acting first.

For $\gamma < \gamma_C$, the patient has no incentives to deceive the doctor, always choosing *Cooperation*. The doctor also prefers to cooperate, since $D_{11} > D_{21}$. As a result, the equilibrium is the strategy pair (C, C). Increasing γ makes the patient feel tempted to choose *No Cooperation*. For $\gamma_C < \gamma < \gamma_{NC_t}$, the patient deceives the doctor if he cooperates, but acts cooperatively if the doctor decides to not cooperate. The doctor cooperates as a reply to a cooperative move ($D_{11} > D_{21}$) but does not cooperate if the patient chooses *No Cooperation* ($D_{22} > D_{12}$). Since $P_{11} > P_{22}$, the patient chooses to cooperate. Once again, the strategy pair (C, C) is the subgame perfect Nash equilibrium. It is possible to extend this reasoning to the remaining intervals of γ ($\gamma_{NC_t} < \gamma < \gamma_{NC_d}$ and $\gamma_{NC_d} < \gamma$). The patient must decide between cooperating and getting the payoff P_{11} or not cooperating and getting the payoff P_{22}. As $P_{11} > P_{22}$, the patient prefers to cooperate no matter what.

Summing up, the only subgame perfect Nash equilibrium for this game is the strategy pair (C, C), regardless of value of γ.

Let us now allow β to take higher values so that the doctor may have more incentives to not cooperate and the strategy *No Cooperation D* may no longer be dominated. Starting with the game in which the doctor decides first, one gets that if $\gamma < \gamma_C$ and $\beta < \beta_C$, then neither player has significant incentive to not cooperate and (C, C) is the only equilibrium solution to this game. Maintaining $\gamma < \gamma_C$ keeps the patient without incentive to not cooperate. Therefore, the doctor decides only by comparing the payoffs D_{11}, D_{21}, and D_{30}. As $D_{11} > D_{21}$ for any β, the doctor does not choose the strategy *No Cooperation T* when dealing with such low values of γ. For the interval $\beta_C < \beta < \beta_{CC}$, the equilibrium solution will depend on the relation between V_1 and β. If $V_1 > \beta$, the doctor's payoff for

mutual cooperation will be higher than the payoff for *No Cooperation D* ($D_{11} > D_{30}$), and the strategy pair (C, C) is a subgame perfect Nash equilibrium. Conversely, if $V_1 < \beta$ ($D_{11} < D_{30}$) the doctor will choose to exclude definitely the possibility of inserting the patient in the list and there is no game. Taking β to its highest values ($\beta > \beta_{CC}$) will make the doctor choose immediately *No Cooperation D*, and again the game is not played. For the last two cases, the players get D_{30} and P_{30} as payoffs. Increasing γ to $\gamma_C < \gamma < \gamma_{NC_t}$, the patient feels tempted to not cooperate more often. If the doctor cooperates, the patient chooses *No Cooperation* and this leads to the payoff combination (D_{12}, P_{12}). As this represents the worst payoff for the doctor, it is possible to conclude that the doctor does not cooperate for $\gamma_C < \gamma$. This leaves us with the choice between *No Cooperation T* and *No Cooperation D*. If the doctor chooses *No Cooperation T*, the patient chooses *Cooperation* (because $P_{21} > P_{22}$). If the doctor chooses *No Cooperation D*, the game does not even begin and the doctor gets $D_{30} = \beta$ as payoff. So, the doctor must take the decision by comparing D_{21} and D_{30}. If $D_{21} > \beta$, the doctor will choose *No Cooperation T* and induce the patient's cooperation. But if β takes values such that $D_{21} < \beta$, the doctor will choose the definitive non-cooperation strategy and there will be no game. Patients associated with higher γ values, high enough so that $\gamma_{NC_t} < \gamma < \gamma_{NC_d}$, tend to be much less cooperative. So it is expected that the doctor assumes a much more cautious attitude towards the patient. This leads us once again to the situation where the doctor must choose between the temporary and the definitive exclusion of the patient from the list. This time, the patient prefers to not cooperate if the doctor chose *No Cooperation T* ($P_{22} > P_{21}$). So, by choosing *No Cooperation T* the doctor gets D_{22}. Yet again, deciding to definitely not insert the patient in the list gets the doctor the payoff D_{30}. In a simple way, the doctor's decision will now be dependent on how the values D_{22} and $D_{30} = \beta$ are related. The doctor will choose *No Cooperation T* whenever $D_{22} > \beta$, and will choose *No Cooperation D* otherwise. Lastly, for even higher values of γ ($\gamma_{NC_d} < \gamma$), the results will be exactly the same.

Let us now perform the previous analysis for the game with the patient as first mover.

