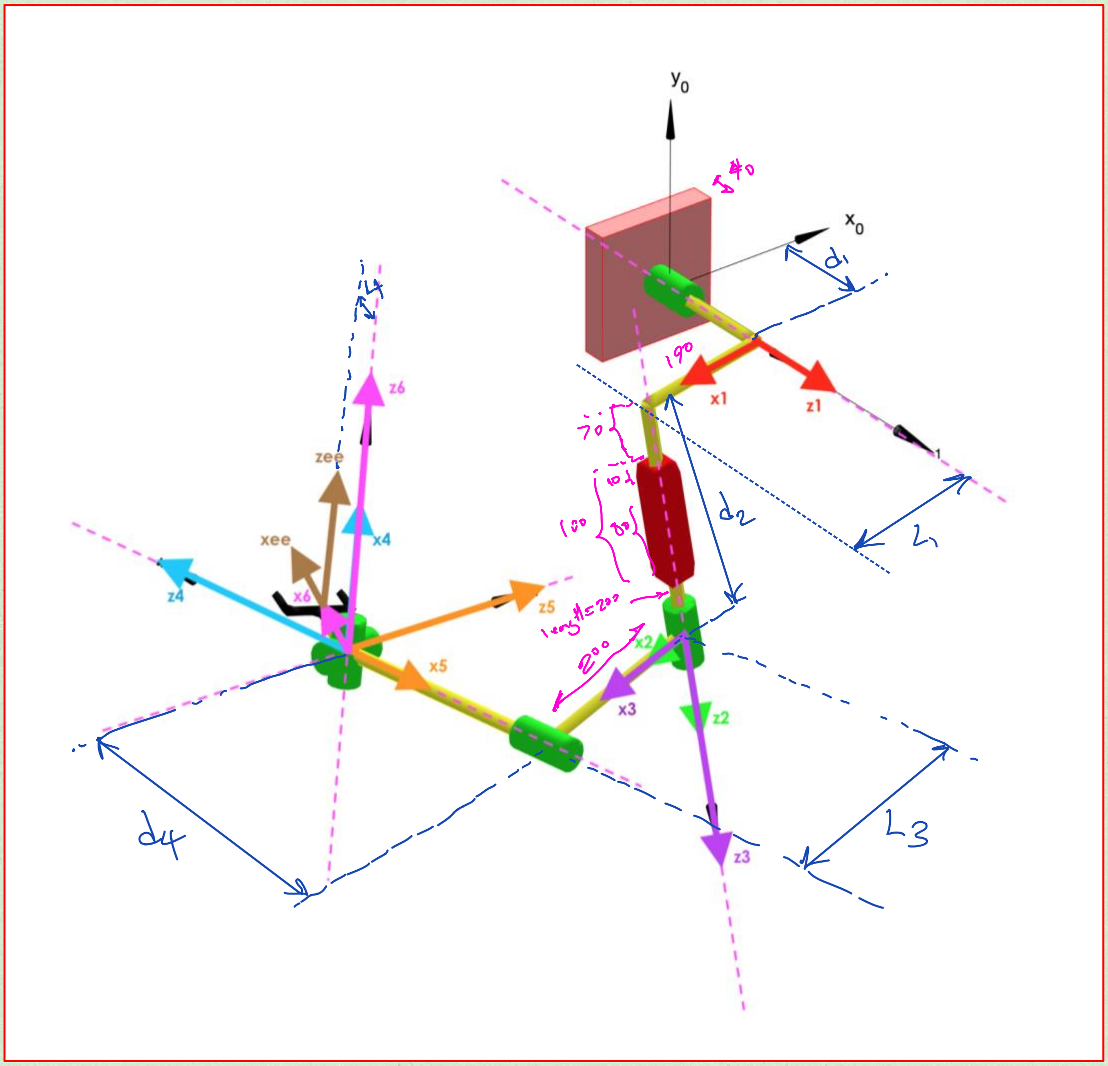
Abstract

For this part of project, path generation and trajectory generation of the end effector was simulated with MATLAB. To do so, forward kinematics and inverse kinematics calculations from parts ones and two were used, and 18 positions and orientations of the end effector were assumed (different points on the path). By assuming alpha, betta, game, Px, Py, and Pz, joints 1-6 variables were calculated, some by hand and the rest by a MATLAB code called “inverse_kin.mat”. Then an animation was also simulated in MATLAB, which roughly shows how the serial manipulator moves to read those 18 desired positions. In order to have reasonable joint speed and acceleration, dt in the main code was changed multiple times, and eventually a desired trajectory generation was simulated.


