MSE 429 - Advanced Kinematics for Robotics Systems

Group Project Report

Due Date: November 23, 2020

Group: 5

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[redacted]

Abstract

In this project, we are designing a 3-RRR planar parallel manipulator. The analysis will be demonstrated through several different properties. Firstly, a potential application of design 3-RRR manipulator will be introduced. Secondly, a CAD model with specified structure and parameters will be performed. Thirdly, Inverse kinematics and forward kinematics will be derived by hand calculation and perform the calculation through MATLAB. In the same file, plotted postures will be generated, which has 12 figures. In a separate file, the function of trajectory generation will animate the path of the platform. Lastly, Jacobians of the manipulator will be calculated by hands. Singularity configurations are considered. And forward velocity problem and forward static problem will be derived by hand.

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Introduction

As shown in Figure 1, A delta manipulator designed and manufactured by BOSCH. It is the fastest delta manipulator in the world. This delta manipulator can achieve more than 200 operations per minute. This robot can be used in the warehouses such as cargo distributor within a short range, which would improves the efficiency of human-machine interactive assembly lines dramatically. In our own design, for the simplicity, the model of a 3-RRR planar [SR1] parallel manipulator will be demonstrated and analyzed. A 3-RRR manipulator only has 3-DOF which means it can only move horizontally [SR2] (3 fewer DOF than delta manipulator). However, the functions of the manipulator, distribute the products, can be conducted successfully even with 3-DOF only. In terms of the cost, 3-RRR manipulators are much cheaper than Delta manipulators which is also essential for product selections. The reason why parallel manipulator is chosen instead of serial manipulators are parallel manipulators works faster, require less space and more durable. The working environment of this manipulator will be at the centre of a distribution junction. Instead of using constant speed belt drive, the 3-RRR manipulator can finish the tasks in a shorter, faster and more accurate way. The specific function of it would be move the product that human put on the platform to destinated location in the workspace.

The designed 3-DOF planar manipulator has features as follows. Firstly, the manipulator will be actuated by three revolute joints on the edge. Secondly, the workspace of our manipulator will be broader than the base which would be easier to reach further locations to distribute the products to human faster. Thirdly, the three base points will be mounted on a triangle frame. It would improve the stability during operation. CAD model of the designed manipulator will be demonstrated later in the report. Kinematics, Trajectory and Jacobian will be discussed analytically in detail though both MATLAB and hand calculations.   [SR3] Inverse kinematics, Forward kinematics and position plot will be delivered in one MATLAB file. Trajectory generation will be delivered in separate MATLAB. Comments can be found in the MATLAB file, which explains the steps and functions. All the derivations of equations will be demonstrated though hand calculation, such as Inverse and forward equations and Jacobians.

Figure 1 - BOSCH delta manipulator [1]

Analysis

This project mainly discusses four parts here: The Forward and Inverse Kinematics, Trajectory, and Jacobian. Both of them will be analyzed in detail through both MATLAB and hand calculations. The hand calculation for each part will be demonstrated later and the MATLAB code will be compressed as a zip file in submission.

SolidWorks Design

3-DOF PPM Designing:

The CAD model of the designed manipulator is demonstrated below:

