Excercise 3

Contents

question

INSTITUTE FOR RESEARCH IN FUNDAMENTAL SCIENCES IPM پژوهشگاه‌دانش‌های‌بنیادی

SNS 2023

Hands-on Exercise: # 3

Build a Hodgkin-Huxley model neuron by numerically integrating the equations for V, m, h, and n given below (see also chapter 5 of the textbook):

$$c_m \frac{dV}{dt} = -i_m + \frac{I_e}{A},$$

where

$$i_m = \overline{g}_L(V - E_L) + \overline{g}_K n^4 (V - E_K) + \overline{g}_{Na} m^3 h (V - E_{Na}).$$

and

$$\tau_n(V)\frac{dn}{dt} = n_\infty(V) - n,$$

$$\tau_n(V) = \frac{1}{\alpha_n(V) + \beta_n(V)}$$

and

$$n_\infty(V) = \frac{\alpha_n(V)}{\alpha_n(V) + \beta_n(V)},$$

(and similar equations for m and h), with

$$\alpha_n = \frac{.01(V+55)}{1-\exp(-.1(V+55))} \qquad \beta_n = 0.125\exp(-0.0125(V+65)),$$

$$\alpha_m = \frac{.1(V+40)}{1-\exp(-.1(V+40))} \qquad \beta_m = 4\exp(-.0556(V+65))$$

$$\alpha_h = .07\exp(-.05(V+65)) \qquad \beta_h = 1/(1+\exp(-.1(V+35))),$$

In these equations, time is in ms and voltage is in mV. Take $c_m = 10$ nF/mm^2, and as initial values take: $V = -65$ mV, $m = 0.0529$, $h = 0.5961$, and $n = 0.3177$. The maximal conductances and reversal potentials used in the model are $\overline{g}_L = 0.3$ mS/mm^2, $\overline{g}_K = 36$ mS/mm^2, $\overline{g}_{Na} = 120$ mS/mm^2, E_L = -54.387 mV, E_K = -77 mV and E_{Na} = 50 mV. Use an integration time step of 0.1 ms.

1) I. Plot the voltage-dependent functions n, m and h in steady state as a function of V .
   II. Plot the voltage-dependent time constants of n, m and h as a function of V .

2) Plot voltage V as function of time , by simulating the injection of a suitable short (try some values!) external current $I_e(t)$, starting at t = 5ms.

3) Apply constant input from 2 ms to 120 ms and see the result. Increase the sodium conductance 5 times and then see the result.

4) Apply inhibitory input from 2 ms to 7 ms and then set the input current to 0. What happens?

Part 1

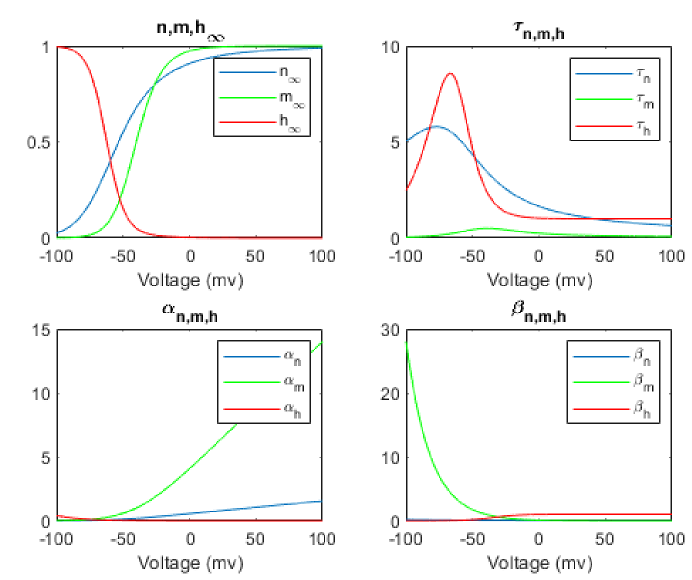
- I. Plot the voltage-dependent functions n, m and h in steady state as a function of V .
- II. Plot the voltage-dependent time constants of n, m and h as a function of V .

```
clc
close all
clear

total_data_points = 2000;
dt = 0.1;              %ms
t = (0:total_data_points) * dt;
cm = 10;              %nF/mm^2
V_init = -65;       %mv
m_init = 0.0529;
h_init = 0.5961;
n_init = 0.3177;
g_L = 0.3;           %mS/mm^2
g_K = 36;            %mS/mm^2
g_Na = 120;        %mS/mm^2
E_L = -54.387;  %mv
E_K = -77;          %mv
E_Na = 50;         %mv

V = linspace(-100, 100, total_data_points);

n(1) = n_init;
m(1) = m_init;
h(1) = h_init;

for i = 1:total_data_points
      alpha_n(i) = 0.01*(V(i) + 55) / (1 - exp(-0.1*(V(i) + 55)));    beta_n(i) = 0.125*exp(-0.0125*(V(i) + 65));
      alpha_m(i) = 0.1*(V(i) + 40) / (1 - exp(-0.1*(V(i) + 40)));     beta_m(i) = 4*exp(-0.0556*(V(i) + 65));
      alpha_h(i) = 0.07*exp(-0.05*(V(i) + 65));                           beta_h(i) = 1/(1 + exp(-0.1*(V(i) + 35)));

      n_inf(i) = alpha_n(i)   / (alpha_n(i) + beta_n(i));
      m_inf(i) = alpha_m(i) / (alpha_m(i) + beta_m(i));
      h_inf(i) = alpha_h(i)  / (alpha_h(i) + beta_h(i));

     Tau_n(i)  = 1 / (alpha_n(i) + beta_n(i));
```

```
    Tau_m(i) = 1 / (alpha_m(i) + beta_m(i));
    Tau_h(i)  = 1 / (alpha_h(i) + beta_h(i));

end
```