y0
x0
d1
190
x1
z1
L1
d2
z6
zee
xee
x4
z4
x6
z5
x5
x2
x3
z2
z3
200
d4
L3
L4

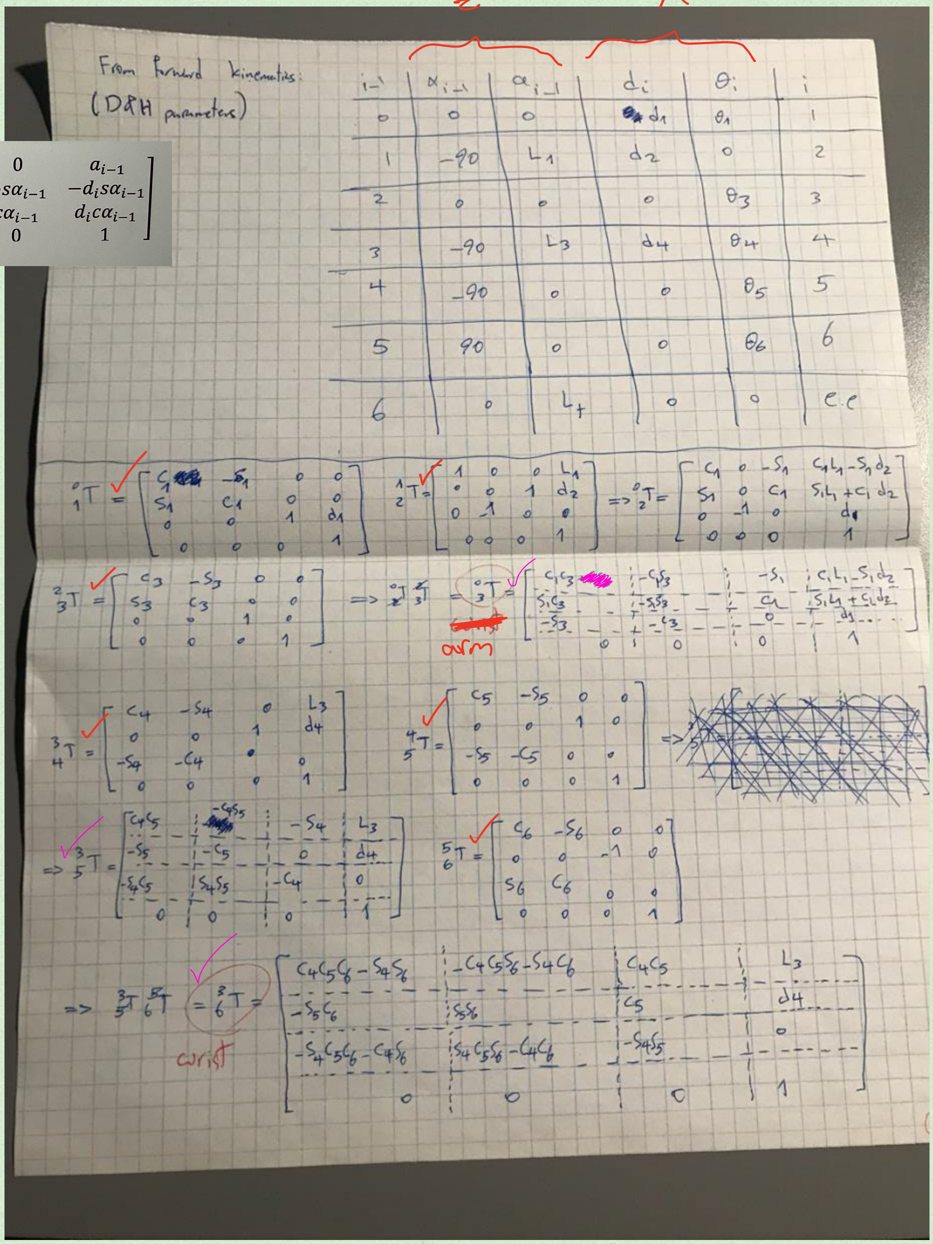
α : Link twist or twist angle

a : Link length

d : Link offset

θ : Joint angle

$$
{}^{i-1}_{\ \ i}\mathbf{T} = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -d_i s\alpha_{i-1} \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & d_i c\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$



From forward kinematics:
(D&H parameters)

	z		x		
$i-1$	α_{i-1}	a_{i-1}	d_i	θ_i	i
0	0	0	d_1	θ_1	1
1	-90	L_1	d_2	0	2
2	0	0	0	θ_3	3
3	-90	L_3	d_4	θ_4	4
4	-90	0	0	θ_5	5
5	90	0	0	θ_6	6
6	0	L_t	0	0	e.e

$$
{}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1_2T = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 0 & 1 & d_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow {}^0_2T = \begin{bmatrix} c_1 & 0 & -s_1 & c_1L_1 - s_1d_2 \\ s_1 & 0 & c_1 & s_1L_1 + c_1d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow {}^0_2T\,{}^2_3T = {}^0_3T = \begin{bmatrix} c_1c_3 & -c_1s_3 & -s_1 & c_1L_1 - s_1d_2 \\ s_1c_3 & -s_1s_3 & c_1 & s_1L_1 + c_1d_2 \\ -s_3 & -c_3 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

arm

$$
{}^3_4T = \begin{bmatrix} c_4 & -s_4 & 0 & L_3 \\ 0 & 0 & 1 & d_4 \\ -s_4 & -c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^4_5T = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_5 & -c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow
$$

$$
\Rightarrow {}^3_5T = \begin{bmatrix} c_4c_5 & -c_4s_5 & -s_4 & L_3 \\ -s_5 & -c_5 & 0 & d_4 \\ -s_4c_5 & s_4s_5 & -c_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^5_6T = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\Rightarrow {}^3_5T\,{}^5_6T = {}^3_6T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4c_5 & L_3 \\ -s_5c_6 & s_5s_6 & c_5 & d_4 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & -s_4s_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

wrist

$${}^0_6T = {}^0_3T \; {}^3_6T =$$

$${}^0_3T = \begin{bmatrix} c_1c_3 & -c_1s_3 & -s_1 & c_1L_1 - s_1d_2 \\ s_1c_3 & -s_1s_3 & c_1 & s_1L_1 + c_1d_2 \\ -s_3 & -c_3 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3_6T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4c_5 & L_3 \\ -s_5c_6 & s_5s_6 & c_5 & d_4 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & -s_4s_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \begin{matrix} c_1c_3(c_4c_5c_6 - s_4s_6) \\ + c_1s_3(s_5c_6) \\ + s_1(s_4c_5c_6 + c_4s_6) \end{matrix} & \begin{matrix} c_1c_3(-c_4c_5s_6 - s_4c_6) \\ - c_1s_3(s_5s_6) \\ - s_1(s_4c_5s_6 - c_4c_6) \end{matrix} & \begin{matrix} c_1c_3(c_4c_5) \\ - c_1s_3(c_5) \\ + s_1(s_4s_5) \end{matrix} & \begin{matrix} c_1c_3(L_3) \\ - c_1s_3(d_4) \\ + (c_1L_1 - s_1d_2) \end{matrix} \\ \begin{matrix} s_1c_3(c_4c_5c_6 - s_4s_6) \\ + s_1s_3(s_5c_6) \\ - c_1(s_4c_5c_6 + c_4s_6) \end{matrix} & \begin{matrix} s_1c_3(-c_4c_5s_6 - s_4c_6) \\ - s_1s_3(s_5s_6) \\ + c_1(s_4c_5s_6 - c_4c_6) \end{matrix} & \begin{matrix} s_1c_3(c_4c_5) \\ - s_1s_3(c_5) \\ - c_1(s_4s_5) \end{matrix} & \begin{matrix} s_1c_3(L_3) \\ - s_1s_3(d_4) \\ + (s_1L_1 + c_1d_2) \end{matrix} \\ \begin{matrix} -s_3(c_4c_5c_6 - s_4s_6) \\ + c_3(s_5c_6) \end{matrix} & \begin{matrix} -s_3(-c_4c_5s_6 - s_4c_6) \\ - c_3(s_5s_6) \end{matrix} & \begin{matrix} -s_3(c_4c_5) \\ - c_3(c_5) \end{matrix} & \begin{matrix} -s_3(L_3) \\ - c_3(d_4) \\ + d_1 \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$${}^{0}_{3}T\,{}^{3}_{6}T = {}^{0}_{6}T = \begin{bmatrix} [(c_1c_3+s_1s_3)(c_4c_5c_6 - s_4s_6)] + [c_1s_3+s_5c_6] + [s_1(s_4c_5c_6+c_4s_6)] & [(c_1c_3+s_1s_3)(-c_4c_5s_6 - s_4c_6)] - [c_1s_3 \times s_5s_6] + [s_1(c_4c_6 - s_4c_5s_6)] & [(c_1c_3+s_1s_3)(c_4c_5)] - [c_1s_3 \times c_5] + [s_1(s_4s_5)] & [(c_1c_3+s_1s_3)L_3] - [(c_1s_3)d_4] + c_1L_1 - s_1d_2 \\ [(s_1c_3)(c_4c_5c_6 - s_4s_6)] + [(s_1s_3)(s_5c_6)] + [c_1(-s_4c_5s_6 - c_4c_6)] & [(s_1c_3)(-s_4c_6 - c_4c_5s_6)] - [(s_1s_3)(s_5s_6)] + [c_1(s_4c_5s_6 - c_4c_6)] & [(s_1c_3)(c_4c_5)] - [(s_1s_3)(c_5)] - [c_1s_4s_5] & [(s_1c_3)L_3] - [(s_1s_3)d_4] + [s_1L_1 + c_1d_2] \\ [-s_3(c_4c_5c_6 - s_4s_6)] + [c_3(s_5c_6)] & [+s_3(c_4c_5s_6 + s_4c_6)] - [c_3s_5s_6] & [-s_3(c_4c_5)] - [c_3c_5] & [-s_3L_3] - [c_3(d_4)] + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