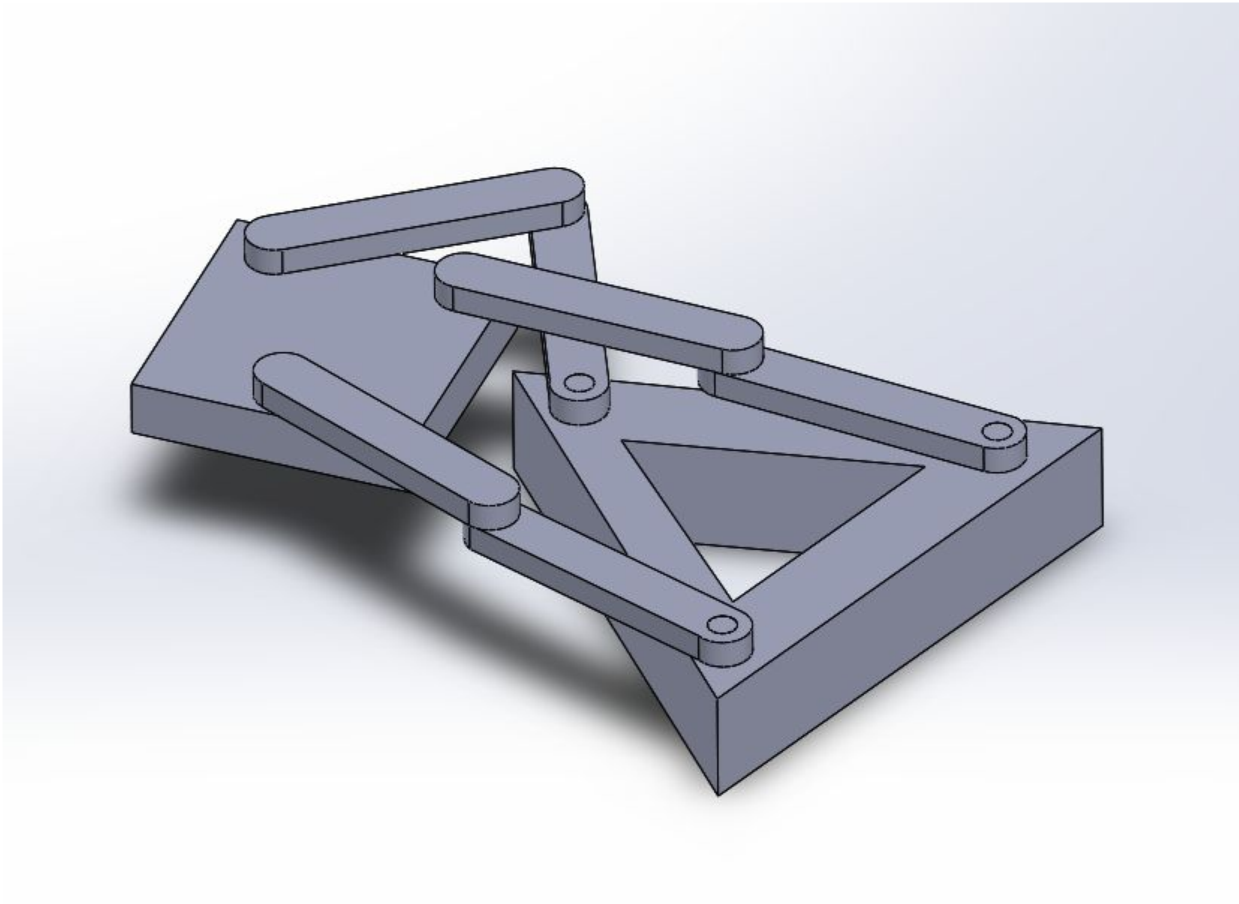


Figure 2 - Designed 3-DOF PPM

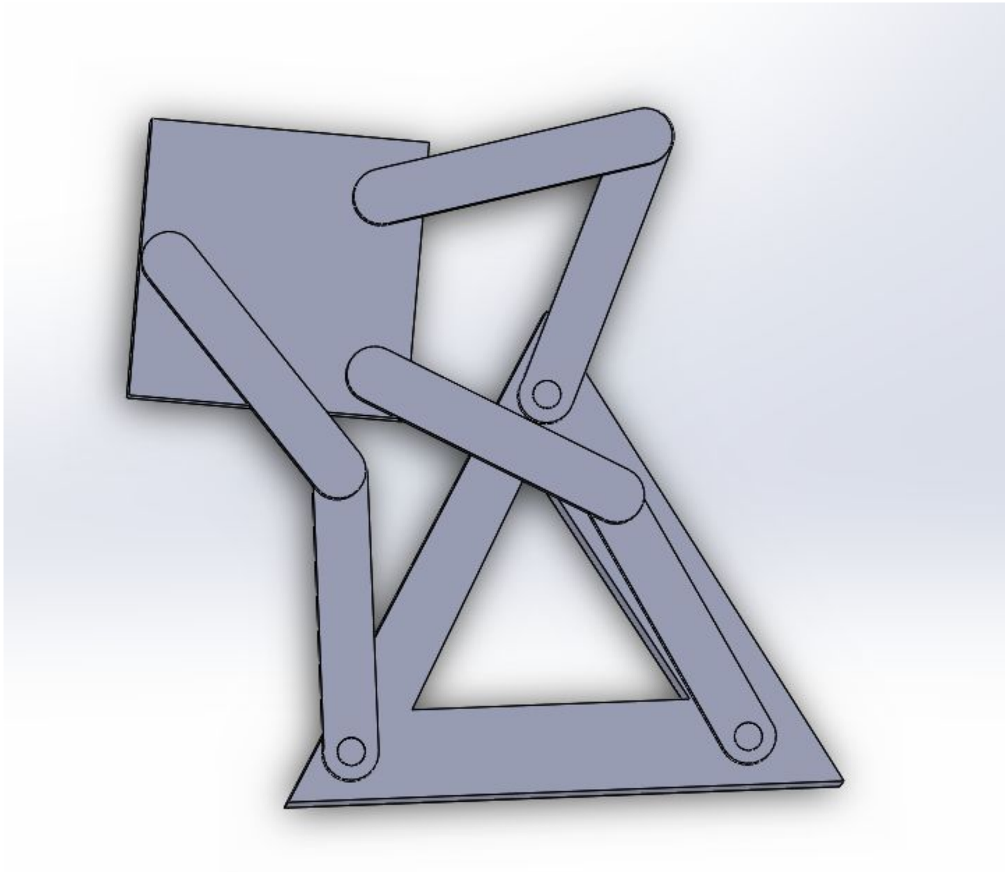


Figure 3 - Designed 3-DOF PPM

Link #	Link Length
R1	1m
R2	1m
R6	1m
R7	1m
R11	1m
R12	1m
a	0.8m
b	0.8m
c	0.8m

Table 1 - The length of the links

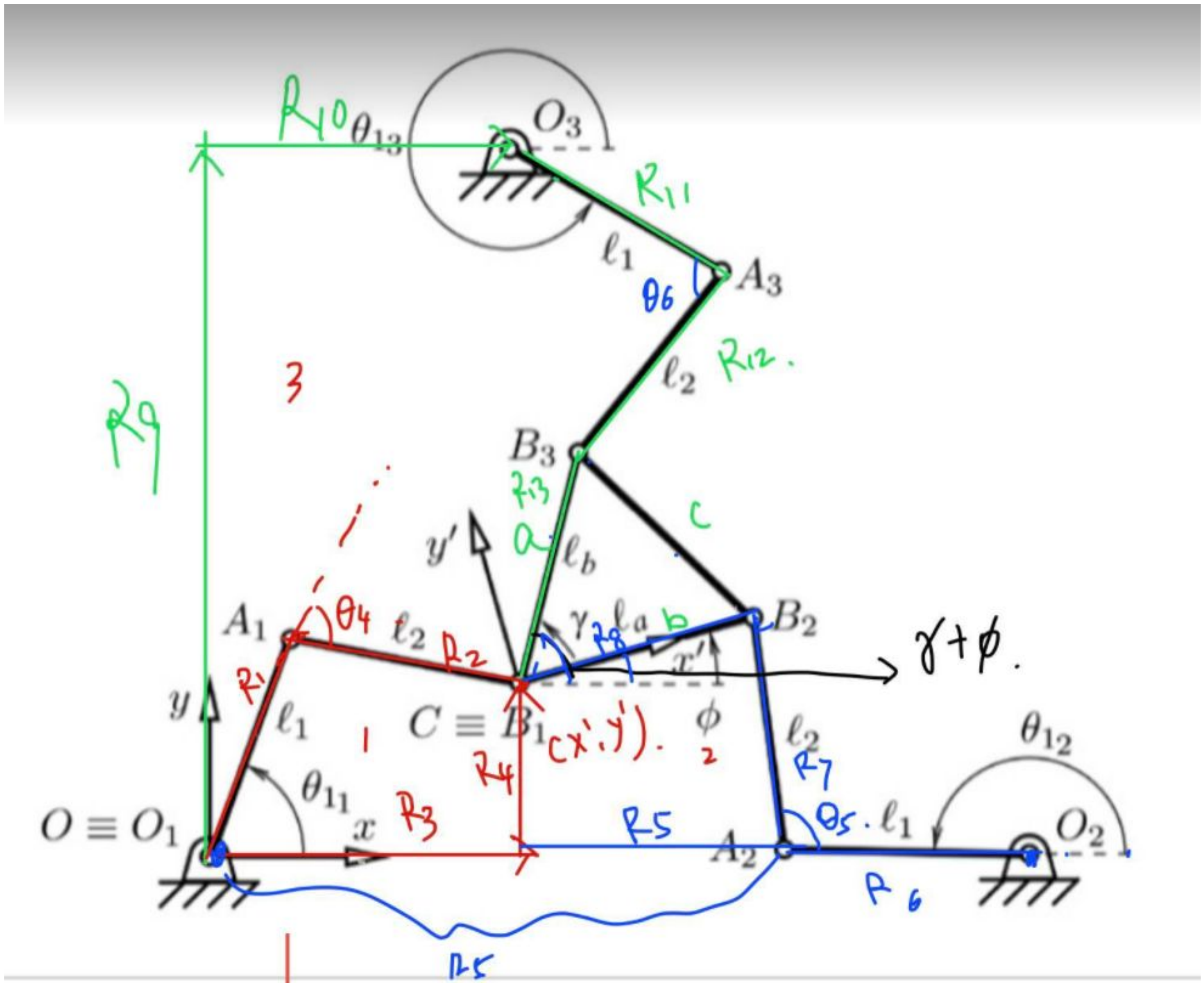


Figure 4 - The parameters of the model

The base of the model is a hollow equilateral triangular prism with 9 meters perimeter and 6 meters inner perimeter. The O1 is joint on the bottom-left corner of the base, O2 is the joint on the bottom-right corner and the O3 is on the top of the base.The value for each link is shown in figure 4 with different R value.

Inverse Kinematics

Inverse calculation

Loop closure equation of Branch 1.

$$R_1 + R_2 = R_3 + R_4$$

x: $l_1 \cos\theta_1 + L_2 \cos(\theta_1 + \alpha) = x'$

y: $l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \alpha) = y'$

$$(l_1 \cos\theta_1 - x')^2 + (l_1 \sin\theta_1 - y')^2 = l_2^2$$

Loop closure equation of Branch 2.

$$R_3 + R_4 + R_8 = R_5 + R_6 + R_7$$

x: $x' + b\cos\phi = (O_{2x} - O_{1x}) + l_1 \cos\theta_2 + l_2 \cos(\beta + \theta_2)$

y: $y' + b\sin\phi = l_1 \sin\theta_2 + l_2 \sin(\beta + \theta_2) + (O_{2y} - O_{1y})$

$$\begin{cases} x' + b\cos\phi - (O_{2x} - O_{1x}) - l_1 \cos\theta_2 = l_2 \cos(\beta + \theta_2) \\ y' + b\sin\phi - l_1 \sin\theta_2 - (O_{2x} - O_{1x}) = l_2 \sin(\beta + \theta_2) \end{cases}$$

$$l_2^2 = (x' + b\cos\phi - (O_{2x} - O_{1x}) - l_1 \cos\theta_2)^2 + (y' + b\sin\phi - l_1 \sin\theta_2 - (O_{2y} - O_{1y}))^2$$

Loop closure equation of Branch 3.