let's plot alpha_n,m,h & n,m,h _ inf & Taw n,m,h

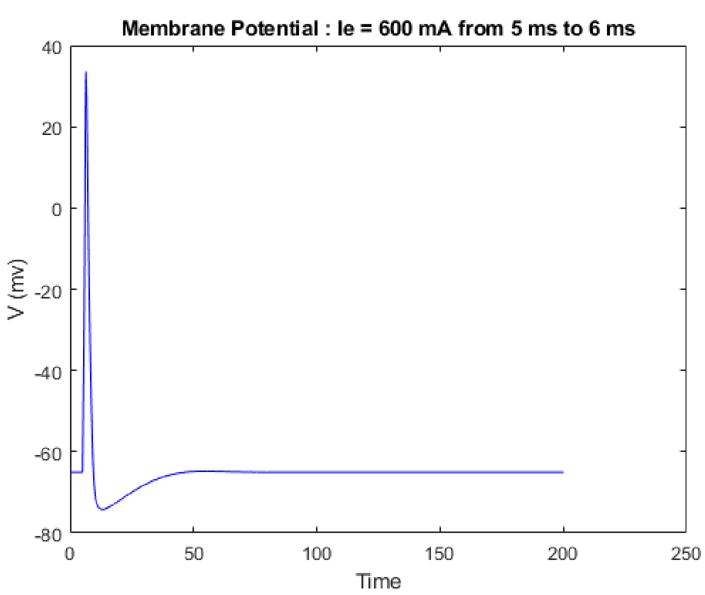
```
figure
subplot(2,2,1)
plot(V , n_inf(:)),     hold on,            plot(V, m_inf (:), 'g'),     hold on,     plot(V, h_inf (:), 'r' ),
title('n,m,h_\infty'),       xlabel('Voltage (mv)'),
legend('n_\infty', 'm_\infty', 'h_\infty');

subplot(2,2,2)
plot(V , Tau_n(:)),     hold on,          plot(V, Tau_m(:), 'g'),      hold on,     plot(V, Tau_h(:) , 'r' )
title('\tau_n_,_m_,_h')
xlabel('Voltage (mv)')
legend('\tau_n', '\tau_m', '\tau_h');

subplot(2,2,3)
plot(V , alpha_n(:)),     hold on,          plot(V, alpha_m(:), 'g'),      hold on,     plot(V, alpha_h(:) , 'r' )
title('\alpha_n_,_m_,_h')
xlabel('Voltage (mv)')
legend('\alpha_n', '\alpha_m', '\alpha_h');

subplot(2,2,4)
plot(V , beta_n(:)),     hold on,          plot(V, beta_m(:), 'g'),      hold on,     plot(V, beta_h(:) , 'r' )
title('\beta_n_,_m_,_h')
xlabel('Voltage (mv)')
legend('\beta_n', '\beta_m', '\beta_h');
```

