Schelling Model on 2D Grid and Erdős–Rényi graph

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INTRODUCTION

Auto-Segregation is a prominent issue in mutli-ethnic cities (1), which has intensive cultural and economical impacts (2); Therefore it would be both interesting and beneficial to study this issue.

Considering the barriers in the way of conducting an actual social experiment with a human population for analyzing such phenomena, theoretical and computational methods have been proposed to predict and describe Auto-Segregation.

Schelling model (3), one such method, is an agent based model which by attributing a certain degree of autonomy to the individuals and devising relatively simple interactions, predicts the patterns of segregation over a network. In this network, each node denotes a house, and the existence of an edge between two nodes, expresses a neighboring relationship between the respective houses. Usually a two-dimensional grid is utilized as a network for this purpose, due to its resemblance of different neighborhoods in a city (which is also a two dimensional surface); But executing this model on an erdős–rényi would also be interesting, and can help us figure out if the lack of spatial features in this graph as well as lower clustering coefficients compared to the grid, will have a substantial effect on the segregation; on the other hand, considering non-spatial networks may be helpful to study the apparent segregation in the social media which manifests itself as filter bubbles (4).

MODEL

Dynamics

There are various implementations of the Schelling model (5), but the baseline is that individuals of two different ethnicities will move to an empty house if there aren't as much people with the same ethnicity neighboring them as they desire. For the purpose of this report, we consider the following setup. The variables we consider are: 1. N: The size of the network. 2. d: Desired similar neighbors. 3. $occupancy\ rate$: Number of houses with a resident. 4. $population\ disparity$: The racial population disparity. Unless stated otherwise, we consider the $occupancy\ rate$ to be 0.95 and $population\ disparity$ to be 0, i.e. ethnicties have equal populations. The initial location of the agents will be chosen randomly.

The dynamic rules at each time-step will be:

1. For each agent i if and only if for $s_i = \frac{similar\ neighbor\#}{different\ neighbor\#+similar\ neighbor\#}$, $s_i < d$ they will be considered satisfied. Agents with 0 different neighbors will also be considered satisfied.
2. A randomly chosen unsatisfied agent will move to a randomly chosen empty house.

The state of the system gets updated using an asynchronous updating scheme, meaning that the nodes' positions won't be updated at the same time.

Also our main order parameters, $S = \overline{s}$ (averaging over all nodes) which can vary between 0 & 1, indicates the degree of segregation in the network.

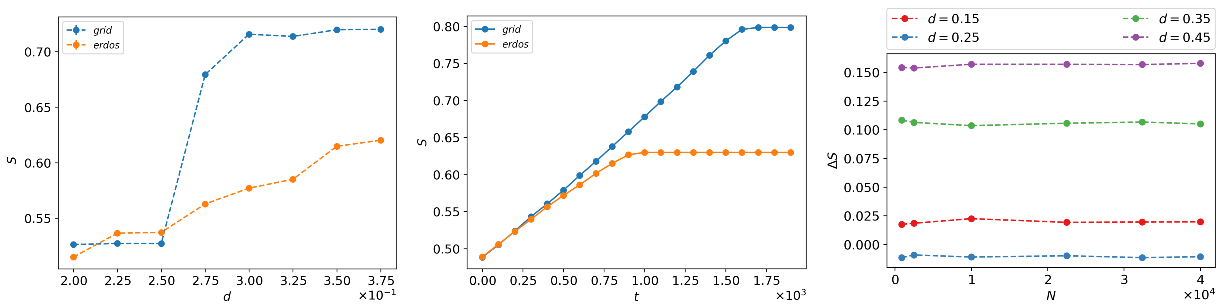
Network Topology

We consider a 2D grid with diagonal edges, to maintain a uniform degree distribution, we make use of the periodical boundary condition.
To obtain comparable results, we set the connectivity probability for the case of Erdős–Rényi graph, such that it also attains a mean degree distribution of 8.

RESULTS

Phase Transition

By employing d (Similar neighbors desired) as a control parameter, and sweeping the range between 0.20 and 0.39, for $N = 10^4$ In Fig. 2a we can observe an abrupt increase in the value of S in the case of the grid graph, which is not present for erdős–rényi. These results can be due to the fact that the spatial features of a grid lay the foundation for segregated communities to get formed. The tendency to have about 0.275 (more than 2 neighbors) of agents' neighbors to be similar to them, will make the unsatisfied agents to move to places with more than 2 similar neighbors. This issue will initiate the formation of ethnic clusters, because *satisfied neighbors create fertile slots for their peers in mutually neighboring houses* and also push the non-similar ones out of those houses by dissatisfying them. The formation of these clusters also leads to a decrease in the density of that particular ethnicity in the rest of the graph, so the remaining agents (which were hard-to-please in the first place) would have to move to the cluster to get satisfied and even if they were initially satisfied they will get dissatisfied by their peers' immigration. Before long we will have two segregated



(1a) **(1b)** **(1c)**

Figure 1. a) Steady state value of S (segregation) for $N = 10^4$. (50 realizations each) b) Time evolution of S for $N = 10^4$ (10 realizations each). c) Discrepancy between steady state S of grid and erdős–rényi graphs. due to the small size, error bars are not visible. (50 realizations each).

neighborhoods, with many agents fully or mostly covered by similar agents, which is far higher than what they aimed for. We take note that even for the initial random state, the value of S is about 50%.