$$R_9 + R_{10} + R_{11} + R_{12} = R_3 + R_4 + R_{13}$$

$$x:\ (O_{3x} - O_{1x}) + l_1 \cos\theta_3 + l_2 \cos(\theta_3 + \gamma) = x' + a\cos(\phi + \theta_4)$$

$$y:\ (O_{3y} - O_{1y}) + l_1 \sin\theta_3 + l_2 \sin(\theta_3 + \gamma) = y' + a\sin(\phi + \theta_4)$$

$$\begin{cases} -l_2 \cos(\theta_3 + \gamma) = (O_{3x} - O_{1x}) + l_1 \cos\theta_3 - x' - a\cos(\phi + \theta_4) \\ -l_2 \sin(\theta_3 + \gamma) = (O_{3y} - O_{1y}) + l_1 \sin\theta_3 - y' - a\sin(\phi + \theta_4) \end{cases}$$

$$l_2^2 = \left((O_{3x} - O_{1x}) + l_1 \cos\theta_3 - x' - a\cos(\phi + \theta_4)\right)^2$$

$$+$$

$$\left((O_{3y} - O_{1y}) + l_1 \sin\theta_3 - y' - a\sin(\phi + \theta_4)\right)^2$$

set $O_{2x} - O_{1x} = d_{2x}$ $\quad O_{2y} - O_{1y} = d_{2y}$

$O_{3y} - O_{1y} = d_{3y}$

$O_{3x} - O_{1x} = d_{3x}$

$$\begin{cases} l_2^2 = (l_1 \cos\theta_1 - x')^2 + (l_1 \sin\theta_1 - y')^2 \\ l_2^2 = (x' + b\cos\phi - d_{2x} - l_1 \cos\theta_2)^2 + (y' + b\sin\phi - l_1 \sin\theta_2 - d_{2y})^2 \\ l_2^2 = (d_{3x} + l_1 \cos\theta_3 - x' - a\cos(\phi + \theta_4))^2 \\ \qquad + (d_{3y} + l_1 \sin\theta_3 - y' - a\sin(\phi + \theta_4))^2 \end{cases}$$

$$\Rightarrow \begin{cases} L_2^2 = (L_1C_1 - x')^2 + (L_1S_1 - y')^2 & ① \\ L_2^2 = (x' + bC_\phi - d_{2x} - L_1C_2)^2 + (y' + bS_\phi - L_1S_2 - d_{2y})^2 & ② \\ L_3^2 = (d_{3x} + L_1C_3 - x' - aC_{\phi 4})^2 + (d_{3y} + L_1S_3 - y' - aS_{\phi 4})^2 & ③ \end{cases}$$

Expand Equation ①

$$L_2^2 = L_1^2C_1^2 + x'^2 - 2x'L_1C_1 + L_1^2S_1^2 + y'^2 - 2L_1S_1y'$$

$$\Rightarrow x'^2 + y'^2 - 2x'L_1C_1 - 2y'L_1S_1 + L_1^2 - L_2^2 = 0$$

Expand Equation ②

$$x'^2 + y'^2 - 2x'd_{2x} - 2y'd_{2y} + d_{2x}^2 + d_{2y}^2 + b^2 + L_1^2 - L_2^2 + 2x'bC_\phi$$
$$+ 2y'bS_\phi - 2x'L_1C_2 - 2y'L_1S_2 - 2L_1bC_\phi C_2 - 2d_{2x}bC_\phi - 2d_{2y}bS_\phi$$
$$+ 2d_{2x}L_1C_2 + 2d_{2y}L_1S_2 - 2L_1bS_\phi S_2 = 0$$

Expand Equation ③

$$x'^2 + y'^2 - 2x'd_{3x} - 2y'd_{3y} + d_{3x}^2 + d_{3y}^2 + a^2 + L_1^2 - L_2^2 + 2x'aC_{\phi\pi/3} + 2y'aS_{\phi\pi/3}$$
$$- 2x'^2L_1C_3 - 2y'L_1S_3 - 2L_1aC_{\phi\pi/3}C_3 - 2d_{3x}aC_{\phi\pi/3} - 2d_{3y}aS_{\phi\pi/3} + 2d_{3x}L_1C_3$$
$$+ 2d_{3y}L_1S_3 - 2L_1aS_{\phi\pi/3}S_3 = 0$$

NB: $a = b = c$, Equilateral triangle.
(theta $4 = \pi/3$)

Inverse calculation.

For theta 1:

$e_1 S_1 + e_2 C_1 + e_3 = 0$

$e_1 = -2L_1 y'$, $e_2 = -2L_1 x'$, $e_3 = x'^2 + y'^2 + L_1^2 - L_2^2$

$S_1 = \frac{2t}{1+t^2}$ $C_1 = \frac{1-t^2}{1+t^2}$ $t = \tan\frac{\theta}{2}$

$e_1 \frac{2t}{1+t^2} + e_2 \frac{1-t^2}{1+t^2} + e_3 = 0$

$e_1 2t + (1-t^2) e_2 + e_3(1+t^2) = 0$

$\theta_1 = 2\operatorname{atan}(t)$

$t^2(e_3 - e_2) + t(2e_1) + e_2 + e_3 = 0$

For theta 2:

$e_4 = x'^2 + y'^2 - 2x' d_{2x} - 2y' d_{2y} + d_{2x}^2 + d_{2y}^2 + b^2 + L_1^2 - L_2^2$
$+ 2x' bC\phi - 2d_{2x} bC\phi - 2d_{2y} bS\phi + 2y' bS\phi$

$e_5 = -2x' L_1$ $e_6 = -2y' L_1$ $e_7 = -2L_1 bC\phi$ $e_8 = 2d_{2x} L_1$

$e_9 = 2d_{2y} L_1$ $e_{10} = -2L_1 bS\phi$

$e_5 C_2 + e_6 S_2 + e_7 C_2 + e_8 C_2 + e_9 S_2 + e_{10} S_2 + e_4 = 0$

$(e_5 + e_7 + e_8) C_2 + (e_6 + e_9 + e_{10}) S_2 + e_4 = 0$

$\Downarrow$

Reset e values

$$e_5 = e_5 + e_7 + e_8 = -2x'l_1 - 2l_1 bC\phi + 2d_{2x}l_1$$

$$= 2l_1(-x' - bC\phi + d_{2x})$$

$$e_6 = e_6 + e_9 + e_{10} = -2y'l_1 + 2d_{2y}l_1 - 2l_1 bS\phi$$

$$= 2l_1(-y' + d_{2y} - bS\phi)$$

$$e_5 C_2 + e_6 S_2 + e_4 = 0$$

$$\hookrightarrow e_6 \frac{2t}{1+t^2} + e_5 \frac{1-t^2}{1+t^2} + e_4 = 0$$

$$t^2(e_4 - e_5) + t(2e_6) + e_5 + e_4 = 0$$

For theta 3:

$$e_7 = x'^2 + y'^2 - 2x'd_{3x} - 2y'd_{3y} + d_{3x}^2 + d_{3y}^2 + a^2 + L_1^2 - l_2^2$$
$$+ 2x'a\cos(\phi + \tfrac{\pi}{3}) + 2y'a\sin(\phi + \tfrac{\pi}{3}) - 2d_{3x}a\cos(\phi + \tfrac{\pi}{3})$$
$$- 2d_{3y}a\sin(\phi + \tfrac{\pi}{3})$$

$$e_8 = -2x'L_1 - 2l_1 a\cos(\phi + \tfrac{\pi}{3}) + 2d_{3x}L_1$$

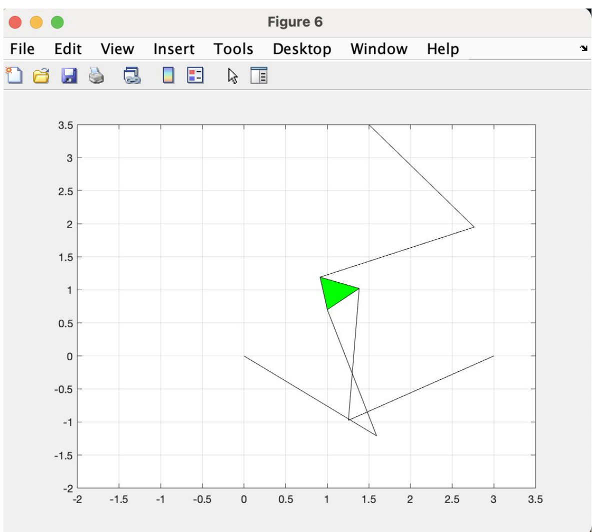
$$e_9 = -2y'L_1 + 2d_{3y}L_1 - 2l_1 a\sin(\phi + \tfrac{\pi}{3})$$

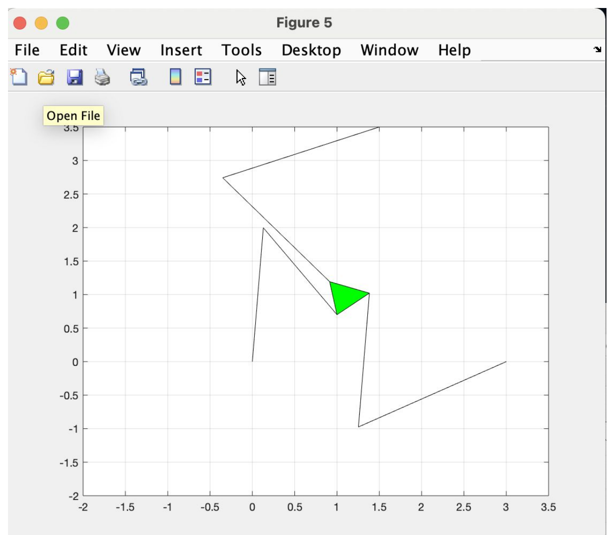
$$e_9 \frac{2t}{1+t^2} + e_8 \frac{1-t^2}{1+t^2} + e_7 = 0$$