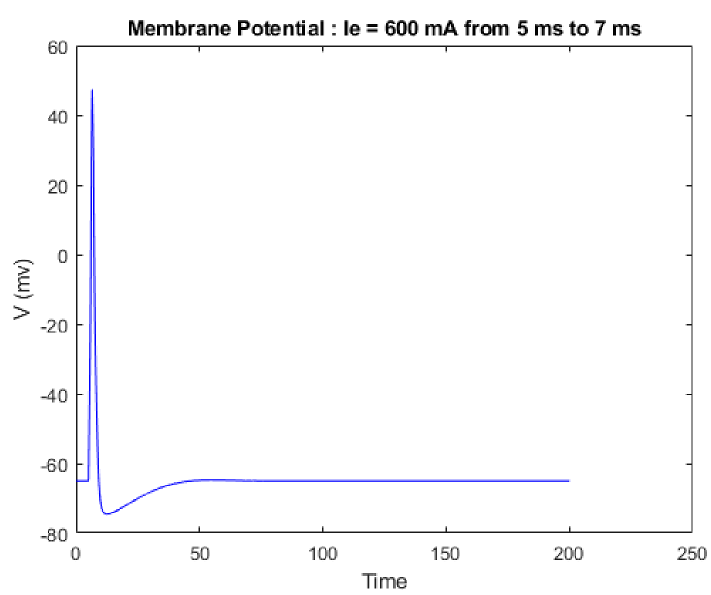


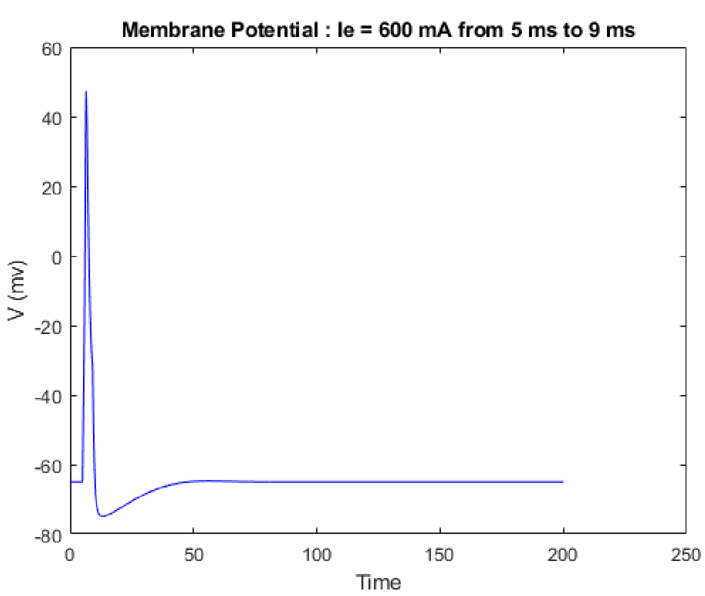
part 2

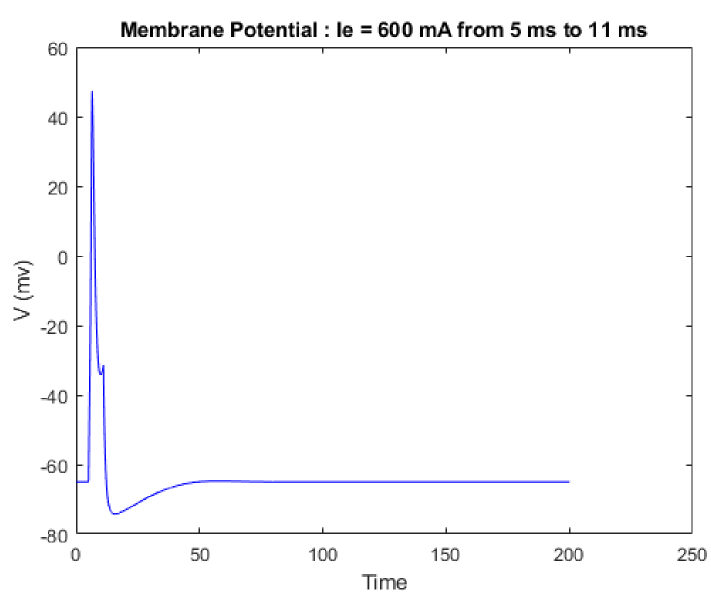
2)Plot voltage V as function of time , by simulating the injection of a suitable short (try some values!) external current Ie ,starting at t = 5ms.

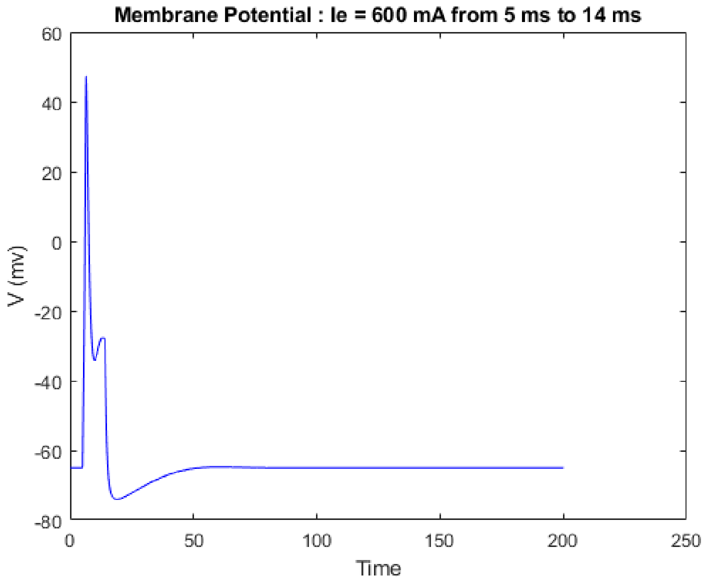
```
close all
Action_Potential_fun(120, 5, 6, 600)
Action_Potential_fun(120, 5, 7, 600)
Action_Potential_fun(120, 5, 9, 600)
Action_Potential_fun(120, 5, 11, 600)
Action_Potential_fun(120, 5, 14, 600)
Action_Potential_fun(120, 5, 18, 600)
```