Starting with $\gamma < \gamma_C$, the subgame perfect Nash equilibrium is once again (C, C), except for $\beta_{CC} < \beta$. As the patient always cooperates for such γ, the doctor's decision is based on the comparison between D_{11}, D_{21}, and D_{31}. For $\beta < \beta_{CC}$, D_{11} is always the best payoff the doctor can get. This way, the strategy pair (C, C) is the equilibrium solution. The exception arises for $\beta_{CC} < \beta$, where $D_{31} > D_{11}$ and the doctor chooses *No Cooperation*

D even if the patient cooperated. For such case, the subgame perfect Nash equilibrium is the strategy pair (C, NC_d). For $\gamma_C < \gamma < \gamma_{NC_t}$, $P_{12} > P_{11}$ so the patient prefers to not cooperate in reply to a cooperative move from the doctor. However, if the patient chooses to not cooperate, the doctor chooses to reply by excluding the patient from the list (temporarily or definitely, depending on β). But if the patient cooperates, the doctor replies with cooperation for any $\beta < \beta_{CC}$. As $P_{11} > P_{22}$ and $P_{11} > P_{32}$, the patient chooses to cooperate and the doctor cooperates back. So, (C, C) is an equilibrium solution for $\gamma_C < \gamma < \gamma_{NC_t}$ and $\beta < \beta_{CC}$. For $\beta_{CC} < \beta$, one obtains the same subgame perfect Nash equilibrium as before - (C, NC_d). Using the previous reasoning in the entire range of γ, one reaches the conclusion that, as long as $\beta < \beta_{CC}$, the strategy pair (C, C) constitutes the only subgame perfect Nash equilibrium. The difference arises when $\beta_{CC} < \beta$, values for which the doctor always chooses the strategy *No Cooperation D*, regardless of what the patient might have chosen before. This way, the patient's decision is only based on the comparison between P_{31} and P_{32}. If $\gamma < \gamma_{NC_d}$, $P_{31} > P_{32}$ and the patient prefers to cooperate, resulting in the equilibrium (C, NC_d). For $\gamma_{NC_d} < \gamma$, $P_{31} < P_{32}$ and the patient chooses *No Cooperation*. Hence, the equilibrium when both parameters take the highest values is (NC, NC_d). Summarizing the results, for $\beta < \beta_{CC}$ the subgame perfect Nash equilibrium is (C, C), for any γ. For $\beta > \beta_{CC}$, the equilibrium will depend on γ. The patient anticipates the doctor's non-cooperation, but prefers to cooperate if $\gamma < \gamma_{NC_d}$. This leads to the equilibrium strategy pair (C, NC_d). Otherwise, if $\gamma_{NC_d} < \gamma$, then the patient prefers to not cooperate after anticipating the doctor's definitive non-cooperation, thus resulting in the subgame perfect Nash equilibrium (NC, NC_d).

4. Discussion

From the equilibrium in pure strategies, one concludes that patients and doctors associated with lower values of γ and β, respectively, tend to be more cooperative. But as these parameters are raised, the cooperation tends to disappear and be replaced by non-cooperation. Some benevolent doctors may temporarily exclude patients with higher levels of pleasure obtained by drinking, in order to focus the treatment on the patient's abstinence.

It would be interesting then, to find ways to shape the parameters in order to increase cooperation rates. It would not be recommendable to decrease β a lot, so that any patient would be maintained in the list, even if not cooperating. This could possibly lead to major losses for society. The ideal setting would be having patients associated with the lowest γ possible, but doctors associated with intermediate

β to maintain selectivity. By trying to decrease γ, one is making an attempt to deviate patients from drinking alcohol and alert them to the importance of following the rules of the process strictly. Patients are very often referred to communities such as Alcoholics Anonymous, but some may quit for several reasons. One could suggest a more incisive approach, such as private lectures where patients were shown shocking images of what alcohol does to health. Additionally, it could possibly be very helpful if patients going through this extremely difficult process could talk and listen to someone who successfully made through it. Having a cured patient, who got the liver transplant, giving speeches could inspire others.

The analysis of the equilibrium in mixed strategies yields the cooperation probability for both players that would make them accept to randomize their choices. For low values of γ the doctor cooperates with higher probability, and for higher values of γ it is the patient who cooperates with higher probability.

From the comparative statics analysis, it was shown that the probability, q, with which the doctor cooperates depends on the γ, as well as on B and H. Higher γ means that the patient has to cooperate with higher probability to maintain the doctors indifference. Both B and H are positively correlated with q, so an increase in each of the parameters (or both) works as an incentive to the patient's cooperation. Bridging the model with reality, this was expected. A doctor cooperates with higher probability if the patient is also more cooperative. The previously policy recommendations may also be applied to increase the patients' perception on how the parameters B and H are influenced by alcohol abstinence or not, and make patients more aware of the harms provoked by alcohol consumption. The value of p indicates the probability with which the patient cooperates. It is dependent basically only on the emotions felt by the doctor. As G increases, the doctor will be much more cautious to avoid being deceived by a patient. So, the patient has to cooperate with higher probability to be kept in the list. The way a doctor interacts with a patient may be a key aspect for the patient's adherence to the recommendations. If the doctor shows real commitment, and makes the patient feel respected and esteemed, the patient will definitely feel worse if ending up disappointing the doctor. Talking to the patient, asking about his personal and professional life is definitely a great hint to make the patient open up with the doctor. But more important than asking these information on the first consultation, is to remember that information the next time the patient comes. This way, the patient feels the doctor cares about him, and is more likely to cooperate.