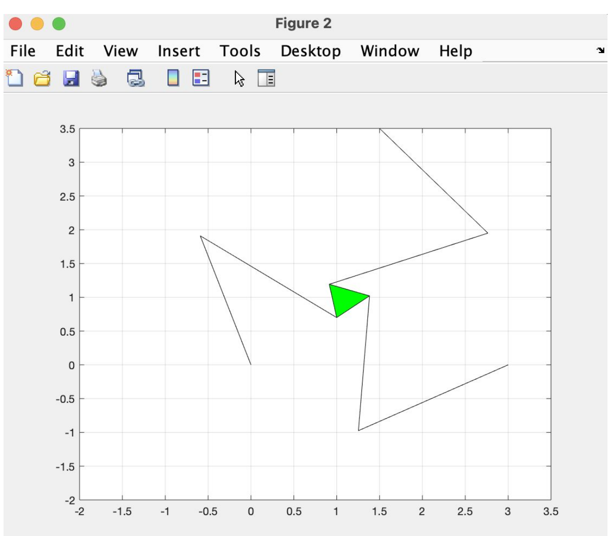
$$(e_7 - e_8)t^2 + 2e_9 t + e_8 + e_7 = 0$$

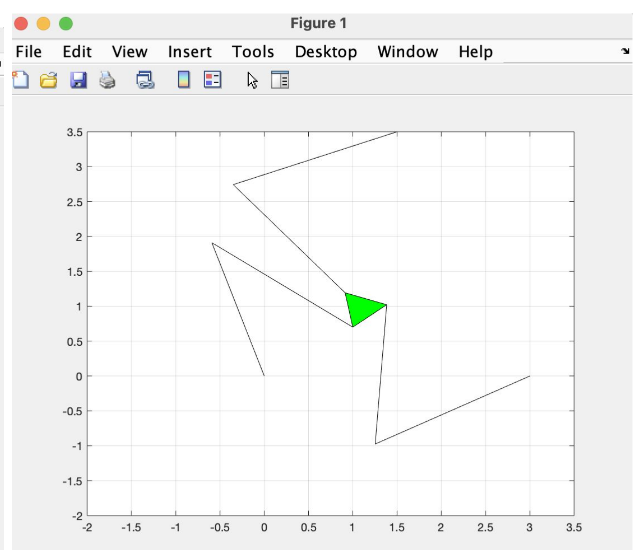
Assembly Modes plot

(Based on MATLAB file: Kinematics)









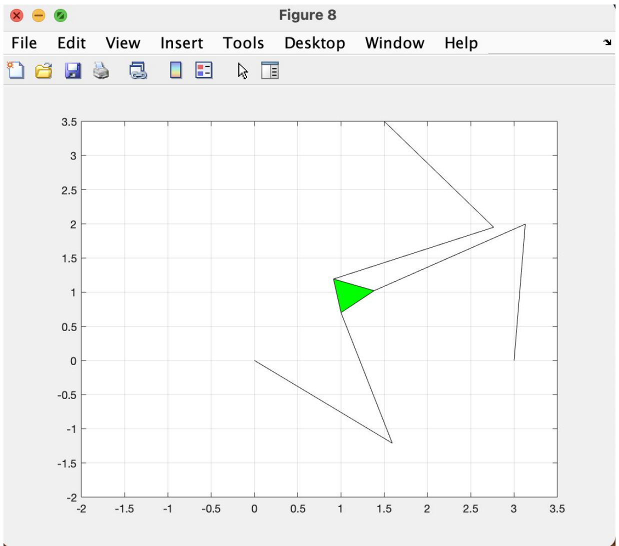


Figure 8
File Edit View Insert Tools Desktop Window Help

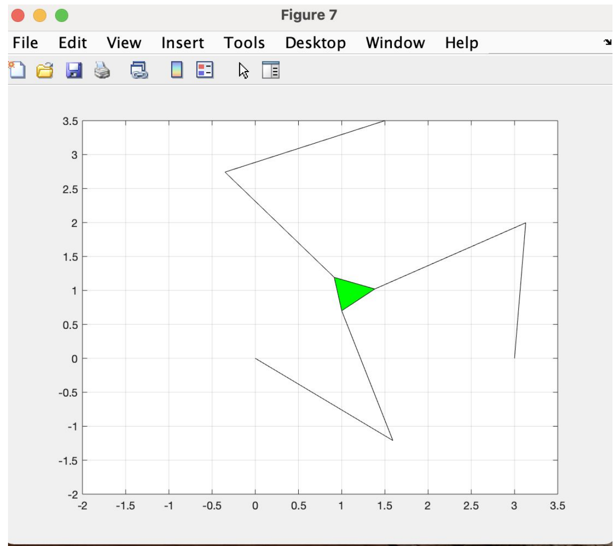


Figure 7
File Edit View Insert Tools Desktop Window Help

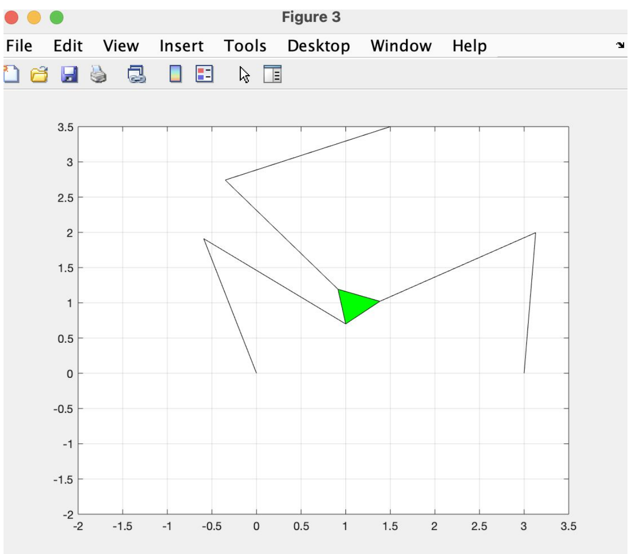
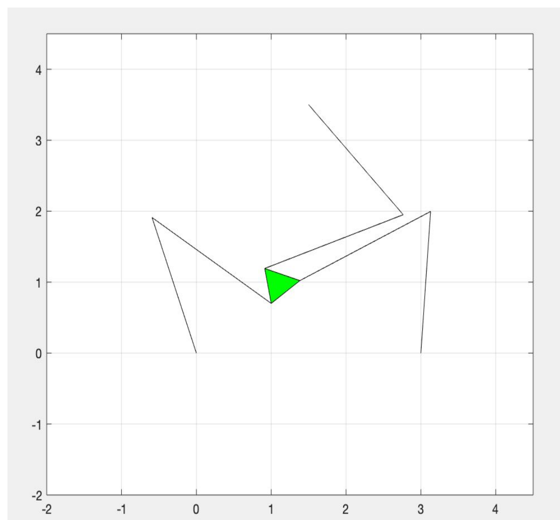
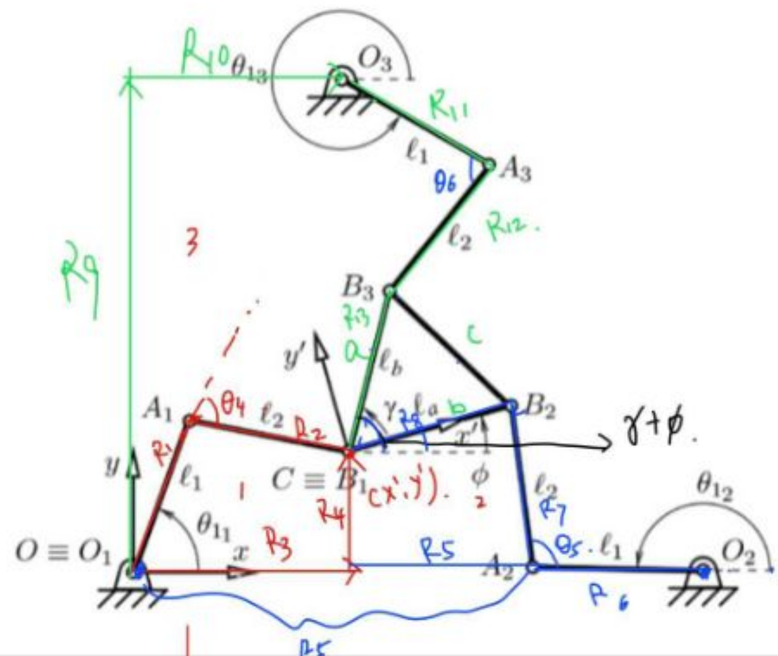