look at last page

Let

$${}^{0}_{6}T = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$a_{11} = [(c_1c_3+s_1s_3)(c_4c_5c_6 - s_4s_6)] + [c_1s_3+s_5c_6] + [s_1(s_4c_5c_6+c_4s_6)]$$

$$= [c_1c_3c_4c_5c_6 - c_1c_3s_4s_6 + s_1s_3c_4c_5c_6 - s_1s_3s_4s_6] + [c_1s_3+s_5c_6] + [s_1s_4c_5c_6 + s_1c_4s_6]$$

$$a_{12} = [(c_1c_3+s_1s_3)(-c_4c_5s_6 - s_4c_6)] - [c_1s_3s_5s_6] + [s_1(c_4c_6 - s_4c_5s_6)]$$

$$= [-c_1c_3c_4c_5c_6 - c_1c_3s_4c_6 - s_1s_3c_4c_5c_6 - s_1s_3s_4s_6] - [c_1s_3s_5s_6] + [s_1c_4c_6 - s_1s_4c_5s_6]$$

$$a_{13} = [(c_1c_3+s_1s_3)(c_4c_5)] - [c_1s_3c_5] + [s_1s_4s_5] = [c_1c_3c_4c_5 + s_1s_3c_4c_5] - [c_1s_3c_5] + [s_1s_4s_5]$$

$$a_{14} = [L_3c_1c_3 + L_3s_1s_3] - [d_4c_1s_3] + [c_1L_1 - s_1d_2]$$

$$a_{21} = [(s_1c_3)(c_4c_5c_6 - s_4s_6)] + [(s_1s_3)(s_5c_6)] + [c_1(-s_4c_5s_6 - c_4c_6)]$$

$$= [s_1c_3c_4c_5c_6 - s_1s_3s_4s_6] + [s_1s_3s_5c_6] + [-c_1s_4c_5s_6 - c_1c_4c_6]$$

2

$$a_{22} = [(s_1c_3)(-s_4c_6 - c_4c_5s_6)] - [s_1s_3s_5s_6] + [c_1(s_4s_5s_6 - c_4c_6)]$$

$$= [-s_1c_3s_4c_6 - s_1c_3c_4c_5s_6] - [s_1s_3s_5s_6] + [c_1s_4s_5s_6 - c_1c_4c_6]$$

$$a_{23} = [s_1c_3c_4c_5] - [s_1s_3c_5] - [c_1s_4s_5]$$

$$a_{24} = [L_3s_1c_3] - [d_4s_1s_3] + [s_1L_1 + c_1d_2]$$

$$a_{31} = [-s_3c_4c_5c_6 + s_3s_4s_6] + [c_3s_5c_6]$$

$$a_{32} = [s_3c_4c_5s_6 + s_3s_4c_6] - [c_3s_5s_6]$$

$$a_{33} = [-s_3c_4c_5] - [c_3c_5]$$

$$a_{34} = [-s_3L_3] - [c_3(d_4)] + d_1$$

$a_{41} = a_{42} = a_{43} = 0$

$a_{44} = 1$

we know this

$$ {}^{6}_{ee}T = \begin{bmatrix} 1 & 0 & 0 & L_t \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow {}^{0}_{ee}T = \underbrace{{}^{0}_{6}T}_{\text{from page 2}}\,{}^{6}_{ee}T = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

Let's say

$$\Rightarrow {}^{0}_{ee}T = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \rightsquigarrow \text{"Symbolic homogeneous Transform"}$$

$b_{11} = a_{11}$ // $b_{12} = a_{12}$ // $b_{13} = a_{13}$ // $b_{14} = (a_{11})(L_t) + a_{14}$

$b_{21} = a_{21}$ // $b_{22} = a_{22}$ // $b_{23} = a_{23}$ // $b_{24} = (a_{21})(L_t) + a_{24}$

$b_{31} = a_{31}$ // $b_{32} = a_{32}$ // $b_{33} = a_{33}$ // $b_{34} = (a_{31})(L_t) + a_{34}$

$b_{41} = a_{41} = 0$ // $b_{42} = a_{42} = 0$ // $b_{43} = a_{43} = 0$ // $b_{44} = (a_{41})(L_t) + a_{44} = 1$

$$ {}^{0}_{ee}T = {}^{0}_{6}T\,{}^{6}_{ee}T = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & L_t \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} & (a_{11})(L_t) + a_{14} \\ a_{21} & a_{22} & a_{23} & (a_{21})(L_t) + a_{24} \\ a_{31} & a_{32} & a_{33} & (a_{31})(L_t) + a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix}$$

$$b_{11} = a_{11} = \begin{pmatrix} c_1c_3(c_4c_5c_6 - s_4s_6) \\ + c_1s_3(s_5c_6) \\ + s_1(s_4c_5c_6 + c_4s_6) \end{pmatrix}$$

$$b_{12} = a_{12} = \begin{pmatrix} c_1c_3(-c_4c_5s_6 - s_4c_6) \\ - c_1s_3(s_5 s_6) \\ - s_1(s_4c_5s_6 - c_4c_6) \end{pmatrix}$$