On the other hand, while it's common for two neighbors to have mutual neighbors (triangle clustering) or to have two different neighbors that are neighbors themselves (square clustering), it's rare in the case of erdős–rényi. (Refer to Fig. 2). Therefore it's not virtually possible to have agents assemble a community in a subset of erdős–rényi since the network topology doesn't allow that. A satisfied agent in the case of erdős-rényi doesn't create new promising slots for its similar friends to land on, because the *satisfied agent and in its "satisfiers" don't share neighboring houses* this matter helps the simulation on this network to reach the steady state at a lower S.

Time Evolution

To get a better insight of the different results for the simulations over the two networks, we calculate the number of satisfied agents at each time-step for both graphs.

As it can be observed in Fig. 1b the grid graph takes more time-steps to get to the equilibrium state. Considering the fact that in each time-step a dissatisfied individual relocates, we observe that it's harder for agents in a grid to find satisfying locations, and also already satisfied agents will be more prone to becoming unsatisfied. This result also verifies the previous conclusion that the effect of clustering of peers, makes it harder for agents to become/stay satisfied outside of the cluster, due to the immigration of their satisfiers to the giant cluster.

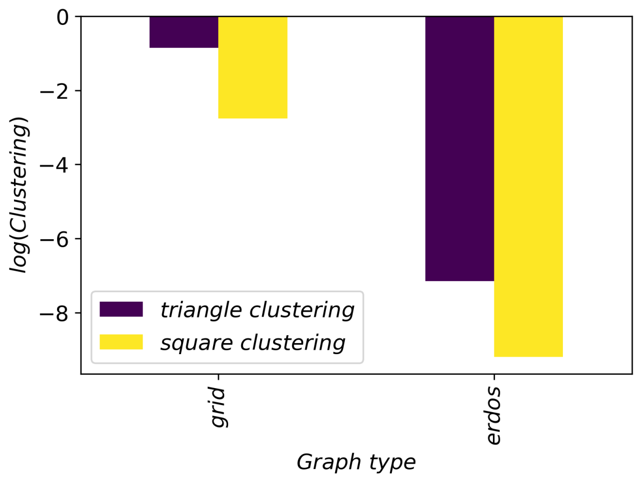


Figure 2. Comparing the logarithm of the average of square and triangle clusters for Grid and Erdős–Rényi graph, for the case of the Erdős–Rényi, 100 different realizations have been averaged, due to the small size, error bars are not visible.

Robustness with respect to the network size

In order to investigate whether our results hold for other network sizes, we conduct the simulation over a range of N values and calculate $\Delta S = S_G - S_E$, where G and E indicate the results for the two different graphs. We perform this analysis for different d values, both lower and higher than the observed abrupt jump in S_G, in Fig. 1c. We observe that the discrepancy between the results of the two graphs gets a spike near the previously observed transition point, for all of the N values. Therefore we conclude that our results are robust with respect to the network size.

DISCUSSION

In summary we have studied the schelling model on grid and erdős–rẽnyi graphs. We have observed that individuals' desire to reside besides their peers d (quantified by the percentage of similar neighbors they wish for) when passed a certain value will lead to an abrupt increase in the average segregation S for the grid network, while erdős–rẽnyi displays a smooth increase.

By calculating the clustering coefficients of these two networks and also studying the time evolution of their dynamics, we conclude that due to the high clustering coefficients of the grid network, satisfied neighboring peers turn their mutual vicinity to fertile spaces for the rest of their peers and push out the non-similar agents, leading to a large segment of agents gaining more similar neighbors than they aimed for, which causes a discrepancy between segregated and non-segregated regimes.

On the other hand, the neighboring satisfied agents in the erdős–eẽnyi graph which rarely share other first or second neighbors, don't show such characteristics and therefore in their case, S increases smoothly.

We also showed that our conclusion holds for different values for graph size N.

REFERENCES

[1]C. U. Rodrigues, "Angolan cities: urban (re) segregation?," in *African cities*, pp. 37–53, Brill, 2009.

[2]S. Lieberson, "The impact of residential segregation on ethnic assimilation," *Social forces*, vol. 40, no. 1, pp. 52–57, 1961.

[3]T. C. Schelling, "Dynamic models of segregation," *Journal of mathematical sociology*, vol. 1, no. 2, pp. 143–186, 1971.

[4]D. Boyd, "Self-segregation: how a personalized world is dividing americans," Jan 2017.

[5]E. Hatna and I. Benenson, "Combining segregation and integration: Schelling model dynamics for heterogeneous population," *arXiv preprint arXiv:1406.5215*, 2014.