Figure 3
File Edit View Insert Tools Desktop Window Help



Forward Kinematics



Forward Kinematics :

For Loop 1:

$$R_1 + R_2 = R_3 + R_4 .$$

$$x' = l_1 \cdot \cos\theta_{11} + l_2 \cdot \cos(\theta_{11} + \theta_4).$$ $\quad \Big\} \; x'^2 + y'^2$

$$y' = l_1 \cdot \sin\theta_{11} + l_2 \cdot \sin(\theta_{11} + \theta_4).$$

$$\therefore \; (l_1 \cdot \cos\theta_{11} - x')^2 + (l_1 \cdot \sin\theta_{11} - y')^2 = l_2^2 .$$

$$\therefore \; l_1^2 \cdot \cos^2\theta_{11} + x'^2 - 2x' \cdot l_1 \cdot \cos\theta_{11} + l_2^2 \sin^2\theta_{11} + y'^2 - 2y' l_2 \cdot \sin\theta_{11} = l_2^2 .$$

$$x'^2 + y'^2 - 2x' l_1 \cdot \cos\theta_{11} - 2y' \cdot l_2 \cdot \sin\theta_{11} = l_2^2 - l_1^2 . \rightarrow (1)$$

For Loop 2:

$$R_3 + R_4 + R_8 = R_5 + R_6 + R_7. \qquad (R_5 + R_6) = O_{2x} - O_{1x}.$$

For x: $x' + b \cdot \cos(\phi) = O_{2x} - O_{1x} + l_1 \cdot \cos(\theta_{12}) + l_2 \cdot \cos(\theta_{12} + \theta_5)$

For y: $y' + b \cdot \sin(\phi) = O_{2y} - O_{1y} + l_1 \cdot \sin(\theta_{12}) + l_2 \cdot \sin(\theta_{12} + \theta_5).$

$$\therefore \begin{cases} l_2 \cdot \cos(\theta_{12} + \theta_5) = x' + b \cdot \cos(\phi) - (O_{2x} - O_{1x}) - l_1 \cdot \cos\theta_{12} \\ l_2 \cdot \sin(\theta_{12} + \theta_5) = y' + b \cdot \sin(\phi) - (O_{2y} - O_{1y}) - l_1 \cdot \sin\theta_{12} . \end{cases}$$

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$$\therefore\ l_2^{2} = \left(x' + \overset{a}{\uparrow}b\cdot\cos(\phi) - (O_{2x} - O_{1x}) - l_1\cdot\cos\theta_{12}\right)^2 + \left(y' + b\cdot\sin(\phi) - \overset{b\cdot}{\uparrow}(O_{2y} - O_{1y}) - l_1\cdot\sin\theta_{12}\right)^2 \rightarrow (2)$$

For Loop 3:

$O_{3y} - O_{1y}$, $O_{3x} - O_{1x}$

$$R_9 + R_{10} + R_1 + R_2 = R_3 + R_4 + R_{13},$$

d_{3x}.

$$\therefore \text{For } x:\ O_{3x} - O_{1x} + l_1\cdot\cos(\theta_{13}) + l_2\cdot\cos(\theta_{13}+\theta_6) = x' + b\cdot\cos(\phi+\gamma)$$

$$\text{For } y:\ \underbrace{O_{3y} - O_{1y}}_{d_{3y}} + l_1\cdot\sin(\theta_{13}) + l_2\cdot\sin(\theta_{13}+\theta_6) = y' + b\cdot\sin(\phi+\gamma).$$

$$\therefore\ l_2^2 = \left(d_{3x} + l_1\cdot\cos(\theta_{13}) - x' - b\cdot\cos(\phi+\gamma)\right)^2$$
$$+ \left(d_{3y} + l_1\cdot\sin\theta_{13}) - y' - b\cdot\sin(\phi+\gamma)\right)^2. \longrightarrow (3)$$

$\therefore$ solve (1) (2) (3). to get the value for x', y', ϕ.

set
$$e_{11} = -2\cdot l_1\cdot\cos\theta_1 \quad ; \quad e_{21} = -2\cdot d_{2x} + 2\cdot b\cdot\frac{1-t^2}{1+t^2} - 2\cdot l_1\cdot\cos\theta_2.$$
$$e_{12} = -2\, l_1\cdot\sin\theta_1 \quad ; \quad e_{22} = -2\cdot d_{2y} + 2b\cdot\frac{2t}{1+t^2} - 2\, l_1\cdot\sin\theta_2.$$
$$e_{13} = l_1^2 - l_2^2$$

$$e_{23} = d_{2x}^2 + d_{2y}^2 + b^2 + l_1^2 - l_2^2 - 2\, l_1\cdot b\cdot\frac{1-t^2}{1+t^2}\cdot\cos\theta_2 - 2 l_1\cdot b\cdot\frac{2t}{1+t^2}\cdot\sin\theta_2$$
$$-2\, d_{2x}\cdot b\cdot\frac{1-t^2}{1+t^2} - 2\cdot d_{2y}\cdot b\cdot\frac{2t}{1+t^2} + 2\cdot(d_{2x})\cdot l_1\cdot\cos\theta_2 + 2\, d_{2y}\cdot l_1\cdot\sin\theta_2$$

$$e_{31} = -2\, d_{3x} + 2\cdot b\cdot\frac{1-t^2}{1+t^2}\cdot\cos(\gamma) - \frac{2t}{1+t^2}\cdot\sin(\gamma) - 2\, l_1\cdot\cos\theta_3.$$

$$e_{32} = -2\, d_{3y} + 2\cdot b\cdot\frac{2t}{1+t^2}\cdot\cos(\gamma) - \frac{1-t^2}{1+t^2}\cdot\sin(\gamma) - 2\, l_1\cdot\sin\theta_3.$$

$$e_{11}' = e_{11} - e_{21}, \quad e_{12}' = e_{12} - e_{22}, \quad e_{13}' = e_{13} - e_{23}.$$
$$e_{21}' = e_{11} - e_{31}, \quad e_{22}' = e_{12} - e_{32}, \quad e_{23}' = e_{13} - e_{33}.$$

set $m = e11' \cdot e22' - e12' \cdot e21'$.

$m_1 = e12' \cdot e23' - e13' \cdot e22'$.

$m_2 = e13' \cdot e21' - e11' \cdot e23'$.

$\therefore$ phi is calculated by solving:

$$m_1^2 + m_2^2 + e11 \cdot m \cdot m_1 + e12 \cdot m \cdot m_2 + e13 \cdot m^2 = 0.$$

$\hookrightarrow$ using matlab to solve t.

then $phi = 2atan(t)$.

$\therefore$ we get the value for phi.

For each phi, we create

$R = e11'$, $S = e12'$, $Q = e13'$, $U = e21'$, $V = e22'$, $W = e23'$.