$$b_{13} = a_{13} = \begin{pmatrix} c_1c_3(c_4c_5) \\ - c_1s_3(c_5) \\ + s_1(s_4s_5) \end{pmatrix}$$

$$b_{14} = (a_{11})(L_7) + a_{14} = (a_{11})(L_7) + \begin{pmatrix} c_1c_3(L_3) \\ - c_1s_3(d_4) \\ + (c_1L_1 - s_1d_2) \end{pmatrix}$$

$$b_{21} = a_{21} = \begin{pmatrix} s_1c_3(c_4c_5c_6 - s_4s_6) \\ + s_1s_3(s_5c_6) \\ - c_1(s_4c_5c_6 + c_4s_6) \end{pmatrix}$$

$$b_{22} = a_{22} = \begin{pmatrix} s_1c_3(-c_4c_5s_6 - s_4c_6) \\ - s_1s_3(s_5 s_6) \\ + c_1(s_4c_5s_6 - c_4c_6) \end{pmatrix}$$

$$b_{23} = a_{23} = \begin{pmatrix} s_1c_3(c_4c_5) \\ - s_1s_3(c_5) \\ - c_1(s_4s_5) \end{pmatrix}$$

$$b_{24} = (a_{21})(L_7) + a_{24} = (a_{21})(L_7) + \begin{pmatrix} s_1c_3(L_3) \\ - s_1s_3(d_4) \\ + (s_1L_1 + c_1d_2) \end{pmatrix}$$

$$b_{31} = a_{31} = \begin{pmatrix} -s_3(c_4c_5c_6 - s_4s_6) \\ + c_3(s_5c_6) \end{pmatrix}$$

$$b_{32} = a_{32} = \begin{pmatrix} -s_3(-c_4c_5s_6 - s_4c_6) \\ - c_3(s_5 s_6) \end{pmatrix}$$

$$b_{33} = a_{33} = \begin{pmatrix} -s_3(c_4c_5) \\ - c_3(c_5) \end{pmatrix}$$

$$b_{34} = (a_{31})(L_7) + a_{34} = (a_{31})(L_7) + \begin{pmatrix} -s_3(L_3) \\ - c_3(d_4) \\ + d_1 \end{pmatrix}$$

$b_{41} = b_{42} = b_{43} = 0$ & $b_{44} = 1$

$${}^{0}_{ee}T_s = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix}$$

$\rightarrow$ Now we have all the components of ${}^{0}_{ee}T_s$.
(Symbolic homogeneous transform

$${}^{0}_{ee}T_n = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \longrightarrow$$ This matrix is known.

$[\hat{n}\ \hat{o}\ \hat{a}]$: Can be found by assuming fixed angle rotation $\longrightarrow {}^{0}R_{XYZ}(\gamma, \beta, \alpha)$
$\Rightarrow$ have to assume γ, β, α.

To find $\bar{P}$, have to assume the components of vector connecting $\{0\}$ to $\{ee\}$

- For now, we continue without inputting values

LHS: Unknown (symbolic) RHS: Known (numeric)

$$\Longrightarrow {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T\,{}^{5}_{6}T\,{}^{6}_{ee}T\,{}^{6}_{ee}T^{-1} = {}^{0}_{ee}T_n\,{}^{6}_{ee}T^{-1}$$

$\longrightarrow$ for It, ${}^{6}_{ee}T$ is known.

$${}^{6}_{ee}T^{-1} = \begin{bmatrix} 1 & 0 & 0 & -L_t \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow {}^{0}_{6}T_s = {}^{0}_{ee}T_n\,{}^{6}_{ee}T^{-1}$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & P_x \\ n_y & o_y & a_y & P_y \\ n_z & o_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -L_t \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & P_x-(L_t)(a_x) \\ n_y & o_y & a_y & P_y-(L_t)(n_y) \\ n_z & o_z & a_z & P_z-(L_t)(n_z) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{Bmatrix} P_x-(L_t)(a_x) \\ P_y-(L_t)(n_y) \\ P_z-(L_t)(n_z) \end{Bmatrix} = \begin{Bmatrix} P_x' \\ P_y' \\ P_z' \end{Bmatrix}$$

$\to$ Now first consider ${}^{0}\bar{P}_{0\to w}$ (symbolic) $= {}^{0}\bar{P}_{0\to w}$ (numeric) $\xrightarrow{\text{Let}}$

$$\begin{cases} {}^{0}\bar{P}_{s_{0\to w}} = \begin{Bmatrix} a_{14} \\ a_{24} \\ a_{34} \end{Bmatrix} \\ {}^{0}P_{n_{0\to w}} = \begin{matrix} P_x' \\ P_y' \\ P_z' \end{matrix} \begin{Bmatrix} P_x-(L_t)(a_x) \\ P_y-(L_t)(n_y) \\ P_z-(L_t)(n_z) \end{Bmatrix} \end{cases}$$

$$(1) \Rightarrow \begin{Bmatrix} P_x' \\ P_y' \\ P_z' \end{Bmatrix} = \begin{Bmatrix} a_{14} \\ a_{24} \\ a_{34} \end{Bmatrix} = \begin{Bmatrix} c_1c_3L_3 - c_1s_3d_4 + c_1L_1 - s_1d_2 \\ s_1c_3L_3 - s_1s_3d_4 + s_1L_1 + c_1d_2 \\ -s_3L_3 - c_3d_4 + d_1 \end{Bmatrix} \to 3E/3U$$

Unknowns $\begin{cases} \theta_1 \\ d_2 \\ \theta_3 \end{cases}$

Side Note

Assume $\to$

$$\begin{cases} d_1 = 20\,m \\ L_1 = 20\,m \\ L_3 = 20\,m \\ d_4 = 40\,m \\ L_t = 10\,m \end{cases}$$

from row 3 of (1) $\longrightarrow$ $\underbrace{P_z' - d_1}_{c} = \underbrace{(-L_3)}_{b} s_3 + \underbrace{(-d_4)}_{a} c_3$