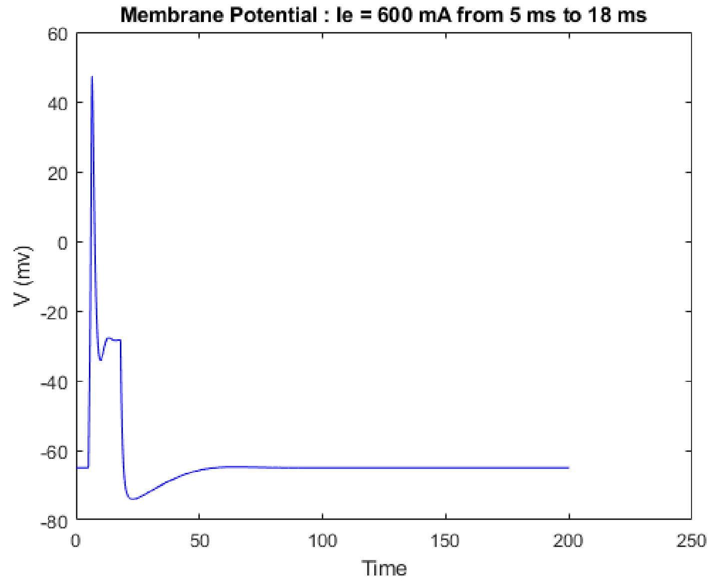








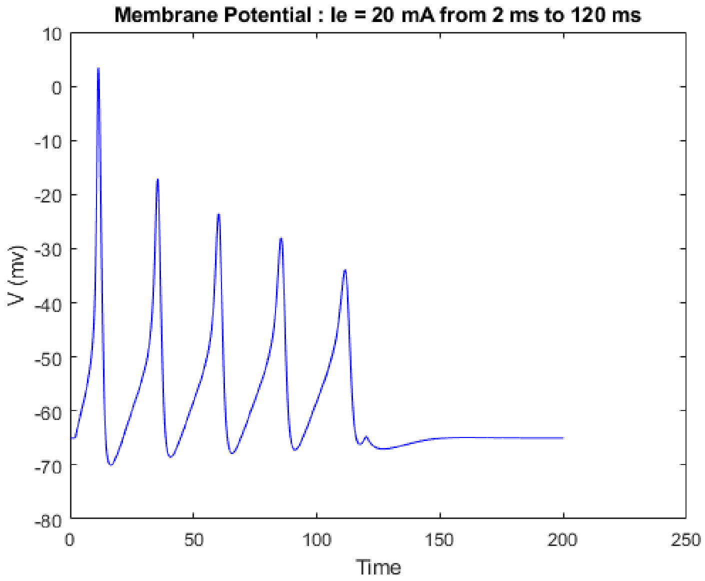


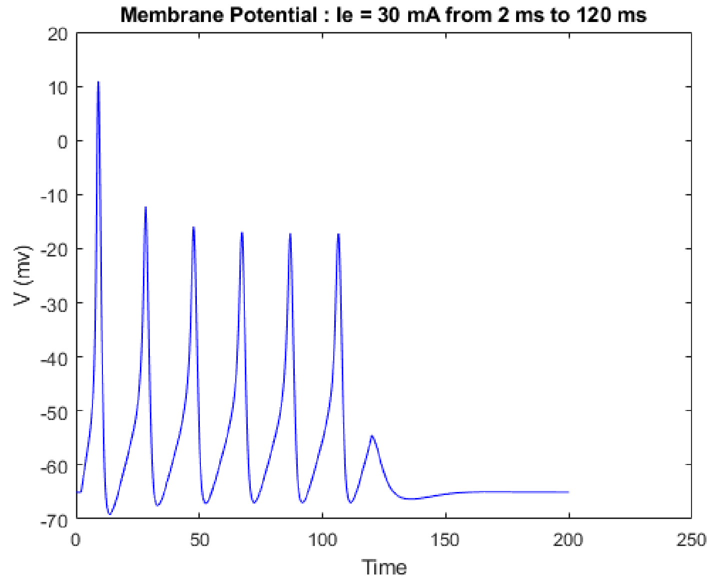


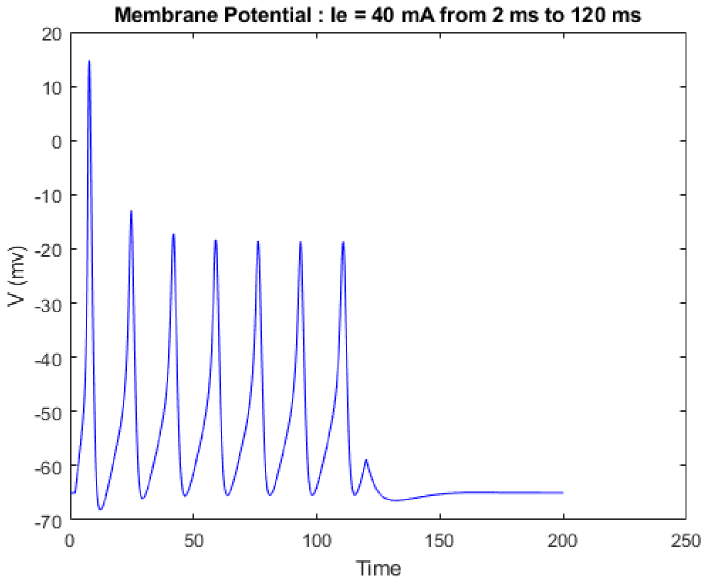
part 3

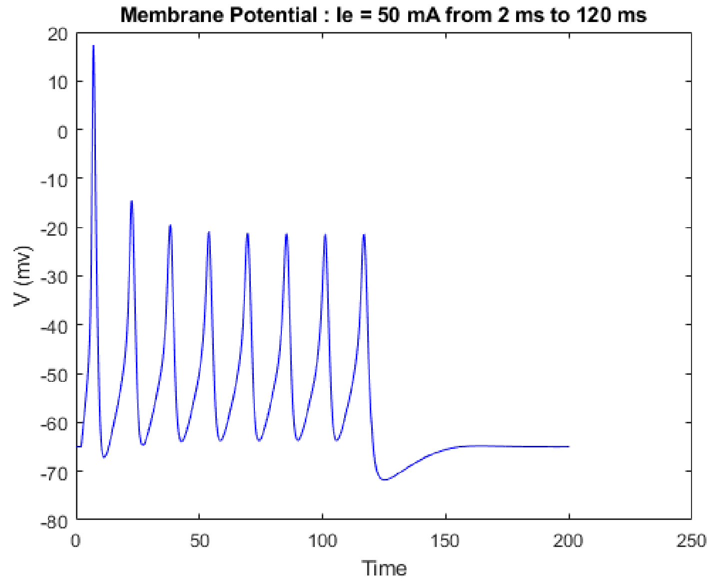
3)Apply constant input from 2 msto 120 msand see the result. Increase the sodium conductance 5 times and then see the result.