$\therefore$ we can calculate x, y by:

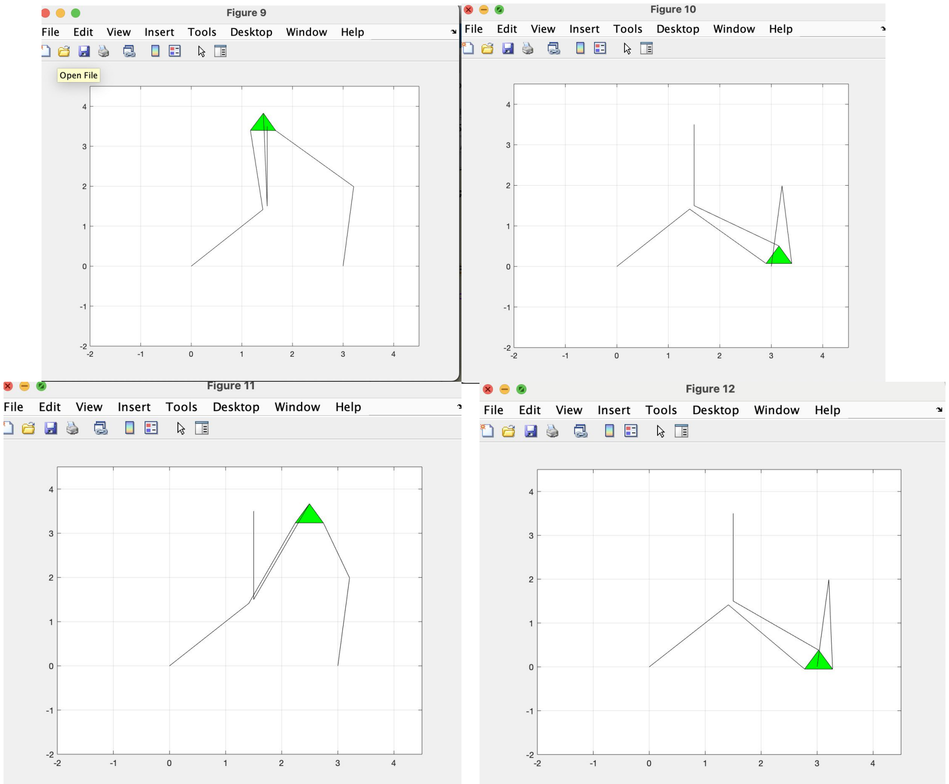
$$x = \frac{S \cdot W - V \cdot Q}{R \cdot V - U \cdot S}$$

$$y = \frac{U \cdot Q - R \cdot W}{R \cdot V - U \cdot S}.$$

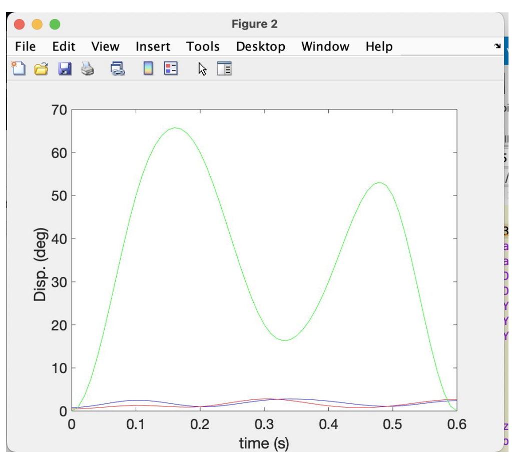
We using matlab to solve the x, y value for each phi value.

Potential Postures Plot

(Based on MATLAB file: Kinematics)

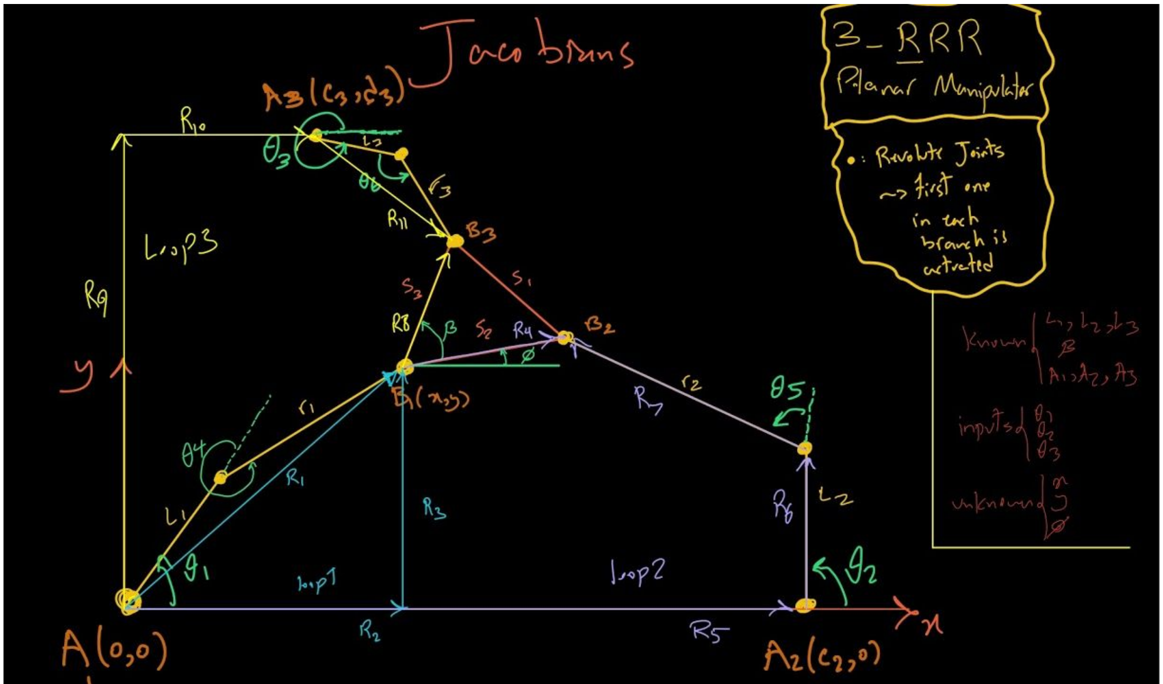


Trajectory Generation



Joint displacements are plotted, the animation can be checked directly in the MATLAB file: trajectory_plot. The explanations can be found in the comment

Jacobians



loop1: $\overline{R_2}+\overline{R_3}=R_1 \Rightarrow \begin{cases} x = L_1\cos\theta_1 + r_1\cos(\theta_1+\theta_4) & (1)\\ y = L_1\sin\theta_1 + r_1\sin(\theta_1+\theta_4) & (2)\end{cases}$

loop 2: $\overline{R_1}+\overline{R_4}=\overline{R_5}+\overline{R_6}+\overline{R_7}$

$$\Rightarrow \begin{cases} x + s_2\cos\phi = c_2 + L_2\cos\theta_2 + r_2\cos(\theta_2+\theta_5) & (3)\\ y + s_2\sin\phi = L_2\sin\theta_2 + r_2\sin(\theta_2+\theta_5) & (4)\end{cases}$$

loop3: $\overline{R_1}+\overline{R_8}=\overline{R_9}+\overline{R_{10}}+\overline{R_{11}}$

$$\Rightarrow \begin{cases} x + s_3\cos(\beta+\phi) = c_3 + L_3\cos\theta_3 + r_3\cos(\theta_3+\theta_6) & (5)\\ y + s_3\sin(\beta+\phi) = d_3 + L_3\sin\theta_3 + r_3\sin(\theta_3+\theta_6) & (6)\end{cases}$$

$$\xrightarrow[\text{(1) \& (2)}]{\text{derivative w.r.t. time}} \begin{cases} \dot{x} = -L_1(\sin\theta_1)\dot{\theta}_1 - r_1\sin(\theta_1+\theta_4)(\dot{\theta}_1+\dot{\theta}_4) \\ \dot{y} = L_1(\cos\theta_1)\dot{\theta}_1 + r_1\cos(\theta_1+\theta_4)(\dot{\theta}_1+\dot{\theta}_4) \end{cases}$$

$$\Rightarrow \quad \dot{\theta}_4 = \frac{-L_1(\sin\theta_1)\dot{\theta}_1 - \dot{x}}{r_1\sin(\theta_1+\theta_4)} - \cancel{\dot{\theta}_1} = \frac{\dot{y} - L_1(\cos\theta_1)\dot{\theta}_1}{r_1\cos(\theta_1+\theta_4)} - \cancel{\dot{\theta}_1}$$

$$\Rightarrow \left(-L_1(\sin\theta_1)\dot{\theta}_1 - \dot{x}\right)\left(r_1\cos(\theta_1+\theta_4)\right) = \left(\dot{y} - L_1(\cos\theta_1)\dot{\theta}_1\right)\left(r_1\sin(\theta_1+\theta_4)\right)$$

$$\Rightarrow \dot{x}\left(r_1\cos(\theta_1+\theta_4)\right) + \dot{y}\left(r_1\sin(\theta_1+\theta_4)\right) = \dot{\theta}_1\left[\left(L_1 r_1\cos(\theta_1)\sin(\theta_1+\theta_4)\right) - \left(L_1 r_1\sin(\theta_1)\cos(\theta_1+\theta_4)\right)\right]$$