$\xrightarrow[\text{c9 \& c10}]{\text{by rule}}$ $\theta_3 = \text{atan2}(b,a) \pm \text{atan2}(\sqrt{a^2+b^2-c^2}, c)$

assume: $\begin{cases} \alpha = 30 \\ \beta = 45 \\ \gamma = 60 \end{cases}$

& assume: $\begin{cases} P_x = -50 \\ P_y = 50 \\ P_z = 0 \end{cases}$

$\Rightarrow a = -40 \,//\, b = -20 \,//\, c = 7.1 - 20 = -12.9$

$$[\hat{n}\ \hat{o}\ \hat{a}] = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$

$$\Rightarrow [\hat{n}\ \hat{o}\ \hat{a}]$$

$$= \begin{bmatrix} 0.61 & 0.28 & 0.74 \\ 0.35 & 0.74 & -0.57 \\ -0.71 & 0.61 & 0.35 \end{bmatrix} \Rightarrow {}^{0}_{ee}T_n = \begin{array}{cccc} \hat{n} & \hat{o} & \hat{a} & \bar{P} \end{array} \begin{bmatrix} 0.61 & 0.28 & 0.74 & -50 \\ 0.35 & 0.74 & -0.57 & 50 \\ -0.71 & 0.61 & 0.35 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{Bmatrix} P_x' \\ P_y' \\ P_z' \end{Bmatrix} = \begin{Bmatrix} -56.1 \\ 46.5 \\ 7.1 \end{Bmatrix} \Rightarrow \theta_3 = \operatorname{atan2}(-20, -40) \pm \operatorname{atan2}(40.8, -17.9)$$

$$\Rightarrow \boxed{\theta_3 = \begin{cases} -45.9^\circ \\ 99^\circ \end{cases}}$$ 2 solutions

$$\begin{Bmatrix} P_x' \\ P_y' \\ P_z' \end{Bmatrix} = \begin{Bmatrix} c_1 c_3 L_3 - c_1 s_3 d_4 + c_1 L_1 - s_1 d_2 \\ s_1 c_3 L_3 - s_1 s_3 d_4 + s_1 L_1 + c_1 d_2 \\ -s_3 L_3 - c_3 d_4 + d_1 \end{Bmatrix}$$

$${}^0_1R^{-1} = {}^0_1R^T = \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{cases} P_x' = L_3(c_1 c_3) + d_4(-c_1 s_3) + L_1 c_1 - s_1 d_2 \\ P_y' = L_3(s_1 c_3) + d_4(-s_1 s_3) + L_1 s_1 + c_1 d_2 \end{cases}$$

$$\Rightarrow \begin{cases} P_x' = c_1(L_3 c_3 - d_4 s_3 + L_1) + s_1(-d_2) \\ P_y' = c_1(d_2) + s_1(\boxed{L_3 c_3 - d_4 s_3 + L_1}) \end{cases}$$ call it k → known

$$\rightarrow k = \begin{cases} 62.6 \rightarrow \theta_3 = -45.9^\circ \\ -22.6 \rightarrow \theta_3 = 99^\circ \end{cases}$$

$$\Rightarrow P_x'^2 + P_y'^2 =$$

$$c_1^2 k^2 + s_1^2 d_2^2 - \cancel{2 s_1 c_1 d_2 k}$$
$$+ \; c_1^2 d_2^2 + s_1^2 k^2 + \cancel{2 s_1 c_1 d_2 k}$$

$$\Rightarrow P_x'^2 + P_y'^2 = k^2 + d_2^2$$

$$\Rightarrow d_2 = \pm\sqrt{P_x'^2 + P_y'^2 - k^2} = \pm\sqrt{(-56.1)^2 + (46.5)^2 - k^2}$$

→ 2 solutions

$\Rightarrow d_2 = \pm\sqrt{1390.7}$

or $\pm\sqrt{4798.7}$

$\Rightarrow \boxed{d_2 = \begin{cases} \pm 37.3 \\ \pm 69.3 \end{cases}}$ $\Rightarrow$ 4 solutions so far

$\rightsquigarrow$ from now $\rightarrow$ pick $+69.3$

$\underbrace{P_x'}_{c} = c_1(\underbrace{L_3c_3 - d_4s_3 + L_1}_{a}) + s_1(\underbrace{-d_2}_{b})$

$\xrightarrow[\text{C9 \& C10}]{\text{by rule}}$ $\theta_1 = \text{atan2}(b, a) \pm \text{atan2}\left(\sqrt{a^2+b^2-c^2}, c\right) \Rightarrow$ 2 Solutions

$a = -22.6 \;//\; b = -69.3 \;//\; c = -56.1$

$\Rightarrow \boxed{\theta_1 = \begin{cases} 32.3^\circ \\ 111.6^\circ \end{cases}}$ $\longrightarrow$ pick this

$\Rightarrow$ So far, $2\times2\times2=8$ solutions

$\begin{cases} d_1 = 20\,m \\ L_1 = 20\,m \\ L_3 = 20\,m \\ d_4 = 40\,m \\ L_4 = 10\,m \end{cases}$

Now, to find θ_4, θ_5 & θ_6:

$${}^3_6R_s = {}^0_3R_n^{-1}\;{}^0_6R_n$$

$\begin{cases} \theta_1 = 32.3^\circ \\ \theta_3 = 99^\circ \end{cases}$

$$\rightarrow {}^0_3R_s = \begin{bmatrix} c_1c_3 & -c_1s_3 & -s_1 \\ s_1c_3 & -s_1s_3 & c_1 \\ -s_3 & -c_3 & 0 \end{bmatrix} = \begin{bmatrix} -0.13 & -0.83 & -0.53 \\ -0.08 & -0.53 & 0.85 \\ -0.99 & 0.16 & 0 \end{bmatrix}$$

$\Rightarrow$

$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4c_5 \\ -s_5c_6 & s_5s_6 & c_5 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & -s_4s_5 \end{bmatrix} = \begin{bmatrix} n_x' & o_x' & a_x' \\ n_y' & o_y' & a_y' \\ n_z' & o_z' & a_z' \end{bmatrix}$$

$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4c_5 \\ -s_5c_6 & s_5s_6 & c_5 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & -s_4s_5 \end{bmatrix} = {}^{0}_{3}R^{T} \begin{array}{c} \begin{array}{ccc} \hat{n} & \hat{o} & \hat{a} \end{array} \\ \begin{bmatrix} 0.61 & 0.28 & 0.74 \\ 0.35 & 0.74 & -0.57 \\ -0.71 & 0.61 & 0.35 \end{bmatrix} \end{array}$$

$$\Rightarrow {}^{0}_{3}R^{T}\,{}^{0}_{6}R_n = \begin{bmatrix} -0.13 & -0.08 & -0.99 \\ -0.83 & -0.53 & 0.16 \\ -0.53 & 0.85 & 0 \end{bmatrix} \begin{bmatrix} 0.61 & 0.28 & 0.74 \\ 0.35 & 0.74 & -0.57 \\ -0.71 & 0.61 & 0.35 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4c_5 \\ -s_5c_6 & s_5s_6 & c_5 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & -s_4s_5 \end{bmatrix} = \begin{bmatrix} 0.60 & -0.70 & -0.40 \\ -0.81 & -0.53 & -0.26 \\ -0.03 & 0.48 & -0.88 \end{bmatrix}$$