The extensive form game may simulate the reality in a closer way, since decisions are not taken without knowing what the other player is doing. For example, the patient usually knows if he was excluded from the list. Two different games were constructed because it is not straightforward to establish a beginning moment for this game. It may be with the doctor's decision of whether to integrate the patient, after analyzing the diagnostic exams; yet, it may also be considered to begin much before that. The patient going to the consultation has a long track record, and it may be considered that the patient was given the chance to cooperate before the first consultation. The results obtained differ in a very interesting way.

Remarkably, the patient deciding first fosters cooperation, and the game ends up having only one subgame perfect Nash equilibrium, (C, C). This can be explained by the fact that deciding first gives the patient a first-mover advantage - the chance to anticipate the doctor's choices. When the doctor is the first to decide, he gets to choose what is best for him taking into account the patient's type (defined by the value of γ). To wrap it up, doctors associated with high values of β are much more prone to not cooperate with patients and the probability that they will not give patients a chance to enter the list is high. Doctors will be much more cautious with patients associated with high values of γ, thus resulting in much less cooperation. For $\gamma_C < \gamma < \gamma_{NC_t}$ and $\beta < D_{21}$, the doctor may induce the patient to cooperate by choosing the temporary non-cooperative strategy. The conclusiosn drawn from the extensive form game suggest that an empowered patient tends to be more cooperative. It could be useful to make the patient understand that her/his behaviors are fundamental for her/his survival. Showing the patient that it is the doctor who is deciding based on what she/he did, and not the other way round, could lead to more cooperative patients. Instead of giving them orders or requisites they must comply with, doctors could give them options to make.

5. Conclusion

Although game theory is widely applied in many study fields, there is still little application in health care issues, particularly to model the medical consultation. This paper applied game theory with the goal of coming up with results that would improve liver transplantation candidates' adherence to doctors' recommendations.

The liver transplantation process is very complex, especially because it involves a specific type of patients going through an extremely delicate period in their lives. The main problem in all this process is the unbalance between the scarce liver grafts available for transplantation and the number of patients

that are in need of one. This unbalance demands that the patients considered eligible for liver transplantation are selected after passing a very strict series of requirements. Knowing this, patients will try as hard as possible to be considered eligible, even if that implies sometimes trying to deceive doctors.

Defining the payoffs for both players is a difficult task because one is trying to quantify abstract concepts. Additionally, and as emotions play a crucial role in processes like this, they had to be taken into account. Moreover, the values for the benefit or harm to the health of the patient are also difficult to measure precisely. Parameters γ and β were inserted to replicate possible incentives to non-cooperative behaviours.

The parameter that can be best managed is γ. It may be possible to help the patient experience lower values of pleasure when drinking, thus reducing the value of γ she/he is associated with. This paper has shown that patient's cooperation (and consequently the cooperative equilibrium) occurs for low values of γ. If it is possible to lower this parameter, then the results obtained advise to do so. As an example, talks with fellow patients who successfully went through the transplantation process could work as an eye-opening experience. Additionally, it is not uncommon to see alcoholic patients totally abandoned by their families, patients who lost their jobs, and so on. A crucial pillar for this type of patients is life stability, and having their time occupied. The more occupied a patient is, the less time she/he will spend drinking or thinking how enjoyable a drink would feel. Many patients suffering from alcoholic liver disease belong to the socioeconomically active age group. These patients can be encouraged to stay abstinent if they are shown they are able to work, or if they are taught a task in order to find a job. In Germany there is a law stating that "All employers (public and private) with a workforce of 20 employees or more are required to fill 5% of their jobs with severely disabled employees" (Kock, 2004). Following this example, one could think of a sort of courses or workshops where alcoholic patients could learn some tasks and then be hired by some specific companies. This way, patients would rebuild their lives, whether it is socially or professionally.

Although the study is completed, there are some suggestions that may lead to very important extensions in future works. These games were analyzed assuming perfect and complete information scenarios. By this, one means that players watch the other player's moves and both players' type and payoffs are common knowledge. Yet, this does not correspond to what happens in real life very often. Medicine is a credence good, and the doctor is more (and better) informed than the patient, and she/he might decide how much of that information to give to the patient. Additionally, the patient may possess information that would be useful for the doctor but decide to hide it for any reason. Thus, there may be asymmetry of information, and Signaling games may be a useful tool to solve such games. As an example, the doctor may not tell the patient she/he was excluded from the list, although this is not very common. Another example could be the patient hiding that she/he drank alcohol, which much more common. In this game, γ and β were common-knowledge for both players, but it might not be what happens in reality.

Lastly, the same procedured applied in this paper could be employed to other diseases, such as obesity, for instance.

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