$$\xrightarrow[3\&4]{\text{time-der}} \begin{cases} \dot{x} - s_2(\sin\phi)\dot{\phi} = -L_2(\sin\theta_2)\dot{\theta}_2 - r_2(\sin(\theta_2+\theta_5))(\dot{\theta}_2+\dot{\theta}_5) \\ \dot{y} + s_2(\cos\phi)\dot{\phi} = L_2(\cos\theta_2)\dot{\theta}_2 + r_2(\cos(\theta_2+\theta_5))(\dot{\theta}_2+\dot{\theta}_5) \end{cases}$$

$$\Rightarrow \dot{\theta}_5 = \frac{s_2(\sin\phi)\dot{\phi} - \dot{x} - L_2(\sin\theta_2)\dot{\theta}_2}{r_2\sin(\theta_2+\theta_5)} - \cancel{\dot{\theta}_2} = \frac{\dot{y} + s_2(\cos\phi)\dot{\phi} - L_2(\cos\theta_2)\dot{\theta}_2}{r_2\cos(\theta_2+\theta_5)} - \cancel{\dot{\theta}_2}$$

$$\left(s_2(\sin\phi)\dot{\phi} - \dot{x} - L_2(\sin\theta_2)\dot{\theta}_2\right)\left(r_2\cos(\theta_2+\theta_5)\right) = \left(\dot{y} + s_2(\cos\phi)\dot{\phi} - L_2(\cos\theta_2)\dot{\theta}_2\right)\left(r_2\sin(\theta_2+\theta_5)\right)$$

$$\dot{x}\left(r_2\cos(\theta_2+\theta_5)\right) + \dot{y}\left(r_2\sin(\theta_2+\theta_5)\right) + \dot{\phi}\left[\left(r_2 s_2\cos(\phi)\sin(\theta_2+\theta_5)\right) - \left(r_2 s_2\sin(\phi)\cos(\theta_2+\theta_5)\right)\right]$$
$$= \dot{\theta}_2\left[\left(r_2 L_2\cos(\theta_2)\sin(\theta_2+\theta_5)\right) - \left(r_2 L_2\sin(\theta_2)\cos(\theta_2+\theta_5)\right)\right]$$

$$\xrightarrow[5\&6]{\text{time-der}} \begin{cases} \dot{x} - s_3\sin(\beta+\phi)\dot{\phi} = -L_3\sin\theta_3\dot{\theta}_3 - r_3\sin(\theta_3+\theta_6)(\dot{\theta}_3+\dot{\theta}_6) \\ \dot{y} + s_3\cos(\beta+\phi)\dot{\phi} = L_3\cos\theta_3\dot{\theta}_3 + r_3\cos(\theta_3+\theta_6)(\dot{\theta}_3+\dot{\theta}_6) \end{cases}$$

$$\Rightarrow \dot{\theta}_6 = \frac{S_3 \sin(\beta+\phi)\dot{\phi} - \dot{x} - L_3 \sin\theta_3 \dot{\theta}_3}{r_3 \sin(\theta_3+\theta_6)} - \cancel{\dot{\theta}_3} =$$

$$\frac{\dot{y} + S_3 \cos(\beta+\phi)\dot{\phi} - L_3 \cos\theta_3\, \dot{\theta}_3}{r_3 \cos(\theta_3+\theta_6)} - \cancel{\dot{\theta}_3}$$

$$\Rightarrow \left(S_3 \sin(\beta+\phi)\dot{\phi} - \dot{x} - L_3 \sin\theta_3 \dot{\theta}_3\right)\left(r_3 \cos(\theta_3+\theta_6)\right)$$

$$= \left(\dot{y} + S_3 \cos(\beta+\phi)\dot{\phi} - L_3 \cos\theta_3\, \dot{\theta}_3\right)\left(r_3 \sin(\theta_3+\theta_6)\right)$$

$$\Rightarrow \dot{x}\left(r_3 \cos(\theta_3+\theta_6)\right) + \dot{y}\left(r_3 \sin(\theta_3+\theta_6)\right) + \dot{\phi}\Big[r_3 S_3 \sin(\theta_3+\theta_6)\cos(\beta+\phi) - r_3 S_3 \sin(\beta+\phi)\cos(\theta_3+\theta_6)\Big]$$

$$= \dot{\theta}_3 \Big[r_3 L_3 \sin(\theta_3+\theta_6)\cos\theta_3 - r_3 L_3 (\sin\theta_3)\cos(\theta_3+\theta_6)\Big]$$

$\rightarrow$ Velocity relation: $J_x \dot{x} = J_q \dot{q} \Rightarrow$

$$\overbrace{\begin{bmatrix} (r_1 \cos(\theta_1+\theta_4)) & (r_1 \sin(\theta_1+\theta_4)) & 0 \\ (r_2 \cos(\theta_2+\theta_5)) & (r_2 \sin(\theta_2+\theta_5)) & [(r_2 S_2 \cos(\phi)\sin(\theta_2+\theta_5)) - (r_2 S_2 \sin(\phi)\cos(\theta_2+\theta_5))] \\ (r_3 \cos(\theta_3+\theta_6)) & (r_3 \sin(\theta_3+\theta_6)) & [r_3 S_3 \sin(\theta_3+\theta_6)\cos(\beta+\phi) - r_3 S_3 \sin(\beta+\phi)\cos(\theta_3+\theta_6)] \end{bmatrix}}^{J_x} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} =$$

$$\underbrace{\begin{bmatrix} [(L_1 r_1 \cos(\theta_1)\sin(\theta_1+\theta_4)) - (L_1 r_1 \sin(\theta_1)\cos(\theta_1+\theta_4))] & 0 & 0 \\ 0 & [(r_2 L_2 \cos(\theta_2)\sin(\theta_2+\theta_5)) - (r_2 L_2 \sin(\theta_2)\cos(\theta_2+\theta_5))] & 0 \\ 0 & 0 & [r_3 L_3 \sin(\theta_3+\theta_6)\cos\theta_3 - r_3 L_3 (\sin\theta_3)\cos(\theta_3+\theta_6)] \end{bmatrix}}_{J_q} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$$\begin{cases} \text{forward velocity problem: } & \bar{\dot{x}} = J^{-1}\bar{\dot{q}} \\ \text{forward static problem: } & \bar{\tau} = J^{T}\bar{F} \end{cases} \longrightarrow J = J_q^{-1} J_x$$

torque applied by actuators to the end effector

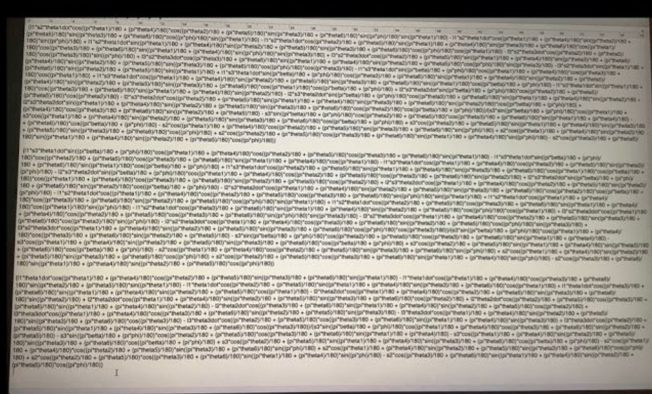
$$\Rightarrow \begin{cases} \text{forward velocity: } & \boxed{\bar{\dot{x}} = J_x^{-1} J_q \bar{\dot{q}}}\ \ast \\ \text{forward static: } & \bar{\tau} = J^{T}\begin{bmatrix} f_x \\ f_y \\ m_z \end{bmatrix} \end{cases}$$

★ Symbolically, it is a huge 3x1 vector → check "Jacobian.m"

Note that values of x, y, ϕ at any instant could be different, depending on initial values of θ_1, θ_2 & θ_3, how long (Δt) it has passed since the actuators on active joints have started and what is the rate of change of active joints ($\dot{\theta}_1$, $\dot{\theta}_2$ and $\dot{\theta}_3$)

$\Rightarrow$ $\bar{\dot{x}}$ (which is velocity of the end-effector) was calculated symbolically in MATLAB. It is a 3x1 vector

$$= \left[\ \ \right]$$



example: for a set of input values, $\dot{\bar{x}} = \begin{bmatrix} 3.1766 \\ 4.8340 \\ 1.2580 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix}$