2 solutions $\Rightarrow$ $8 \times 2 = 16$ solutions so far

$$\Rightarrow \theta_5 = \operatorname{atan2}\left(\pm\sqrt{1-(-0.26)^2},\ -0.26\right) \Rightarrow \boxed{\theta_5 = \begin{cases} 105.1^\circ \\ -105.1^\circ \end{cases}}$$

$$\Rightarrow S_6 = \frac{-0.53}{\sin(105.1^\circ)} \Rightarrow \theta_6 = \pm\operatorname{atan2}\left(-0.55,\ \sqrt{1-(-0.55)^2}\right) \Rightarrow \boxed{\theta_6 = \begin{cases} 33.3^\circ \\ -33.3^\circ \end{cases}}$$

$\Rightarrow$ 2 solutions $\rightarrow$ $16 \times 2 = 32$ solutions so far.

$\Rightarrow S_4 = \dfrac{0.88}{\underbrace{S_5}_{0.911}} \Rightarrow \theta_4 = \pm \operatorname{atan2}\left(0.911, \sqrt{1-(0.911)^2}\right)$

$$\Rightarrow \boxed{\theta_4 = \begin{cases} 65.7^\circ \\ -65.7^\circ \end{cases}}$$

$\Rightarrow$ 2 solutions

$\Rightarrow 32 \times 2 = 64$ possible solutions/combination of joint displacements, will give the desired position + orientation of the end-effector.

Now for path/trajectory generation, same process holds true.

The difference would be to change $\begin{Bmatrix} P_x \\ P_y \\ P_z \end{Bmatrix}$ & $\begin{Bmatrix} \alpha \\ \beta \\ \gamma \end{Bmatrix}$ for each desired point.

Point 1 assume: $\begin{cases} \alpha = 30 \\ \beta = 45 \\ \gamma = 60 \end{cases}$

& assume: $\begin{cases} P_x = -50 \\ P_y = 50 \\ P_z = 0 \end{cases}$

for the rest → we assume nump matrix the same way.

variables \ points	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
α	30	30	30	30	30	30	0	0	0	0	0	0	30	30	30	0	0	0
β	45	45	45	45	45	45	0	0	0	0	0	0	45	45	45	0	0	0
γ	60	60	60	60	60	60	0	0	0	0	0	0	60	60	60	0	0	0
P_x	-50	50	0	0	50	-50	-50	50	0	0	50	-50	0	0	100	-100	100	-100
P_y	50	-50	50	-50	0	0	50	-50	50	-50	0	0	100	-100	0	0	-100	100
P_z	0	0	-50	50	-50	50	0	0	-50	50	50	-50	-100	100	-100	100	0	0

$$[\hat{n}\ \hat{o}\ \hat{a}] = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$

$$\begin{Bmatrix} P_x - (L_t)(n_x) \\ P_y - (L_t)(n_y) \\ P_z - (L_t)(n_z) \end{Bmatrix} = \begin{Bmatrix} P_x' \\ P_y' \\ P_z' \end{Bmatrix}$$

$$\begin{cases} d_1 = 20\,m \\ L_1 = 20\,m \\ L_3 = 20\,m \\ d_4 = 40\,m \\ L_t = 10\,m \end{cases}$$

$$\overset{c}{P_z' - d_1} = \overset{b}{(-L_3)}s_3 + \overset{a}{(-d_4)}c_3$$

$$\theta_3 = \mathrm{atan2}(b, a) \pm \mathrm{atan2}\left(\sqrt{a^2+b^2-c^2}\,,\ c\right)$$

$$\boxed{L_3c_3 - d_4s_3 + L_1} \rightarrow k \qquad d_2 = \pm\sqrt{P_x'^2 + P_y'^2 - k^2}$$

$$\underbrace{P_x'}_{c} = c_1\underbrace{(L_3c_3 - d_4s_3 + L_1)}_{a=k} + s_1\underbrace{(-d_2)}_{b}$$

$$\theta_1 = \mathrm{atan2}(b, a) \pm \mathrm{atan2}\left(\sqrt{a^2+b^2-c^2}\,,\ c\right)$$

Find $\theta_4 - \theta_6$

$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & -s_4s_5 \end{bmatrix} = \begin{bmatrix} c_1c_3 & -c_1s_3 & -s_1 \\ s_1c_3 & -s_1s_3 & c_1 \\ -s_3 & -c_3 & 0 \end{bmatrix} [\hat{n}\ \hat{o}\ \hat{a}]$$

Point 2: $\begin{cases} 30 \\ 45 \\ 60 \end{cases}$ & $\begin{cases} 50 \\ -50 \\ 0 \end{cases}$ $\Rightarrow \begin{bmatrix} \hat{n} & \hat{o} & \hat{a} \end{bmatrix} =$

$$\begin{array}{ccc} \hat{n} & \hat{o} & \hat{a} \\ \end{array}$$
$$\begin{bmatrix} 0.61 & 0.28 & 0.74 \\ 0.35 & 0.74 & -0.57 \\ -0.71 & 0.61 & 0.35 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \theta_3 = \text{atan2}(\overset{-20}{b}, \overset{-40}{a}) \pm \text{atan2}\left(\sqrt{a^2+b^2-c^2}, c\right)$$

$$\begin{Bmatrix} P_x' \\ P_y' \\ P_z' \end{Bmatrix} = \begin{Bmatrix} 43.9 \\ -53.5 \\ 7.1 \end{Bmatrix}$$