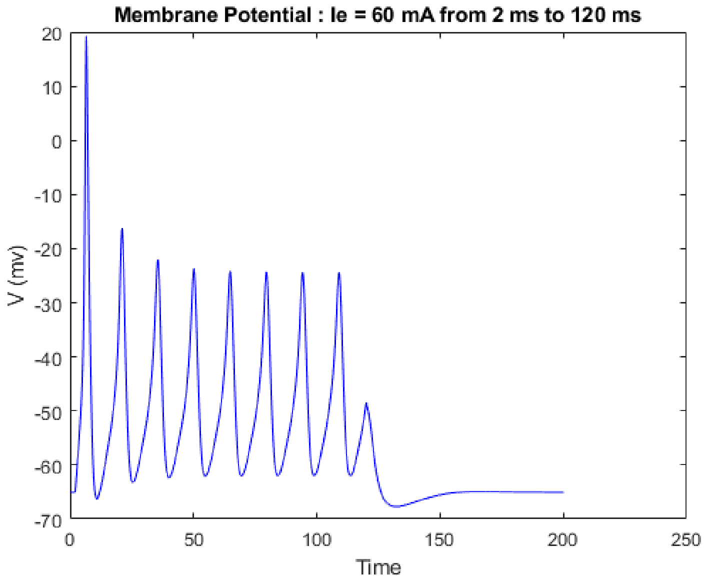
```
close all
Action_Potential_fun(120, 2, 120, 20)
Action_Potential_fun(120, 2, 120, 30)
Action_Potential_fun(120, 2, 120, 40)
Action_Potential_fun(120, 2, 120, 50)
Action_Potential_fun(120, 2, 120, 60)
Action_Potential_fun(120, 2, 120, 200)
```