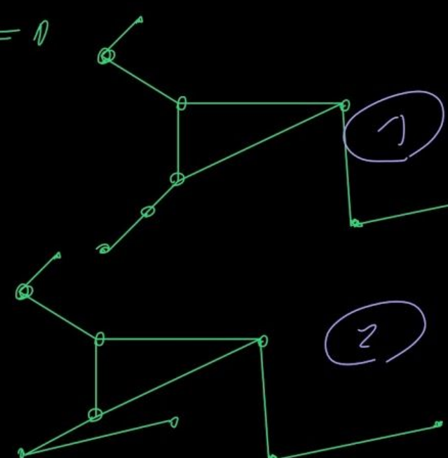
→ input values
$$\begin{cases} r_1 = r_2 = r_3 = 2 \\ l_1 = l_2 = l_3 = 0.6 \\ \theta_1 = \theta_2 = \theta_3 = 60^\circ \\ \theta_4 = 60^\circ \,//\, \theta_5 = 88^\circ \,//\, \theta_6 = 310^\circ \\ s_2 = s_3 = 5 \,//\, \beta = 60^\circ \,//\, \phi = 45^\circ \,// \end{cases} \begin{cases} \dot{\theta}_1 = 5^\circ/s \\ \dot{\theta}_2 = 10^\circ/s \\ \dot{\theta}_3 = 5^\circ/s \end{cases}$$

Singularities:

We have 3 types
$$\begin{cases} J_q \text{ is singular} \\ J_x \text{ " "} \\ \text{Both } J_q \text{ \& } J_x \text{ are singular} \end{cases}$$

∘ first type: $\det(J_q) = 0$

2 possibilities → manipulator loses 1DOF

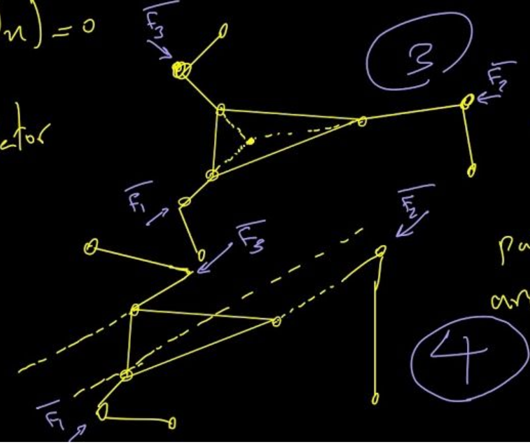


$\theta_4 = 0^\circ$ ⇒ one leg is fully extended

$\theta_4 = 180^\circ$ ⇒ one leg is fully folded

∘ Second type: $\det(J_x) = 0$

2 possibilities → manipulator gains 1DOF

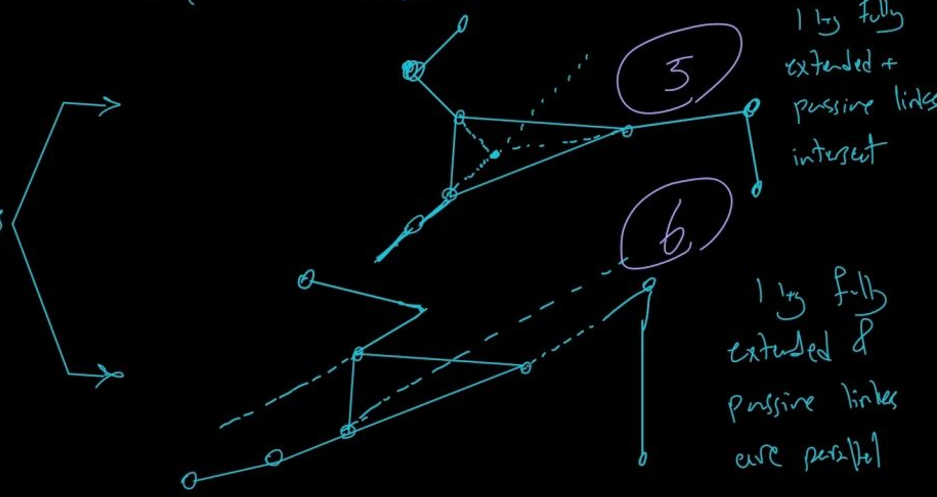


passive links intersect at a common point

passive links are parallel

○ third type: $\det(J_q) = \det(J_x) = 0$

2 possibilities



Note: For case 3 and case 4 (type 2 singularity),

we have actuators forming Planar Pencil (where $\overline{F_1}$, $\overline{F_2}$ & $\overline{F_3}$ either in intersect at a common point or are parallel.)

=> Those forces cannot produce a moment

=> Moment being applied to the manipulator cannot be constrained

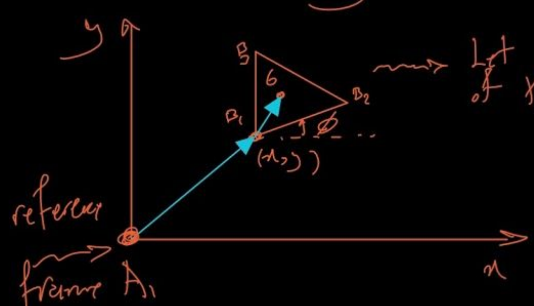
=> The e.e. will gain 1 DoF

→ forces have direction of passive links.

∘ Forward velocity problem (vel @ center of mobile platform)

$$\dot{x} = J^{-1}\dot{q} = \underbrace{J_x^{-1} J_q \dot{q}}_{\text{we found this earlier.}}$$

→ this is speed of corner of the mobile platform ⇒ to find velocity at center of the mobile platform, have to perform velocity transformation



Let point G be center of the mobile platform

$$\therefore\ {}^{A_1}\bar{V}_G = {}^{A_1}\bar{V}_{B_1} + \cancel{{}^{A_1}_{B_1}R\ {}^{B_1}\bar{V}_G}$$

absolute velocity of point G

absolute velocity of point B_1

relative velocity of point G to B_1

$$\Rightarrow \boxed{\dot{x} = J^{-1}\dot{q} = J_x^{-1} J_q \dot{q}}$$

is also equal to velocity of center of mobile platform relative to joints rates ($\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3$)

∘ Forward static problem:

→ Going back to 3 types of singularities discussed, for cases 3, 4, 5 and 6, where active joint forces form planar pencils

⇒ they cannot produce moment

→ but for other scenarios:

$$\tau = J^{T}\bar{F} = \underbrace{J_q^{-1} J_x}_{\text{found earlier}} \bar{F}$$

$$\bar{F} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$ → forces that are applied by θ_1, θ_2 and θ_3

example → Calculation: $\bar{M}_1 = \bar{r}_1 \times \bar{F}_1$

Active Joint Moment

along active joint link $\{L_1, L_2, L_3\}$

force will be along passive links $\{r_1, r_2, r_3\}$

Conclusions

In this report, different characteristics of 3-R'RR planar parallel manipulators were analyzed. Parallel manipulators in general are very popular for industry applications. For our purposes, we considered appropriate scale planar manipulators that can work accurately and precisely. For example, something as simple as designing food and beverages for special occasions, or engraving electronic products like personal computers, cell phones or tablets.

This report can be divided into three main parts: first, assuming different joint displacement and velocities for theta1-theta3 (active joints or joints that have actuators) and finding position and orientation of the end-effector (mobile platform). Second, assuming x, y (coordinates of triangle's corner) and phi (angle between platform and horizontal axis) values in order to find active joint values and check whether our forward and inverse kinematics have been correct. Lastly, finding the relationship between active joint rates and velocity of the end-effector by finding Jx and Jq matrices. MATLAB codes are included in separate zip files, and here we only have hand calculations.

While calculation singularities, we found three different positions where it is referred to as singular configuration. Depending on singularity type, one of the following or a combination of both might happen. First, Jq matrix singular, which means, the mobile platform is stationary, even though active joints have movements. Second Situation where Jx matrix is singular, and in this situation, the mobile platform has movement even though all three active joints are locked. The last type could be a combination of both. Overall, this singular configuration is not desired, but it is important that 1 degree of freedom is lost in the first type, while 1 degree of freedom will be added in the second type of singularity. That means forces in active joints cannot produce moment, that results in moment being applied to the manipulator cannot be constrained.

References

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