Finite automata and formal languages
Assignment 1

DIT323 (Finite automata and formal languages)
at Gothenburg University, Spring 2024

Sebastian Pålsson

Last updated: **January 25, 2024**

1

Question: Prove using induction that, for every finite alphabet, Σ, $\forall n \in \mathbb{N}.\ |\Sigma^n| = |\Sigma|^n$.

2

Question: Define a language S containing words over the alphabet $\Sigma = \{a, b\}$ inductively in the following way:

- The empty word is in S: $\varepsilon \in S$
- If $u, v \in S$, then $auavb \in S$.
- If $u, v, w \in S$, then $buavaw \in S$.

2.1

Question: Use recursion to define two functions $\#a, \#b \in \Sigma^* \to \mathbb{N}$ that return the number of occurrences of a and b, respectively, in their input.

2.2

Question: Use induction to prove that $\forall w \in S.\#a(w) = 2\#b(w)$.

Hint: You might want to show that $\#_a(auavb) = 2 + \#_a(u) + \#_a(v)$. How do you prove this? This property follows from a lemma that you can perhaps prove by induction: $\forall u, v \in \Sigma^*.\#_a(uv) = \#_a(u) + \#_a(v)$

3

Question: Let $\Sigma = \{0\}$ and define $f, g, h \in \Sigma^* \to \mathbb{N}$ recursively in the following way:

- $g(\varepsilon) = 1$
  $g(0w) = |w| + g(w) + -h(w)$
- $h(\varepsilon) = 0$
  $h(0w) = |w| + g(w)$
- $f(\varepsilon) = 1$
  $f(0w) = h(w) = 2g(w)$

3.1

Question: Compute the values of $f(00), g(00), h(00), f(000), g(000), h(000), f(0000), g(0000), h(0000)$

3.2

Question: Prove that $\forall n \in \mathbb{N}.f(0^n) = 1 + n$.

Hint: First prove that $\forall n \in \mathbb{N}.g(0^n) = 1 \land h(0^n) = n$.