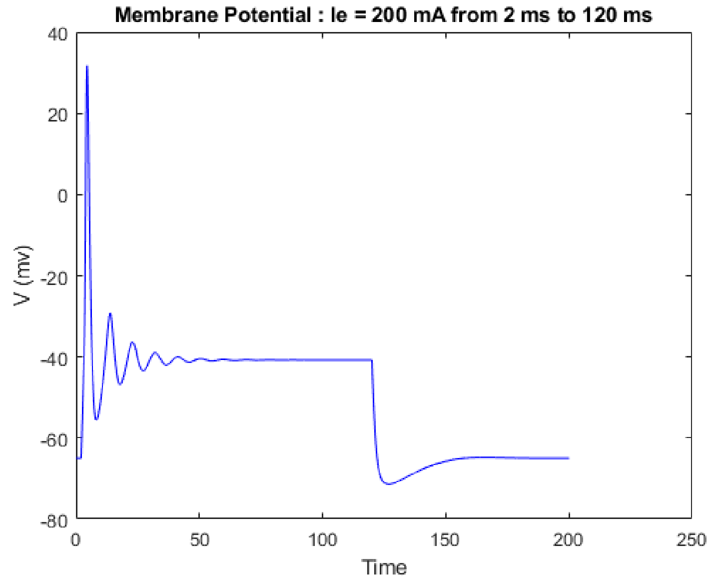






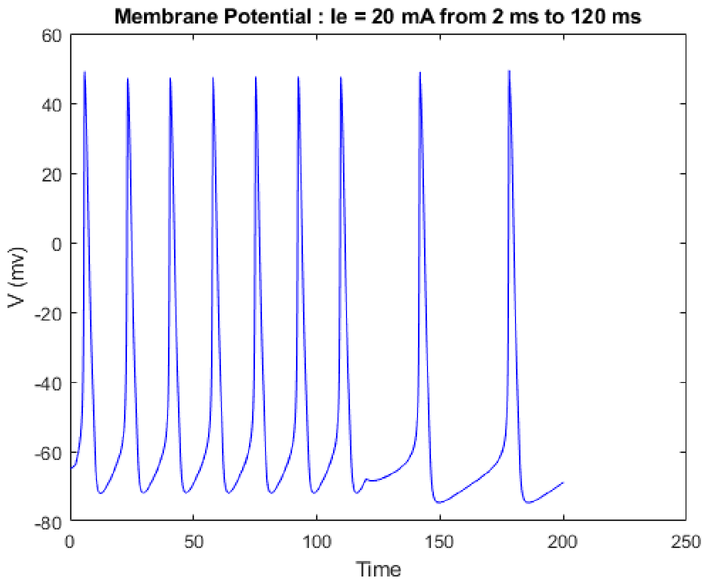


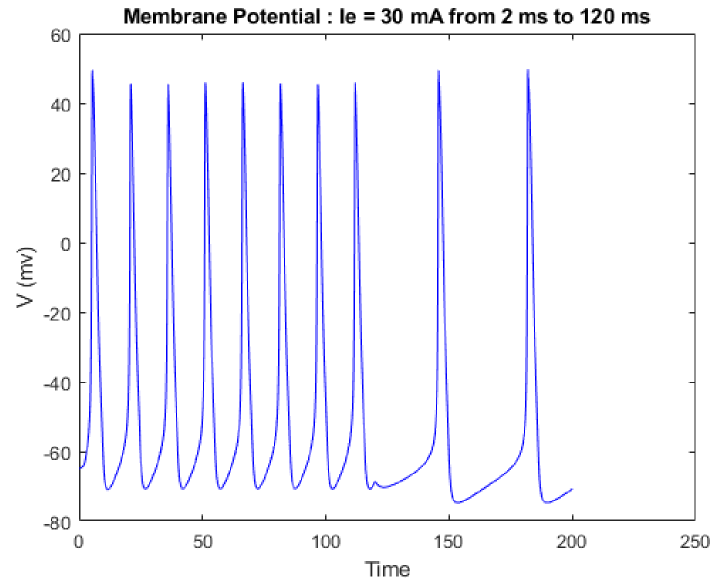


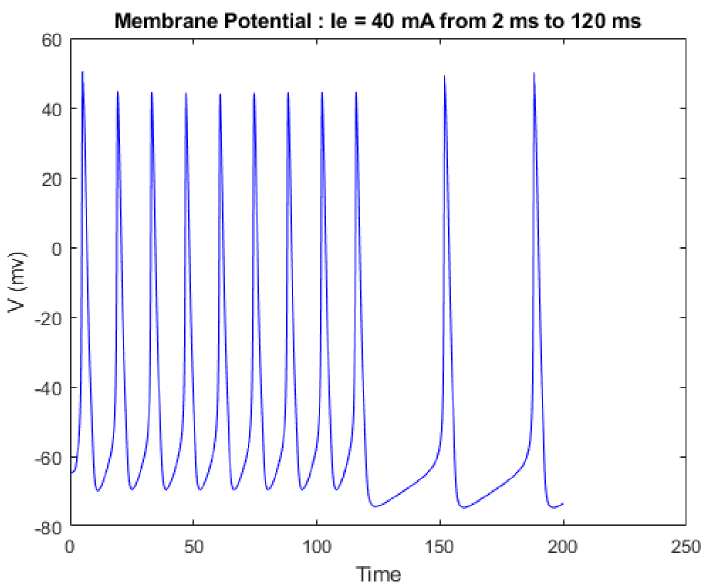


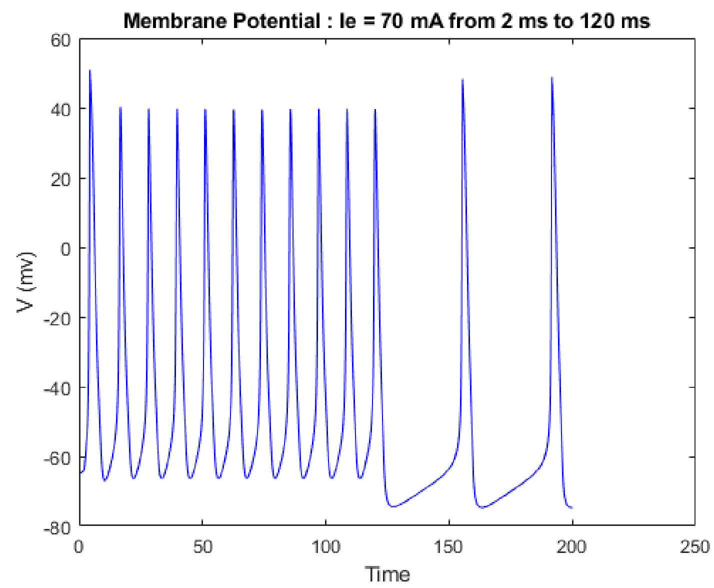
Now Let's multiply the Na conductance by 5

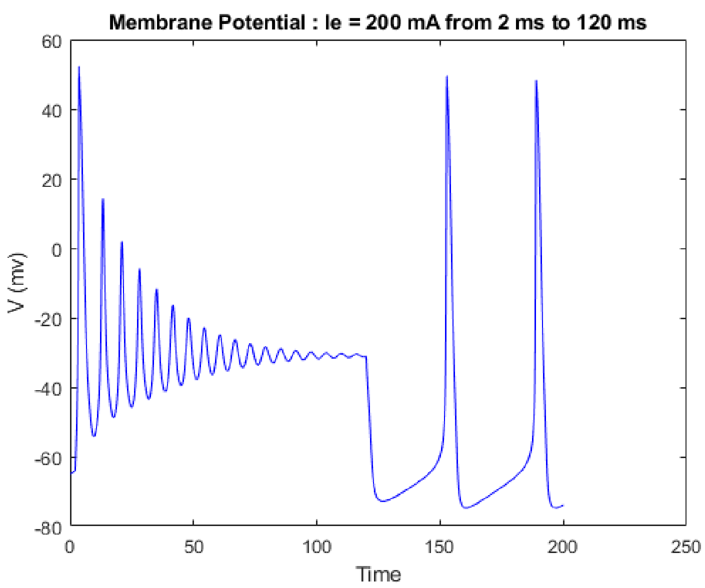
```
close all
Action_Potential_fun(600, 2, 120, 20)
Action_Potential_fun(600, 2, 120, 30)
Action_Potential_fun(600, 2, 120, 40)
Action_Potential_fun(600, 2, 120, 70)
Action_Potential_fun(600, 2, 120, 200)
Action_Potential_fun(600, 2, 120, 600)
```