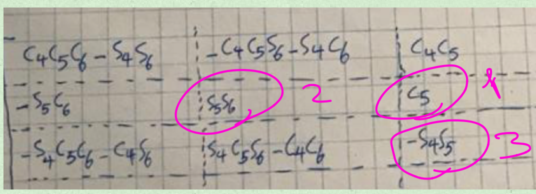
$$c = P_z' - d_1 = -72.9 \Rightarrow \theta_3 = \begin{cases} -45.9 \\ 99^\circ \end{cases}$$

$$d_2 = \pm\sqrt{P_x'^2 + P_y'^2 - k^2} = \{\pm 65.4$$

$$\theta_1 = \text{atan2}(\underset{-65.4}{b}, \underset{-22.6}{a}) \pm \text{atan2}\left(\sqrt{a^2+b^2-c^2}, \underset{43.9}{c}\right)$$

$$= \begin{cases} -58.4 \\ -159.7 \end{cases}$$

$\theta_3 = 99 \,//\, \theta_1 = -58.4$



$c_4c_5c_6 - s_4s_6$	$-c_4c_5s_6 - s_4c_6$	c_4s_5
$-s_5c_6$	s_5s_6 2	c_5 1
$-s_4c_5c_6 - c_4s_6$	$s_4c_5s_6 - c_4c_6$	$-s_4s_5$ 3

$$= \begin{bmatrix} c_1c_3 & s_1c_3 & -s_3 \\ -c_1s_3 & -s_1s_3 & -c_3 \\ -s_1 & c_1 & 0 \end{bmatrix} \begin{bmatrix} 0.61 & 0.28 & 0.74 \\ 0.35 & 0.74 & -0.57 \\ -0.71 & 0.61 & 0.35 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad = \begin{bmatrix} -0.08 & 0.13 & -0.99 \\ 0.08 & -0.13 & 0.15 \\ 0.85 & 0.52 & 0 \end{bmatrix} \begin{bmatrix} 0.61 & 0.28 & 0.74 \\ 0.35 & 0.74 & -0.57 \\ -0.71 & 0.61 & 0.35 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\theta_5 = \text{atan2}\left(\pm\sqrt{1-(0.19)^2}, 0.19\right)$$

$$\Rightarrow \theta_5 = \pm 79^\circ$$

$$= \begin{bmatrix} 0.72 & -0.53 & -0.48 \\ -0.11 & 0.02 & 0.19 \\ 0.70 & 0.62 & 0.33 \end{bmatrix}$$

$$s_6 = \frac{0.02}{s_5} \Rightarrow \theta_6 = \pm\text{atan2}\left(s_6, \sqrt{1-s_6^2}\right) = \pm 1.16$$

$$s_4 = \frac{0.33}{s_5} \Rightarrow \theta_4 = \pm\text{atan2}\left(s_4, \sqrt{1-s_4^2}\right) = \pm 19.63$$

Point3: $\begin{cases}30\\45\\60\end{cases}$ & $\begin{cases}5^0\\50\\-50\end{cases} \Rightarrow [\hat n\ \hat o\ \hat a] = \begin{array}{ccc} \hat n & \hat o & \hat a \end{array}\begin{bmatrix} 0.61 & 0.28 & 0.74 \\ 0.35 & 0.74 & -0.57 \\ -0.71 & 0.61 & 0.35 \\ 0 & 0 & 0 \end{bmatrix}$

$$\Rightarrow \theta_3 = \text{atan2}(\overset{-20}{b}, \overset{-40}{a}) \pm \text{atan2}\left(\sqrt{a^2+b^2-c^2}, \overset{-62.9}{c}\right)$$

$$\Rightarrow \theta_3 = \begin{cases} 26.6^\circ \\ 27^\circ \end{cases}$$

$$\begin{Bmatrix} P_x' \\ P_y' \\ P_z' \end{Bmatrix} = \begin{cases} -6.1 \\ 46.5 \\ -42.9 \end{cases}$$

$$d_2 = \pm\sqrt{P_x'^2 + P_y'^2 - k^2} = \begin{cases} \oplus\ 42.6 \end{cases}$$

$$\theta_1 = \text{atan2}(b, a) \pm \text{atan2}\left(\sqrt{a^2+b^2-c^2}, c\right)$$

-65.4 k -6.1

$$= \begin{cases} 21.9^\circ \\ 190^\circ \end{cases}$$

$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4c_5 \\ -s_5c_6 & s_5s_6\ (2) & c_5\ (1) \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & -s_4s_5\ (3) \end{bmatrix} = \begin{bmatrix} c_1c_3 & s_1c_3 & -s_3 \\ -c_1s_3 & -s_1s_3 & -c_3 \\ -s_1 & c_1 & 0 \end{bmatrix} \begin{array}{ccc} \hat n & \hat o & \hat a \end{array}\begin{bmatrix} 0.61 & 0.28 & 0.74 \\ 0.35 & 0.74 & -0.57 \\ -0.71 & 0.61 & 0.35 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad = \begin{bmatrix} -0.88 & -0.15 & -0.45 \\ 0.88 & 0.15 & -0.9 \\ 0.17 & -0.98 & 0 \end{bmatrix} \begin{array}{ccc} \hat n & \hat o & \hat a \end{array}\begin{bmatrix} 0.61 & 0.28 & 0.74 \\ 0.35 & 0.74 & -0.57 \\ -0.71 & 0.61 & 0.35 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\theta_5 = \text{atan2}\left(\pm\sqrt{1-(0.16)^2}, 0.16\right)$$

$$\Rightarrow \theta_5 = \oplus\ 80.7$$

$$= \begin{bmatrix} -0.08 & 0.13 & -0.99 \\ 0.08 & -0.13 & 0.16 \\ 0.85 & 0.52 & 0 \end{bmatrix}$$

$$S_6 = \frac{-0.13}{S_5} \Rightarrow \theta_6 = \pm\text{atan2}\left(S_6, \sqrt{1-S_6^2}\right) = \oplus\ 7.5$$

$$S_4 = \frac{0}{S_5} \Rightarrow \theta_4 = \pm\text{atan2}\left(S_4, \sqrt{1-S_4^2}\right) = \begin{cases} 0 \\ -80 \end{cases}$$

Point 4: $\begin{cases} 30 \\ 45 \\ 60 \end{cases}$ & $\begin{cases} 5 \\ -50 \\ 50 \end{cases}$ $\Rightarrow \begin{bmatrix} \hat{n} & \hat{o} & \hat{a} \end{bmatrix} = \begin{array}{ccc} \hat{n} & \hat{o} & \hat{a} \end{array} \begin{bmatrix} 0.61 & 0.28 & 0.74 \\ 0.35 & 0.74 & -0.57 \\ -0.71 & 0.61 & 0.35 \\ 0 & 0 & 0 \end{bmatrix}$

$$\Rightarrow \theta_3 = \operatorname{atan2}(\overset{-20}{b}, \overset{-40}{a}) \pm \operatorname{atan2}\left(\sqrt{a^2+b^2-c^2}, \overset{57.1}{c}\right)$$

$$\begin{Bmatrix} P_x' \\ P_y' \\ P_z' \end{Bmatrix} = \begin{cases} -6.1 \\ -53.5 \\ 57.1 \end{cases}$$

