Finite automata and formal languages
Assignment 3

DIT323 (Finite automata and formal languages)
at Gothenburg University

Sebastian Pålsson

Last updated: **February 11, 2024**

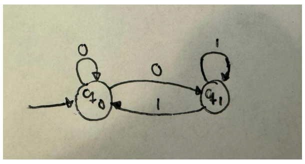
1

Let $\Sigma = \{0, 1\}$

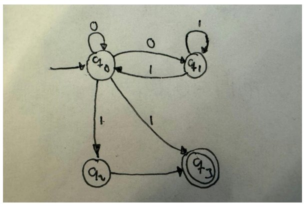
Let $R = (0 + 01^*)^*(\varepsilon + 1)1(\varepsilon + 0 + 1)^*$

Let's construct a NFA for R:

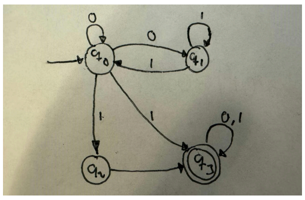
1. Construct a NFA for $(0 + 01^*)^*$



2. Add $(\varepsilon + 1)1$ to the NFA



3. Add $(\varepsilon + 0 + 1)^*$ to the NFA



The NFA for R is the thus following:

```
      0       1
→ q0 {q0 q1} {q2 q3}
  q1 {}      {q0 q1}
  q2 {}      {q3}
* q3 {q3}    {q3}
```

2

The regular expression is: $R = 1(1+01)^+$

- The "1" in the beginning makes it so that the string must start with a 1.
- The "+" makes it so $|w| \geq 2$
- $(1+01)$ makes it so it always ends with a 1 and two zeroes never appear after one another.

3

Let the following table represent the NFA:

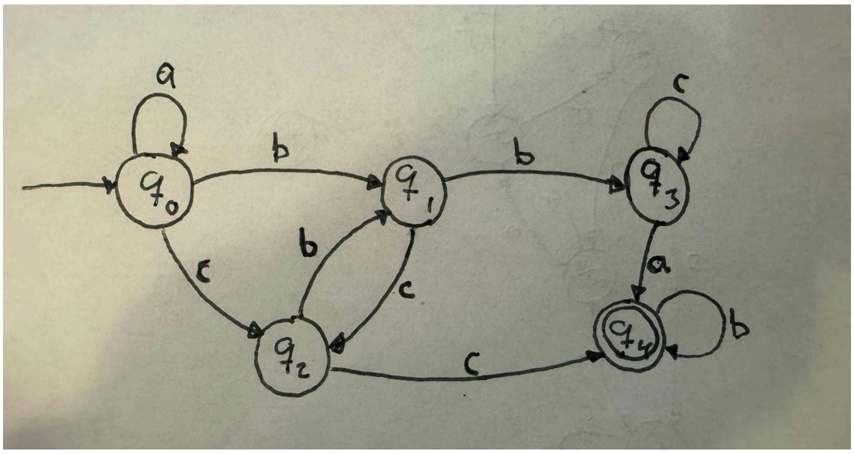
State	ε	a	b
$\rightarrow s_0$	$\emptyset$	s_1	$\{s_0, s_2\}$
s_1	s_2	s_4	s_3
s_2	$\emptyset$	$\{s_1, s_4\}$	s_3
s_3	s_5	$\{s_4, s_5\}$	$\emptyset$
s_4	s_3	$\emptyset$	s_5
$*s_5$	$\emptyset$	s_5	s_5

We then define the following DFA by subset construction:

State	a	b
$\rightarrow s_0$	$\{s_1, s_2\}$	$\{s_0, s_2\}$
$\{s_1, s_2\}$	$\{s_1, s_2, s_3, s_4, s_5\}$	$\{s_3, s_5\}$
$\{s_0, s_2\}$	$\{s_1, s_2, s_3, s_4, s_5\}$	$\{s_0, s_2, s_3, s_5\}$
$*\{s_1, s_2, s_3, s_4, s_5\}$	$\{s_1, s_2, s_3, s_4, s_5\}$	$\{s_3, s_5\}$
$*\{s_3, s_5\}$	$\{s_3, s_4, s_5\}$	$\{s_5\}$
$\{s_0, s_2, s_3, s_5\}$	$\{s_1, s_2, s_3, s_4, s_5\}$	$\{s_0, s_2, s_3, s_5\}$
$*\{s_3, s_4, s_5\}$	$\{s_3, s_4, s_5\}$	s_5
s_5	s_5	s_5

4

Transition diagram for the NFA:



Let’s define the following equations for the NFA:

$$\begin{aligned} q_0 &= \varepsilon + q_0 a \\ q_1 &= q_0 b + q_2 b \\ q_2 &= q_0 c + q_1 c \\ q_3 &= q_1 b + q_3 c \\ q_4 &= q_3 a + q_4 b + q_2 c \end{aligned} \tag{1}$$

The goal is to find the regular expression for the final state, q_4.

For q_0 we have:

$$\begin{aligned} q_0 &= \varepsilon + q_0 a \\ &= \varepsilon a^* \qquad \text{(Arden's lemma)} \end{aligned} \tag{2}$$

For q_1 we have:

$$\begin{aligned} q_1 &= q_0 b + q_2 b \\ &= q_0 b + (q_0 c + q_1 c) b \qquad \text{(Substitute } q_2\text{)} \\ &= q_0 b + q_0 cb + q_1 cb \\ &= (q_0 b + q_0 cb)(cb)^* \qquad \text{(Arden's lemma)} \end{aligned} \tag{3}$$

For q_2 we have:

$$\begin{aligned} q_2 &= q_0 c + q_1 c \\ &= q_0 c + (q_0 b + q_0 cb)(cb)^* c \qquad \text{(Substitute } q_1\text{)} \end{aligned} \tag{4}$$

For q_3 we have:

$$\begin{aligned} q_3 &= q_1 b + q_3 c \\ &= q_1 b c^* \end{aligned} \qquad [5]$$

Now we solve q_4:

$$\begin{aligned}
q_4 &= q_3 a + q_4 b + q_2 c \\
&= (q_3 a + q_2 c) b^* && \text{(Arden's lemma)} \\
&= ((q_1 b c^*) a + (q_0 c + (q_0 b + q_0 c b)(cb)^* c) c) b^* && \text{(Substitute } q_3 \text{ and } q_2\text{)} \\
&= (q_1 b c^* a + q_0 cc + q_0 b (cb)^* cc + q_0 cb (cb)^* cc) b^* && \text{(Simplify)} \\
&= (((q_0 b + q_0 cb)(cb)^*) b c^* a + q_0 cc + q_0 b (cb)^* cc + q_0 cb (cb)^* cc) b^* && \text{(Substitute } q_1\text{)} \quad [6] \\
&= (q_0 b (cb)^* b c^* a + q_0 cb (cb)^* b c^* a + q_0 cc + q_0 b (cb)^* cc + q_0 cb (cb)^* cc) b^* && \text{(Simplify)} \\
&= (\varepsilon a^* b (cb)^* b c^* a + \varepsilon a^* cb (cb)^* b c^* a + \varepsilon a^* cc + \varepsilon a^* b (cb)^* cc + \varepsilon a^* cb (cb)^* cc) b^* && \text{(Substitute } q_0\text{)} \\
&= (a^* b (cb)^* b c^* a + a^* cb (cb)^* b c^* a + a^* cc + a^* b (cb)^* cc + a^* cb (cb)^* cc) b^* && \text{(Remove } \varepsilon\text{)} \\
&= a^* (b (cb)^* b c^* a + cb (cb)^* b c^* a + cc + b (cb)^* cc + cb (cb)^* cc) b^* && \text{(Simplify)}
\end{aligned}$$