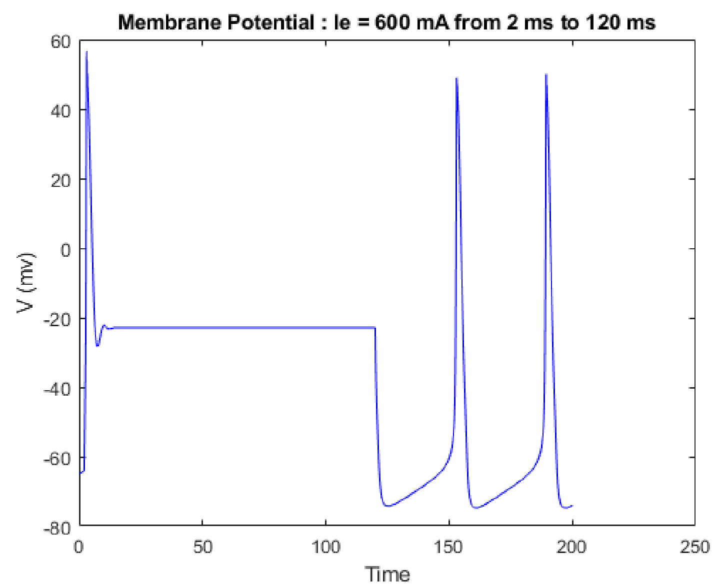








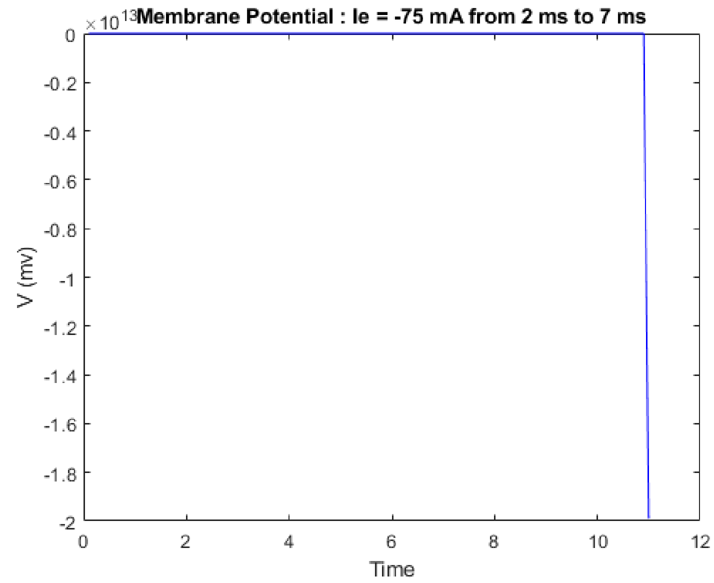


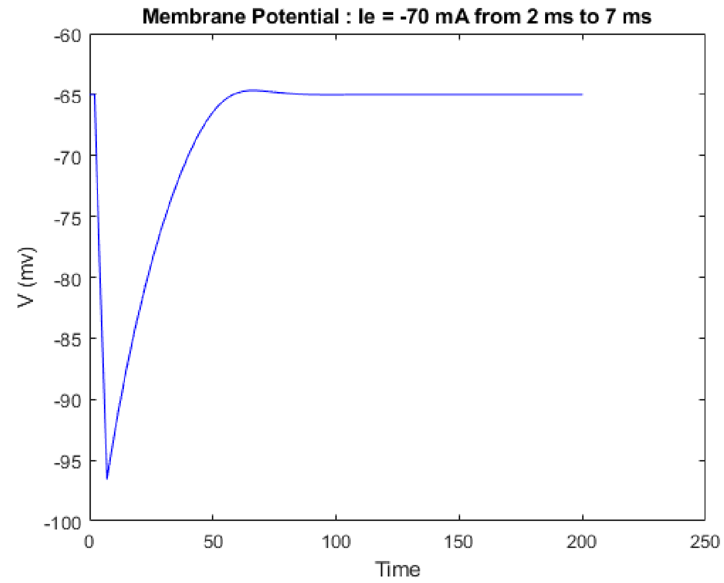


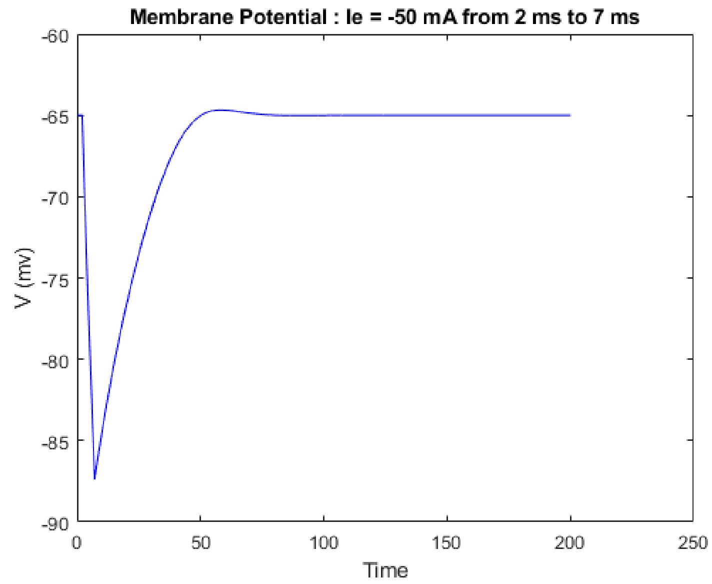
part 4

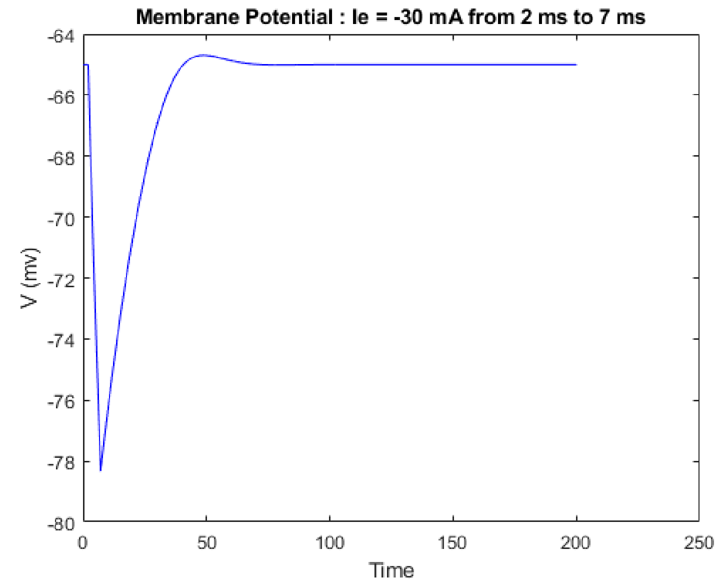
4)Apply inhibitory input from 2 ms to 7 ms and then set the input current to 0. What happens?

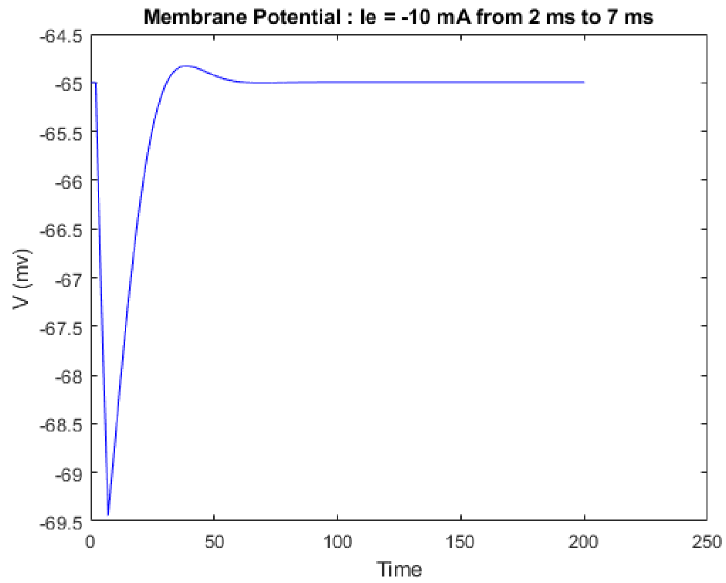
```
close all
Action_Potential_fun(120, 2, 7, -75)
Action_Potential_fun(120, 2, 7, -70)
Action_Potential_fun(120, 2, 7, -50)
Action_Potential_fun(120, 2, 7, -30)
Action_Potential_fun(120, 2, 7, -10)
Action_Potential_fun(120, 2, 7, 0)
```