$k = 40$

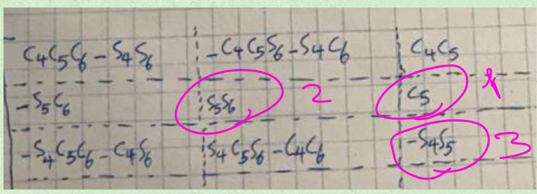
$$\Rightarrow \theta_3 = \begin{cases} 0 \\ -80.7 \end{cases}$$

$$d_2 = \pm\sqrt{P_x'^2 + P_y'^2 - k^2} = \{ + \; 36.05$$

$$\theta_1 = \operatorname{atan2}(b, \overset{k}{a}) \pm \operatorname{atan2}\left(\sqrt{a^2+b^2-c^2}, \overset{-6.1}{c}\right)$$

-65.4

$$= \begin{cases} 36 \\ -153 \end{cases}$$



$$\begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4c_5 \\ -s_5c_6 & s_5s_6 \;(2) & c_5 \;(1) \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & -s_4s_5 \;(3) \end{bmatrix}$$

$$= \begin{bmatrix} c_1c_3 & s_1c_3 & -s_3 \\ -c_1s_3 & -s_1s_3 & -c_3 \\ -s_1 & c_1 & 0 \end{bmatrix} \begin{array}{c} \begin{array}{ccc} \hat{n} & \hat{o} & \hat{a} \end{array} \\ \begin{bmatrix} 0.61 & 0.28 & 0.74 \\ 0.35 & 0.74 & -0.57 \\ -0.71 & 0.61 & 0.35 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$\Rightarrow$

$$= \begin{bmatrix} 0.81 & 0.56 & 0 \\ -0.81 & -0.56 & -1 \\ -0.59 & 0.81 & 0 \end{bmatrix}^T \begin{array}{c} \begin{array}{ccc} \hat{n} & \hat{o} & \hat{a} \end{array} \\ \begin{bmatrix} 0.61 & 0.28 & 0.74 \\ 0.35 & 0.74 & -0.57 \\ -0.71 & 0.61 & 0.35 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\theta_5 = \operatorname{atan2}\left(\pm\sqrt{1-(0.13)^2}, 0.13\right)$$

$$\Rightarrow \theta_5 = \pm\; 82.5$$

$$= \begin{bmatrix} 0.41 & -0.92 & -0.13 \\ 0.07 & 0.25 & 0.13 \\ 0.26 & 0.18 & -0.52 \end{bmatrix}$$

$$S_6 = \frac{0.25}{S_5} \Rightarrow \theta_6 = \pm\operatorname{atan2}\left(S_6, \sqrt{1-S_6^2}\right) = +\; 14.5$$

$$S_4 = \frac{-0.52}{S_5} \Rightarrow \theta_4 = \pm\operatorname{atan2}\left(S_4, \sqrt{1-S_4^2}\right) = \{ \pm\; 31.5$$

The following are done in MATLAB → "inverse kinemat"

Point 1 ✓

Command Window

theta3 = -260.2390

d2 = 69.1986

theta1 = 32.1338

theta5 = 104.8610

theta6 = 32.3797

theta4 = -65.3667

Point 2: ✓

Command Window

theta3 = -260.2390

d2 = 65.3514

theta1 = -58.5793

theta5 = 143.3142

theta6 = -76.7274

theta4 = 33.7570

Point 3 ✓

Command Window

theta3 = -274.2792

d2 = 43.1049

theta1 = -15.6037

theta5 = 152.8870

theta6 = 14.8317

theta4 = -50.8173

Point 4 ✓

Command Window

theta3 = -187.4455

d2 = 53.6508

theta1 = 1.1856

theta5 = 75.1469

theta6 = -36.0560

theta4 = -37.4981

Point 5 ✓

Command Window

theta3 = -301.4424

d2 = 43.8632

theta1 = -90.2067

theta5 = 132.1600

theta6 = -24.8980

theta4 = 89.6896

Point 6 ✓

Command Window

theta3 = -187.4455

d2 = 56.0109

theta1 = 81.2793

theta5 = 65.8284

theta6 = -33.7622

theta4 = -63.6545

Point 7 ✓

Command Window

theta3 = -270

d2 = 75.4983

theta1 = 35.3572

theta5 = 90.0000

theta6 = 35.3572

theta4 = -7.2038e-31

Point 8 ✓

5 - Rn= [cos(

Command Window

theta3 = -270

d2 = 60.8276

theta1 = -56.8606

theta5 = 90.0000

theta6 = -56.8606

theta4 = -1.2410e-31

Point 9

```
Command Window
theta3 =
  -306.8699
d2 =
   50.9902
theta1 =
   11.3099
theta5 =
  126.8699
theta6 =
   11.3099
theta4 =
   5.6745e-17
fx
```

Point 10

```
Command Window
theta3 =
  -201.3045
d2 =
   49.2610
theta1 =
   -3.6540
theta5 =
   21.3045
theta6 =
   -3.6540
theta4 =
   4.1107e-16
fx
```

Point 11

```
Command Window
theta3 =
  -201.3045
d2 =
   37.7710
theta1 =
  -109.2176
theta5 =
   21.3045
theta6 =
  -70.7824
theta4 =
  -9.8007e-16
fx
```

Point 12

```
4 -    px= -50; p
Command Window
theta3 =
  -306.8699
d2 =
    60
theta1 =
   90.0000
theta5 =
  126.8699
theta6 =
    90
theta4 =
   3.5311e-30
fx
```

Point 13

```
Command Window
theta3 =
  -274.2792
d2 =
   94.8919
theta1 =
   -7.3391
theta5 =
  146.1502
theta6 =
   24.2512
theta4 =
  -58.3360
fx >>
```

Point 14

```
Command Window
theta3 =
  -247.1901
d2 =
  100.7507
theta1 =
  -10.3500
theta5 =
  128.9154
theta6 =
   -7.8016
theta4 =
  -33.6452
fx >>
```

Point 15

```
Command Window
theta3 =
  -274.2792
d2 =
   92.1240
theta1 =
  -99.1360
theta5 =
  118.2756
theta6 =
  -55.5793
theta4 =
   68.7567
fx >>
```

Point 16

```
theta3 =
  -256.3559
d2 =
  107.4410
theta1 =
   77.6170
theta5 =
   76.3559
theta6 =
   77.6170
theta4 =
  -2.4162e-16
fx >>
```

Point 17

```
  -270
d2 =
  133.0413
theta1 =
  -50.5364
theta5 =
   90.0000
theta6 =
  -50.5364
theta4 =
   9.1917e-32
fx >>
```

Point 18

```
Command Window
theta3 =
  -270
d2 =
  147.3092
theta1 =
   39.9946
theta5 =
   90.0000
theta6 =
   39.9946
theta4 =
  -3.8175e-32
```