Stochastic Prisoner's Dilemma

January 1, 2020

Game Rules

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D	b_1	0

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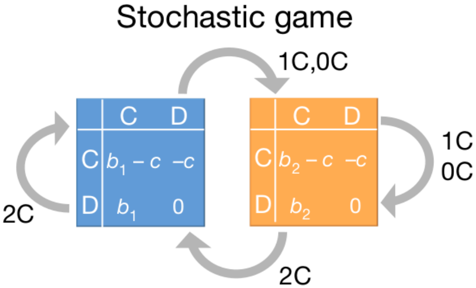

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5. ϵ chance to screw up.

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2. L learns from R, with $\rho = \frac{1}{1+e^{-\beta(\Pi_R - \Pi_L)}}$.
3. Or Mutates with γ

Projects

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4. "What happens when an unstoppable force meets an immovable object?"

Guideline

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3. Team Work!

Analytical Calculation for repeated games

States

1. We define a state $v_t = \{w_t, {a_t}^{(1)}, {a_t}^{(2)}\}$ (t : time-step) based on: The world we are currently in, the action of the first player and the action of the second player.
2. For example 1CD is when where are in the good world, player one cooperates and player two, defects.
3. Overall we have 8 states: [1CC, 1CD, ..., 2CC, 2CD, ..., 2DD]

Analytical Calculation for repeated games

Transition Matrix

1. The probability of going from $v_1 = \{w_1, {a_1}^{(1)}, {a_1}^{(2)}\}$ to $v_2 = \{w_2, {a_2}^{(1)}, {a_2}^{(2)}\}$ is dependent on the strategies ($S^{(1)}$ & $S^{(2)}$) of the two individuals, and also world transition Q (how the world reacts).
2. $P_{\{w_1,{a_1}^{(1)},{a_1}^{(2)}\}\rightarrow\{w_2,{a_2}^{(1)},{a_2}^{(2)}\}} = S^{(1)}({a_2}^{(1)}|{a_1}^{(1)}, {a_1}^{(2)}) \times S^{(2)}({a_2}^{(2)}|{a_1}^{(1)}, {a_1}^{(2)}) \times Q(w_2|{a_1}^{(1)}, {a_1}^{(2)})$
3. P can be described as a matrix.
4. State v can be described as a vector.
5. Multiplying v by P will take the state to the next time-step.

$$
P = \begin{array}{c} \\ 1CC \\ 1CD \\ ... \\ 2DD \end{array}
\begin{array}{c} \begin{array}{cccc} 1CC & 1CD & ... & 2DD \end{array} \\
\begin{bmatrix}
P_{\{1CC \to 1CC\}} & P_{\{1CD \to 1CC\}} & \cdot & P_{\{2DD \to 1CC\}} \\
P_{\{1CC \to 1CD\}} & P_{\{1CD \to 1CD\}} & \cdot & P_{\{2DD \to 1CD\}} \\
. & . & \cdot & . \\
P_{\{1CC \to 2DD\}} & P_{\{1CD \to 2DD\}} & \cdot & P_{\{2DD \to 2DD\}}
\end{bmatrix} \end{array} \tag{1}
$$

$$
v_{t+1} = P v_t \tag{2}
$$

1. Where is the final State?
2. When we get to state $v_* = P v_*$ (so we are at a stationary state.)
3. Because of the ϵ it can be proven that such state exists.
4. v_* is the eigenvector of P for eigenvalue 1.
5. After we get to v_* we will stay there forever, so all of the reward should be calculated based on v_*

Analytical Calculation for repeated games

Overview

1. We construct transition matrix P based on two strategies.
2. We calculate its eigenvector for eigenvalue 1 and call it v_*
3. v_* is the stationary state, so we calculate each player's reward based on this state.

References

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