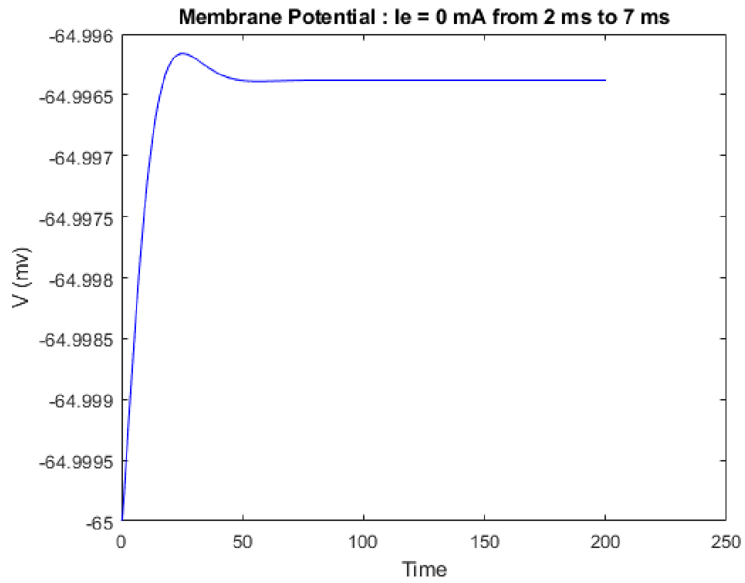













```
clc
```

Explanation of the plots:

Regarding the effect of duration and input amplitude value, the input value is important, in low values only depolarization occurs, but in stronger amplitude values, the neuron spikes.

Regarding the duration, if the input is applied for 120 milliseconds, after that the neuron enters the sustained periodic response phase and spikes regularly even though the stimulation is stopped.

These show that the duration of stimulation and the amount of stimulation amplitude both have an effect on the behavior of the neuron, and sometimes they have non-obvious effects: as in the fourth part, when an inhibitory stimulus is applied to a neuron, after the stimulation is removed, we might expect the neuron's voltage to change. but on the contrary, the neuron spikes regularly. This effect is called post inhibitory rebound.

This exercise demonstrates the complex and non-trivial behavior of the HH model.

```
function Action_Potential_fun(g_Na, start_time, end_time, Ie_val)
    total_data_points = 2000;
    dt = 0.1;             %ms
    cm = 10;              %nF/mm^2
    V_init = -65;      %mv
    m_init = 0.0529;
    h_init = 0.5961;
    n_init = 0.3177;
    g_L = 0.3;           %mS/mm^2
    g_K = 36;            %mS/mm
    E_L = -54.387;  %mv
    E_K = -77;          %mv
    E_Na = 50;         %mv
    A = 1;

    n(1) = n_init;
    m(1) = m_init;
    h(1) = h_init;

    V(1) = -65; %mv

    Ie = zeros (total_data_points ,1);
    Ie(start_time/dt: end_time/dt) = Ie_val;
    for i =1:total_data_points
            alpha_n(i) = 0.01*(V(i) + 55) / (1 - exp(-0.1*(V(i) + 55)));    beta_n(i) = 0.125*exp(-0.0125*(V(i) + 65));
            alpha_m(i) = 0.1*(V(i) + 40) / (1 - exp(-0.1*(V(i) + 40)));     beta_m(i) = 4*exp(-0.0556*(V(i) + 65));
            alpha_h(i) = 0.07*exp(-0.05*(V(i) + 65));                           beta_h(i) = 1/(1 + exp(-0.1*(V(i) + 35)));

            n_inf(i) = alpha_n(i)   / (alpha_n(i) + beta_n(i));
            m_inf(i) = alpha_m(i) / (alpha_m(i) + beta_m(i));
            h_inf(i) = alpha_h(i)  / (alpha_h(i) + beta_h(i));

            Tau_n(i)  = 1 / (alpha_n(i) + beta_n(i));
            Tau_m(i) = 1 / (alpha_m(i) + beta_m(i));
            Tau_h(i)  = 1 / (alpha_h(i) + beta_h(i));
```

```
            % cm dV/dt= -im + Ie/A
            % im=gl ( V - El) + gk n^4 (V-Ek) + gNa m^3 h (V-ENa)
            im (i+1) = g_Na * m(i)^4  * (V(i) - E_Na) + g_K * n(i)^4 * (V(i) - E_K) + g_L * (V(i) - E_L);
            V(i+1)= V(i)- dt/cm * (g_Na * m(i)^3 *  h(i) * (V(i) - E_Na) + g_K * n(i)^4 * (V(i) - E_K) + g_L * (V(i) - E_L))+ dt/cm * Ie(i)/A;
            n(i+1) = n(i)   + dt*(n_inf(i) - n(i))/Tau_n(i);
            m(i+1) = m(i) + dt*(m_inf(i) - m(i))/Tau_m(i);
            h(i+1) = h(i)  + dt*(h_inf(i) - h(i))/Tau_h(i);

    end

    % Let's plot the membrane potential vursus time
    figure
    time = dt * (1:total_data_points+1);
    plot(time , V,'b'),     title(sprintf("Membrane Potential : Ie = %d mA from %d ms to %d ms",Ie_val, start_time, end_time)),
    xlabel('Time'),         ylabel('V (mv)')
end
```

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